Dark Matter, Gauge Theories and the String Landscape (Recent progress in the Physics of Special Holonomy Spaces)

- Simons Collaboration on Special Holonomy in Geometry, Analysis and Physics
  3rd Annual Meeting.
- 12-13th September 2019, New York
- B.S. ACHARYA (ICTP, Trieste and King's College London)

Outline

- Introduction / overview of some aspects of string/M theory on special holonomy spaces
  - singularities of G_2 holonomy spaces
- Complexity of the Dark Universe and special holonomy
- 4d Gauge Theories and New G_2-holonomy spaces
- Ricci flat manifolds and the String Landscape

Special Holonomy Spaces in String/M theory

- Ubiquitous in string/M theory
- Arise as models of the extra dim^{ns} of space
- Their properties play a huge role in physics models in string/M theory
- Especially Ricci flat special holonomy (Parallel spinors $\nabla\eta=0$ supersymmetry)

G_2-holonomy spaces are a FRAMEWORK for particle physics and cosmology, much like QFT.

But G_2-holonomy spaces are <u>much</u> more constrained compared to QFT.

(Many QFT's are not in the string landscape)

cf C. Vafa's talk

M theory on G_2-holonomy Spaces:

- A geometric picture of the Universe:
- The basic ingredients of the Standard Model of Particle physics, Chiral Fermions and Non-Abelian Gauge fields, arise from special kinds of singularities of G_2-holonomy spaces.
- Physically, the picture was understood using string dualities ...

String Dualities and G_2-holonomy

- M-theory – Heterotic duality:

  "K3-fibered compact G_2-hol spaces"

  (cf S. Donaldson's talk
  S.T. Yau's talk)

- M-theory – Type IIA duality:

  "6d Collapsed limits and G_2-hol spaces from S^1-bundles over Calabi-Yau's"

  cf M. Haskins and S. Salamons talk

Review: 4d gauge theories from codim-4 orbifold singularities

- M-theory on $X \times \mathbb{R}^{3,1}$

  $\uparrow$ G_2 holonomy metric g_X

- A codim-4 orbifold singularity supported on $\underline{M^3 \times \mathbb{R}^{3,1}} \subset X \times \mathbb{R}^{3,1}$

  $\Updownarrow$

  7d gauge theory on $M^3 \times \mathbb{R}^{3,1}$

  $\downarrow$ Low energies $E << \frac{1}{\mathrm{diam}(M^3)}$

  * 4d gauge theory on $\mathbb{R}^{3,1}$

Codim-4 orbifold singularities in a G_2-holonomy space are always of A-D-E type, modelled locally on $\frac{\mathbb{R}^4}{\Gamma_{ADE}} \times M^3 \subseteq N(M^3)$

A-D-E is the gauge group supported along $M^3 \times \mathbb{R}^{3,1} \subset X \times \mathbb{R}^{3,1}$.

(cf BSA hep-th/9812205)

Remark: the precursor to this was: a situation with much more symmetry, namely M theory on $K3 \times \mathbb{R}^7$ (Hull/Townsend, Witten) which was conjectured to be dual to Heterotic string theory on $T^3 \times \mathbb{R}^7$.

This is important for the physical applications of "K3-fibered" G_2-manifolds/spaces to physics. (cf talks by S. Donaldson, M. Haskins and S.T. Yau)

Codimension 7 Singularities and Chiral fermions

Quarks and Leptons (like electrons or neutrinos or quarks) are chiral fermions (S^+ and S^- interact differently)

In M theory on G_2-holonomy spaces they arise from codim 7 singularities like

e.g. Bryant-Salamon G_2 cones $C(\mathbb{CP}^3)$ or $C\left(\frac{SU(3)}{T^2}\right)$

or conjectural new metrics on $C(W\mathbb{CP}^3_{ppqq})$ (BSA/Witten)

(cf Talk by S. Salamon)

Physics of M theory on a G_2-holonomy Space

- Einstein (super) gravity plus
- Yang-Mills fields from codim 4 singularities
- Chiral fermions from codim 7 singularities
- Interaction from Associative 3-cycles
- Moduli fields (G_2-moduli, $H^3(X,\mathbb{R})$)
- Axions $\left(\dfrac{H^3(X,\mathbb{R})}{H^3(X,\mathbb{Z})}\right)$ $\Rightarrow$ combine to

$$\frac{H^3(X,\mathbb{C})}{H^3(X,i\mathbb{Z})}$$

In particular, Beyond the Yang-Mills fields of the Standard Model one will have additional gauge fields, matter fields and interactions since the G_2-hol space is expected to have more than ONE A-D-E singularity

$\Longrightarrow$

In particular, Beyond the Yang-Mills fields of the Standard Model one will have additional gauge fields, matter fields and interactions

$\Longrightarrow$ Rich and Complex DARK sector motivated by special holonomy.

SU(4) holonomy:

For Calabi-Yau fourfolds this has been observed "experimentally" by:

- A. Grassi, J. Halverson, J. Shaneson, W. Taylor
- J. Halverson, W. Taylor
- J. Halverson, C. Long, B. Sung
- D. Morrison, W. Taylor

. . . . . .

$\boxed{\langle \# ADE \rangle = 762 \pm 11}$

- DARK matter could have several, even MANY components.
- DARK matter will have interactions

Mention briefly some studies done in this direction

- Glueball Dark Matter
  (BSA, M. Fairbairn, E. Hardy)
  Here DM is a bound state of gluons (H.S.) at a codim four singularity produced from the decay in the early universe of a G_2 or CalabiYau modulus field.

- Categorisation and Detection of Dark Matter Candidates from String/M theory
  (BSA, S. Ellis, G. Kane, M. Perry, B. Nelson)

- Multiple Moduli in Cosmology

(BSA, M. Dhuria, D. Ghosh, A. Maharana, F. Muia)

Since G_2 and Calabi-Yau's tend to have "lots" of moduli, we considered the impact of this, leading to a solution of the Dark Radiation and DM overabundance problems.

- Decays of slightly heavier moduli dilute abundances

Goal: to obtain/develop
a "complete" cosmological
history of a "typical"
G_2-holonomy or $SU(4)$ holonomy space
including all the moduli, axions,
and <u>many</u> dark sectors.
Generic physical predictions???

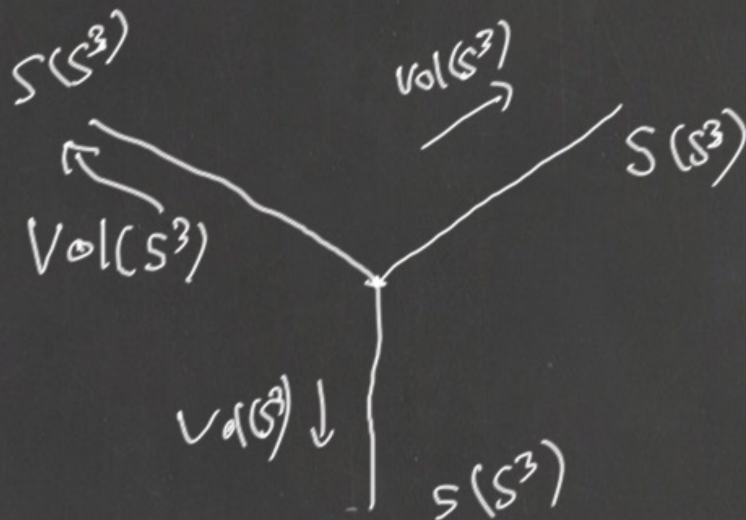
Part II

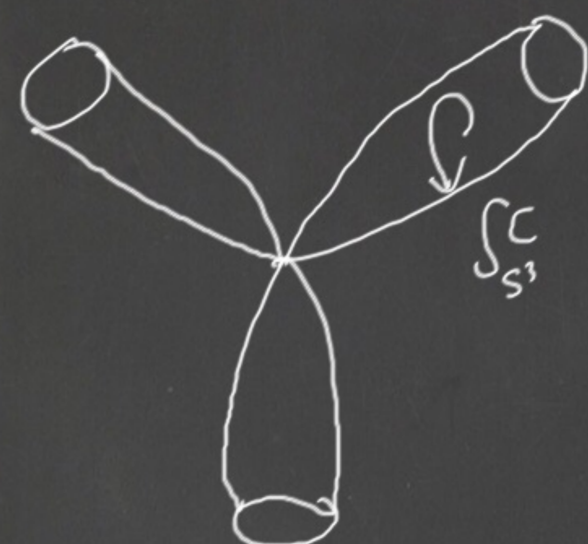
M-theory/Type IIA duality, New G_2-holonomy Spaces and 4d Gauge theories

Work to appear with
L. Foscolo,
M. Najjar,
E. Svanes

Classical Bryant-Salamon Moduli Space

Real G_2-moduli space $\longmapsto$ Complexified in M-theory



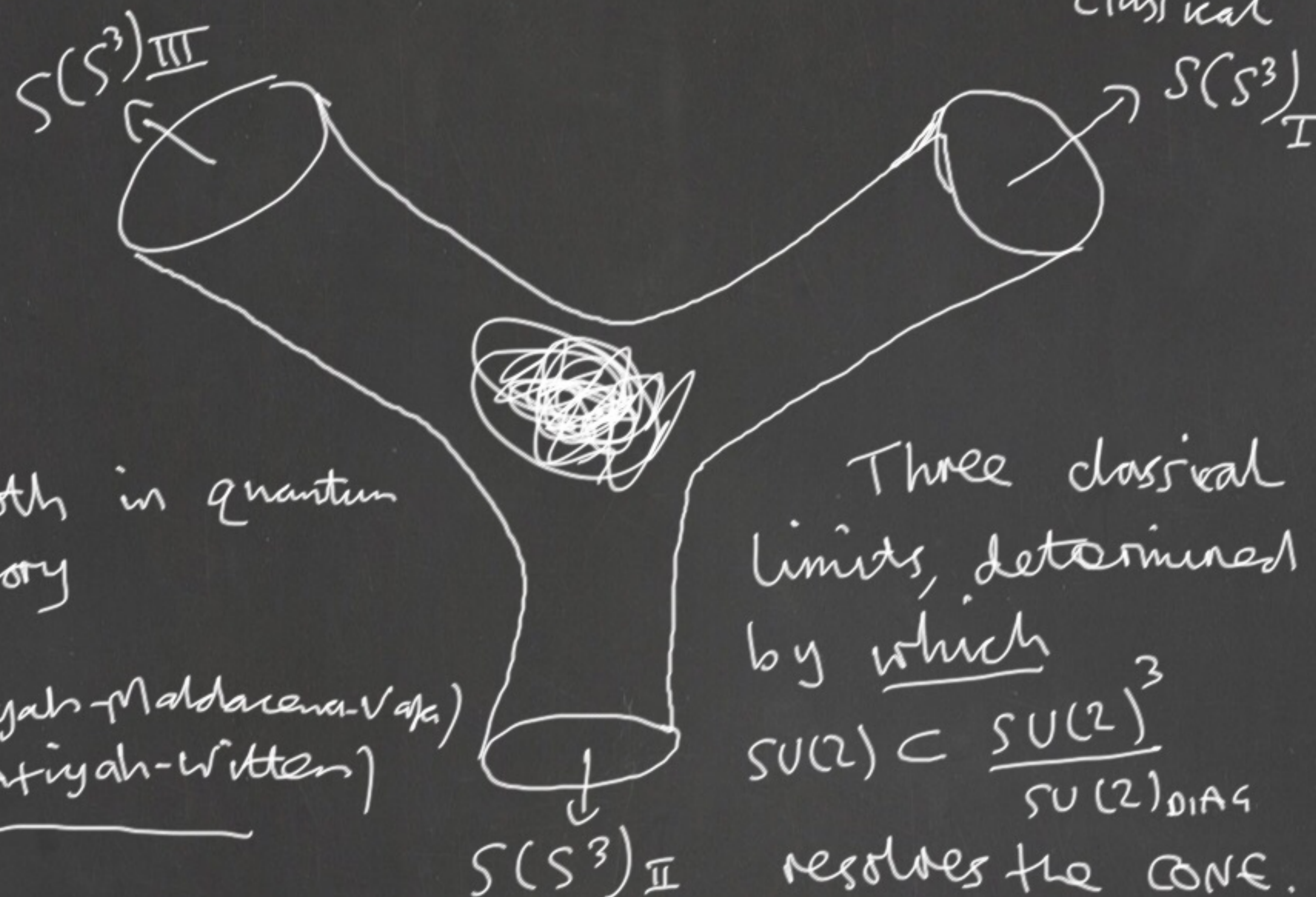


Three AC manifolds, filled in by a choice of $SU(2)$ $\subset SU(2)^3$ at ∞

Complex moduli space

$$Z \sim \int_{S^3} (\varphi + iC)$$

Quantum Bryant-Salamon Review



Smooth in quantum theory

(Atiyah-Maldacena-Vafa)
(Atiyah-Witten)

Three classical limits, determined by which

$$SU(2) \subset \frac{SU(2)^3}{SU(2)_{DIAG}}$$

resolves the CONE.

M-theory flop: quotients and gauge theory

UV $S(S^3)/\Gamma_{ADE} \equiv (\mathbb{R}^4/\Gamma_{ADE}) \times S^3$

$S(S^3)/\Gamma_{ADE} = \mathbb{R}^4 \times (S^3/\Gamma_{ADE})$

smooth

· Same cones at ∞

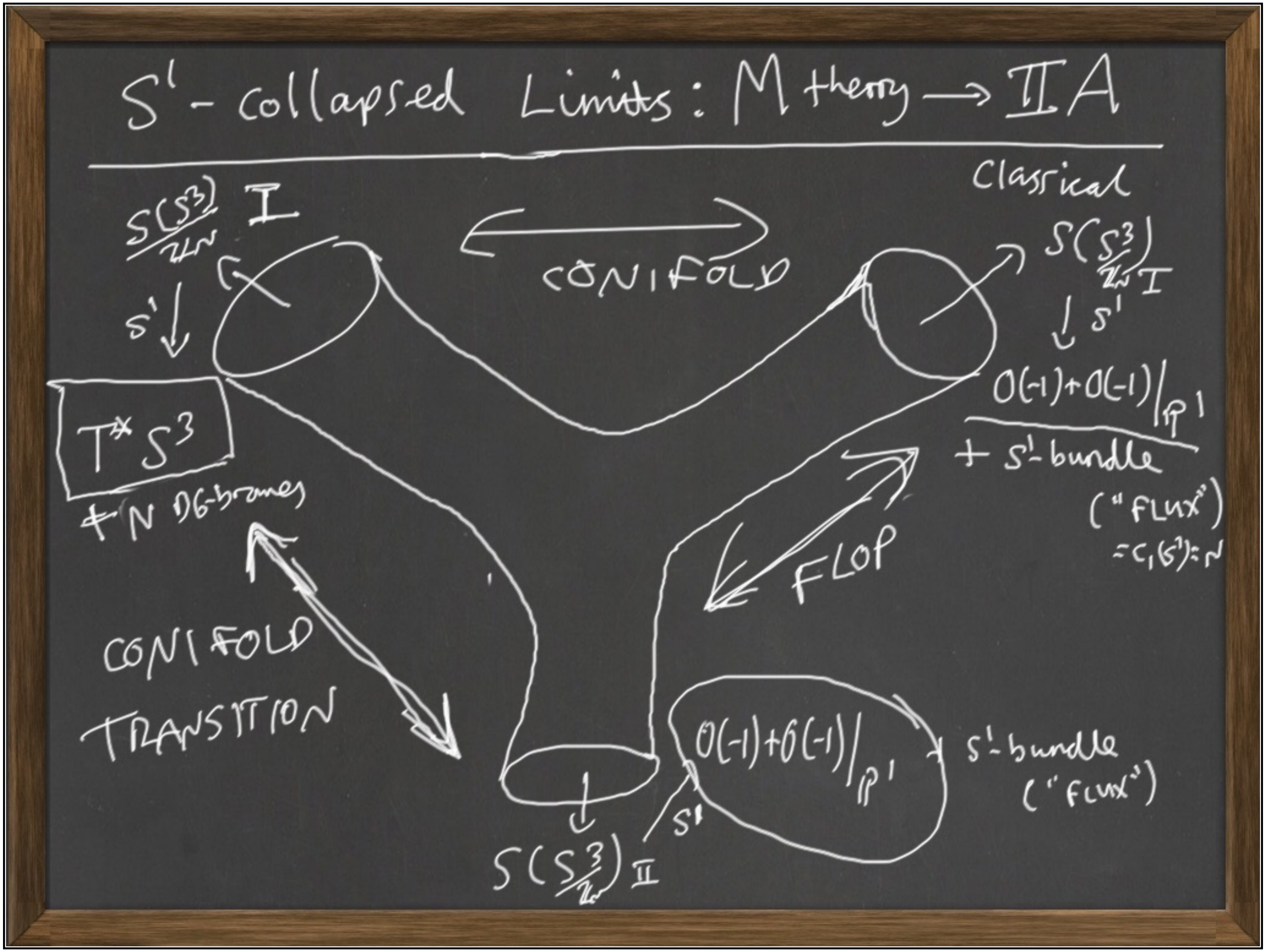
· Different "smoothings"

(BSA, Atiyah-Maldacena-Vafa,
Atiyah-Witten)

$S(S^3)/\Gamma_{ADE} = \mathbb{R}^4 \times (S^3/\Gamma_{ADE})$ ← smooth

Type IIA limits ie Examples which collapse to 6-mflds

- These generalise Bryant-Salamon, but instead of AC, they are ALC:
  - At ∞, S^1-bundles over Calabi Yau Cone
- For metrics on $S(S^3)$ we have the $\mathbb{B}_7$ and $\mathbb{D}_7$ metrics (Brandhuber et al) (Cvetic et al 2002)
- Have $SU(2)^2 \times U(1)$ symmetry instead of $SU(2)^3$


S¹ - collapsed Limits: M theory → IIA
Classical
S(S³)/ℤN I
S¹
T*S³
+ N D6-branes
CONIFOLD
S(S³/ℤN) I
S¹
O(-1)+O(-1)/ℙ¹
+ S¹-bundle
("FLUX")
= c₁(S¹) = N
FLOP
CONIFOLD
TRANSITION
O(-1)+O(-1)/ℙ¹
S¹
S¹-bundle
("FLUX")
S(S³/ℤN) II

One can consider more general Type IIA backgrounds:

If (B^6, ω, Ω) is a complete Calabi-Yau threefold, Type IIA theory on (B, ω, Ω) is equivalent to M-theory on

$$\left(S^1 \times B \,,\, \varphi = \omega \wedge R\,d\theta + \mathrm{Re}\,\Omega^{3,0} \right)$$ and R is "small".

What about more general S^1-bundle $S^1 \to M^7 \longrightarrow B^6$??

Physically, they are described for small R as Type IIA on (B, ω, Ω) plus a superpotential.

$$W = \int_B c_1(M) \wedge \omega_{\mathbb{C}} \wedge \omega_{\mathbb{C}}$$

$$(\omega_{\mathbb{C}} = \omega + i B_2)$$

Supersymmetric, Minkowski solutions satisfy $\partial W = W = 0$

$$\Longrightarrow \quad \boxed{c_1(M) \wedge \omega = 0}$$

In M theory this is described by a G_2-holonomy metric on

$$S^1 \to M^7 \to B$$

Foscolo-Haskins-Nordström proved the following...

Thm (Foscolo-Haskins-Nordström):

If (B, ω, Ω) is an AC CY 3-fold and M^7 an S^1-bundle over (B, ω, Ω) with $c_1(M) \wedge \omega = 0$, then $\exists$ a complete G_2-holonomy metric on M^7, which collapses to (B, ω, Ω) as $R \to 0$.

- This IS M-theory/Type IIA duality.

Foscolo-Haskins-Nordström Thm allows one to construct (infinitely) many complete G_2-manifolds and G_2-hol spaces with singularities.

I will v. briefly discuss one class of examples which extends the M-theory flop to an infinite number of G_2-manifolds ...

(w/ Foscolo, Najjar + Svanes).

- $\mathcal{B} = \widetilde{\mathcal{C}/\mathbb{Z}_N} = \underline{\underline{\overbrace{\frac{\mathcal{O}(-1)+\mathcal{O}(-1)}{\mathbb{Z}_N}}\Big|_{\mathbb{P}^1}}}$

  ie crepant resolutions of a class of "$\underline{\underline{\text{hyperconifolds}}}$" ≡ quotient of conifold $\mathcal{C}$.

- One "simple" class of resolutions, 'resolves each fiber', so one has N-1 fiber curves plus the base.

So M is a 7-orbifold labelled by N integers $(k_0, k_1 \dots k_{N-1})$

$$S^1 \longrightarrow M_{k_0 \dots k_{N-1}} \longrightarrow \tilde{\mathcal{C}}/\mathbb{Z}_N$$

At ∞, all of these G_2 spaces are S^1-bundles over $\boxed{S^2 \times S^3/\mathbb{Z}_N}$

ie $\dfrac{S^3 \times S^3}{\Gamma(k, N, q)}$ $k = k_0 + k_1 + 2k_2 + 3k_3 + \dots k_{N-1}$

$$q = \sum_{1}^{N-1} j k_j \mod N$$

So, at ∞ looks like ∞

in $\dfrac{S(S^3)}{\Gamma(k,N,q)}$,

In fact, $M_{K00000..} = \dfrac{S(S^3)}{\mathbb{Z}_K \times \mathbb{Z}_N}$

ie a quotient of the original M-theory flop we began with.

In that case, which is $N=1$,

$M_k = S(S^3/\mathbb{Z}_k)$ is a smooth

G_2-manifold which describes

IR (= Infra Red = low energy) of a

strongly coupled, confining, $SU(k)$

gauge theory. (BSA, Atiyah-Maldacena-Vafa,
4d

Atiyah-Witten)

M_k can be "flopped" via a conifold transition to another G_2-space, M_k^{uv} which describe the UV (= ultraviolet = high energy) physics of this gauge theory

M_k^{uv} has an $\underline{A_{k-1}}$ singularity along S^3

$\Rightarrow$ $\underline{\text{II}}$A limit is $T^*S^3 + k$ D6-branes

In fact, $M_{k0000\ldots} = \dfrac{S(S^3/\mathbb{Z}_k)}{\mathbb{Z}_N}$,

Under a conifold transition in Type IIA (ie an M-theory flop) we get another G_2-orbifold, which is

$$M^{uv}_{(N,k)} = \frac{S(S^3/\mathbb{Z}_N)}{\mathbb{Z}_k}$$

ie the $S^3 \times S^3$ factors exchanged.

$M^{4v}_{(N,K)}$ has an A_{K-1} singularity along $S^3/\mathbb{Z}_N$ which has a Type IIA limit as $T^* S^3/\mathbb{Z}_N$ plus K D6-branes.

There is a subtlety: one must specify the flat S^1 at ∞. There are thus N IIA backgrounds on $T^* S^3/\mathbb{Z}_N$

$SU(k)$ gauge theory on $S^3/\mathbb{Z}_N$

actually has lots of vacua in the UV, labelled by FLAT $SU(k)$ connections.

$$\mathrm{Hom}(\mathbb{Z}_N, SU(k)).$$

Each of these has an IR limit which seem to correspond to the 7-manifolds

$$S^1 \to M^7_{k_0 \dots k_{N-1}} \to \widetilde{\mathcal{C}} / \mathbb{Z}_N$$

(4, 3) example

S^1-bundles over $\dfrac{\mathcal{O}(-1)+\mathcal{O}(-1)}{\mathbb{Z}_3}\Big|_{\mathbb{P}^1} \cong M_{p_1 p_2 p_3}$

- Fix $\sum p_i = 4 \Rightarrow 15$ S^1-bundles (fluxes)

$q=0$

$c_1(\mathcal{L})$	Flat $SU(4)$
4 0 0	$SU(4)$
1 3 0	$SU(3)\times U(1)$
1 0 3	$SU(3)\times U(1)$
0 2 2	$SU(2)\times SU(2)\times U(1)$
2 1 1	$SU(2)\times U(1)^2$

$q=1$

$c_1(\mathcal{L})$	Flat $SU(4)$
0 4 0	$SU(4)$
0 1 3	$SU(3)\times U(1)$
3 1 0	$SU(3)\times U(1)$
2 0 2	$SU(2)\times SU(2)$
1 2 1	$SU(2)\times U(1)^2$

$q=2$

$c_1(\mathcal{L})$	Flat $SU(4)$
0 0 4	$SU(4)$
3 0 1	$SU(3)\times U(1)$
0 3 1	$SU(3)\times U(1)$
2 2 0	$SU(2)\times SU(2)\times U(1)$
1 1 2	$SU(2)\times U(1)^2$

Note: not all cases are covered by FHN theorem, so the IR G_2-manifold is only known to exist in an (∞) subset of cases.

For those that do exist we can make non-trivial checks of the UV/IR correspondence....

The # of unbroken $U(1)$ factors will be the same in UV and IR $\Rightarrow$ $(M_{k_0 \ldots k_{r-1}}, \varphi)$ will have $\# U(1)$ L^2-normalisable harmonic 2-forms.

- This is true in "all cases where the IR manifold is known.

(Hausel, Hunsicker, Mazeo).

Earlier work on M theory on quotients of $(S(S^3), Q_{BS})$

by T. Friedmann

Hosomichi-Page

Supersymmetry, Ricci flat Manifolds and the String Landscape.

(BSA, arXiv/1906.06886)

(also work to appear with G. Aldazabal, A. Font, K. Narain and I. Zadeh)

Does superstring/M-theory predict supersymmetry below the Kaluza-Klein or GUT scale?

- This has been a longstanding question.
- Most models of particle physics/cosmology in superstring theory start with a COMPACTIFICATION TO MINKOWSKI SPACE.
- Usually based on Ricci flat metric

Compact Ricci flat manifolds are remarkable objects!

Since the '80's they have had lots of significant roles and applications in string/M theory

String/M theory $\Longleftrightarrow$ compact Ricci flat mflds

World-sheet instantons in Calabi-Yau $\Longleftrightarrow$ Gromov-Witten

$N=2$ SCFT's $c=9$ $\Longleftrightarrow$ Mirror-Symmetry

BPS _ -branes and Black Holes $\Longleftrightarrow$ Gopakumar-Vafa

Donaldson-Thomas

Gauge Theory on G_2 and Spin7 mflds

SUPERSYMMETRY ?

Compact, simply connected Ricci flat manifolds & special holonomy: remarkable, rather magical objects.

Are they the only such Ricci flat manifolds?

All known examples are of this type.

A necessary condition for superstring/M-theory to predict low energy supersymmetry is:

$$\boxed{\text{Conjecture: All stable, compact, Ricci flat manifolds have special holonomy}}$$

i.e. ALL semi-classical, stable, Minkowski solutions are supersymmetric.

A more physical statement is:

Conjecture: Minkowski vacua of superstring/M-theory are exactly supersymmetric.

ie Non-SUSY Minkowski 'is' 'in' the swampland.

Berger/Simons Holonomy Classification

Dim(X)	Hol(g_X)	Name
n	$SO(n)$	Riemannian
$n=2k$	$U(k)$	Kähler
$n=2k$	$SU(k)$	Calabi-Yau
$n=4k$	$Sp(1)\cdot Sp(k)$	Quaternionic Kähler
$n=4k$	$Sp(k)$	HyperKähler
$n=7$	G_2	Exceptional
$n=8$	$Spin(7)$	Exceptional

- X irreducible, $\pi_1(X)=0$

Ricci flatness and Holonomy

Dim(X)	$Hol(g_X)$	Ricci flat? $Ric(g_X) = 0$?
n	$SO(n)$	? ? ?
$n = 2k$	$U(k)$	NO
$n = 2k$	$SU(k)$	YES
$n = 4k$	$Sp(1) \cdot Sp(k)$	NO
$n = 4k$	$Sp(k)$	YES
$n = 7$	G_2	YES
$n = 8$	$Spin(7)$	YES

Holonomy and Ricci flatness.

Dim(X)	Hol(g_X)	Name	Parallel Spinor? $\nabla\psi=0$	Ricci flat? $Ric=0$.
n	$SO(n)$	Riemannian	×	???
$n=2k$	$U(k)$	Kähler	×	×
$n=2k$	$SU(k)$	Calabi-Yau	✓	✓
$n=4k$	$Sp(1)\cdot Sp(k)$	Quaternionic Kähler	×	×
$n=4k$	$Sp(k)$	HyperKähler	✓	✓
$n=7$	G_2	Exceptional	✓	✓
$n=8$	$Spin(7)$	Exceptional	✓	✓

All known compact simply connected Ricci flat manifolds are <u>supersymmetric</u> $\Leftrightarrow \nabla_s \eta = 0$

Would <u>not</u> be true for <u>generic</u> holonomy Ricci flat manifolds if they existed.

The Structure of Compact Ricci flat Manifolds

Cheeger-Gromoll Splitting Theorem:

A compact Ricci flat manifold is ISOMETRIC to $X = \underline{Y_1 \times Y_2 \times \ldots Y_k \times T^d} / \Gamma$, where $\pi_1(Y_i) = 1$ and Γ is $\underline{\text{finite}}$.

$\Rightarrow$ suffices to consider

A) X simply connected

B) X/Γ ie π_1 finite

C) T^d/Γ $\leftarrow$

D) $\dfrac{X \times T^d}{\Gamma}$

A) We will discuss what is known and how to prove the conjecture in a variety of cases

C) For non-susy examples we demonstrate a "Witten instability"

X is simply connected

Interesting in dim 4

Topologically $X(p,q) = p\,\mathbb{CP}^2 \# q\,\overline{\mathbb{CP}^2}$ Non-Spin

or

$X(n,m) = n\,K3 \# m\,S^2 \times S$ Spin

- Hitchin Thorpe: $X(p,q)$, $q \gg p$, no Einstein metric.
- Lichnerowicz-Hitchin: $\not{D}^2 = \nabla^2 + \frac{R}{4}$

So $R = 0$ implies harmonic spinors are parallel

$\Rightarrow X(n>0, m)$ Ricci flat iff $n=1, m=0$

- Seiberg-Witten theory can also be used

- Explicit attempts to construct generic holonomy Ricci flat metrics can fail in interesting ways. (Brendle-Kapouleas)

Is there a Ricci flat metric on $X(0,m) = \#_m S^2 \times S^2$?

X is flat ie T^n/Γ

Conjectured to be unstable when:

$\mathrm{Hol}(\Gamma) \not\subset$ Ricci flat holonomy group

ie T^n/Γ has <u>no</u> parallel spinor.

Non-trivial instability even for $n=1$ with odd spin structure.

Witten Instability of $\mathbb{R}^4 \times S^1$ (1983)

- Witten gave alternative proof of Schoen-Yau the positive mass theorem for $\mathbb{R}^n$ using spinors
- Around same time he showed that flat $\mathbb{R}^4 \times S^1$ is unstable
- $\exists$ a Euclidean instanton (based on Euclidean Schwarzschild)
- $\Rightarrow$ A bubble can be nucleated which eats the space from inside (v. quickly)
- S^1 has odd spin structure.

Witten

Euclidean Schwarzschild $d=5$

$$ds^2 = \frac{dr^2}{1-\frac{R^2}{r^2}} + r^2 d\Omega_3^2 + \left(1-\frac{R^2}{r^2}\right) d\phi^2$$

$d\Omega_3^2$: round S^3

ϕ: $(2\pi R)$

$r: (R, \infty)$

At $r=\infty$, radius of $S^1 = R$.

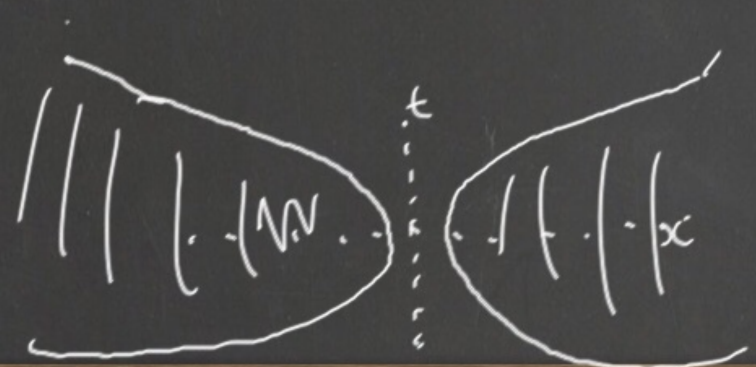
At $r \to R$ S^1 collapses and the geometry "ends".

Analytically continuing along a polar angle in S^3 gives a metric asymptotic to flat $\mathbb{R}^{3,1} \times S^1$ with an expanding "bubble of nothing".

$$ds^2 = \frac{dr^2}{(1-R^2/r^2)} + r^2\underbrace{(d\theta^2 + \cos^2\theta\, d\Omega_2^2)} + \underline{(1-R^2/r^2)\, d\phi^2}$$

$$\underline{d\bar{s}^2 = \frac{dr^2}{(1-R^2/r^2)} + r^2(-dt^2 + \cosh^2 t\, d\Omega_2^2)} + (1-R^2/r^2)\, d\phi^2$$

Bubble of <u>nothing</u> which eats space from inside out. (Witten)



<u>Nothing</u> inside.

Crucial point: the $\mathbb{R}^4 \times S^1$ at $r=\infty$ has ODD spin structure.

We will see that many non-susy flat manifolds have a similar instability.

- Consider only orientable, spin T^d/Γ
- $d=1,2$ $\quad \mathrm{Rot}(\Gamma)=\mathbb{1}$ ie S^1 or T^2 only
- $d=3$, $\exists$ six manifolds:

X_3	Lattice, $\Delta = \{a_1, a_2, a_3\}$	Γ	$\mathrm{Hol}(g) = \mathrm{Rot}(\Gamma)$
$G1 = T^3$	$(1,0,0)$ $(0,1,0)$ $(0,0,1)$	$\mathbb{1}$	$\mathbb{1}$
$G2 = T^3/\mathbb{Z}_2$	$(1,0,0)$ $(0,1,0)$ $(0,0,1)$	$\alpha = (R_3(\pi), \frac{a_3}{2})$	$\mathbb{Z}_2$
$G3 = T^3/\mathbb{Z}_3$	$(1,0,0)$ $(0,0,1)$ $(\frac{-1}{2}, \frac{\sqrt{3}}{2}, 0)$	$\alpha = (R_3(\frac{2\pi}{3}), \frac{a_3}{3})$	$\mathbb{Z}_3$
$G4 = T^3/\mathbb{Z}_4$	$(1,0,0)$ $(0,1,0)$ $(0,0,1)$	$\alpha = (R_3(\frac{2\pi}{4}), a_3/4)$	$\mathbb{Z}_4$
$G5 = T^3/\mathbb{Z}_6$	$(1,0,0)$ $(0,1,0)$ $(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$	$\alpha = (R_3(\frac{2\pi}{6}), a_3/6)$	$\mathbb{Z}_6$
$G6 = \frac{T^3}{\mathbb{Z}_2 \times \mathbb{Z}_2}$	$(1,0,0)$ $(0,1,0)$ $(0,0,1)$	$\alpha = (R_3(\pi), a_3/2)$, $\beta = (R_1(\pi), \frac{a_1+a_2}{2})$	$\mathbb{Z}_2 \times \mathbb{Z}_2$

Spin Structures on Flat 3-manifolds (Pfäffle)

X_3	Spin Structure		#
G_1	$a_1 \to \delta_1 \quad a_2 \to \delta_2 \quad a_3 \to \delta_3$	$\delta_i \in \pm\mathbb{1}$	8
G_2	$a_1 \to \delta_1;\ a_2 \to \delta_2;\ a_3 \to -\mathbb{1};\ \alpha \to \delta_3 \hat{\alpha}$	$\delta_i \in \pm\mathbb{1}$	8
G_3	$a_1 \to \mathbb{1};\ a_2 \to \mathbb{1};\ a_3 \to -\delta_3;\ \alpha \to \delta_3 \hat{\alpha}$	$\delta_3 = \pm\mathbb{1}$	2
G_4	$a_1 \to \delta_1;\ a_2 \to \delta_2;\ a_3 \to -\mathbb{1};\ \alpha \to \delta_2 \hat{\alpha}$	$\delta_i = \pm\mathbb{1}$	4
G_5	$a_1 \to \mathbb{1};\ a_2 \to \mathbb{1};\ a_3 \to -\mathbb{1};\ \alpha \to \delta \hat{\alpha}$	$\delta = \pm\mathbb{1}$	2
G_6	$a_i \to -\mathbb{1} \quad \alpha \to \delta_1 i\sigma_3 \quad \beta \to \delta_2 i\sigma_2 \quad \gamma \to \delta_3 i\sigma_1$	$\delta_1\delta_2\delta_3 = \mathbb{1}$, $\delta_i = \pm\mathbb{1}$	4
			28

- 27 NON-SUSY SPIN STRUCTURES ON $\{T^3/\Gamma\}$
- 26 of 27 descend from ODD SPIN STRUCTURES ON T^3.

$$\hat{\alpha} = \begin{pmatrix} e^{i\pi/N} & 0 \\ 0 & -e^{i\pi/N} \end{pmatrix}$$

Consider now the metric on $\mathbb{R}^2 \times S^3 \times T^2$:

$$ds^2 = \frac{dr^2}{\left(1-\frac{R^2}{r^2}\right)} + r^2\left(d\theta^2 + \cos^2\theta \, d\Omega_2^2\right) + \left(1-\frac{R^2}{r^2}\right)\underbrace{R^2 dx_3^2 + dx_1^2 + dx_2^2}_{S^1 \times T^2}$$

Can also be regarded as a metric on

$$\mathbb{R}^2 \times S^3 \longrightarrow \frac{(\mathbb{R}^2 \times S^3 \times T^2)}{\Gamma} \longrightarrow T^2/\Gamma$$

This describes an instability for 26 of the 27 NON-SUSY spin structures in dimension 3.

Clearly generalises to higher dimensions

(w/ Aldazabal, Font, Narain and Zadeh)

T^n/Γ can be studied exactly as a 2d string theory. Many, possibly <u>all</u> have a <u>tachyon</u> at small enough $R \ll 1$.

This may be related to the Witten instability at large R.

Rmk:

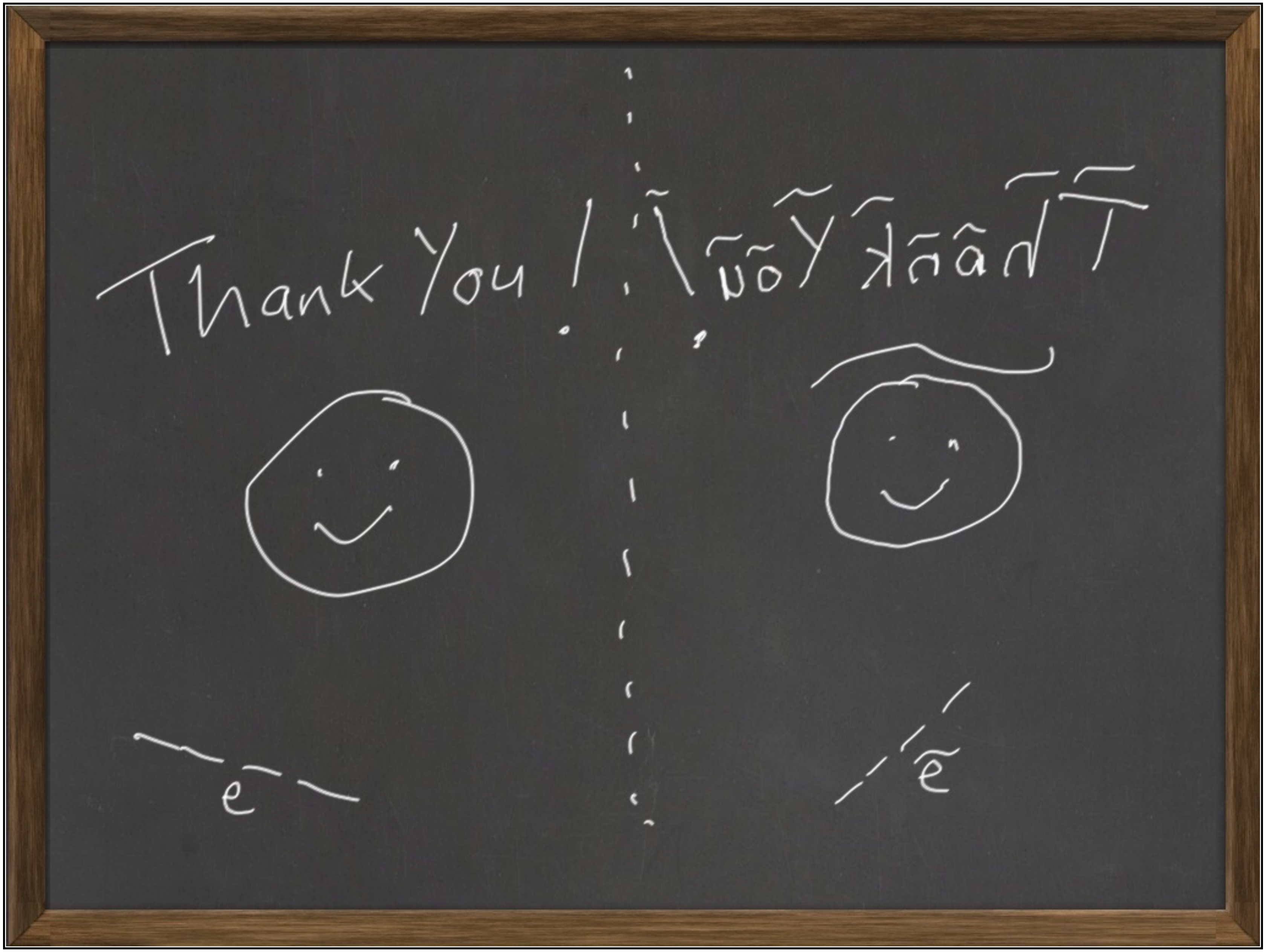
One interesting question which arises here is how to formulate superstring theory at the worldsheet level on manifolds with different spin structures.

Leads to interesting results even for S^1

(BSA, G. Aldazabal, A. Font, K. Narain, I. Zadeh)

So, lots of progress in applications of special holonomy to physics, I presented just a snapshot here. (See S. Schaefer-Nameki talk for more)

Very healthy exchange of ideas between mathematics and physics.


Thank You!