

$$M = G/K$$

G = split real form of E_{n-d}

K = max compact = R-sym

d	G/K	dim(G/K)	2-brane	string	particle
9	$GL(2, \mathbb{R})/U(1)$	3	1	2	$2 \oplus 1$
8	$SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$	7	$\frac{1}{2}(1, 2)$	$(3, 1)$	$(3, 2)$
7	$SL(5, \mathbb{R})/Spin(5)$	14	—	5	10
6	$Spin(5, 5)/Spin(5) \times Spin(5)$	25	—	$\frac{1}{2}(10)$	16
5	$E_{6(6)}/Sp(4)$	42	—	—	27

4	$E_{7(7)}/SU(8)$	70	—	—	$\frac{1}{2}(56)$
3	$E_{8(8)}/Spin(16)$	128	—	—	—

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = 2\delta^a_b \sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu \quad a,b=1,\dots,N$$

$$R\text{-sym} = U(N) \\ (SU(N))$$

$$N=4 \checkmark$$

$$N=2$$

Hypermultiplets - same as 6d (1,0)
 vector multiplets : gauge field, cplx scalar
 + fermions

IF

If gauge group Abelian, scalar manifold need not be flat

M_{Scherk} = "(rigid) special Kähler MFD"

hep-th/9712042

• Kähler

• There's a holomorphic cubic sym form $\Omega \in H^3(M, \text{Sym}^3 T^*M)$

s.t.

$$R_{j\bar{k}\ell}(\phi) = -g_{j\bar{k}}(\phi) \frac{\partial}{\partial \phi^{\bar{\ell}}} \frac{\partial}{\partial \phi^{\ell}} \Omega(\phi)$$

$$\{Q_{\vec{a}b}\} = 2\delta_{\vec{a}b} \sigma_{\vec{a}b} P_n \quad a,b=1,\dots,N$$

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$M_{\text{scalar}} = \text{"(rigid) special Kähler Mfd"}$

• Kähler

• There's a holomorphic cubic sym form $\Omega \in H^3(M, \text{Sym}^3 T^*M)$

s.t.
$$R_{j\bar{k}\ell} = -g_{(k} \bar{n}_{\ell)} \Omega_{j\bar{k}}(\phi) \Omega_{\ell\bar{j}}(\phi)$$

locally, $\exists$ special coords s.t.

$$\Omega_{j\bar{k}}(z) = \frac{\partial^3 \mathcal{F}(z)}{\partial z^j \partial \bar{z}^k \partial z^\ell}$$

$$g_{j\bar{k}} = \text{Im}(\gamma_{j\bar{k}}(z))$$

$$\gamma_{j\bar{k}} = \frac{\partial^2 \mathcal{F}(z)}{\partial z^j \partial \bar{z}^k}$$

$N=1$: vector multiplets gaugefield & gaugino

chiral multiplets cplx scalar & fermion

$M_{\text{scalar}} = \text{Kähler}$

superpotential $W(\phi)$: scalar potential $V(\phi) = g^{\bar{i}j}(\phi) \frac{\partial W}{\partial \phi^i} \frac{\partial \bar{W}}{\partial \bar{\phi}^{\bar{j}}}$

locally, $\exists$ special coords s.t.

$$\Gamma_{ijk}(z) = \frac{\partial^3 \mathcal{F}(z)}{\partial z^i \partial z^j \partial z^k}$$

$$g_{i\bar{j}} = \text{Im } \gamma_{i\bar{j}}(z)$$

$$\gamma_{i\bar{j}} = \frac{\partial^2 \mathcal{F}(z)}{\partial z^i \partial \bar{z}^j}$$

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$D=11$ SUGRA

little group = $so(9)$

traceless sym tensor

bosons: $44 \oplus 84$ for W

fermions: 128

$$16 \otimes 9 = 16 \oplus 128$$

$$\int d^9 x \, e \left(R(W) - 2i \bar{\Psi}_m \gamma^{\mu\nu\lambda} D_\nu(W) \Psi_\lambda - \frac{1}{2 \cdot 4!} G^2 \right) + \frac{1}{6} \int C \wedge G \wedge G + 4\text{-fermi terms.}$$

$$K = \max \text{ compact} = R\text{-sym}$$

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9	$GL(2, \mathbb{R})/U(1)$	3	1	2	$2 \oplus 1$
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$$G = dC$$

$$D_\mu(w) \bar{\Psi}_\nu = \partial_\mu \bar{\Psi}_\nu + \frac{1}{L_1} w_\mu^{AB} \Gamma_{AB} \bar{\Psi}_\nu$$

$$g_{\mu\nu}(x) = e_\mu^A(x) e_\nu^B(x) \eta_{AB}$$

$$\omega = de^A + \omega^A_B \wedge e^B$$

$$\bar{\Psi}_\mu = \bar{\Psi}_\mu^\dagger \rho^0$$

locally, $\exists$ special coords s.t.

$$E_{\mu\nu}(z) = \frac{\partial^2 \mathcal{F}(z)}{\partial z^\mu \partial z^\nu}$$

$$g_{\mu\bar{\nu}} = \frac{\partial^2 \mathcal{F}(z)}{\partial z^\mu \partial \bar{z}^\nu}$$

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chiral multiplets cplx scalar & fermion

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superpotential, $W(\phi)$: scalar potential $V(\phi) = g^{\bar{i}\bar{j}}(\phi) \frac{\partial W}{\partial \phi^i} \frac{\partial W}{\partial \bar{\phi}^{\bar{j}}}$

$D=11$ SUGRA

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bosons: $44 \oplus 84$ traceless sym tensor for m

fermions: 128

$$\int e^{\star} = -\frac{i}{2} \bar{\Psi}(x) \Gamma^a \Psi$$

$$\int C_{\mu\nu\lambda} = \frac{2}{3} \bar{\Psi}(x) \Gamma_{\mu\nu} \Psi_\lambda$$

$$\int \bar{\Psi} = 2 D_\mu \Psi + \frac{i}{144} (\Gamma^{\mu\nu\lambda} \rho_\mu - 8 \Gamma^{\lambda\rho\sigma} \delta_\mu^\nu) \Psi$$

$$\int d^9 x e [R(w) - 2i \bar{\Psi}_\mu \Gamma^{\mu\nu\lambda} \Psi_\nu]$$

$$+ \frac{1}{6} \int C \wedge G \wedge G + 4 -$$

$$- G_{\mu\nu\lambda} \rho_\sigma + \dots$$

$$M = mG/K$$

G = split real form of E_{d-1}

K = max compact = R-sym

Λ = p-brane charges

d	G/K	$d \dim(G/K)$	2-brane	string	particle
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$\Lambda_m = \Lambda_b^*$ (d-p-4) brane charges

For $p = \frac{d-4}{2}$

$\Lambda_{em} = \Lambda_{em}^*$

4	$E_{7(7)} / SU(8)$	70	—	—	$\frac{1}{2}(56)$
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$$\omega = de^A + w^A_B \wedge e^B$$

$$\bar{\Psi}_\mu = \bar{\Psi}_\mu^\dagger \Gamma_0$$

IIB = (2,0) SUGRA in $d=10$

(1,0) SUGRA

spin(8)

1 real scalar

$$G = E_8 \times E_8 \quad \text{or} \quad SO(32)$$

Grav $\begin{cases} \text{bosons} & 35_v + 28 + 1 \\ \text{fermions} & 56_c + 8_c \end{cases}$

Yang Mills $\begin{cases} \text{boson} & 8_v \\ \text{fermion} & 8_s \end{cases}$

s.t.

$$g_{i\bar{j}} = \text{Im}(\tau_{i\bar{j}}(z)) \rightarrow U(1,1)$$

$$\tau_{i\bar{j}} = \frac{\partial^2 \mathcal{F}(z)}{\partial z^i \partial \bar{z}^j}$$

gauge field & gaugino

scaly scalar & fermion

$M_{\text{scalar}} = \text{Kähler}$

potential $V(\phi) = g^{i\bar{j}}(\phi) \frac{\partial W}{\partial \phi^i} \frac{\partial W}{\partial \bar{\phi}^j}$

IIB = (2,0) SUGRA in $d=10$

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Yang Mills $\left\{ \begin{array}{l} \text{boson: } 8_v \\ \text{fermion: } 8_s \end{array} \right.$

dim red to $d < 10$

Scalars in adjoint of Yang-Mills multiplet $\Rightarrow$ Higgs down to Cartan.

Again M_{scalar} is homogeneous $U(1)^{\text{rank}}$

$$\mathbb{R} / O(26-d, 10-d) / O(26-d) \times O(10-d) \times \mathbb{R}$$

$$O(n, 10-d) / O(n) \times O(10-d) \times \mathbb{R} \quad \text{for any } n$$

$$d=6$$

(p,q) SUSY

$$p+q \leq 4$$

$$(2,2) \checkmark$$

$$(1,1) \checkmark$$

Remaining cases

$$(2,0)$$

$$(2,1)$$

$$(1,0)$$

$$(2,0)$$

$$\text{Spin}(4) \times \text{Sp}(2)_R$$

gravity

$$\text{bosons } (3,3;1) + (1,3;5)$$

$$\text{fermions } (2,3;4)$$

tensor

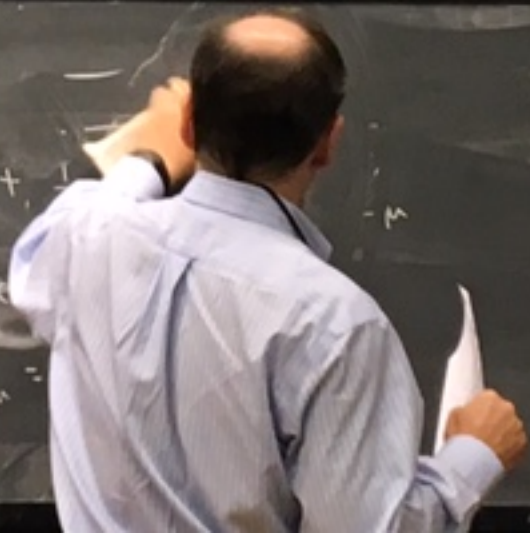
$$\text{bosons } (3,1;1) + (1,1;5)$$

$$\text{fermions } (2,1;4)$$

Only consistent theory is $n_T=21$

$$M_{\text{sector}} = \mathcal{O}(2,5) / \mathcal{O}(2,1) \times \mathcal{O}(5)$$

4



$$(p, q) \text{ SUSY } p+q \leq 4$$

$$(2, 2) \checkmark$$

$$(1, 1) \checkmark$$

Remaining cases $(2, 0)$ $(2, 1)$ $(1, 0)$

$(2, 0)$

$$\text{Spin}(4) \times \text{Sp}(2)_R$$

gravity

$$\text{bosons } (3, 3; 1) + (1, 3; 5)$$

$$\text{fermions } (2, 3; 4)$$

tensor

$$\text{bosons } (3, 1; 1) + (1, 1; 5)$$

$$\text{fermions } (2, 1; 4)$$

Only consistent theory is $n_T = 21$

$$m_{\text{scalar}} = \sqrt{\frac{\mathcal{O}(2, 5)}{\mathcal{O}(2, 1) \times \mathcal{O}(5)}}$$

$(1, 0)$

$$\text{Spin}(4) \times \text{Sp}(1)_R$$

gravity

bosons

$$(3, 3; 1) + (1, 3; 1)$$

fermions

$$(2, 3; 2)$$

quaternionic Kähler

$$n_T \text{ tensors} + n_V \text{ vectors} + n_H \text{ hypers}$$

$$d = 4$$

N - extended sugra

$$N = 2(p+q)$$

$$(U(N)_R)$$

$$SU(N)_R$$

IIB = (2,0) SUGRA in d=10

(1,0) SUGRA

spin(8)

1 real scalar

Grav { bosons
fermions

$$35_v + 28 + 1$$

$$56_c + 8_c$$

$$G = E_8 \times E_8 \quad \text{or} \quad SO(32)$$

Yang

boson:

$$8_v$$

Mills

fermion:

$$8_s$$

$B_{\mu\nu} \rightsquigarrow$ scalar

N=4

$$M_{\text{scalar}} = \sqrt{\mathcal{O}(n,6)/\mathcal{O}(n) \times \mathcal{O}(6)} \times SU(4)/U(1)$$

N=2

grav

$g_{\mu\nu}$

A_μ

ψ^a_μ

vector

A_μ

ϕ_c

ψ^a

$su(2)$ doublet of gravitini

scalars "projective special

Kähler manifold

hypers

$M_{\text{hypers}} = \text{quaternion Kähler}$

$$d=6$$

(p,q) SUSY

$$p+q \leq 4$$

$$(2,2) \checkmark$$

$$(1,1) \checkmark$$

Remaining cases (2,0) (2,1) (1,0)

$$(2,0)$$

$$\text{Spin}(4) \times \text{Sp}(2)_R$$

gravity

$$\text{bosons } (3,3;1) + (1,3;5)$$

$$\text{fermions } (2,3;4)$$

tensor

$$\text{bosons } (3,1;1) + (1,1;5)$$

$$\text{fermions } (2,1;4)$$

Only consistent theory is $n_T=21$

$$M_{\text{scalar}} = \sqrt{\Theta(2,5)/\Theta(2,1) \times \Theta(5)}$$

$$N=1$$

gravity multiplet

$$g_{\mu\nu}, \psi_\mu$$

$$(M, L, w)$$

vector

$$A_\mu, \psi$$

$$[w] = J(C, L)$$

chiral

$$\phi_i, \psi$$

$$M_{\text{chiral}} = \text{Hodge}$$

$$V(\phi, \bar{\phi}) = e$$

$$K(\phi, \bar{\phi})$$

$$\left(g^{\mu\bar{\nu}}(\phi) \frac{\partial w}{\partial \phi^i} \frac{\partial \bar{w}}{\partial \bar{\phi}^{\bar{j}}} \right)$$

$$-3|w|^2$$