Higgs Bundles for M-theory on G_2-Manifolds

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Motivation

M-theory on G_2-manifolds is in theory a perfect place to construct 4d $\mathcal{N} = 1$ SYM coupled to matter, with interactions, and coupling to (super-)gravity:

$$
\begin{aligned}
SO(7) &\to G_2 \\
\mathbf{8} &\to \mathbf{7} \oplus \mathbf{1}\,.
\end{aligned}
$$

- Interesting 4d gauge theories: non-compact G_2s with codimension 4 and 7 singularities [Acharya, Witten, Atiyah, Maldacena, Vafa...] $\Rightarrow$ Main challenge: compact G_2s with codim 4 and 7 conical singularities
- Compact G_2-manifolds:
  - Joyce orbifolds T^7/Γ
  - $CY_3 \times S^1/\Omega$
  - Twisted Connected Sums: Kovalev; Corti, Haskins, Nordstrom, Pascini, $\Rightarrow$ codim 4 & 6 singularities but not codim 7

A useful way to guide the search: Higgs bundles
Proposed first by [Pantev, Wijnholt, 2009]

Some Lessons from F-theory

The framework of choice in recent years for geometric engineering, e.g. 4d $\mathcal{N}=1$, is F-theory (i.e. Type IIB with varying axio-dilaton τ) on elliptic Calabi-Yau four-folds (CY4). Lessons we learned there:

- Start with 'local' models, i.e. Higgs bundles, encoding gauge sector of 7-branes on M_4 inside CY4:

  7-branes on $M_4 \times \mathbb{R}^{1,3} \equiv \{(\phi, A) : \omega \wedge F_A + i[\phi, \bar{\phi}] = 0, \bar{\partial}\phi = 0\,,\ F^{(0,2)} = 0\}$

  VEV for adjoint valued Higgs field $\langle \phi \rangle \neq 0$ breaks $\widetilde{G} \to G \times G_{\perp}$.

- Spectral cover description for $[\phi, \bar{\phi}] = 0$:
  The local ALE-fibration over M_4 is encoded in the eigenvalues of $\phi \sim \text{diag}(\lambda_1, \cdots, \lambda_n)$.

- Most importantly: these spectral cover models opened up the systematic study of global F-theory compactifications. $\Rightarrow$ Precise connection between elliptic fibrations (+ flux) and Higgs bundles

Higgs bundles/Hitchin systems ubiquitous in the description of the gauge sectors in string theory.

Dp-branes on calibrated cycles M_d in reduced holonomy manifolds X: partial topological twist of the $p+1$ dimensional supersymmetric Yang-Mills theory on M_d always yields an equation on M_d of the type

$$F + [\phi, \phi] = 0\,, \qquad D\phi = D^\dagger \phi = 0$$

The specific details of this depend on the characteristics of X and M_d.

For the gauge sector of M-theory compactifications a similar argument holds, as we shall see, using the Super-Yang-Mills (SYM) arising from twisted dimensional reduction

M-theory on ALE-space $\mathbb{C}^2/\Gamma_{ADE}$ $\Rightarrow$ 7d SYM with gauge group G

Further reduction from 7d to 4d $\Rightarrow$ Higgs bundle on M_3, which reconstructs ALE-fibration over M_3

Plan

1. Gauge sector of G_2-compactifications:
   Local Higgs bundles for G_2s
2. Twisted Connected Sum (TCS) G_2
3. From TCS to chiral models.

4d $\mathcal{N} = 1$ Gauge Theories from G_2 Holonomy

Gauge Sector of M-theory on G_2 Manifolds

- M-theory on $\mathbb{C}^2/\Gamma_{ADE}$ gives 7d SYM with G=ADE: gauge connection A, adjoint scalars ϕ_i, $i = 1, \cdots, 3$, and fermions λ

$$
\begin{aligned}
S =& \frac{1}{g_7^2} \int d^7x \left[-\frac{1}{4} \mathrm{Tr} F_{MN} F^{MN} - \frac{1}{2} \mathrm{Tr} \left(D_M \phi_i D^M \phi^i \right) + \frac{1}{4} \mathrm{Tr} \left([\phi_i, \phi_j][\phi^i, \phi^j] \right) \right] \\
& + \frac{1}{g_7^2} \int d^7x \left[+ \frac{i}{2} \mathrm{Tr} \left(\bar{\lambda}^{\alpha\hat{\alpha}} (\hat{\gamma}^M)_\alpha{}^\beta D_M \lambda_{\beta\hat{\alpha}} \right) - \frac{i}{2} \mathrm{Tr} \left(\bar{\lambda}^{\alpha\hat{\alpha}} (\sigma^i)_{\hat{\alpha}}{}^{\hat{\beta}} [\phi_i, \lambda_{\alpha\hat{\beta}}] \right) \right] ,
\end{aligned}
$$

- ADE-singularity fibered over a three-manifold:

$$
\mathbb{C}^2/\Gamma_{ADE} \to M_3
$$

  This can be given a local G_2-structure.

- Adiabatic picture: 7d SYM on M_3

$$
SO(1,6)_L \times SU(2)_R \quad \to \quad SO(1,3)_L \times \underline{SO(3)_M \times SU(2)_R}
$$

  To retain susy in 4d, topologically twist $SO(3)_M$ with $SU(2)$ R-symmetry: $SO(3)_{\text{twist}} = \mathrm{diag}(SO(3)_M \times SU(2)_R)$

Higgs bundle on M_3

The supersymmetric field configurations on M_3 are characterized by the BPS equations

$$\langle \delta\lambda \rangle = 0$$

where

$$\delta\lambda_{\alpha\hat{\alpha}} = -\frac{1}{4} F_{MN} (\hat{\gamma}^{MN})_\alpha{}^\beta \epsilon_{\beta\hat{\alpha}} + \frac{i}{2} D_M \phi_i (\hat{\gamma}^M)_\alpha{}^\beta (\sigma^i)_{\hat{\alpha}}{}^{\hat{\beta}} \epsilon_{\beta\hat{\beta}} - \frac{1}{4} [\phi_i, \phi_j] \epsilon^{ij}{}_k (\sigma^k)_{\hat{\alpha}}{}^{\hat{\beta}} \epsilon_{\alpha\hat{\beta}}$$

After the twist: background fields are one-forms **3** of $SO(3)_{\text{twist}}$:

- ϕ twisted scalars are adjoint valued one-forms, i.e. $\Omega^1(M_3) \otimes \text{Ad}(G_\perp)$
- $\mathcal{A}$ gauge field for principal $G_\perp$ bundle, components along M_3

$$0 = F_{\mathcal{A}} - i[\phi, \phi]\,, \qquad 0 = D_{\mathcal{A}}\phi$$
$$0 = D_{\mathcal{A}}^\dagger \phi\,.$$

[Pantev, Wijnholt][Braun, Cizel, Huebner, SSN]

$\langle\phi\rangle \neq 0$ breaks $\widetilde{G} \to G \times G_\perp$, e.g. $SU(N+1) \to SU(N) \times U(1)$.

Solutions

Higgs bundle $(\phi, \mathcal{A})$:

$$0 = F_{\mathcal{A}} - i[\phi, \phi]\,, \qquad 0 = D_{\mathcal{A}}\phi$$
$$0 = D^{\dagger}_{\mathcal{A}}\phi\,.$$

Consider first $[\phi, \phi] = 0$ and so $F_{\mathcal{A}} = 0$. If ϕ regular:

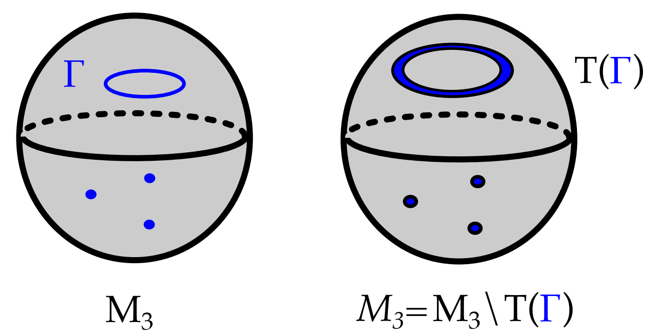
- $\pi_1(M_3) = 0$ then $\phi = 0$
- $\pi_1(M_3) \neq 0$: ϕ can have non-trivial solutions

Relax regularity: allow ϕ to have poles. Model by electrostatics

$$\phi = df\,, \qquad \Delta f = \rho$$

$\rho =$ charge distribution on M_3

$f =$ electrostatics potential



ϕ singular along support of ρ: Γ. Excise a tubular neighborhood $T(\Gamma)$ and consider instead manifold with boundary

$$M_3 \to M_3 = M_3 \backslash T(\Gamma)$$

In summary: we consider solutions to the Hitchin equations on M_3 that satisfy:

- $\partial M_3 \neq \emptyset$
- $\phi \in \Gamma(\Omega^1(M_3, \mathrm{Ad}\tilde{G})$ with non-trivial entries along $G_\perp$
- ϕ regular, $\phi = df$ and $\Delta f = 0$ and suitable boundary conditions on ∂M_3

$\phi = (\phi_1, \phi_2, \phi_3)$ vanishes generically in codim 3 in M_3, where gauge group is unhiggsed from G to $\tilde{G}$.

ALE-fibration

As per usual: Higgs bundles define ALE-fibrations over the base, here M_3. Local geometry

$$\phi = \phi_{i,\alpha} dx^i T_\alpha \,, \qquad T_\alpha = \text{generators of Lie } (G_\perp)$$

then the vevs of $\phi_{i,\alpha}$ give the volume of the rational curves in the ALE fiber with HK structure $\omega_1, \omega_2, \omega_3$

$$\phi_{i,\alpha} = \int_{\mathbb{P}^i_\alpha} \omega_i \,.$$

E.g. for $f = c + \sum_i x_i^2$ then $z_1^2 + z_2^2 + z_3^2 = \sum_i x_i^2$ in $\mathbb{C}^3 \times \mathbb{R}^3$ gives a local ALE fibration where fiber collapses at $\mathbf{x} = \mathbf{0}$.
$\Rightarrow$ Critical points of f correspond to collapse of cycles in the fiber. Defines a local G_2: ALE-fibration over M_3.

Spectrum

Consider ϕ $U(1)$-Higgs field, Higgsing

$$\mathrm{Ad}SU(N+1) \to \mathrm{Ad}SU(N) \oplus \mathrm{Ad}U(1) \oplus \mathbf{R}_q \oplus \overline{\mathbf{R}}_{-q}\,.$$

Given background values "vevs" for $(\phi, \mathcal{A})$, i.e. a local G_2, what is 4d matter content? 7d SYM dimensionally reduced along M_3 yields:

Fermions: $\chi_\alpha \in H^3_{\mathcal{D}}(M_3)$, $\psi_\alpha \in H^1_{\mathcal{D}}(M_3)$ where $\mathcal{D} = d + [\varphi \wedge \cdot]\,, \quad \varphi = \phi + i\mathcal{A}\,.$

Compute twisted cohomology for $\mathcal{D} = d + [\varphi \wedge \cdot]$ and $\mathcal{D}^\dagger = d - [\bar{\varphi} \wedge \cdot]$ with $\phi = df$, or harmonic forms for twisted Laplacian

$$\Delta_f = \mathcal{D}\mathcal{D}^\dagger + \mathcal{D}^\dagger\mathcal{D} = d^\dagger d + dd^\dagger + q^2|df|^2 + q\sum_{i,j=1}^{3}(H_f)_{ij}[(a^i)^\dagger, a^j]\,.$$

where H_f= Hessian of f, $(a^i)^\dagger = dx^i\wedge$ and $a^i = \iota_{\partial_i}$.

Zero-Modes

Boundary conditions: $\mathcal{D}$ and $\mathcal{D}^\dagger$ acting on forms are not adjoints unless we impose on the boundary

$$\int_{\partial M_3} \bar{\alpha} \wedge \star\beta = 0$$

$\alpha_{t,n}$ be the tangent (i.e. pullback of α to the boundary) and normal components $\alpha = \alpha_t + \alpha_n$, of the forms and $\partial M_3 = \Sigma_+ \cup \Sigma_-$:

$$\begin{aligned} \text{Dirichlet b.c. on } \Sigma_-: \quad & \alpha_t|_{\Sigma_-} = 0 \\ \text{Neumann b.c. on } \Sigma_+: \quad & \star\,\alpha_n|_{\Sigma_+} = 0 \end{aligned}$$

Then the twisted cohomologies are computed by the relative cohomology wrt Σ_-

$$H^*_{\mathcal{D}}(M_3) = H^*(M_3, \Sigma_-)$$

Example

$M_3 = S^3 \backslash T(\Gamma)$, where Γ= points, links.

$n_\pm = $ #components with charge $\pm$

$\ell_\pm = $ #loops with charge $\pm$

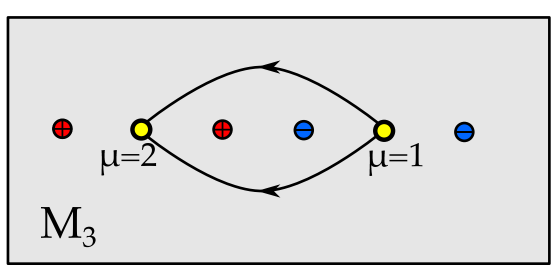
$r = $ #- charged looops that are independent in homology in $S^3 \backslash \Gamma_+$

Then the zero-mode spectrum is

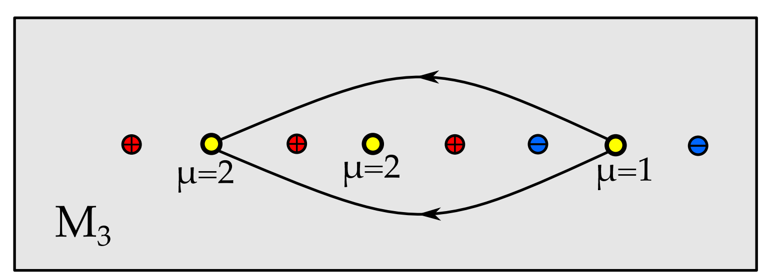
$$b^1(M_3, \Sigma_-) = \ell_+ + n_- - r - 1\,, \qquad b^2(M_3, \Sigma_-) = \ell_- + n_+ - r - 1\,,$$

and the chiral index is simply

$$\chi = (n_+ - \ell_+) - (n_- - \ell_-)$$



$\chi = 0$



$\chi \neq 0$

Next: Interactions

However to describe the interactions we first need to take an alternative, but equivalent description, of the zero-mode spectrum, using Super-Quantum Mechanics (SQM) and Morse/Morse-Bott theory (cf. Witten)

4d Effective Theory	SQM
Matter fields	State Space
$\mathcal{D}$, $\mathcal{D}^\dagger$	Supercharges
Δ_f	Hamiltonian
Higgs field $\phi = df$	f=Superpotential
Matter zero modes	Ground states

For $U(1)$ Higgs field and $f = c + \frac{1}{2}\sum_{i=1}^{3} c_i (x^i)^2 + \cdots$ with isolated critical points
$\Rightarrow$ f Morse.
Let $\mu(p)$ be the Morse index of the critical point p, i.e. $\# c_i < 0$. Then

$$\begin{aligned}\Delta_f &= d^\dagger d + dd^\dagger + q^2 |df|^2 + q\{d, \iota_{\text{grad } f}\} + q\{d^\dagger, df \wedge\} \\ &= \sum_{i=1}^{3} -\frac{\partial^2}{\partial (x^i)^2} + q^2 c_i^2 (x^i)^2 + q c_i [dx^i, \iota_{\partial/\partial x^i}] + \cdots\end{aligned}$$

So that zero modes are to this order ("perturbative zero-modes") are essentially harmonic oscillator wave-functions:

$$\mu(p) = 1: \quad \psi_{(p,q)} = \psi \exp\left(-q \sum_{i=1}^{3} |c_i| (x^i)^2\right) dx^1$$

$$\mu(p) = 2: \quad \bar{\psi}_{(p,q)} = \bar{\psi} \exp\left(-q \sum_{i=1}^{3} |c_i| (x^i)^2\right) dx^1 \wedge dx^2$$

ψ take care of the spinor nature of the fields.

Instanton Corrections

In the 7d SYM: $\mathrm{Tr}(\psi \wedge \mathcal{D}\psi)$ coupling, which descends to a mass term (p_a, p_b critical points of f)

$$\begin{aligned} M^{ab} &= \langle \psi_{(p_a,q)} | \mathcal{D}\psi_{(p_b,q)} \rangle \\ &= \frac{1}{qf(p_a) - qf(p_b)} \int_{\gamma(+\infty)=p_a}^{\gamma(-\infty)=p_b} D\gamma D\eta D\bar{\eta}\, [\mathcal{D}, f] e^{-S_{\text{SQM}}} \,, \end{aligned}$$

where the action for the SQM is the sigma model into M_3, with the fields being paths $\gamma : p_a \to p_b$

$$\begin{aligned} S_{\text{SQM}} = \int_{\mathbb{R}} ds \bigg(& \frac{1}{2} g_{ij} \frac{d\gamma^i}{ds} \frac{d\gamma^j}{ds} + \frac{q^2 t^2}{2} g^{ij} \partial_i f \partial_j f \\ & + g_{ij} \bar{\eta}^i D_s \eta^j + q D_i \partial_j f \bar{\eta}^i \bar{\eta}^j + \frac{1}{2} R_{ijkl} \eta^i \bar{\eta}^j \eta^k \bar{\eta}^l \bigg) \,, \end{aligned}$$

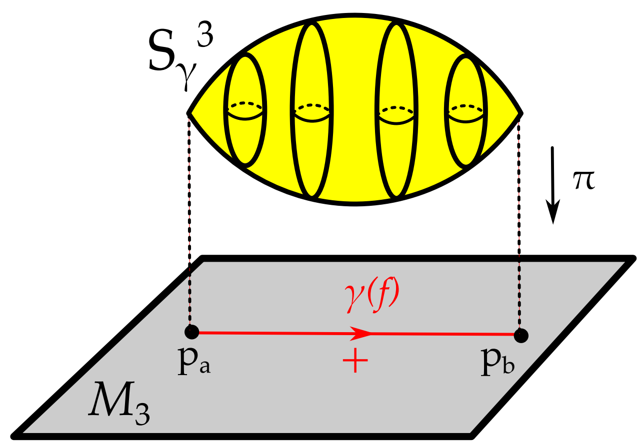
This localizes on gradient flow trajectories for f

$$\frac{d\gamma^i}{ds} = q g^{ij} \partial_j f$$

Zero-mode counting gets correct by

$$M^{ab} = \sum_{\mathrm{gradientflow}\,\gamma:\mathrm{p_a}\to\mathrm{p_b}} n_\gamma e^{-q(f(p_a)-f(p_b))}$$

where $n_\gamma = \pm 1$ depending on orientation on the moduli space of gradient flows.



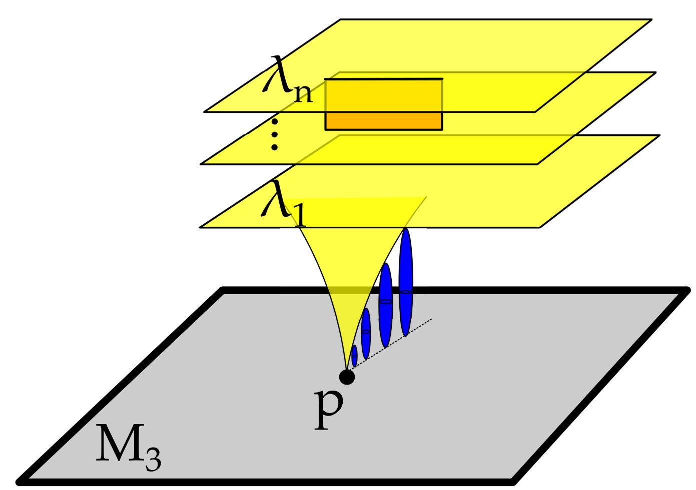
S_3^γ are associatives iff γ is a gradient flow line
$\Rightarrow$ non-trivial M2-instanton contributions from associatives in G_2 (cf. [Harvey, Moore]), depending on # of γ from p_a to p_b, and n_γ.
Upshot: This reproduces $H^*(M_3, \Sigma_-)$.

Spectral Cover

Consider $[\phi, \phi] = 0$, diagonalizable ϕ in $U(1)^n$

$$\mathcal{C}: \qquad 0 = \det(\phi - s) = \sum_{i=0}^{n} b_{n-i} s^i = b_0 \prod_{i=1}^{n} (s - \lambda_i)$$

$\phi = df = 0$ becomes $\lambda_i = 0$ loci, i.e. when one of the covers intersects the zero-section M_3.



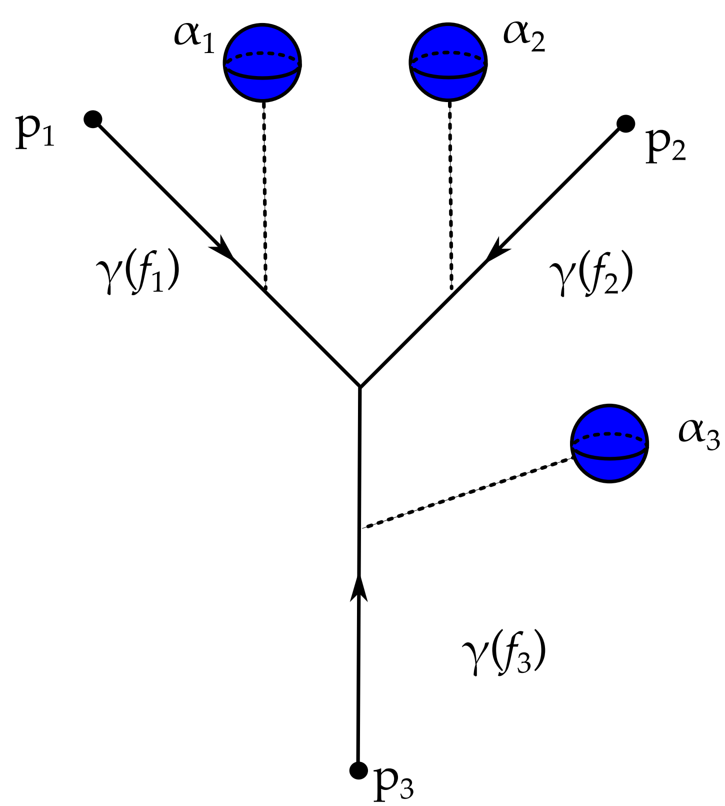
If p is connected by a flow line to another critical point, there is a corresponding associative three-cycle which is built by fibering the collapsing S^2 (blue) over the flow line.

Couplings

From the 7d SYM the following coupling decends:

$$Y^{abc}_{pqr} = \int_{M_3} \psi^{(a,p_1)} \wedge \varphi^{(b,p_2)} \wedge \psi^{(c,p_3)} \,, \qquad Q_1 + Q_2 + Q_3 = 0$$

p_i are the points where matter is localized; a, b, c labels the modes.



This localizes along gradient flows $\gamma(f)$

$$\frac{d\gamma(f)^i}{ds} = qg^{ij}\partial_j f$$

which emanuate from each critical point. The S^2s in ALE-fiber fibered over the gradient flow tree gives rise to a supersymmetric three-cycle
$\Rightarrow$ M2-instanton contribution.

Building of Models

$\widetilde{G} \to G \times U(1)^n$, $\mathfrak{t}^i$ generate $U(1)$s, and consider a charge configuration

$$i = 1, \ldots, n\ : \qquad \phi = \mathfrak{t}^i df_i\,, \qquad \rho = \mathfrak{t}^i \rho_i\,, \qquad \Delta f_i = \rho_i\,, \qquad \int_{M_3} \rho_i = 0\,.$$

Then for $Q = (q_1, \cdots, q_n)$

$$\rho_Q = \sum_{i=1}^{n} q_i \rho_i\,, \qquad f_Q = \sum_{i=1}^{n} q_i f_i$$

At every point in M_3 where $df_Q = 0$, there is a localized chiral multiplet transforming in $\mathbf{R}_Q$.

Example: Top Yukawa

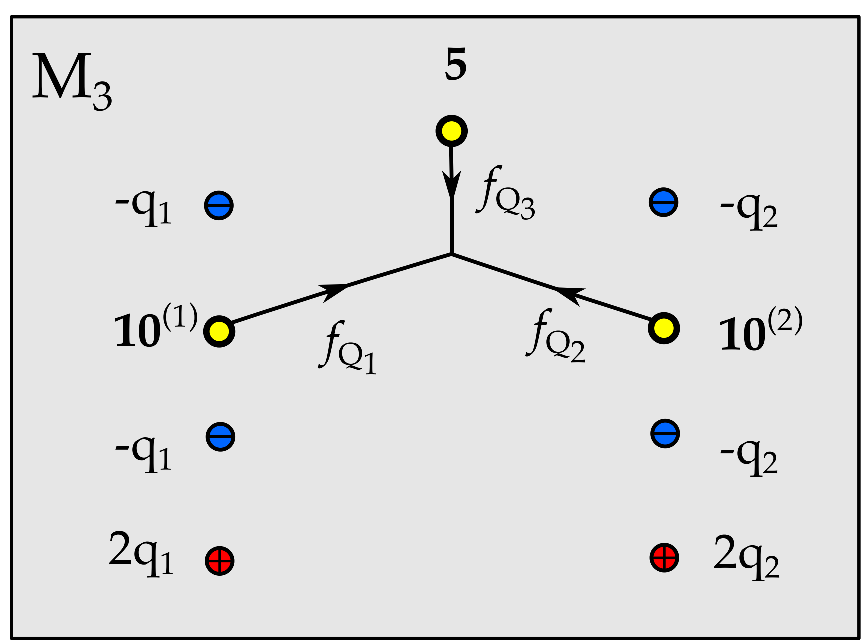
$$E_6 \quad \to \quad SU(5)\times U(1)_a \times U(1)_b \,,$$

Let the matter be localized along the critical loci of the following Morse functions, i.e. f:

$$\mathbf{5}_{-3,3}: \qquad f_5 = -3f_a + 3f_b\,,$$

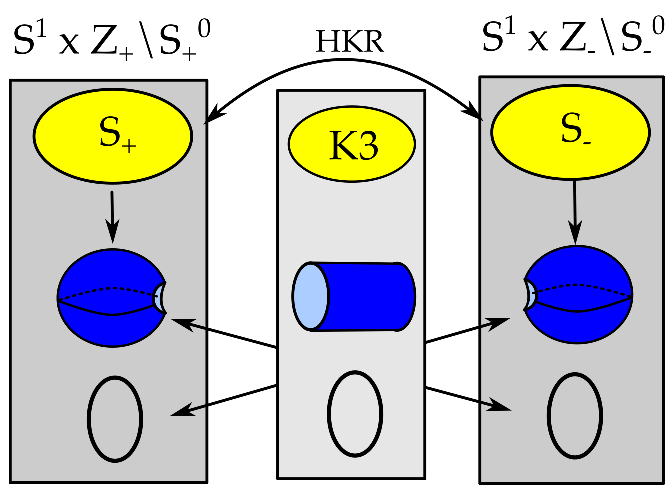
$$\mathbf{10}_{-1,-3}: \qquad f_{10}^{(1)} = -f_a - 3f_b\,,$$

$$\mathbf{10}_{4,0}: \qquad f_{10}^{(2)} = 4f_a\,.$$



2. Local Models for TCS G_2-Manifolds

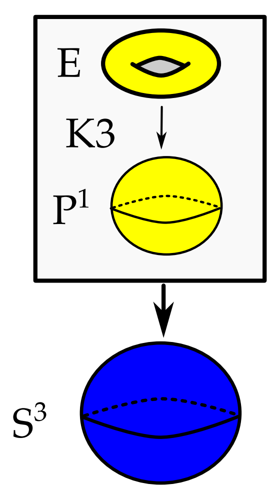
Twisted Connected Sums



Building blocks: Calabi-Yau three-folds = K3s $S_\pm$ over $\mathbb{P}^1$. Remove a fiber ($S_0^\pm$), take a product with S^1 and glue $S_\pm$ with a hyper-Kähler rotation (HKR)

$$\omega_\pm \leftrightarrow \mathrm{Re}\,\Omega_\mp^{(2,0)}\,, \qquad \mathrm{Im}\,\Omega^{(2,0)} \leftrightarrow -\,\mathrm{Im}\,\Omega^{(2,0)}$$

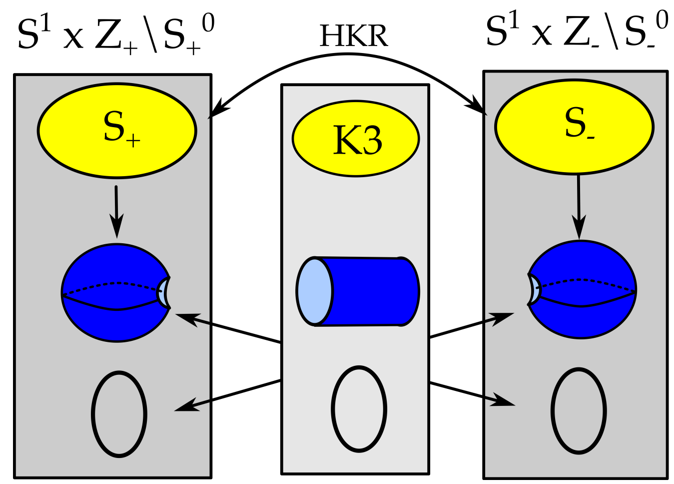
[Kovalev; Corti, Haskins, Nordström, Pacini]



Let $S_\pm$ be elliptically fibered K3 with sections, i.e. Weierstrass models over $\mathbb{P}^1$, and e.g.
S_+: smooth elliptic fibration
S_-: two II^* singular fibers
Singular K3-fibers result in non-abelian gauge groups, e.g. E_n

[Braun, SSN]

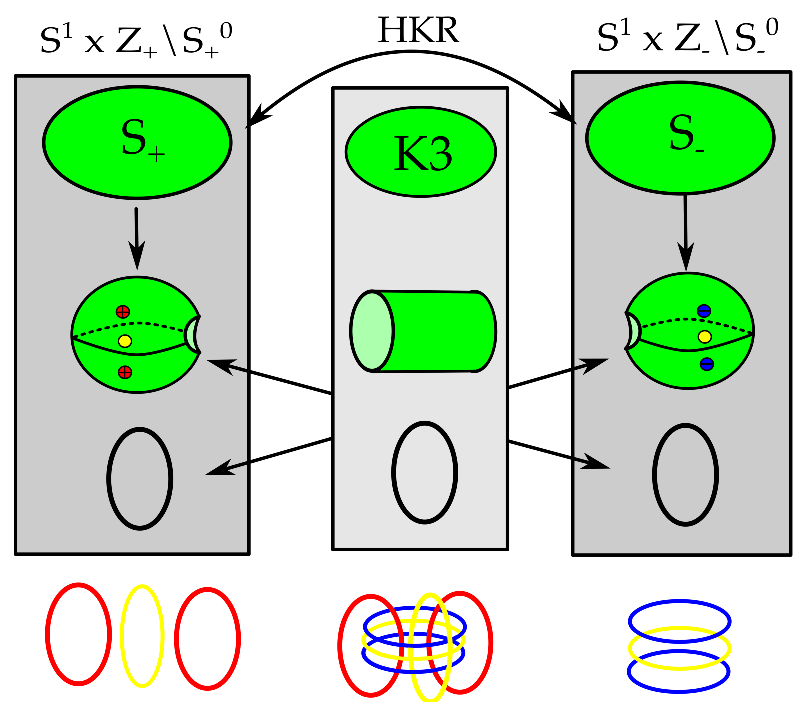
Field Theoretic Interpretation of TCS



- M-theory on Calabi-Yau $Z_{\pm} \times S^1$ preserves $\mathcal{N} = 2$ in 4d.
- Central region: $K3 \times T^2 \times$interval preserves $\mathcal{N} = 4$ in 4d.
- HyperKähler rotation and gluing retains only a common $\mathcal{N} = 1$ susy.
- Key: building blocks have algebraic models.
- TCS are globally $K3 \to S^3$. Apply M on K3/het on T^3 duality; and even het/F-theory duality to e.g. understand instantons [Braun, SSN; Braun, del Zotto, Halverson, Larfors, SSN; Acharya, Braun, Svanes, Valandro]

TCS Higgs-Bundle

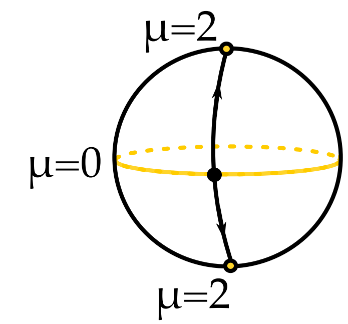
Local Higgs bundle model for Calabi-Yau threefolds in each building block is a spectral cover model over $\mathbb{P}^1$ (with charge loci excised). Charges: circles (red/blue), and critical loci are circles (yellow).



Due to product structure of each building block the critical loci of f, and so matter loci, are always 1d! Requires generalization to Morse-Bott theory. Upshot: Matter Spectrum is always non-chiral.

Morse-Bott generalization for TCS

Example: $f(x,y,z) = z^2$: two critical points and one critical line.
Gradient "curves", connect the critical loci (black lines)



SQM analysis generalizes to gradient trajectories between N_μ=critical submanifolds of Morse index μ

$$\mathcal{M}(N_m, N_n) = \left\{ \gamma : \mathbb{R} \to M \,\middle|\, \lim_{t \to \pm\infty} \gamma(t) \in N_{n,m}\,, \quad \frac{d\gamma^i}{ds} = tqg^{ij}\partial_j f \right\} \Big/ \mathbb{R}.$$

Applied to M_3 we have N_1, N_2 only. The Morse-Bott complex is built from

$$C^1 = \Omega^0(N_1)\,, \qquad C^2 = \Omega^1(N_1) \oplus \Omega^0(N_2)\,.$$

Applied to critical loci in the TCS

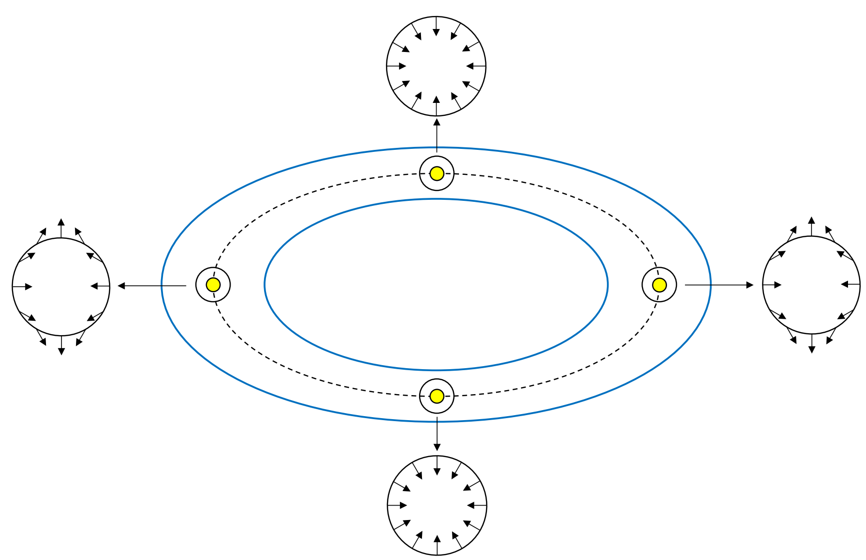
$$C^1 = \Omega^0(S^1)^k\,, \qquad C^2 = \Omega^1(S^1)^k$$

$$H^1(M_3, \Sigma_-) \cong \mathbb{R}^k\,, \qquad H^2(M_3, \Sigma_-) \cong \mathbb{R}^k\,.$$

Singular Transitions in TCS G_2-manifolds

Can TCS be deformed to yield chiral 4d theories?

Deformation of concentric circular charge configurations to e.g. ellipses: gives 4 critical points with equal chiral and conjugate-chiral matter:



Singular Transitions in TCS G_2-manifolds

To change chirality, recall:

$n_\pm = $ #components with charge $\pm$

$\ell_\pm = $ #loops with charge $\pm$

$r = $ #- charged looops that are independent in homology in $S^3 \backslash \Gamma_+$

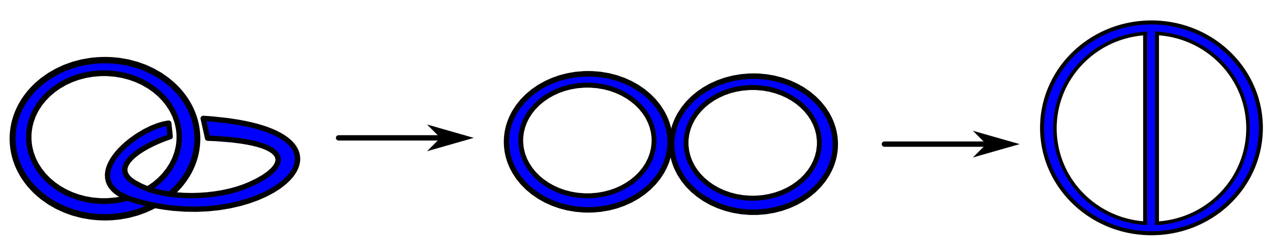
Then the zero-mode spectrum is

$$b^1(M_3, \Sigma_-) = \ell_+ + n_- - r - 1\,, \qquad b^2(M_3, \Sigma_-) = \ell_- + n_+ - r - 1\,,$$

and the chiral index is simply

$$\chi = (n_+ - \ell_+) - (n_- - \ell_-)$$

Singular transitions in the local model that will generate chirality:



$Spin(7)$

– See Andreas Braun's Talk

Recent resurgence of insights in 3d $\mathcal{N} = 1$ theories and dualities.
Geometric engineering of these in M-theory: $Spin(7)$ 8-manifold.

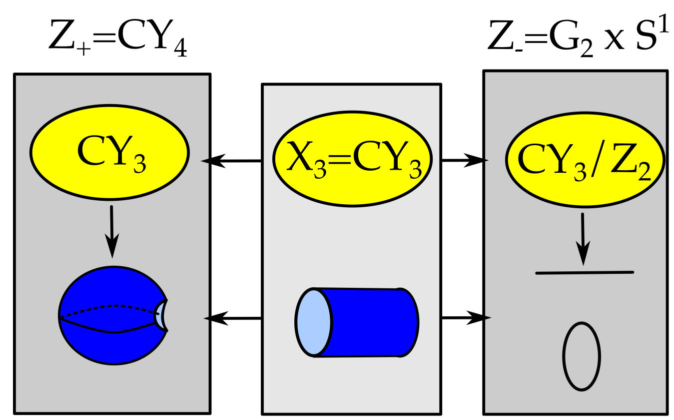
[Alternatively: M5-branes on associative three-cycles in G_2 [Eckhard, SSN, Wong]]

Compact $Spin(7)$ manifolds are equally sparse:

- [Joyce (2000)] orbifold T^8/Γ
- Calabi-Yau four-fold orientifold [Kovalev (2018?)]
- Inspired by TCS for G_2 we developed a Generalized Connected Sum construction. [Braun, SSN (2018)]

Generalized Connected Sum $Spin(7)$-manifolds

Generalized Connected Sum (GCS): [Braun, SSN (2018)]



Field theoretic construction: $Z_\pm$ preserves 3d $\mathcal{N}=2$. Central region preserves 3d $\mathcal{N}=4$, but gluing retains only common 3d $\mathcal{N}=1$. Examples of new compact $Spin(7)$ manifolds [Braun, SSN].

Higgs bundle for Spin(7): [Heckman, Lawrie, Lin, Zoccarato]

Summary and Outlook

- G_2 manifolds provide a purely geometric way of engineering gauge theories in 4d with minimal susy.
- Local Higgs bundle model gives insights into the structure of the gauge sector
- Future: using insights into deformations of TCS form local model, try to construct compact G_2 with codim 7 singularities