5d SCFTs:
Geometry, Graphs and Gauge Theories

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Special Holonomy and SCFTs

M/String theory on manifolds with special holonomy $M \times \mathbb{R}^d$ preserves supersymmetry in d dimensions.

M compact, then the theory in $\mathbb{R}^d$ is generically a supersymmetric gauge theory coupled to supergravity.

M non-compact, $M_{\text{Planck}} \to \infty$ decouples gravity.

Compact case is well-motivated as a theory of quantum gravity (see talk by Cumrun Vafa), albeit supersymmetry remaining experimentally elusive
$\Rightarrow$ main challenge: construct compact singular G_2 holonomy manifolds.

Alternative motivation: application to formal quantum field theory (QFT), i.e. the study of QFT properties, dynamics, moduli spaces, symmetries, etc. without an immediate real world application.

Special Holonomy and SCFTs

In this framework there is a natural, very central question in string theory:

Question: can we embed and even classify superconformal field theories (SCFTs)?

Nahm classified the superconformal algebras: $d \leq 6$.

- 6d $(2,0)$: Type IIB on non-compact K3, $\mathbb{C}^2/\Gamma_{ADE}$
- 6d $(1,0)$: Putative classification in the last few years as F-theory on singular
$$\mathbb{E}_\tau \hookrightarrow \text{CY}_3 \to B_2$$
- 5d $\mathcal{N}=1$: M-theory on CY_3 with canonical singularity
$\Rightarrow$ how to systematically proceed?

5d $\mathcal{N} = 1$ Gauge Theories and SCFTs

5d $\mathcal{N} = 1$ gauge theories are IR-free. Many can be thought of as deformations of 5d $\mathcal{N} = 1$ SCFTs in the UV, which are are intrinsically non-perturbative.

Weakly-coupled description:
Consider G_{gauge} and classical flavor symmetry $G_{\text{F, cl}}$ and

$$\begin{aligned} \text{Vector multiplet:} \quad & \mathcal{A} = (A_\mu, \phi^i, \lambda) \\ \text{Hyper-multiplet in } \mathbf{R}_{\text{F}} \text{ of } G_{\text{F, cl}}: \quad & \mathbf{h} = (h \oplus h^c, \psi)\,. \end{aligned}$$

Coulomb branch (CB) is parametrized by $\langle \phi^i \rangle \neq 0$.

Effective dynamics on CB is governed by the pre-potential:

$$\begin{aligned}\mathcal{F} =&\mathcal{F}_{\text{classical}} + \mathcal{F}_{\text{1-loop}} \\ =&\left(\frac{1}{2g_{YM}^2}\,C_{ij}^{\text{gauge}}\phi^i\phi^j + \frac{k}{6}\,d_{ij\ell}\phi^i\phi^j\phi^\ell\right) + \frac{1}{12}\left(\sum_{\alpha\text{ roots}}|\phi\cdot\alpha|^3 - \sum_{\lambda_{\text{F}}\in\mathbf{R}_{\text{F}}}|\lambda_{\text{F}}\cdot\phi + m_{\text{F}}|^3\right)\end{aligned}$$

which determines the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = G_{ij}\,d\phi^i\wedge\star d\phi^j + G_{ij}\,F^i\wedge\star F^j + \frac{c_{ij\ell}}{24\pi^2}\,A^i\wedge F^j\wedge F^\ell$$

where G_{ij} and CS-levels $c_{ij\ell}$ are derivatives of $\mathcal{F}$.

Example: Rank 1 Theories [Seiberg]

- $G_{\text{gauge}} = SU(2)$ with $N_F\mathbf{F}$ fundamental flavors, $N_F = 1,\cdots,8$
- 1d Coulomb branch, weakly coupled flavor symmetry $G_{\text{F},cl} = SO(2N_F)$
- In the UV: $G_{\text{F},cl}\hookrightarrow G_{\text{F}} = E_{N_F+1}$ 'super-conformal flavor symmetry'.

In general: difficult to come by properties of the UV fixed points from CB.
$\Rightarrow$ Framework which encodes UV and IR descriptions?

Strategy

- Classification of 6d $(1,0)$ SCFTs: F-theory on elliptically fibered non-compact Calabi-Yau threefold [Heckman, Morrison, (Rudelius), Vafa][Bhardwaj]

- 6d to 5d:
  ⇒ Compactification on S^1: 5d marginal theory (UV completes in 6d).
  ⇒ Add masses to hypermultiplets: flows to a 5d UV fixed point.
  ⇒ M-theory on same CY3 provides setting to interpolate between UV and IR description.

- Can we identify and characterize all such 5d SCFTs?
  ⇒ Yes, we propose a combinatorial way, based on graphs, to do so.

Proposal

[Apruzzi, Lawrie, Lin, SSN, Wang]

Given a 5d marginal theory:

We propose a graph-based approach, using ("Combined Fiber Diagrams" (CFDs)) which describe all descendant 5d SCFTs including

- ⋆ strongly coupled (usually enhanced) flavor symmetry
- ⋆ Mass deformations, i.e. descendant SCFTs
- ⋆ BPS states

Extended Coulomb branch phases ($\langle\phi\rangle$ + masses m_F, i.e. Coulomb branch parameters to weakly gauge the classical flavor symmetry) are matched with geometric moduli space (partial resolutions of singular CY3)

Computationally, useful tool: 5d SCFTs from M-theory on elliptic CY3 with non-flat resolutions.

M theory on Calabi-Yau threefold — 5d Dictionary

1. S_i compact divisors $i = 1, \cdots, r$, give rise to $C_3 = A_i \wedge \omega_i$
   $\Rightarrow$ $U(1)^r$ gauge bosons, gauge coupling $1/\text{vol}(S_i)$
   $\star$ $S_i \to C_i$ (collapse to curve) $\Rightarrow U(1)^r$ enhances to G_{gauge}
   $\star$ $S_i \to$ point (collapse to point) $\Rightarrow$ strong coupling
2. Prepotential:
   $$\mathcal{F}(\phi, m_{\text{F}}) \quad \leftrightarrow \quad \mathcal{F}_{\text{geo}} = (S_i \cdot S_j \cdot S_k)\phi^i \phi^j \phi^k$$
   Extended Coulomb branch (ϕ, m_F) (weakly gauging flavor sym) identified with extended Kähler cone of the CY3 singularity.
3. Flavor symmetry in UV G_{F}: fibers of ADE singularities over non-compact curves, that are contained in S_i
4. Mass deformations: flop transitions of curves out of S_i.
5. M2-wrapping modes on curves: BPS states (M5s: strings)

Elliptic Calabi-Yau three-folds

Starting point: Marginal 5d theory, which is based on 6d SCFT on S^1. 6d theory is F-theory on a non-compact elliptic CY3:

- Elliptic CY3 $\mathbb{E}_\tau \hookrightarrow Y_3 \to B$, with a section has Weierstrass form

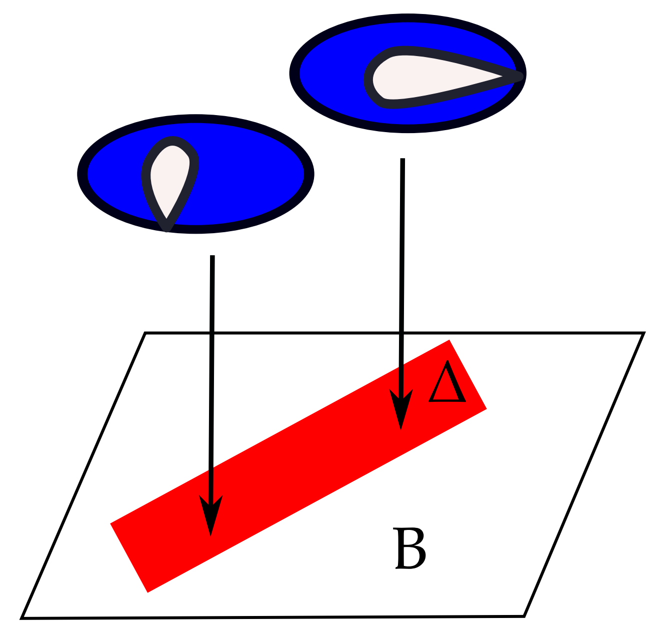
$$y^2 = x^3 + fx + g$$

  Noncompact base B: locally $\mathbb{C}^2$, with coordinates u, v

- Discriminant: Singular fiber above $u = 0$:

$$\Delta = 4f^3 + 27g^2 = O(u^n)$$

- Kodaira fiber above $u = 0$, i.e. in codim 1.



Classification of Singular Fibers

Codim 1 in base: Kodaira classified singular fibers

Singular fibers	$\longleftrightarrow$	ADE affine Dynkin diagram

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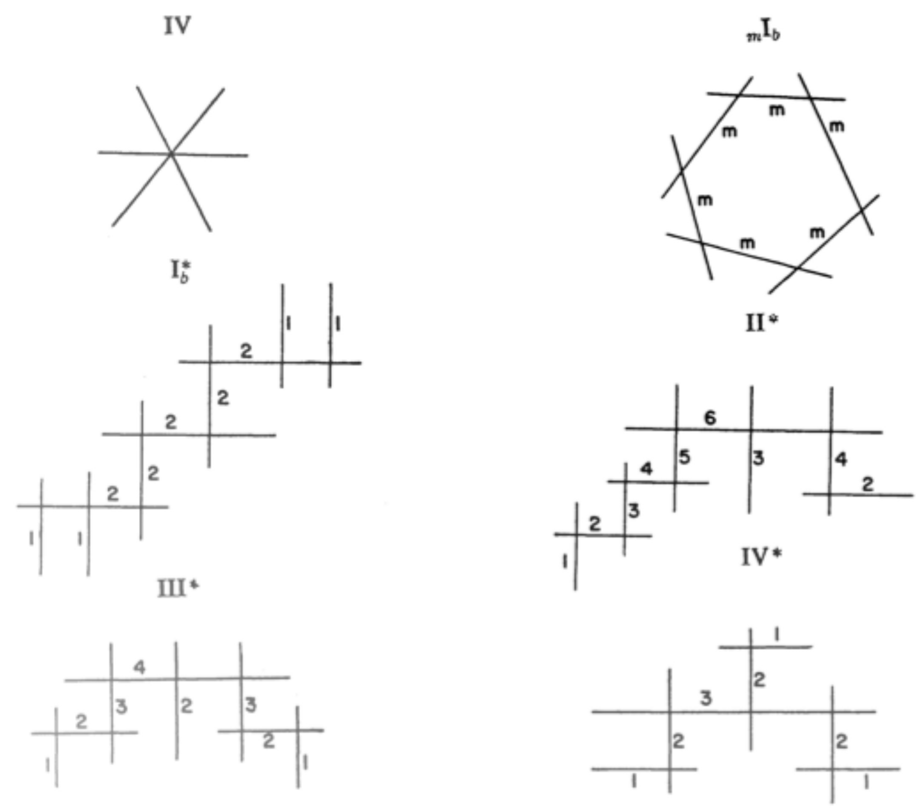


FIGURE 1. Each line represents $\Theta_{\rho s}$; the integer attached to the line gives $n_{\rho s}$.

	$\mathrm{ord}_S(f)$	$\mathrm{ord}_S(g)$	$\mathrm{ord}_S(\Delta)$	singularity	local gauge group factor
I_0	≥ 0	≥ 0	0	none	–
I_1	0	0	1	none	–
I_2	0	0	2	A_1	$SU(2)$
$I_m, m \geq 1$	0	0	m	A_{m-1}	$Sp([\frac{m}{2}])$ or $SU(m)$
II	≥ 1	1	2	none	–
III	1	≥ 2	3	A_1	$SU(2)$
IV	≥ 2	2	4	A_2	$Sp(1)$ or $SU(3)$
I_0^*	≥ 2	≥ 3	6	D_4	G_2 or $SO(7)$ or $SO(8)$
$I_m^*, m \geq 1$	2	3	$m+6$	D_{m+4}	$SO(2m+7)$ or $SO(2m+8)$
IV^*	≥ 3	4	8	E_6	F_4 or E_6
III^*	3	≥ 5	9	E_7	E_7
II^*	≥ 4	5	10	E_8	E_8
non-minimal	≥ 4	≥ 6	≥ 12	non-canonical	–

Kodaira's classification of singular fibers and gauge groups

Minimal versus non-minimal: codimension 2

In codimension 2: $u = v = 0$ in the base:

1. Minimal: ordinary bifundamental matter, and codimension two fiber are (monodromy-reduced) Kodaira fibers (collection of rational curves).

2. Non-mininal: Weierstrass model

$$y^2 = x^3 + fx + g\,, \qquad \text{ord}_{u=v=0}(f, g, \Delta) \geq (4, 6, 12)$$

   Does not have a Calabi-Yau resolution of the fiber that keeps it complex 1d. Two options:

   (1) Blowup the base (6d approach)

   (2) Fiberal resolution, with non-flat fibers

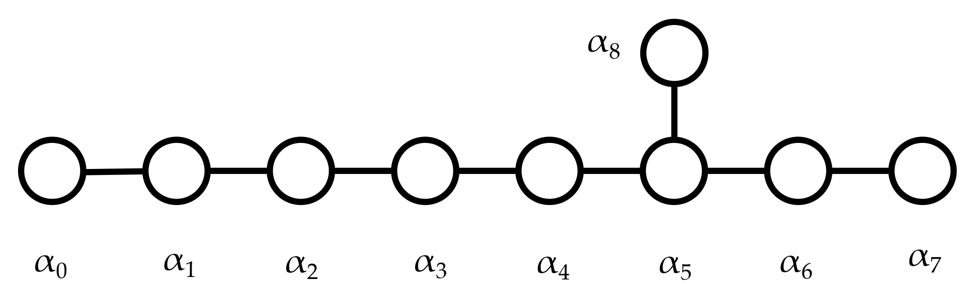
Example: The 6d E-string and 5d rank 1 SCFTs

1. Starting point 6d: $II^* - I_1$ collision: Tate model:
   $\text{ord}_u(b_i) = (1,2,3,4,5)$ and $\text{ord}_v(b_i) = (0,0,0,0,1)$

$$y^2 + b_1 uxy + b_3 u^3 = x^3 + b_2 u^2 x^2 + b_4 u^4 x + b_6 u^5 v$$

2. Geometry after blowup:

   1. $\widehat{E}_8$ worth of non-compact surfaces, fibered by $\mathbb{P}^1 = S^2$

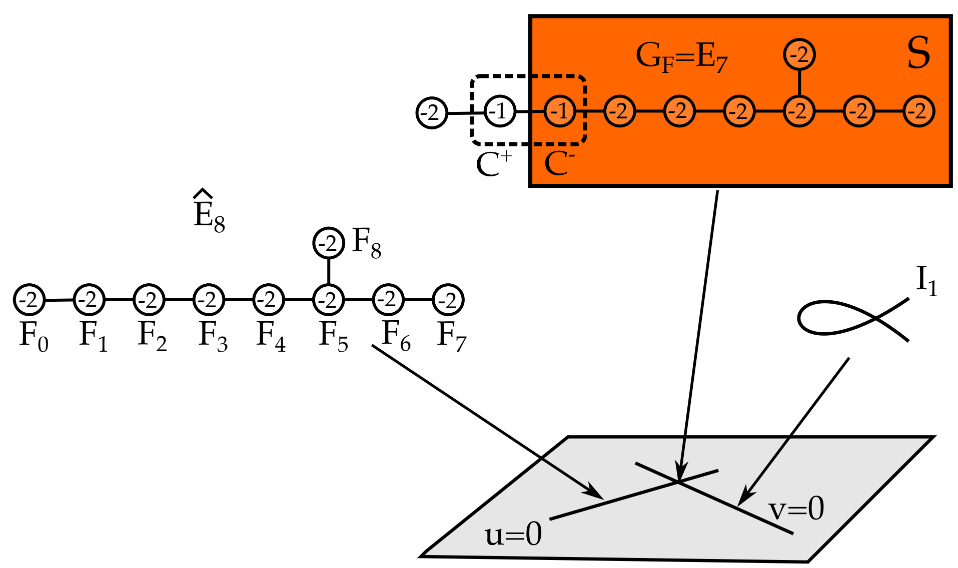


   2. $S \longleftrightarrow$ Compact surface component

   $\Rightarrow$ Rank 1 gauge group

E-string: Non-Flat Resolution

Non-flat fiber resolution



Physics:

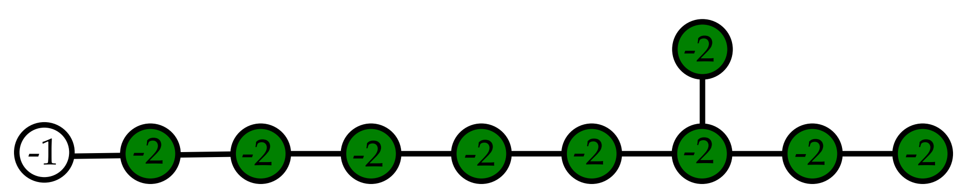
Non-flat fiber surface is compact $S \leftrightarrow U(1)$ gauge field.

Flavor $\mathbb{P}^1$s contained in S_i remain flavor symmetries as $\mathrm{vol}(S_i) \to 0$ limit.

Geometry:

- Different resolution sequences of the E_8 and codim 2 non-flat locus, yield different 5d theories.
- They are related by flops: shinking -1 curves and transforming them out of the surface S.
- Strategy: start with "marginal" theory, i.e. $\mathbb{P}^1_i \subset S_1$ for all $i = 0, \cdots, 8$, and descend to other models by flops.

Marginal model for rank one theories: $S_1 = gdP_9$ showing curves $\mathbb{P}^1_i = S_1 \cdot D_i$



Non-flat Resolutions from Graph-Transitions

Start with diagram of curves that are contained in the compact surface S.
For higher rank: Consider Mori cone generators of reducible surface $\cup_k S_k$.

Combined Fiber Diagram (CFD):

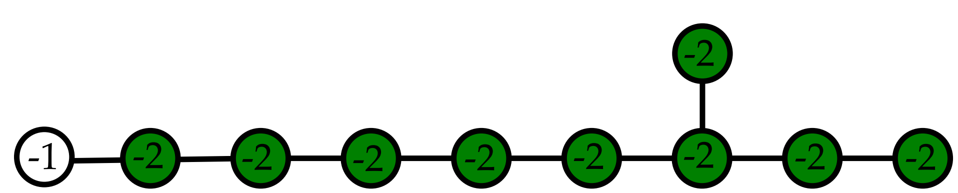
CFDs are graphs with:

Vertices: Curves C_i, labeled by $C_i^2 = n_i$

Edges: $C_i \cdot C_j = m_{i,j}$ edges connecting vertices

Marked vertices: $n_i = -2$ colored
$\Rightarrow$ subgraph = Dynkin of superconformal flavor symmetry

Marginal rank 1 CFD:



CFD Transitions

Given a CFD, the descendant CFDs are obtained by removing $n_i = (-1)$ vertex and updating

$$n'_j = n_j + m_{i,j}^2$$
$$m'_{j,k} = m_{j,k} + m_{i,j} m_{i,k}$$

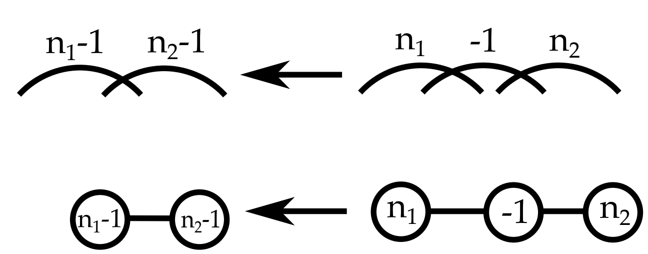
Any (-2)-vertex whose n_j changes becomes unmarked.

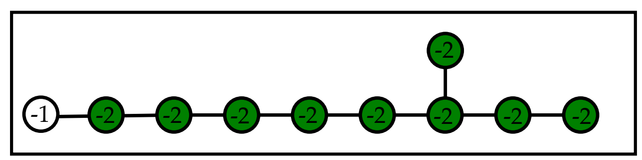
$\Rightarrow$ Network of CFDs/SCFTs

What are these?

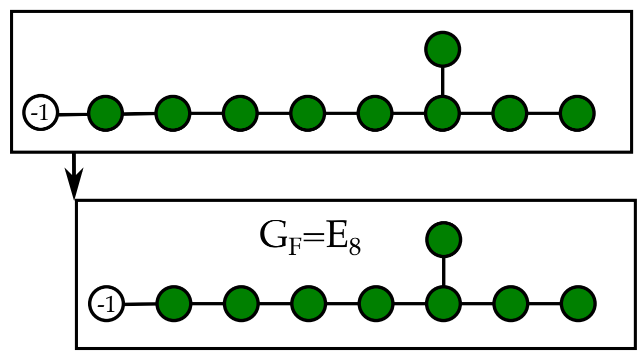
5d SCFT: gives mass to flavors, decoupling them and leads to weakly coupled theory for another SCFT

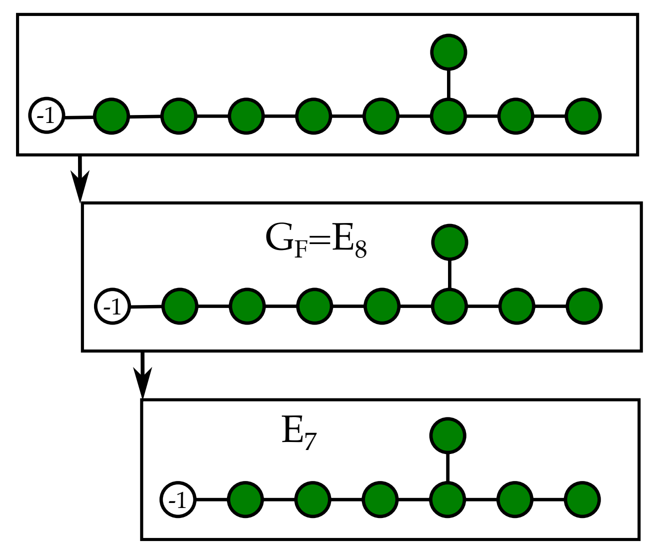
Geometry: (-1) curves can be contracted and flopped out of S_i:



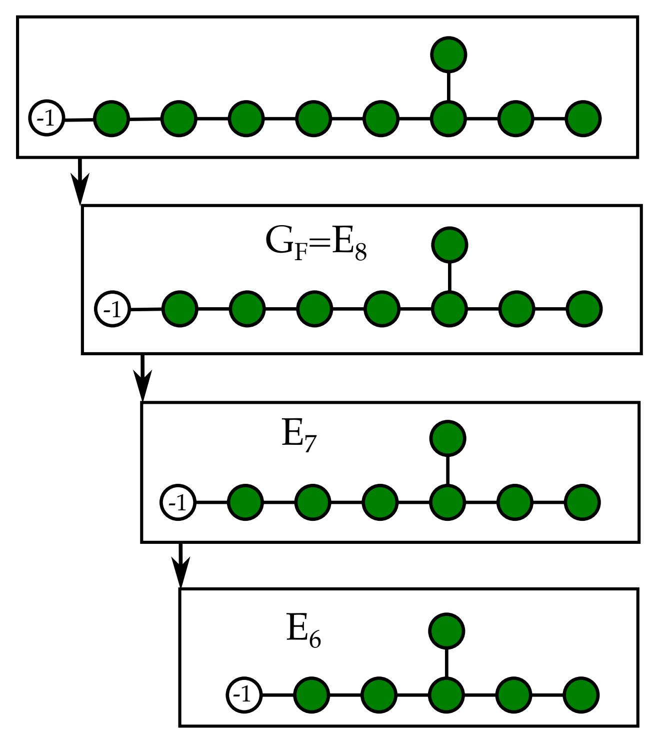


-2
-1
-2
-2
-2
-2
-2
-2
-2
-2

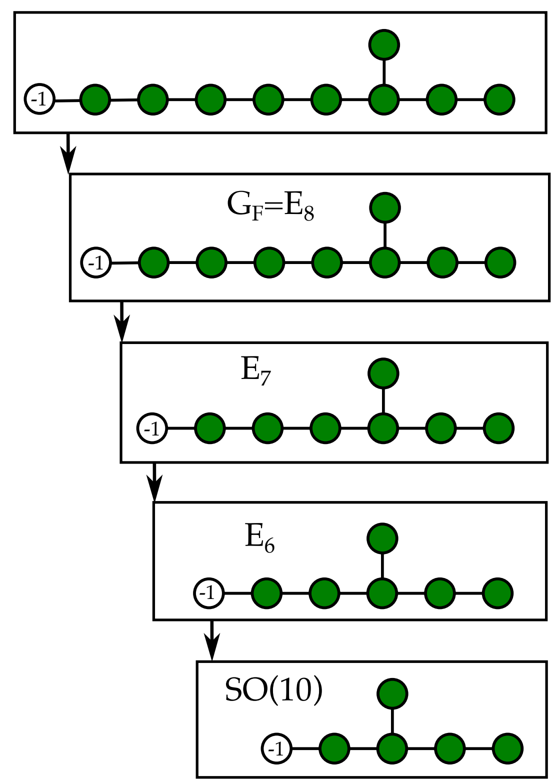

-1
$G_F=E_8$
-1



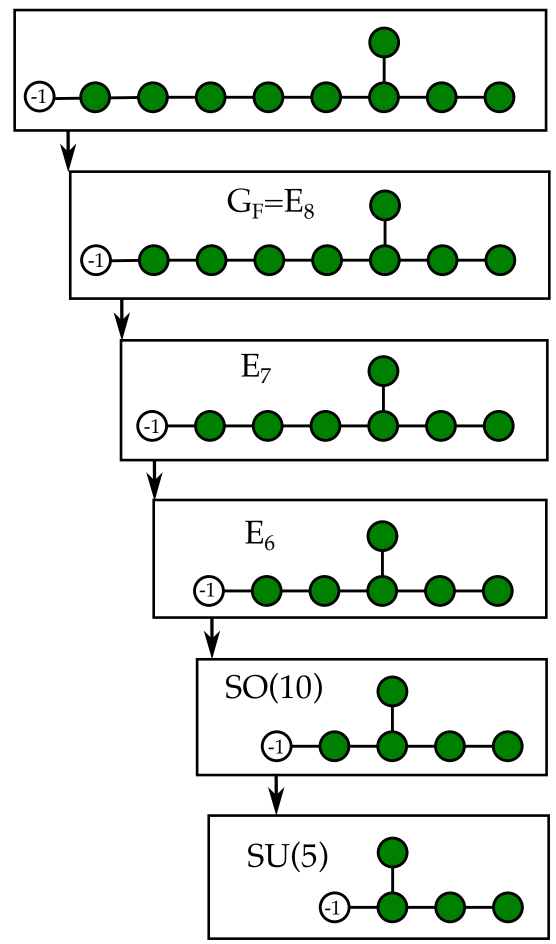
-1
G_F=E_8
-1
E_7
-1

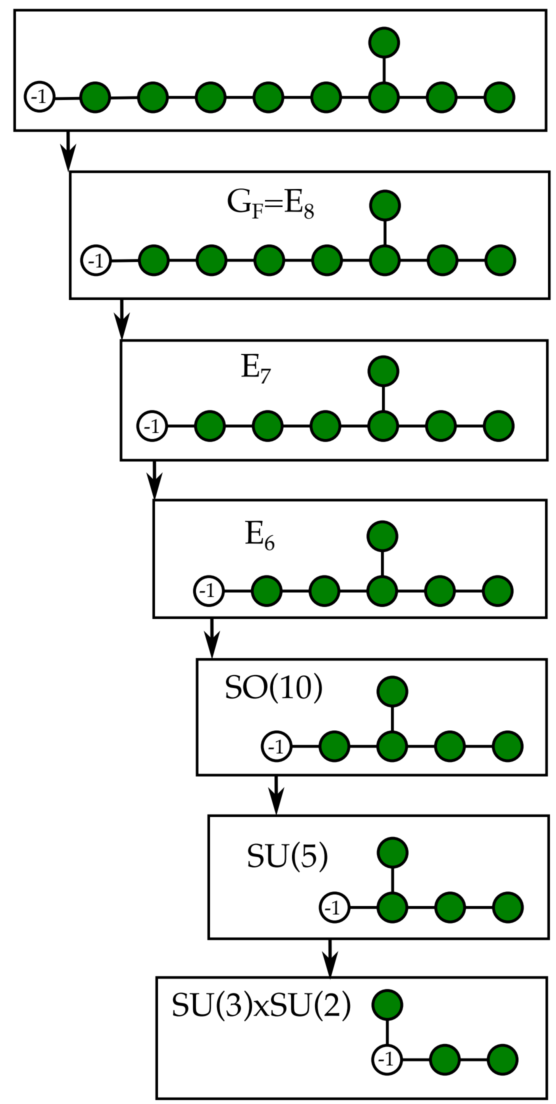


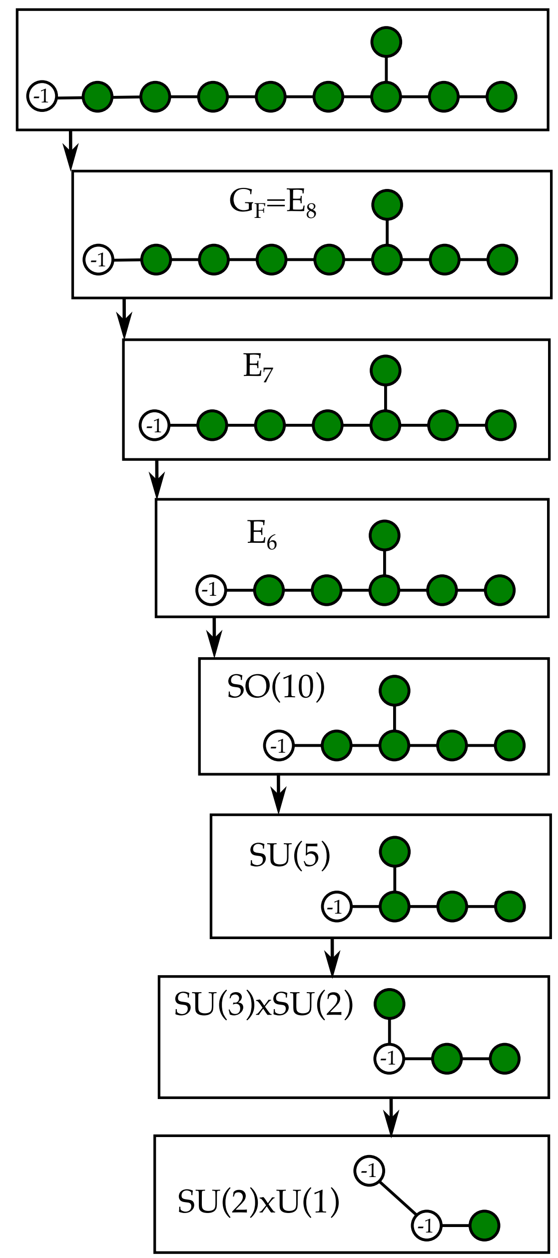
-1
G$_F$=E$_8$
-1
E$_7$
-1
E$_6$
-1



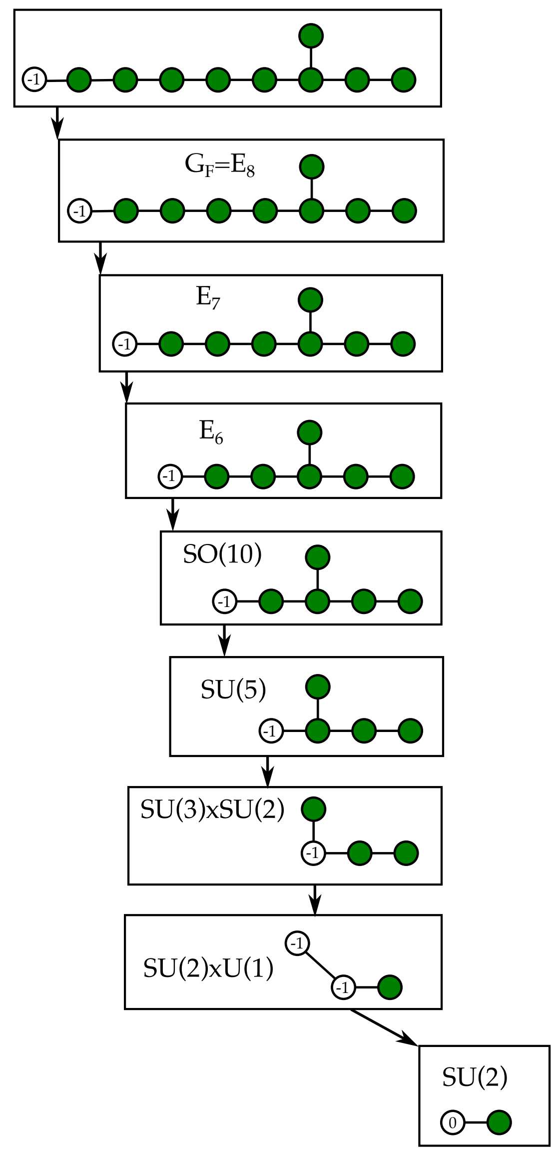
-1
G_F=E_8
E_7
E_6
SO(10)

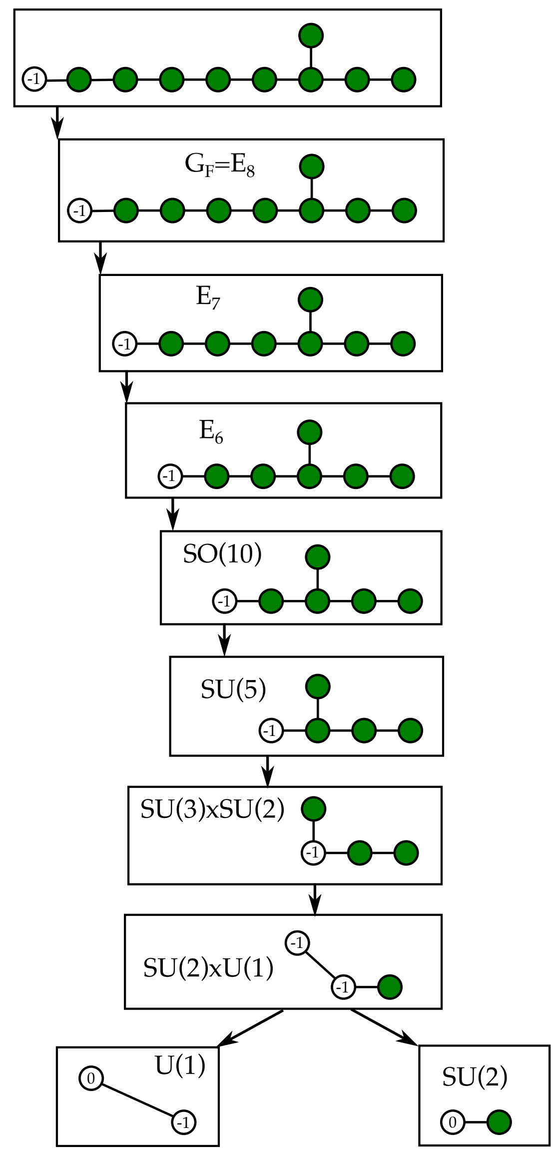

-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1


-1
G_F=E_8
E_7
E_6
SO(10)
SU(5)
SU(3)xSU(2)

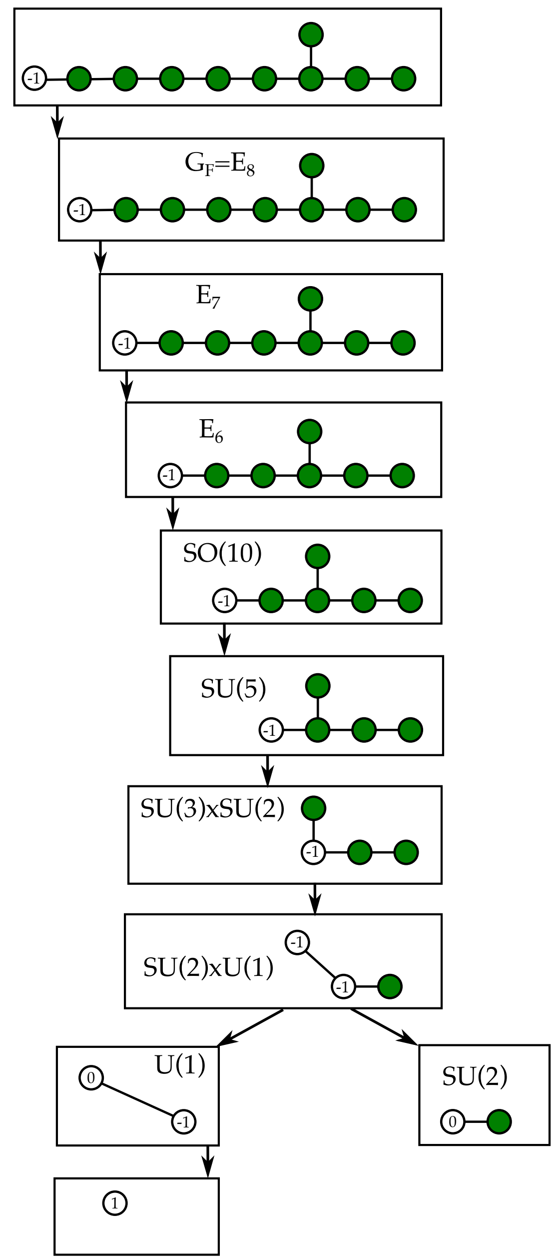


-1
G_F=E_8
E_7
E_6
SO(10)
SU(5)
SU(3)xSU(2)
SU(2)xU(1)

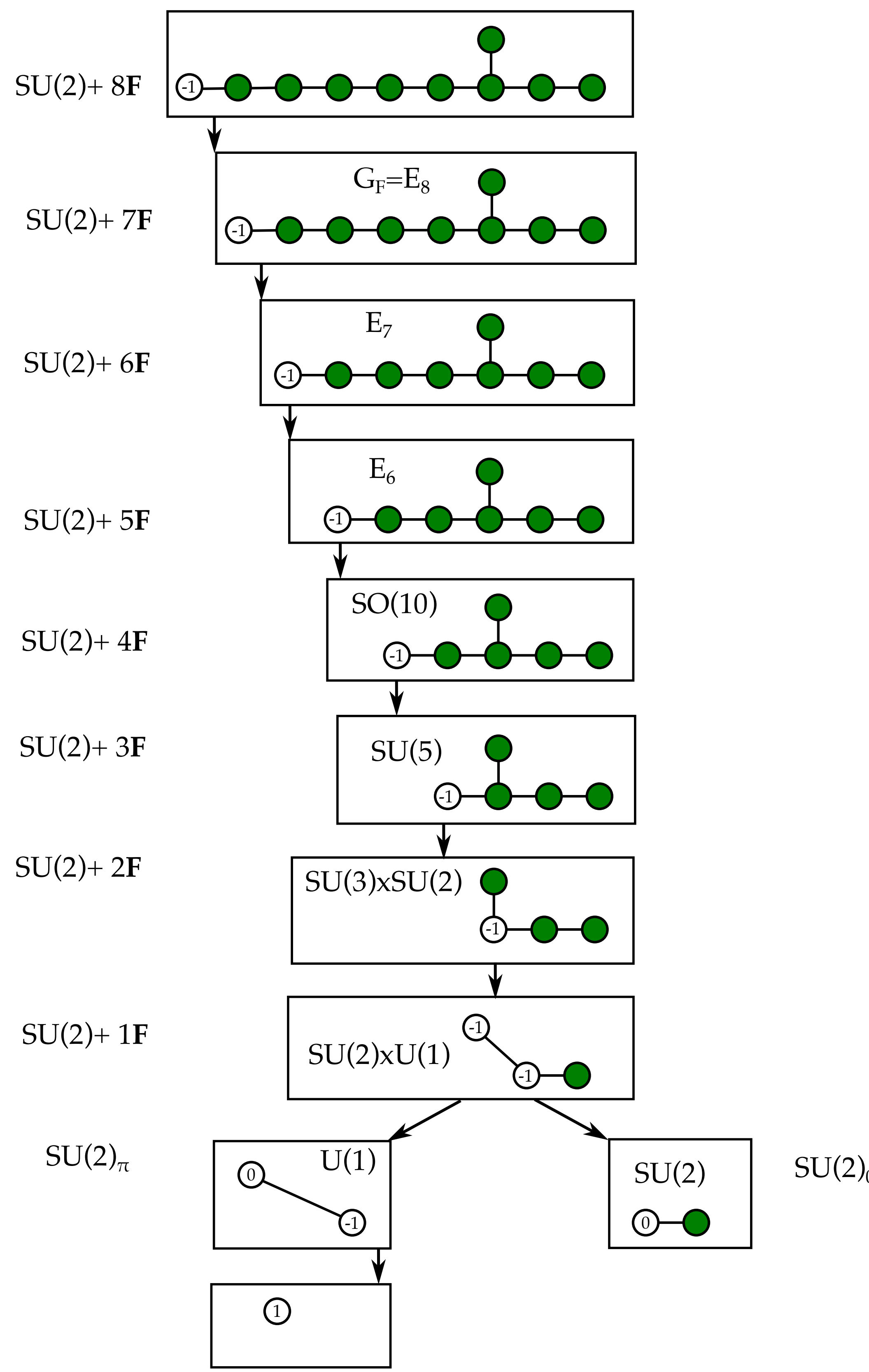

-1
G_F=E_8
E_7
E_6
SO(10)
SU(5)
SU(3)xSU(2)
SU(2)xU(1)
SU(2)
0



-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1
SU(3)xSU(2)
-1
SU(2)xU(1)
-1
-1
U(1)
0
-1
SU(2)
0



-1
G_F=E_8
-1
E_7
-1
E_6
-1
SO(10)
-1
SU(5)
-1
SU(3)xSU(2)
-1
SU(2)xU(1)
-1
-1
U(1)
0
-1
SU(2)
0
1



SU(2)+ 8F
SU(2)+ 7F
GF=E8
SU(2)+ 6F
E7
SU(2)+ 5F
E6
SU(2)+ 4F
SO(10)
SU(2)+ 3F
SU(5)
SU(2)+ 2F
SU(3)xSU(2)
SU(2)+ 1F
SU(2)xU(1)
SU(2)π
U(1)
SU(2)
SU(2)0

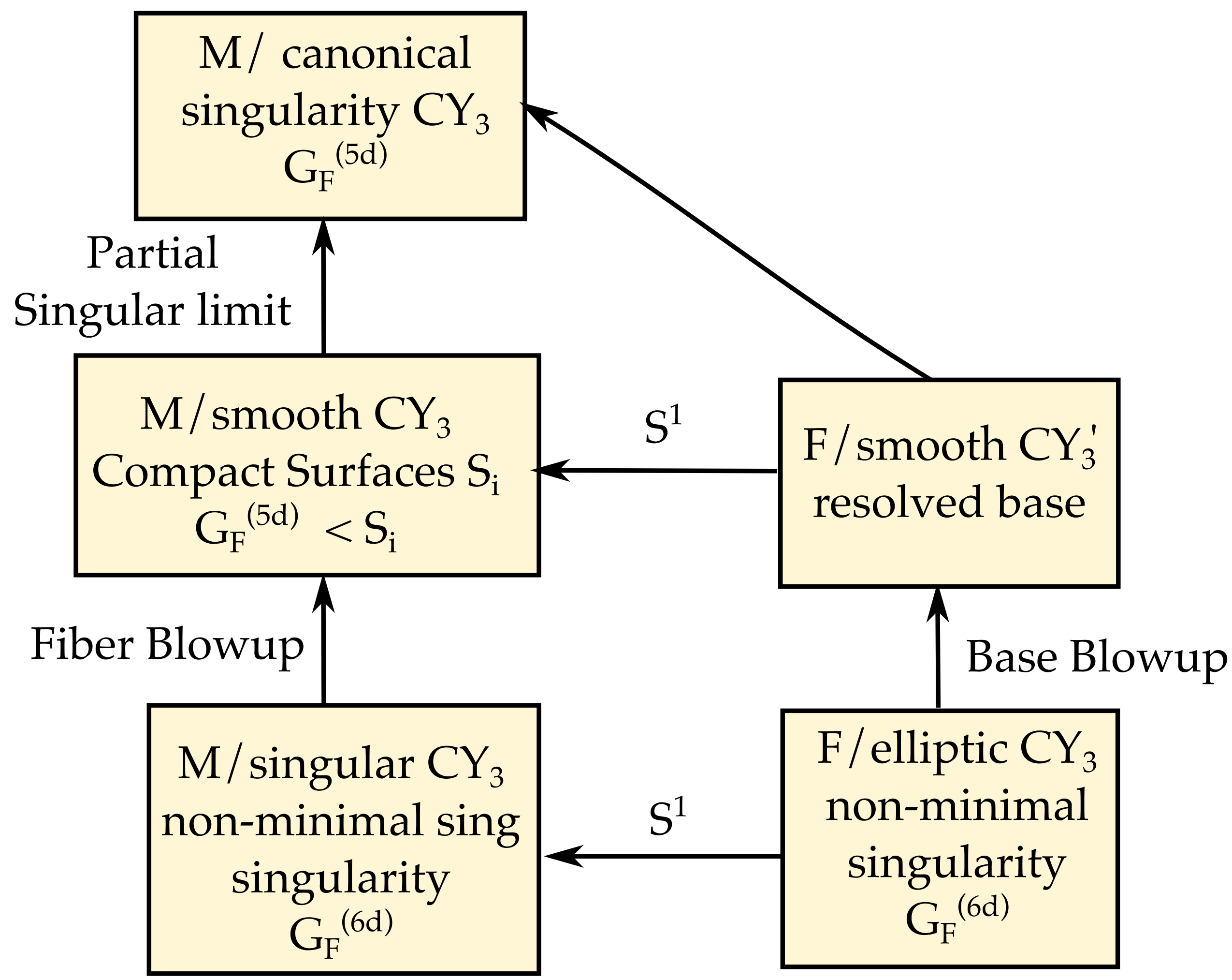
Rank 1 CFDs

This constructs precisely the known theories:

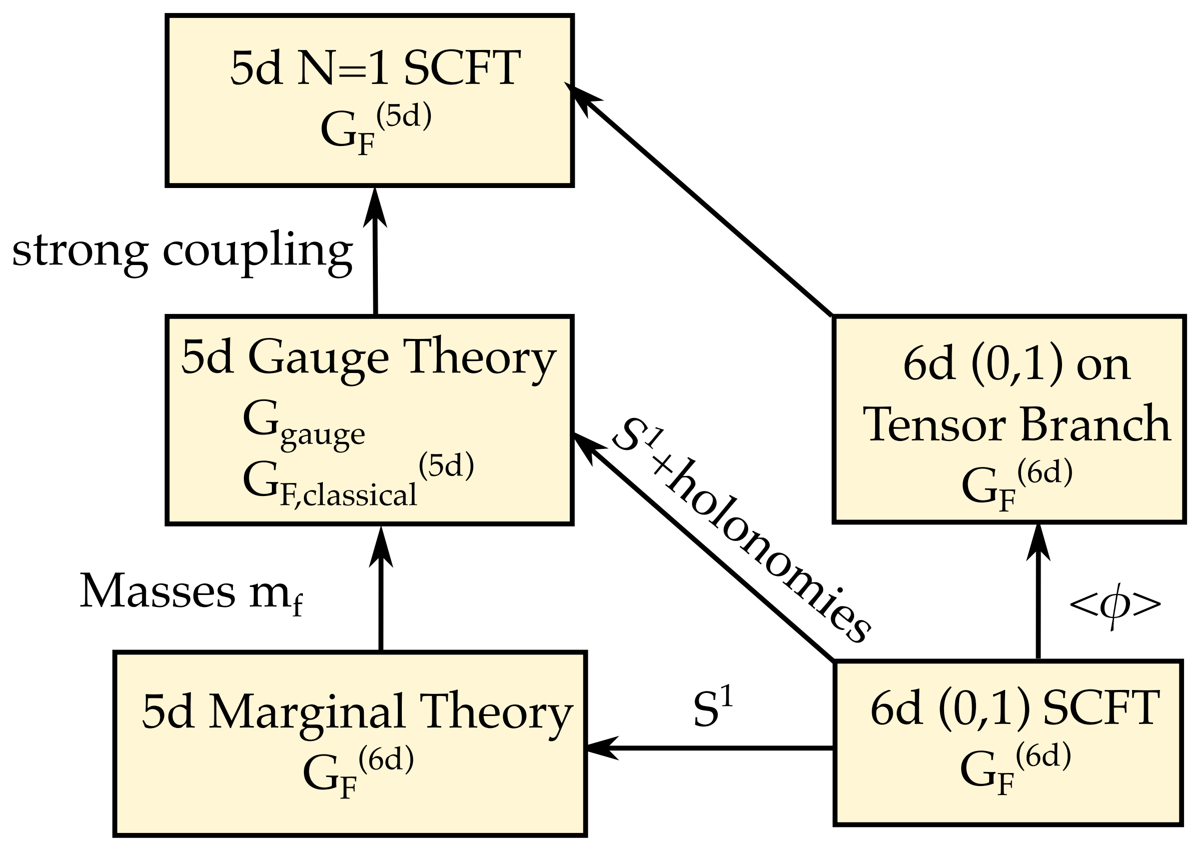
- ⋆ $SU(2)$ gauge theory with N_F flavors, which enhances to E_{N_F+1} flavor symmetry at SCFT point
  $\Rightarrow$ this reproduces classic results for rank 1 [Seiberg]
- ⋆ From non-flat fiber: $\mathbb{P}^1$s that are contained in S (green) encode superconformal flavor symmetry G_F
- ⋆ Includes 5d SCFT without weakly coupled gauge theory description, geometry of $S = \mathbb{P}^2$ (no ruling)

General Strategy: Geometry

[Apruzzi, Lawrie, Lin, SSN, Wang]



General Strategy: 6d to 5d



What about higher rank? Rank 2

Rank 2 theories:
In 2018, a purely geometric classification using geometry of surfaces [Jefferson, Katz, Kim, Vafa], and from pq-5-brane webs by [Hayashi, Kim, Lee, Yagi]. However these approaches do not *manifestly* encode G_{F}, BPS states, etc.

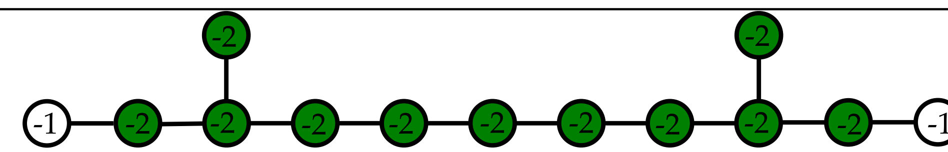
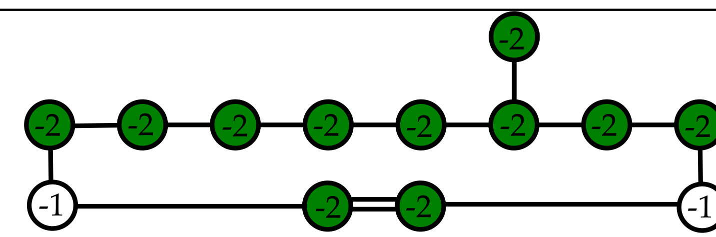
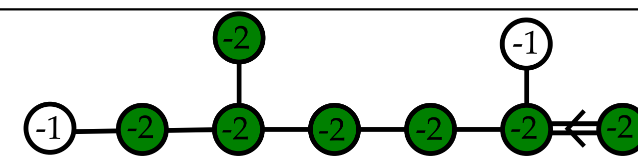
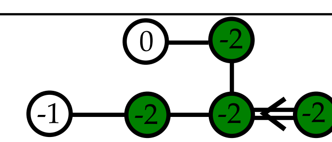
Using CFDs we obtained an independent derivation, which in addition keeps track of the full superconformal flavor symmetry and BPS states. [Apruzzi, Lawrie, Lin, SSN, Wang]

Strategy: determine the marginal theories and apply CFD-transitions.

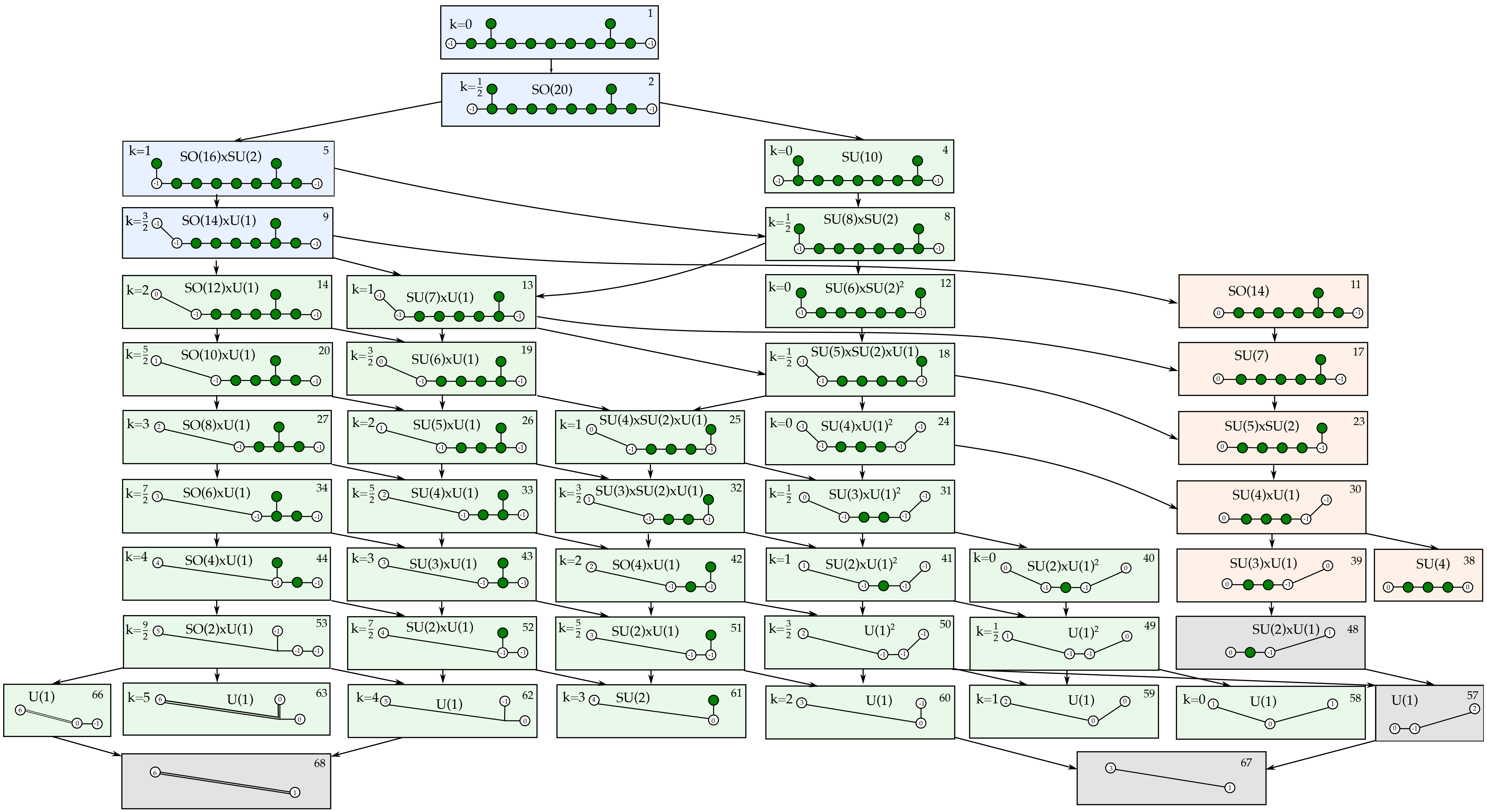
CFDs for the marginal theories have to be computed by doing a geometric resolution.

CFDs for Rank 2 Theories

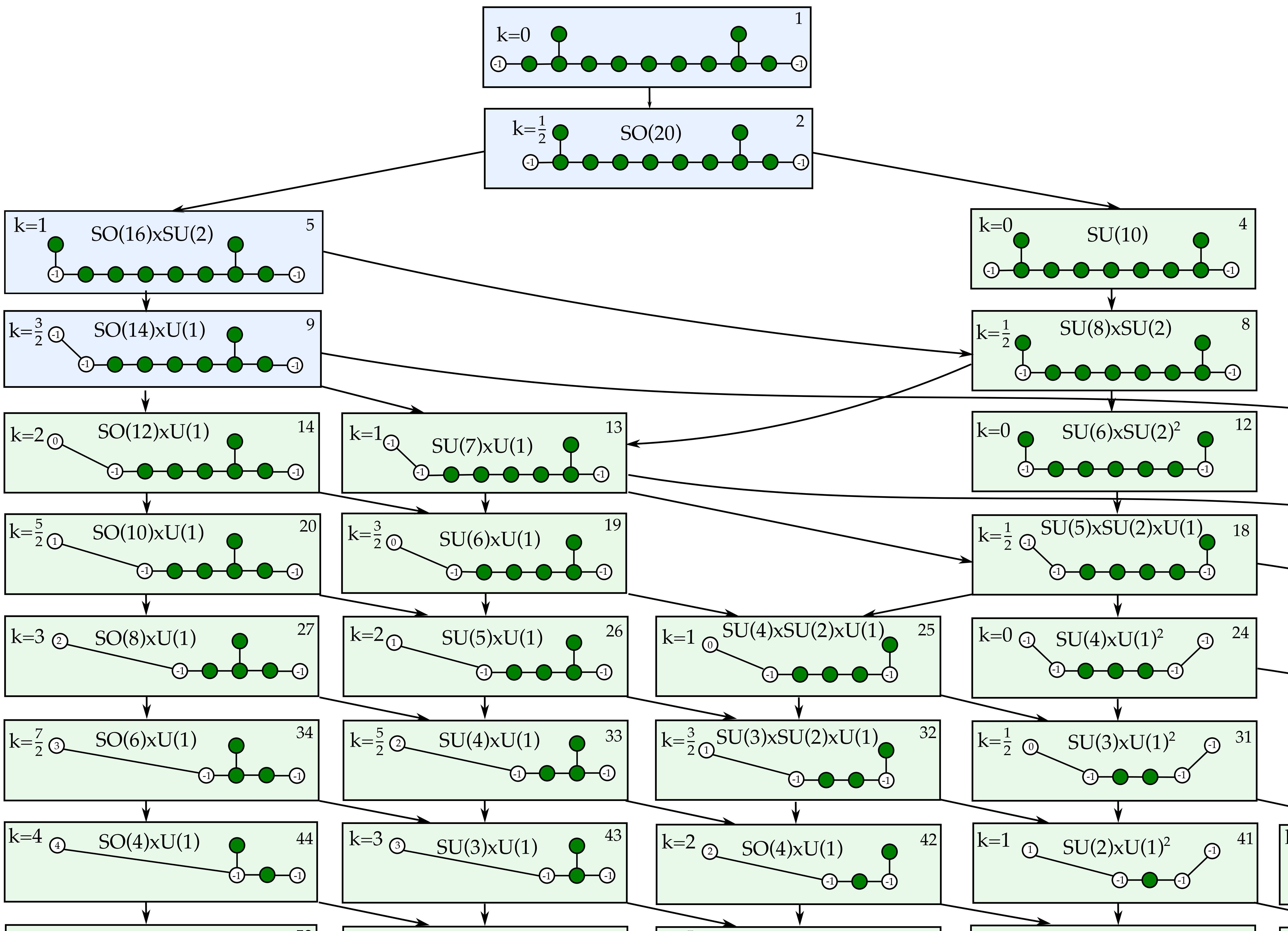
CFDs for marginal theories computed geometrically:

6d SCFT	CFD of the 5d Marginal Theory
(D_5, D_5) Conformal Matter	
Rank 2 E-string: $II^* - I_2$	
$SU(3)$ on a (-1)-curve + 12 hypers	
$SU(3)$ on a (-2)-curve + 6 hypers	

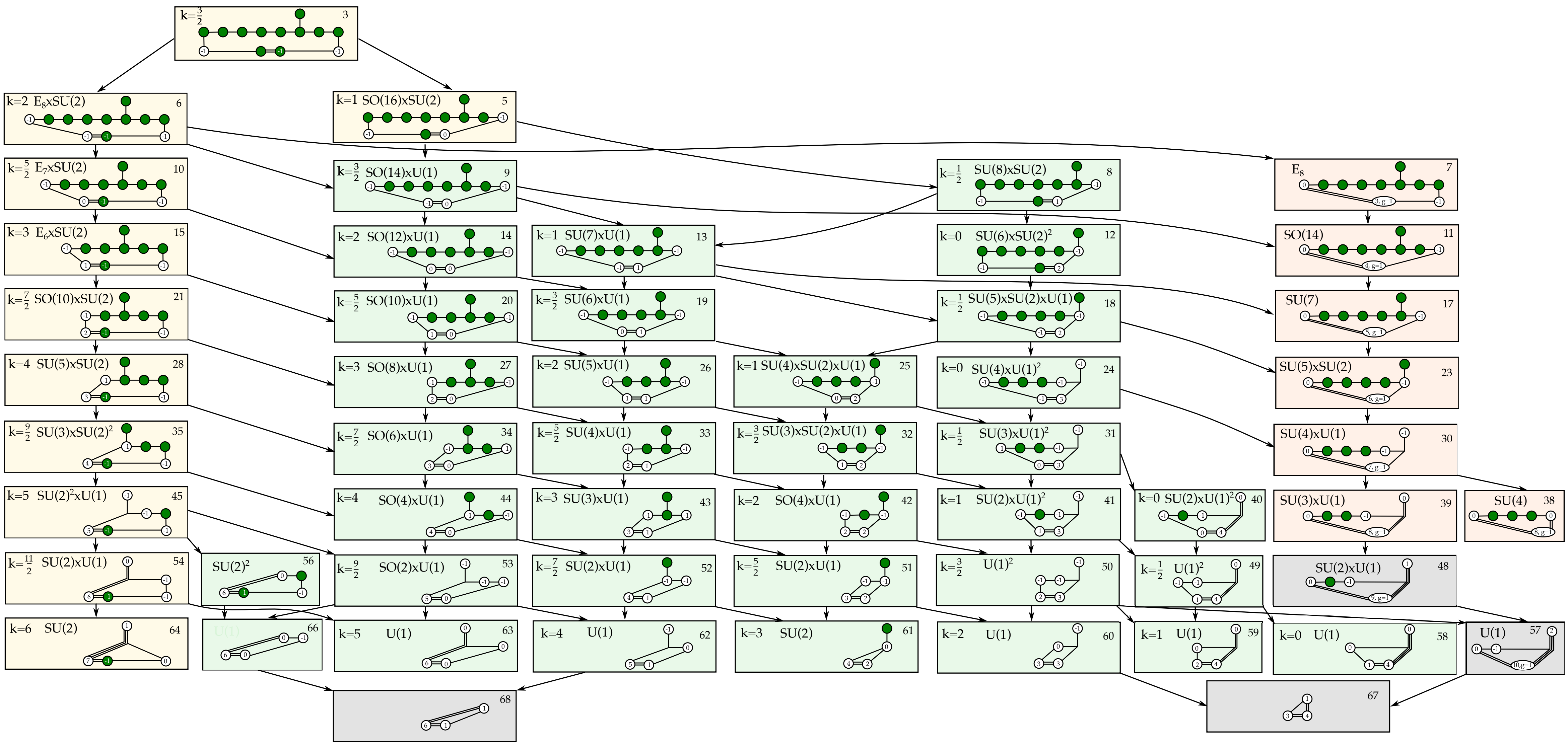
Next figures: blue: only D_{10} realization; green: also rank 2 E-string realization; levels: $SU(3)$; pink: $SU(2)^2$; grey: no weakly-coupled gauge theory realization.



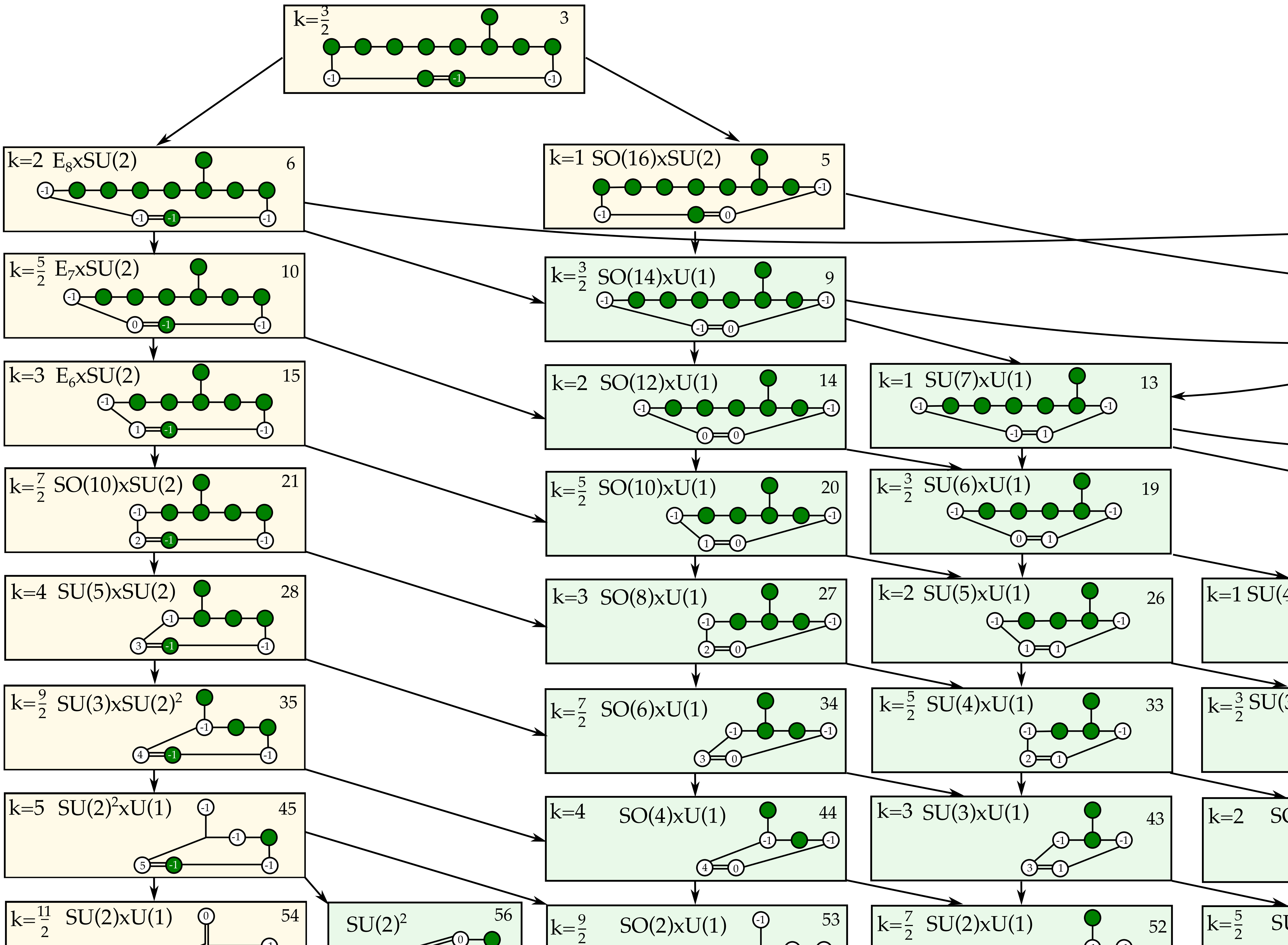
k=0
1
k=½
SO(20)
2
k=1
SO(16)xSU(2)
5
k=0
SU(10)
4
k=3/2
SO(14)xU(1)
9
k=½
SU(8)xSU(2)
8
k=2
SO(12)xU(1)
14
k=1
SU(7)xU(1)
13
k=0
SU(6)xSU(2)²
12
SO(14)
11
k=5/2
SO(10)xU(1)
20
k=3/2
SU(6)xU(1)
19
k=½
SU(5)xSU(2)xU(1)
18
SU(7)
17
k=3
SO(8)xU(1)
27
k=2
SU(5)xU(1)
26
k=1
SU(4)xSU(2)xU(1)
25
k=0
SU(4)xU(1)²
24
SU(5)xSU(2)
23
k=7/2
SO(6)xU(1)
34
k=5/2
SU(4)xU(1)
33
k=3/2
SU(3)xSU(2)xU(1)
32
k=½
SU(3)xU(1)²
31
SU(4)xU(1)
30
k=4
SO(4)xU(1)
44
k=3
SU(3)xU(1)
43
k=2
SO(4)xU(1)
42
k=1
SU(2)xU(1)²
41
k=0
SU(2)xU(1)²
40
SU(3)xU(1)
39
SU(4)
38
k=9/2
SO(2)xU(1)
53
k=7/2
SU(2)xU(1)
52
k=5/2
SU(2)xU(1)
51
k=3/2
U(1)²
50
k=½
U(1)²
49
SU(2)xU(1)
48
U(1)
66
k=5
U(1)
63
k=4
U(1)
62
k=3
SU(2)
61
k=2
U(1)
60
k=1
U(1)
59
k=0
U(1)
58
U(1)
57
68
67



k=0
k=1/2 SO(20)
k=1 SO(16)xSU(2)
k=0 SU(10)
k=3/2 SO(14)xU(1)
k=1/2 SU(8)xSU(2)
k=2 SO(12)xU(1)
k=1 SU(7)xU(1)
k=0 SU(6)xSU(2)^2
k=5/2 SO(10)xU(1)
k=3/2 SU(6)xU(1)
k=1/2 SU(5)xSU(2)xU(1)
k=3 SO(8)xU(1)
k=2 SU(5)xU(1)
k=1 SU(4)xSU(2)xU(1)
k=0 SU(4)xU(1)^2
k=7/2 SO(6)xU(1)
k=5/2 SU(4)xU(1)
k=3/2 SU(3)xSU(2)xU(1)
k=1/2 SU(3)xU(1)^2
k=4 SO(4)xU(1)
k=3 SU(3)xU(1)
k=2 SO(4)xU(1)
k=1 SU(2)xU(1)^2

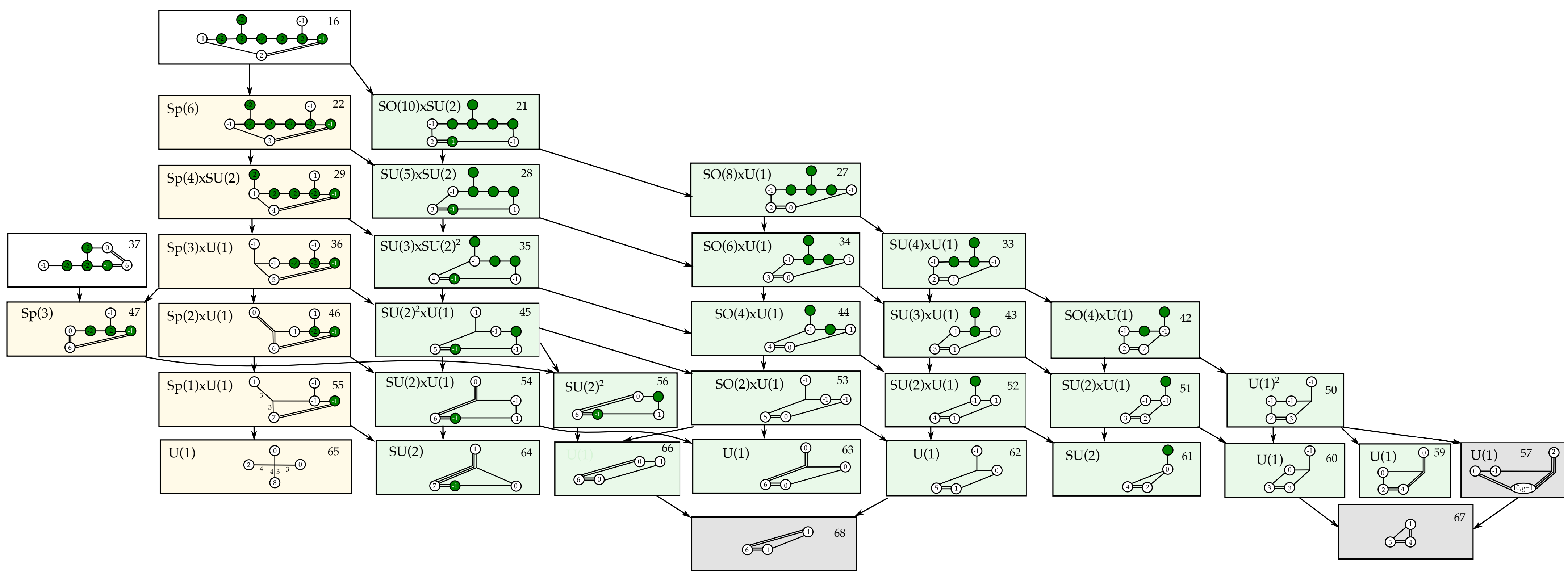


k=3/2 3
k=2 E8xSU(2) 6
k=1 SO(16)xSU(2) 5
k=5/2 E7xSU(2) 10
k=3/2 SO(14)xU(1) 9
k=1/2 SU(8)xSU(2) 8
E8 7
k=3 E6xSU(2) 15
k=2 SO(12)xU(1) 14
k=1 SU(7)xU(1) 13
k=0 SU(6)xSU(2)2 12
SO(14) 11
k=7/2 SO(10)xSU(2) 21
k=5/2 SO(10)xU(1) 20
k=3/2 SU(6)xU(1) 19
k=1/2 SU(5)xSU(2)xU(1) 18
SU(7) 17
k=4 SU(5)xSU(2) 28
k=3 SO(8)xU(1) 27
k=2 SU(5)xU(1) 26
k=1 SU(4)xSU(2)xU(1) 25
k=0 SU(4)xU(1)2 24
SU(5)xSU(2) 23
k=9/2 SU(3)xSU(2)2 35
k=7/2 SO(6)xU(1) 34
k=5/2 SU(4)xU(1) 33
k=3/2 SU(3)xSU(2)xU(1) 32
k=1/2 SU(3)xU(1)2 31
SU(4)xU(1) 30
k=5 SU(2)2xU(1) 45
k=4 SO(4)xU(1) 44
k=3 SU(3)xU(1) 43
k=2 SO(4)xU(1) 42
k=1 SU(2)xU(1)2 41
k=0 SU(2)xU(1)2 40
SU(3)xU(1) 39
SU(4) 38
k=11/2 SU(2)xU(1) 54
SU(2)2 56
k=9/2 SO(2)xU(1) 53
k=7/2 SU(2)xU(1) 52
k=5/2 SU(2)xU(1) 51
k=3/2 U(1)2 50
k=1/2 U(1)2 49
SU(2)xU(1) 48
k=6 SU(2) 64
U(1) 66
k=5 U(1) 63
k=4 U(1) 62
k=3 SU(2) 61
k=2 U(1) 60
k=1 U(1) 59
k=0 U(1) 58
U(1) 57
68
67



k=3/2 3
k=2 E8xSU(2) 6
k=1 SO(16)xSU(2) 5
k=5/2 E7xSU(2) 10
k=3/2 SO(14)xU(1) 9
k=3 E6xSU(2) 15
k=2 SO(12)xU(1) 14
k=1 SU(7)xU(1) 13
k=7/2 SO(10)xSU(2) 21
k=5/2 SO(10)xU(1) 20
k=3/2 SU(6)xU(1) 19
k=4 SU(5)xSU(2) 28
k=3 SO(8)xU(1) 27
k=2 SU(5)xU(1) 26
k=9/2 SU(3)xSU(2)^2 35
k=7/2 SO(6)xU(1) 34
k=5/2 SU(4)xU(1) 33
k=5 SU(2)^2xU(1) 45
k=4 SO(4)xU(1) 44
k=3 SU(3)xU(1) 43
k=2
k=11/2 SU(2)xU(1) 54
SU(2)^2 56
k=9/2 SO(2)xU(1) 53
k=7/2 SU(2)xU(1) 52

$SU(3)$ on a (-1)-curve + 12 hypers and $SU(3)$ on (-2)-curve + 6 hypers



Summary:

- Rank 1 and 2: complete agreement with expected network and subset of known enhanced flavor symmetries.
- In addition to the classification, the CFD-approach predicts new flavor symmetry enhancements, and encodes G_{F} manifestly, and deformations (-1)-vertices. Also: BPS states.
- Requires one geometric resolution computation for CFD of the marginal theory.

Cross-checks:

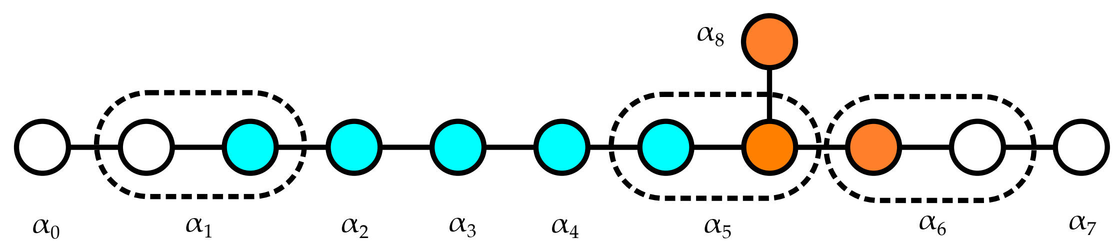
1. Geometry
2. Coulomb branch of weakly-coupled gauge theory.

Cross-Check 1: Geometric Resolutions

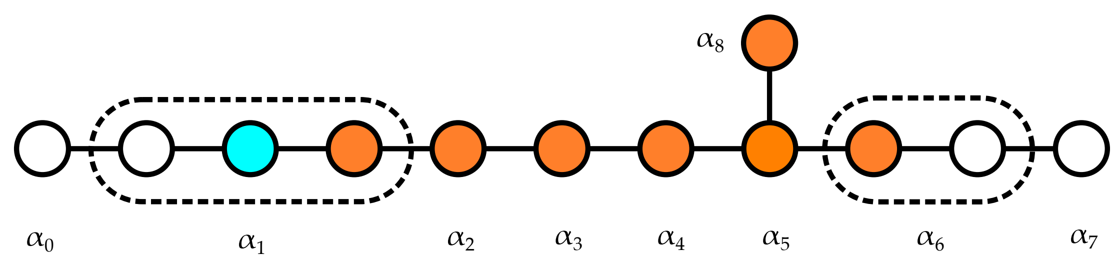
Fiber geometry of non-flat resolutions reproduces the descendant CFDs (using methods from [Lawrie, SSN][Tian, Wang]).

Rank 2 E-string codim 2 fibers and wrappings by S_1 and S_2

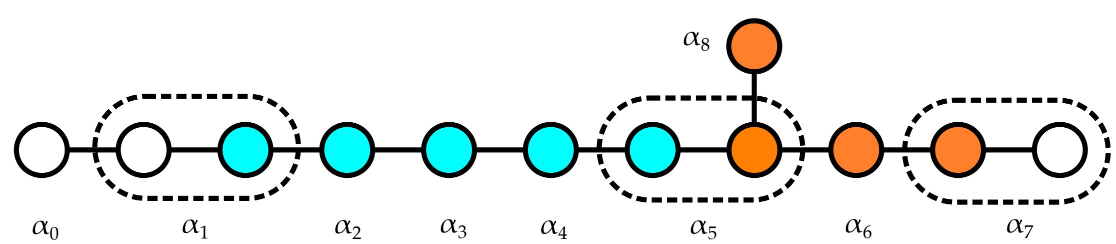
$G_{\mathrm{F}} = SU(6) \times U(1)$



$G_{\mathrm{F}} = SU(6) \times U(1)$



$G_{\mathrm{F}} = SO(12) \times U(1)$



CFDs are an efficient way to package the codim 2 fiber data that is relevant for the SCFT (first two models have same CFD).

Cross-Check 2: Gauge Theory

[Apruzzi, Lawrie, Lin, SSN, Wang, to appear Part II]

Whenever an SCFT has a weakly-coupled description:
study extended Coulomb branch using 'box graphs' in [Hayashi, Lawrie, Morrison SSN].

⋆ Rank 1: Coulomb branch phases of $SU(2) \times SO(16)_{\text{cl, F}}$ with $(\mathbf{2}, \mathbf{16})$
⋆ Rank 2: $G_{\text{gauge}} = SU(3),\ Sp(2),\ SU(2) \times SU(2),\ G_2$.
E.g. for rannk 2 E-string:

$$SU(3) \times U(9)_{\text{cl, F}} \quad \text{with} \quad (\mathbf{3}, \mathbf{9})$$

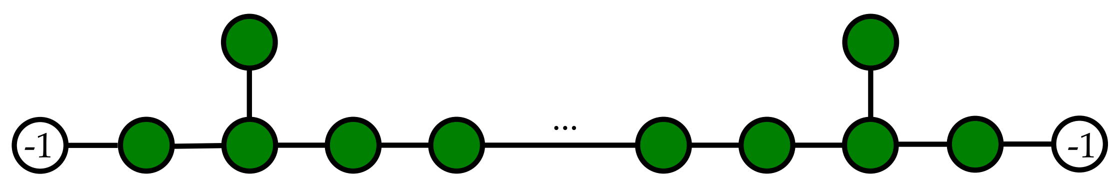
Matches all CFD-tree and consistent with G_{F} [...] that admit an $SU(3)$ gauge theory description at weak coupling.

However, there are theories with no weakly-coupled description (geometrically: surfaces do not admit a ruling), then geometric realization is only evidence.

Higher Rank

For any 6d SCFT, we only need to determine the marginal CFD. The rest is algorithmic.

Infinite class: (D_k, D_k) minimal Conformal Matter. Marginal CFD is:

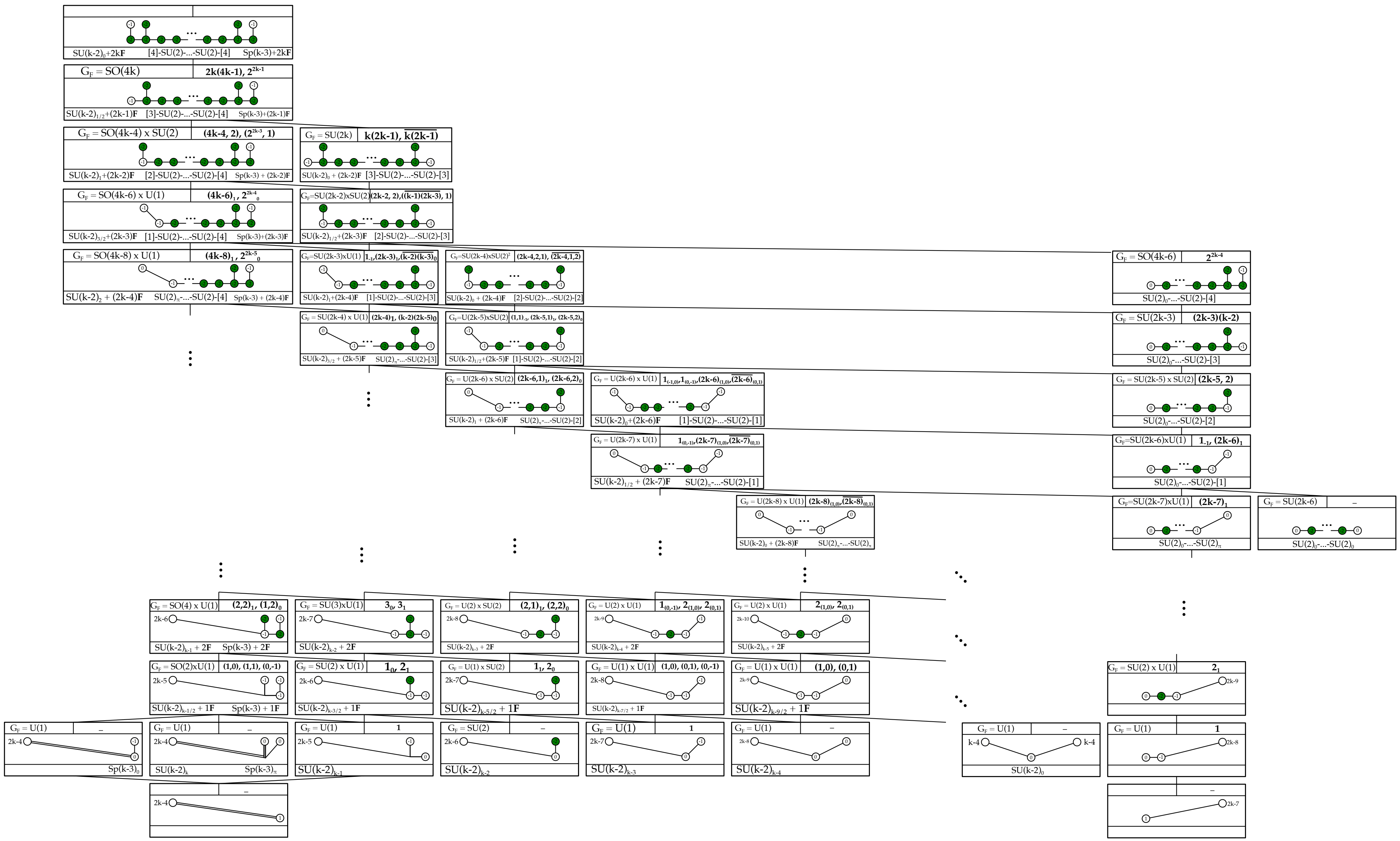


Weakly coupled gauge theory descriptions of marginal theory:

- $SU(k-2)_0$ with $2k\mathbf{F}$
- $4\mathbf{F} - SU(2) - ... - SU(2) - 4\mathbf{F}$, with $k-5$ $SU(2)$s nodes and theta angle 0
- $Sp(k-3)$ with $2k\mathbf{F}$.

To determine the daughter CFDs, run algorithm:

[Full CFDs at: `https://people.maths.ox.ac.uk/schafernamek/CFD/`]



SU(k-2)0+2kF
[4]-SU(2)-...-SU(2)-[4]
Sp(k-3)+2kF
GF = SO(4k)
2k(4k-1), 2^2k-1
SU(k-2)1/2+(2k-1)F
[3]-SU(2)-...-SU(2)-[4]
Sp(k-3)+(2k-1)F
GF = SO(4k-4) x SU(2)
(4k-4, 2), (2^2k-3, 1)
SU(k-2)1+(2k-2)F
[2]-SU(2)-...-SU(2)-[4]
Sp(k-3) + (2k-2)F
GF = SU(2k)
k(2k-1), k(2k-1)
SU(k-2)0 + (2k-2)F
[3]-SU(2)-...-SU(2)-[3]
GF = SO(4k-6) x U(1)
SU(k-2)3/2+(2k-3)F
[1]-SU(2)-...-SU(2)-[4]
Sp(k-3)+(2k-3)F
SU(k-2)1/2+(2k-3)F
[2]-SU(2)-...-SU(2)-[3]
GF = SO(4k-8) x U(1)
SU(k-2)2 + (2k-4)F
Sp(k-3) + (2k-4)F
SU(k-2)1+(2k-4)F
[1]-SU(2)-...-SU(2)-[3]
SU(k-2)0 + (2k-4)F
[2]-SU(2)-...-SU(2)-[2]
GF = SO(4k-6)
SU(2)0-...-SU(2)-[4]
GF = SU(2k-3)
(2k-3)(k-2)
SU(2)0-...-SU(2)-[3]
SU(k-2)3/2 + (2k-5)F
SU(k-2)1/2+(2k-5)F
[1]-SU(2)-...-SU(2)-[2]
GF = U(2k-6) x SU(2)
SU(k-2)1 + (2k-6)F
GF = U(2k-6) x U(1)
SU(k-2)0+(2k-6)F
[1]-SU(2)-...-SU(2)-[1]
GF = SU(2k-5) x SU(2)
(2k-5, 2)
SU(2)0-...-SU(2)-[2]
GF = U(2k-7) x U(1)
SU(k-2)1/2 + (2k-7)F
SU(2)π-...-SU(2)-[1]
GF=SU(2k-6)xU(1)
SU(2)0-...-SU(2)-[1]
GF = U(2k-8) x U(1)
SU(k-2)0 + (2k-8)F
GF=SU(2k-7)xU(1)
(2k-7)1
GF = SU(2k-6)
SU(2)0-...-SU(2)0
GF = SO(4) x U(1)
(2,2)1, (1,2)0
SU(k-2)k-1 + 2F
Sp(k-3) + 2F
GF = SU(3)xU(1)
30, 31
SU(k-2)k-2 + 2F
GF = U(2) x SU(2)
(2,1)1, (2,2)0
GF = U(2) x U(1)
GF = SO(2)xU(1)
(1,0), (1,1), (0,-1)
SU(k-2)k-1/2 + 1F
Sp(k-3) + 1F
GF = SU(2) x U(1)
10, 21
SU(k-2)k-3/2 + 1F
GF = U(1) x SU(2)
11, 20
SU(k-2)k-5/2 + 1F
GF = U(1) x U(1)
(1,0), (0,1), (0,-1)
(1,0), (0,1)
SU(k-2)k-9/2 + 1F
GF = SU(2) x U(1)
21
GF = U(1)
Sp(k-3)0
SU(k-2)k
Sp(k-3)π
SU(k-2)k-1
GF = SU(2)
SU(k-2)k-2
SU(k-2)k-3
SU(k-2)k-4
SU(k-2)0

(D_k, D_k) cont'd.

$(k-2)^2 - 3$ descendant SCFTs, $2k-6$ w/o weakly coupled description

Flavor enhancement at UV fixed point, e.g. $SU(k-2)_\kappa + m\mathbf{F}$

κ SCFT Flavor Symmetry G_F

$$k - \frac{m}{2} : \begin{cases} SO(4k) & m = 2k-1 \\ SO(4k-4) \times SU(2) & m = 2k-2 \\ SO(2m) \times U(1) & m = 0, ..., 2k-3 \end{cases}$$

$$k - 1 - \frac{m}{2} : \begin{cases} SU(2k) & m = 2k-2 \\ SU(2k-2) \times SU(2) & m = 2k-3 \\ SU(m+1) \times U(1) & m = 0, ..., 2k-4 \end{cases}$$

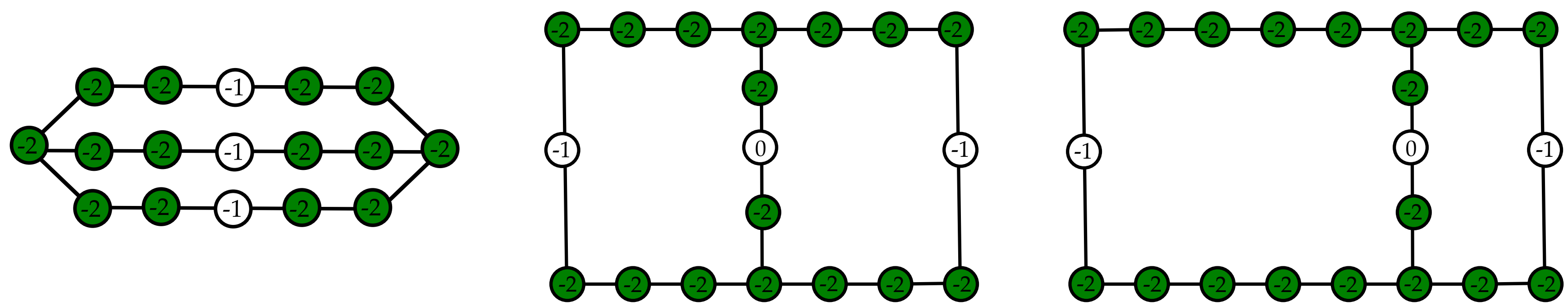
$$k - 2 - \frac{m}{2} : \begin{cases} SU(2k-4) \times SU(2)^2 & m = 2k-4 \\ U(m) \times SU(2) & m = 0, ..., 2k-5 \end{cases}$$

Agrees with 10/2018 results from 'magnetic quivers' [Cabrera, Hanany, Zajac]

$\rightarrow$ Ami Hanany's talk

Higher Rank: (E_n, E_n) Conformal Matter

(E_n, E_n) minimal Conformal Matter are 6d SCFTs with E_n^2 flavor symmetry.
We computed the CFDs for these 5d marginal theories to be:



(E_6,E_6) (E_7,E_7) (E_8,E_8)

Applying our algorithm, we construct all descendant 5d SCFTs, including flavor symmetry etc:

$$\begin{aligned} (E_6, E_6): &\qquad 93 \text{ descendant SCFTs} \\ (E_7, E_7): &\qquad 56 \text{ descendant SCFTs} \\ (E_8, E_8): &\qquad 127 \text{ descendant SCFTs}\,. \end{aligned}$$

Checks? The only weakly coupled quiver descriptions:

$\underline{(E_6, E_6):}$

$$\begin{array}{c} [2] \\ | \\ SU(2) \\ | \\ [2] - SU(2) - \; SU(3)_0 \; - SU(2) - [2]\,. \end{array}$$

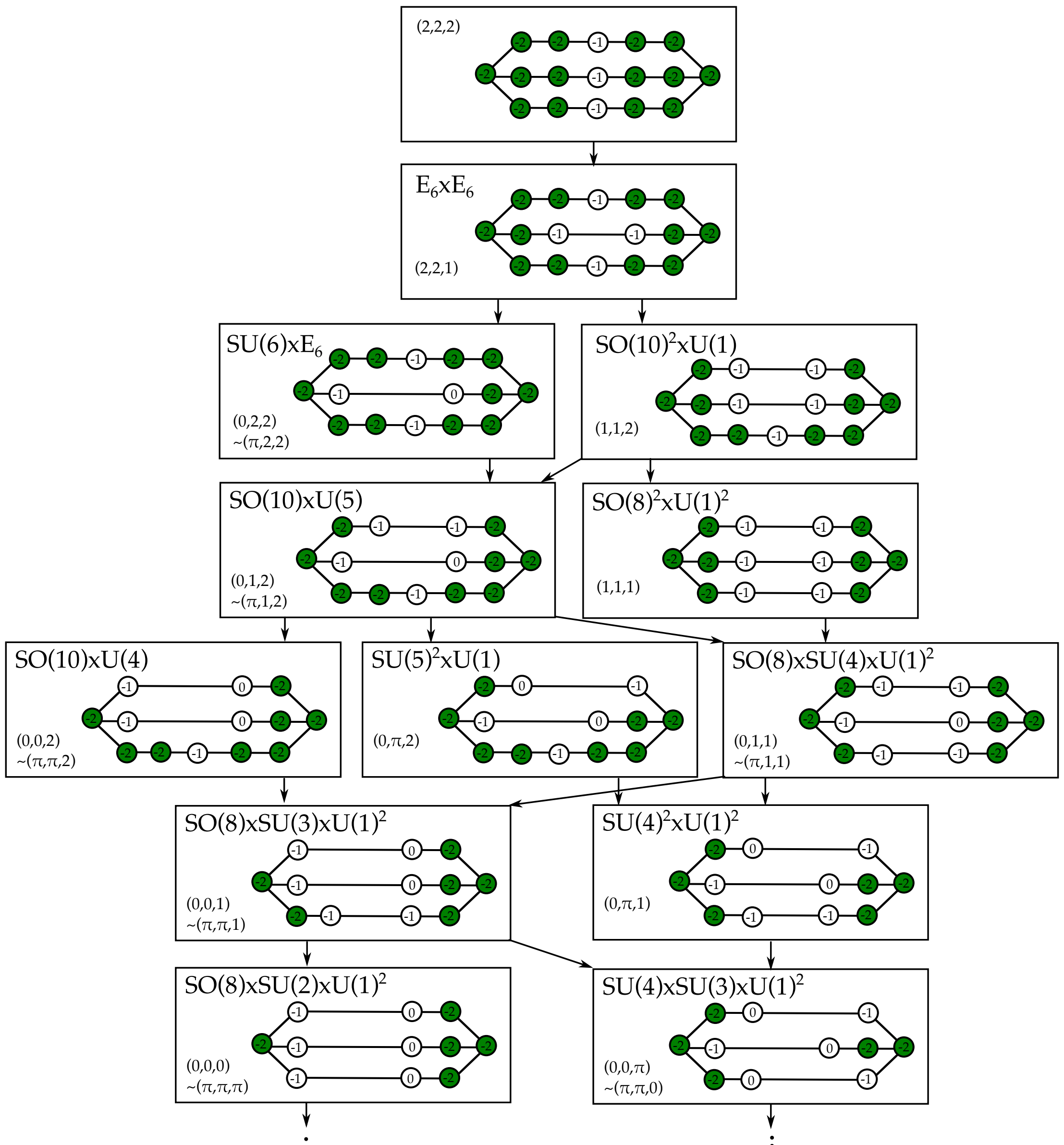
$\underline{(E_7, E_7):}$

$$\begin{array}{c} SU(2)_{\theta=0} \\ | \\ [2] - SU(2) - SU(3)_{k=0} - \; SU(4)_{k=0} \; - SU(3)_{k=0} - SU(2) - [2]\,. \end{array}$$

$\underline{(E_8, E_8):}$

$$\begin{array}{ccc} & SU(3)_{k=0} & \\ & | & \\ [2] - SU(2) - SU(3)_{k=0} - SU(4)_{k=0} - SU(5)_{k=0} - & SU(6)_{k=0} & - SU(4)_{k=0} - SU(2)_{\theta=0} \end{array}$$

5d SCFTs from weakly coupled quiver description of the marginal theory:
(E_6, E_6):

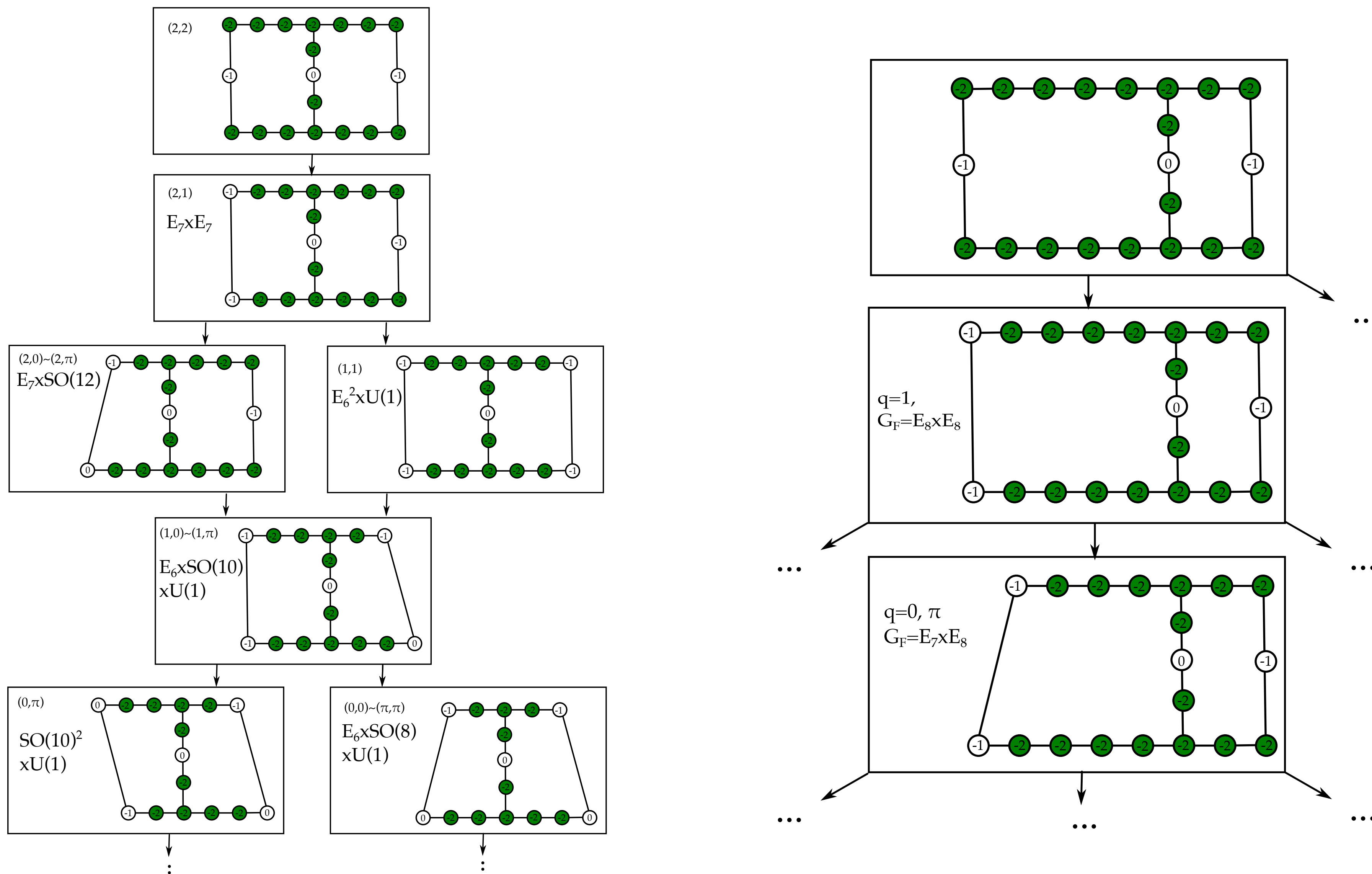


Descendants from quiver description: 12 SCFTs. From CFDs we find 81 additional ones.

Geometric realization possible. We are developing alternative quiver descriptions for these theories, but expectation is that large subset will not have a weakly couple description at all.

Full CFDs at: `https://people.maths.ox.ac.uk/schafernamek/CFD/`

5d SCFTs from weakly coupled quiver description of the marginal theory: (E_7, E_7) and (E_8, E_8):



From CFDs: we find in total 56 and 127 5d SCFTs arising from these!

Summary

Classification of all 5d SCFTs that descend from 6d:

1. Compute the marginal CFD from the 6d geometry
2. Determine the descendant using CFD-transitions
3. Read off enhanced flavor symmetry of the SCFT (green vertices)
4. Weakly coupled description: ruling of surfaces in marginal model [or embed ”gauge theory CFDs”].

Dualities and weakly coupled gauge theory descriptions:
Different rulings of reducible surfaces give rise to dual weakly coupled descriptions.

⇒ Given a 6d SCFT, we provide a systematic exploration of all descendant 5d SCFTs.

Outlook: 4d $\mathcal{N}=1$ SCFTs and G_2

4d $\mathcal{N}=1$ SCFTs can in principle be obtained by further dimensional reduction. However, there are non-perturbative corrections that can spoil this analysis. Alternatives:

- F-theory on elliptic CY_4: requires inclusion of fluxes, D3-instanton corrections.
- M-theory on non-compact singular G_2:
  Construction of TCS G_2 manifolds is reminiscent of the $\mathcal{N}=1$ class S construction of [Bah, et al], i.e. two $\mathcal{N}=2$ building blocks are connected by $\mathcal{N}=1$ gauging. Are there quiver-like constructions of non-compact G_2, engineering 4d SCFTs?