M theory / Heterotic / Type IIA Dualities

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Simons Collaboration Meeting on Special Holonomy

Sep 10–13, 2017, Simons Center Stony Brook

Introduction

- M theory on K3 is dual to Heterotic String theory on T^3
- G_2-manifolds which are K3-fibred are conjecturally dual to Calabi Yaus which are T^3 fibred, together with HYM connections
- Donaldson recently investigated the conditions for G_2 holonomy in adiabatic K3 fibrations over $\mathbb{R}^3$.

- Natural to consider "Heterotic adiabatic limit".
- This is work in progress partly with E. Svanes.
- Will report on new, simple solutions of heterotic equations (Hull-Strominger System)

- Will also try to say something about Heterotic duals of Joyce's G_2-manifolds (close to flat limit).

w/ D. Morrison and A. Kinsella

M theory / Heterotic duality in 6+1 D

— a ~~review~~ reminder:

$$\boxed{\begin{array}{c}\text{M theory on } K3 \times \mathbb{R}^{6,1} \\ \cong \\ \text{Heterotic strings on } T^3 \times \mathbb{R}^{6,1}\end{array}}$$

M theory zero modes:

$C_3 \}$ $b^2(K3)$ $U(1)$ gauge fields

$g_{\mu\nu} \}$ 58 scalars $= \left(Vol(K3), \int_{\Sigma_\alpha} \omega_I \equiv \phi_{I\alpha} \right)$

Einstein metrics on K3. "SD cycles"

$\psi_{3/2} \}$ fermions which make everything supersymmetric

Heterotic zero modes on T^3:

- g_{10} }: flat metrics on T^3 $\longrightarrow$ $\underline{6 \text{ scalars}}$ $\quad \frac{\mathbb{R}^+ \times SL(3,\mathbb{R})}{SO(3)}$

  Isometries of $\underline{T^3}$ $\longrightarrow$ 3 $\underline{U(1)}$ gauge fields

- B }: $\cdot\ H^1(T^3,\mathbb{R}) \longrightarrow$ 3 $U(1)$ gauge fields

  $\cdot\ H^2(T^3,\mathbb{R}) \longrightarrow$ 3 scalars

- $A^{E_8\times E_8}$ }: $\cdot$ Flat connections on T^3 $\longrightarrow$ $\underline{48 \text{ scalars}}$ $\quad \frac{(\hat{T}^3)^{rk(E_8\times E_8)}}{W_{E_8\times E_8}}$

  $\cdot\ H^0(T^3,\mathbb{R}) \longrightarrow$ 16 $\underline{U(1)}$ gauge fields

- Dilaton ϕ. $\longrightarrow$ 1 scalar

Both theories have same zero modes

i.e. 22 $U(1)$ gauge fields and

58 scalars

In fact there is strong evidence that they are <u>equivalent</u> physically at low energies.

Moduli space: $\quad \mathbb{R}^+ \times \dfrac{SO(3,19;\mathbb{R})}{SO(3)\times SO(19)} \Big/ SO(3,19,\mathbb{Z})$

Einstein metrics on K3

Narain moduli space

Remark: Heterotic theory on T^3 clearly has non-Abelian gauge symmetries.

These can arise from "special" connections, whose commutants in $E_8 \times E_8$ are non-Abelian.

eg trivial connection commutes with $E_8 \times E_8$

In M theory, these are K3's with orbifold singularities.

M theory on $K3 \times T^3$ and Heterotic on $T^3 \times T^3$.

Regard $K3 \times T^3$ as a G_2 manifold:

G_2 structure: $\varphi = \omega_I \wedge dx_I + dx_{123}$

ω_I : HyperKähler structures on $(K3, g)$

dx_I : coords on $\mathbb{R}^3/\mathbb{Z}^3$.

$SO(3)$ clearly preserves this structure.

In 6+1 d this $SO(3)$ is a physical symmetry (ie preserves the Lagrangian density).

From 3+1 d pt of view, scalar fields in supersymmetric theories are often "complex". The $SO(3)$ symmetry helps ensure that here, because the

3×19 scalars in $\dfrac{SO(3,19;\mathbb{R})}{SO(3) \times SO(19)}$

naturally become

$(\mathbb{R}^3 + i\mathbb{R}^3) \times 19$ complex scalars

"$x_i + \sqrt{-1}\, y_i$"

$$\int_{\Sigma_L \times S^1_I} \left(C_3 + \sqrt{-1}\, \mathcal{C} \right)$$ naturally gives

rise to complex scalars in 3+1d which can be regarded as a complex gauge field on (the second) T^3. More generally B^3

Heterotic on $T^3 \times T^3$

Metric $ds^2 = dy_i^2 + dx_i^2$ $\qquad \begin{pmatrix} x_i \\ y_i \end{pmatrix} \sim \begin{pmatrix} x_i+1 \\ y_i+1 \end{pmatrix}$

Calabi-Yau structure $\qquad z_j = dx_i + i\,dy_i$

$$\Omega = dz_1 \wedge dz_2 \wedge dz_3$$

$$W = -\tfrac{i}{2}\, dz_i \wedge d\bar{z}_i \qquad (\text{sum } i)$$

$E_8 \times E_8$ connection $A^{E_8 \times E_8}$ on a bundle $E \longrightarrow T^3 \times T^3$ with

$$ch_2(E) = ch_2(T(T^3 \times T^3)) = 0 .$$

A obeys Hermitian YM equations.

$F^{0,2} = 0$ and $\omega \wedge \omega \wedge F = 0$:

Dimensional reduction along T^3, $\frac{\partial}{\partial y_i} = 0$

$$\Longleftrightarrow \quad A \equiv (A_{\underline{x}}, A_{\underline{y}})$$

$$\begin{cases} dA_{\underline{x}} + A_{\underline{x}} \wedge A_{\underline{x}} = A_{\underline{y}} \wedge A_{\underline{y}} \\ d_{A_x} A_{\underline{y}} = 0 \\ d_{A_{\underline{x}}}^{\cdot} A_{\underline{y}} = 0 \quad (d^{*}_{A_x} A_y = 0) \end{cases}$$

(Stability in sense of Kempf-Nees)

$$\mathcal{A} \equiv A_x + i A_y$$

$$F_{\mathcal{A}} = 0$$

So for solutions dimensionally reduced,

$\Rightarrow$ Flat Complex ($E_8 \times E_8$)

connections on T^3. These should be stable: $d^{\dagger}_{A_x} A_y = 0$ $*$.

Exactly same system arises at a codim ~~four~~ $E_8 \times E_8$ singularity in a G_2 manifold.

Key point: in general the components of curvature $F \in \Lambda^{1,1} \otimes \mathcal{L}(E_p \times E_p)$ are non-zero in general.

$\Rightarrow$ The M-theory geometry is not a metric product in general, but a K3 fibration over T^3/(possible singularities).

Since $d_{A_x} A_y = 0$, take

$$A_y = d_{A_x} \psi \qquad d_{A_x} A_x = 0.$$

ψ is "Harmonic"

$$\psi : (T^3 \longrightarrow \mathcal{L}(E_8 \times E_8)).$$

- Related observations by

- Leung, Yau, Zaslow (2000) (mirror symmetry)

- Pantev, Wijnholt .

LYZ : in semi-flat T^3 fibration with Calabi-Yau metric, Hermitian Yang-Mills connections $\longleftrightarrow$ A-branes on dual $\hat{T}^3$-fibration.

Panter / Vijnholt : spectral covers to understand non-Abelian configurations and chiral fermions in IIA limit.

IIA to Type I = IIB/Ω = Heterotic

on

$$\begin{tikzcd}
\hat{T}^3/\mathbb{Z}_2 \arrow[d] & & T^3 \arrow[d] \arrow[r, leftrightarrow, "S\text{ duality}"] & T^3 \arrow[d] \\
\hat{Z} \arrow[d] \arrow[rr, leftrightarrow, "\text{Mirror}", "\text{Symmetry}"'] & & Z \arrow[d] & Z \arrow[d] \\
B^3 \arrow[rr, leftrightarrow] & & B^3 & B^3
\end{tikzcd}$$

$\underbrace{\hat{T}^3/\mathbb{Z}_2 \to \hat{Z} \to B^3}$: orientifolds and D6-branes

$\underbrace{T^3 \to Z \to B^3}$: Stable Holomorphic bundles

M theory

G_2

$$\underbrace{\begin{array}{ccc} K3 & \longrightarrow & X_7 \\ & & \downarrow \\ & & B_3 \end{array}}$$

"Adiabatic K3 fibrations"
(Donaldson)

$$=$$

Heterotic
Calabi-Yau

$$\begin{array}{ccc} T^3 & \longrightarrow & Z \\ & & \downarrow \\ & & B_3 \end{array}$$

"Adiabatic T^3 fibrations with Hermitian Yang Mills connections".

Local Heterotic solutions?

This was considered in (BSA, E. Witten) to try and say something about codim seven singularities in (X_7, φ).

There, a family of adiabatic flat $E_8 \times E_8$ connections along B^3 with isolated zeroes $\Longleftrightarrow$ codim 7 holonomy G_2 singularities in X_7.

With Eirik Svanes we try to be a bit more explicit than this, to find solutions of Heterotic equations on $T^3 \times \mathbb{R}^3$.

Hull-Strominger equations:

SU(3) structure (ω, Ω), dilaton ϕ, connection A, satisfying:

$$d(e^{-2\phi}\Omega) = d(e^{-2\phi}\omega\wedge\omega) = 0$$

$$i\partial\bar\partial\omega = \frac{\alpha'}{4}(\mathrm{tr}\, F\wedge F + \text{higher order}); \quad \omega\wedge\omega\wedge F = 0 = \omega$$

$$d\left(e^{-2\phi}\,\omega \wedge \omega\right) = 0$$

$$i\,\partial\bar{\partial}\,\omega = \frac{\alpha'}{4}\left(\mathrm{tr}\,F \wedge F - \underbrace{\mathrm{tr}\,R \wedge R}_{\text{Higher order}}\right)$$

HYM : $F \wedge \Omega = 0 \quad \leftarrow$ Holom

$F \wedge \omega \wedge \omega = 0 \quad \leftarrow$ Stable.

On $T^3 \times \mathbb{R}^3$ we set
$y_i \quad x_i$

$\frac{\partial}{\partial y_i} = 0$ for all fields.

Let $A \in \Omega^1(\mathfrak{C}artan(\mathcal{L}(E_8 \times E_8)))$

ie "Abelian". Pick one direction in $Cartan(\mathcal{L}(E_8 \times E_8))$,

$A = A_x + A_y$ is a 1-form on $T^3 \times \mathbb{R}^3$.

$dA_x = 0$ so set $A_x = 0$.

$$\Rightarrow \quad dA_y = d^*A_y = 0$$

$* = *$ on $\mathbb{R}^3_{(x_1, x_2, x_3)}$

So A_y is a closed, co-closed 1 form on $\mathbb{R}^3$.

Choose $A_y = d\psi$ then ψ is a harmonic function on Euclidean $\mathbb{R}^3$.

Two solutions

a) "Monopole" $\psi = \frac{1}{r}$

$dz_j = d(x_j + i y_j)$ $\quad \omega = e^{\phi}\, d\underset{\sim}{x} \wedge d\underset{\sim}{y}$

$d\underset{\sim}{z} = d\underset{\sim}{x} + i\, d\underset{\sim}{y}$ $\quad \Omega = e^{-2\phi}\, dz_1 dz_2 dz_3$

$$e^{2\phi} \sim \frac{c}{r^4}$$

Remark:
- Solves Hull-Strominger system
- obviously singular
- believe $\exists$ a smooth non-Abelian solution $\sim$ t'Hooft-Polyakov

b) Near a <u>zero</u> connection

(BSA, Witten): zeros of A_y at pts on $\mathbb{R}^3 \Longleftrightarrow$ zero modes of Dirac operator on $T^3 \times \mathbb{R}^3$

So if $A_y = d\psi \Rightarrow$ isolated critical points.

Near these $\psi \sim x_1 x_2 + x_1 x_3 + x_2 x_3$ + linear + const.

Again can solve the full set of eq^{n}s.

Remark / Question

What are natural boundary conditions for general solutions on $T^3 \times \mathbb{R}^3$?

General solutions (without critical pts)

$\Longleftrightarrow$ smooth holonomy

G_2 metrics on $K3 \times \mathbb{R}^3$.

at least in a small region in $\mathbb{R}^3$.

In general though, one can write an adiabatic limit equations for "collapsed" T^3 fibration limit in which leading terms are "flat" Calabi Yau on $T^3 \times \mathbb{R}^3$ plus closed, co-closed 1-form $\in \Omega^1(\mathcal{L}(E_F \times E_F))$

Conjecture:

As one expands this $\Longrightarrow$ should arrive at a heterotic dual description of S. Donaldson's adiabatic picture of certain G_2-manifolds as co-associative fibration

This will "prove"
M-theory / Heterotic duality
adiabatically.

(cf Gukov, Yau, Zaslow)

M-theory on some Joyce orbifolds and their Heterotic duals

i.e. close to flat limit

w D. Morrison, A. Kinsella

earlier work (BSA 1996) $\leftarrow$

recent work Braun/Schafer-Nameki
2017

T^7, with Euclidean coordinates
$(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$

Introduce various $\mathbb{Z}_2$ isometries of T^7 preserving a G_2-structure

$$\varphi = (dx_{12} + dx_{34})\, dx_5 + (dx_{13} - dx_{24})\, dx_6 + (dx_{14} + dx_{23})\, dx_7 - dx_{567}$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7
α	$-$	$-$	$-$	$-$	$+$	$+$	$+$
β	$-$	$\frac{1}{2}-$	$+$	$+$	$-$	$-$	$+$
γ	$\frac{1}{2}-$	$+$	$\frac{1}{2}-$	$+$	$-$	$+$	$-$

Joyce's original example.

α, β, γ each fix 16 T^3's $\subset T^7$ but These are each 4 T^3's in $\frac{T^7}{\langle\alpha,\beta,\gamma\rangle}$.

The 12 T^3's of orbifold singularities are modelled on $\frac{\mathbb{R}^4}{\mathbb{Z}_2} \times T^3$ and desingularised to $\widetilde{\frac{\mathbb{R}^4}{\mathbb{Z}_2}} \times T^3$ with Eguchi-Hanson metric

$\Rightarrow$ G_2-structure with small torsion can then be perturbed to a nearby torsion free structure (Joyce).

$\dfrac{T^7}{\langle \alpha \rangle}$ is $K3 \times T^3_{567}$ à la Kummer

In orbifold limit M theory has

$(A_1)^{16} = SU(2)^{16}$ gauge symmetry

In Heterotic string on

$\underbrace{T^3}_{y_1 y_2 y_3} \times T^3_{567}$ need a.

stable holomorphic $E_8 \times E_8$ connection

$\langle A^{E_8 \times E_8} \rangle$ whose commutant is

$SU(2)^{16} \subset E_8 \times E_8$.

$\langle A^{E_8 \times E_8} \rangle$

Take connection to be trivial on T^3_{567}.

Along T^3_{123} can take $A_i^{E_8 \times E_8}$ to all be flat connections in centre of $SU(2)^{16} \subset E_8 \times E_8$.

Centre $= (\mathbb{Z}_2)^{16}$ where the 16 generators are "aligned" with the Cartan of $E_8 \times E_8$.

What about $\frac{\tilde{T}^7}{\langle \alpha, \beta, \gamma \rangle}$?

How do they act on Heterotic on $T^3_y \times T^3_{567}$?

Examine the action of β, γ on T^3_{567}:

$$\begin{array}{cccc} & x_5 & x_6 & x_7 \\ \beta & - & - & + \\ \gamma & - & + & - \end{array}$$

SO(3) invariant Calabi Yau structure

on $T^3_{y_1 y_2 y_3} \times T^3_{567}$:

The CY structure has $z_i = x_i + \sqrt{-1}\, y_i$

and $\omega = -\frac{i}{2}\, dz_j \wedge d\bar{z}_j$

$\Omega = dz_1 dz_2 dz_3$

is SO(3) invariant

The flat CY structures on $T^3 \times T^3$ which are $\mathbb{Z}_2^\rho \times \mathbb{Z}_2^\sigma$ invariant and $SO(3)$ invariant are in correspondence with flat oriented orbifolds:

$$\frac{T^3}{\mathbb{Z}_2 \times \mathbb{Z}_2}$$

Smooth cases (Bieberbach)

1) T^3 itself

2) $\dfrac{T^3}{\langle \beta \rangle} = \dfrac{T^3}{\mathbb{Z}_2}$ $\quad \beta \; (- \;\; - \;\; +\tfrac{1}{2})$

3) $\dfrac{T^3}{\langle \beta, \gamma \rangle}$

$\beta \; (\tfrac{1}{2} \;\; - \;\; - \;\; +\tfrac{1}{2})$

$\gamma \; (- \;\; +\tfrac{1}{2} \;\; -)$

$\beta\gamma \; (+\tfrac{1}{2} \;\; -\tfrac{1}{2} \;\; -\tfrac{1}{2})$

Singular cases

4) $\dfrac{T^3}{\langle \beta \rangle}$ $\quad \beta \quad (- - +)$.

$$\frac{T^3}{\langle \beta \rangle} \cong S^2 \times S^1 \text{ flat metric}$$

5) $\dfrac{T^3}{\langle \beta, \gamma \rangle}$

$\beta \quad (- - +)$

$\gamma \quad (\tfrac{1}{2} - \; + \tfrac{1}{2} \; -)$

$\beta\gamma \quad (+\tfrac{1}{2} \; -\tfrac{1}{2} \; -)$

6) $\dfrac{T^3}{\langle \beta, \gamma \rangle}$

$\beta \quad (- - +)$ $\quad S^3$

$\gamma \quad (- + -)$

flat metric locally.

The extension of these to the Calabi Yau uses $SO(3) \subset SU(3)$ as the <u>real</u> subgroup.
This determines the action up to translations of the y_i.

In the end one finds

1') $T^3 \times T^3$ $\qquad (y_1\, y_2\, y_3\, x_5\, x_6\, x_7)$

2') $\dfrac{T^3 \times T^3}{\langle \beta \rangle}$ $\qquad \beta \quad (- - + - - \tfrac{1}{2})$

3') $\dfrac{T^3 \times T^3}{\langle \beta, \gamma \rangle}$ $\qquad \beta \quad (- - + - - \tfrac{1}{2})$
$\qquad\qquad\qquad\qquad \gamma \quad (- - + -\tfrac{1}{2} + \tfrac{1}{2} -)$

these are the smooth cases.

3') is a CY with $Hol = \mathbb{Z}_2 \times \mathbb{Z}_2$ and $h^{1,1} = h^{2,1} = 3$

4') $\dfrac{T^3 \times T^3}{\langle \beta \rangle}$ $\beta\,(-\,-\,+\;\;-\,-\,+)$

orbifold limit of $K3 \times S^1_y \times S^1_x$

5') $\dfrac{T^3 \times T^3}{\langle \beta, \gamma \rangle}$ $\begin{array}{l} \beta\,(-\,-\,+\;\;-\,-\,+) \\ \gamma\,(-\,-\,+\;\;-\tfrac{1}{2}\;+\tfrac{1}{2}\;-) \end{array}$

CY with $\mathrm{Hol} = SU(2) \ltimes \mathbb{Z}_2$

$h^{1,1} = 11 = h^{2,1}$

6') $\dfrac{T^3 \times T^3}{\langle \beta, \gamma \rangle}$ β $(\tfrac{1}{2}-\,-\,+\,-\,-\,+)$
γ $(-\,+\,-\,-\,+\,-)$.

Orbifold limit of Schoen "Double Elliptic fibration" $Hol = SU(3)$

$h^{1,1} = 19 = h^{2,1}$

7') $\dfrac{T^3 \times T^3}{\langle \beta, \gamma \rangle}$ β $(-\,-\,+\,-\,-\,+)$
γ $(-\,+\,-\,-\,+\,-)$

$h^{1,1} = 51 \quad h^{2,1} = 3.$

$Hol = SU(3)$

Would like to understand detailed Heterotic dual in all these cases.

Here: explain 3') the M theory model has holonomy $\underbrace{SU(2) \ltimes (\mathbb{Z}_2 \times \mathbb{Z}_2)}_{\subset G_2}$

and $X_7 \cong \dfrac{K3 \times T^3}{\langle \beta, \gamma \rangle}$

Here (β, γ) act freely and exchange the 16 A_1 singularities in pairs $\longrightarrow$ 4 A_1 " remain in quotient.

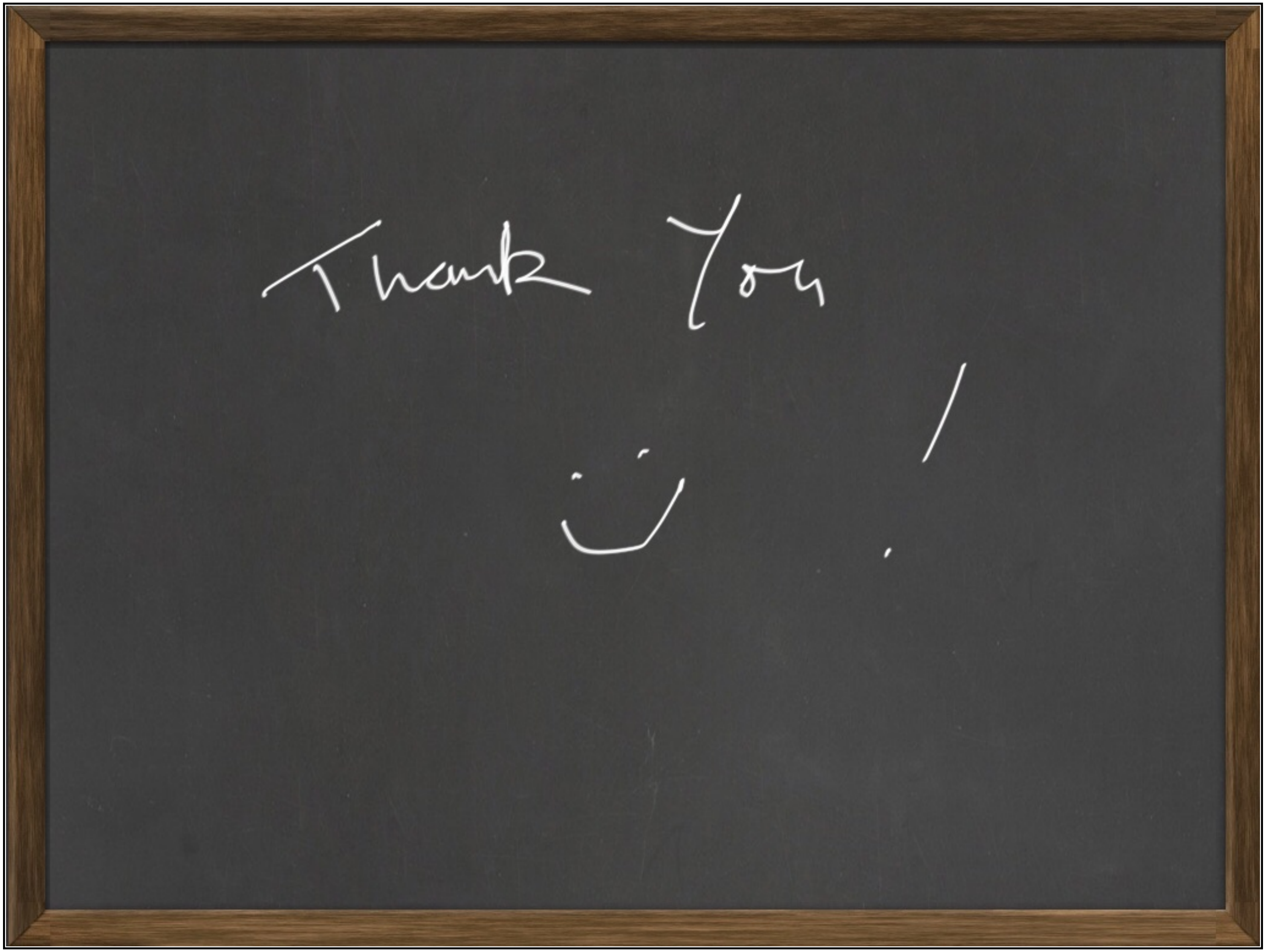
$$b_2(X_7) = 4 \quad b_3(X_7) = 19 \ .$$

On Heterotic side we get a (β, γ) invariant projection of the 16 flat connections

This matches precisely
the spectrum of the
Joyce orbifold in M theory.

Some of the other examples
can also be worked out...

Braun/Schafer-Nameki 2017 !
recently proposed a
"TCS heterotic dual"
to TCS G_2 manifolds
and some of these examples
arise there also.


Thank You
!