Nonabelian Gauge Symmetry in String Theory and its Cousins, including an Introduction to Branes

David R. Morrison

University of California, Santa Barbara

Special Holonomy and Branes

American Institute of Mathematics (virtual meeting)

26 October 2020

This is a review talk which I hope helps the partcipants in this meeting understand the basic framework being discussed. (There is nothing new or original in this lecture.) Some suggestions for further reading:

- D. Z. Freedman and A. Van Proyen, *Supergravity*, Cambridge University Press 2012.
- K. Becker, M. Becker, and J. H. Schwarz, *String Theory and M-Theory*, Cambridge University Press 2007.
- C. Johnson, *D-Branes*, Cambridge University Press 2003.
- P. S. Aspinwall et al., *Dirichlet Branes and Mirror Symmetry*, Clay Mathematics Institute and the American Mathematical Society, 2009.
- D. R. Morrison, *TASI Lectures on Compactification and Duality*, in: *Strings, Branes, and Gravity (TASI '99)*, World Scientific 2001.

Superstring theories

Superstring theories are quantum versions of the 9+1 dimensional supergravity theories, based on the notion of a propagating one-dimensional object known as a string. The string sweeps out a "worldsheet" in the 9+1 dimensional spacetime, and the "super" in superstring refers to supersymmetry of the worldsheet physical theory. Remarkably, the quantized superstring reproduces the spectrum of a 9+1 dimensional physical theory with spacetime supersymmetry and with gravity, and so corresponds (in the "classical limit") to a supergravity theory.

The string in a superstring theory can either be open or closed, and if it is open one should think about boundary conditions for the endpoints. They can be of Neumann or Dirichlet type, and the Dirichlet strings must end on a particular sub-spacetime known as a Dirichlet brane, or D-brane.

D-branes, supergravity theories, superstring theories

It is conventional to label the D-brane by its spatial dimension, and call the resulting object a Dp-brane when it has spacetime dimension p+1, i.e;, when it contains p spatial and one time dimension.

There are three or four supergravity theories in 9+1 dimensions (depending on how you count), known as type I, type IIA, and type IIB. In the case of type I, there is also a choice of 10-dimensional gauge algebra, which can be either $so(32)$ or $e_8 \oplus e_8$. (This latter restriction is due to the celebrated result of Green and Schwarz from 1984, which restricts type I theories if one is interested in theories which might have a quantum version).

On the other hand, there are five superstring theories, known as type I, type IIA, type IIB, and two “heterotic” theories (depending on the choice of gauge algebra $so(32)$ or $e_8 \oplus e_8$). D-branes are only relevant for the first three.

Gauge fields

An open superstring must couple to a gauge field at each endpoint, typically with gauge algebra $u(N)$, in other words, there should be a gauge field on the D-brane worldvolume with gauge algebra $u(N)$. It is common to regard N as a "multiplicity" of the D-brane, and to think of this as a stack of N D-branes on top of each other. If the D-branes in the stack are slightly separated, each D-brane contributes a $u(1)$ gauge field (the diagonal elements in a $U(N)$ matrix) whereas the short strings connecting the i^{th} D-brane to the j^{th} D-branes are slightly massive, and represent the off-diagonal elements in a $U(N)$ matrix.

BPS

D-branes are charged under certain of the k-form fields in the supergravity theory, and the most frequently studied D-branes satisfy an inequality between charge and mass (or charge and volume) known as a BPS bound. Now, in a supersymmetric theory, objects which saturate a BPS bound have different supersymmetry representation properties than objects which do not saturate the bound. This leads to the conclusion that, provided supersymmetry remains unbroken, the BPS D-branes (those that saturate the bound) will be stable objects which can be followed as parameters change. More is true: the D-branes serve as "sources" for some of the k-form fields in the theory, analogous to how electrons are sources for the familiar electromagnetic field. BPS D-branes will necessarily break some of the supersymmetry of the theory (since they break infinitesimal translation invariance) but they typically preserve some fraction of it, such as $1/2$ or $1/4$.

Supergravity solutions; types of branes

In super-Minkowski spacetime, there are explicit supergravity solutions (with singularities on the brane world-volume) which realize all the properties of the brane. These solutions are expected to provide the correct asymptotics for 1/2-BPS D-branes in more general spacetimes.

In the type IIA theory, one has D0, D2, D4, and D6-branes, while in the type IIB theory, one has D1, D3, D5, and D7-branes. It is natural to extend this a bit, and consider also D9-branes in IIB (which would fill all of spacetime), D(-1)-branes (also known as D-instantions) in type IIB, and D8-branes (which may be boundaries or may separate regions) in type IIA.

There is another kind of 5-brane in type II theories known as an NS5-brane which couples to the "B-field" 2-form (and its dual 6-form) and has a supergravity solution.

Type I and Type I'

One way to realize type I is as a quotient of type IIB by an "orientifold" symmetry, which reverses orientation on the worldsheet (and also acts on spacetime). When this is done, the gauge fields naturally become either Sp or SO rather than U. In paticular, one needs 32 D9-branes to have a consistent quotient, and these give rise to the $so(32)$ gauge field in type I. D5-branes and D1-branes are also possible in type I theory.

The fixed point locus of an orientifold symmetry is known as an "orientifold plane," and in counts of Dp-branes, an orientifold p-plane usually counts negatively.

A type IIA theory with D8-branes has similar properties to type I, and is sometimes called "type I' " or "type IA." Compactifying this theory on the interval S^1/Z_2 gives enhanced gauge symmetry of SO type at the endpoints (due to the orientifold); it is also possible to take a "strong coupling limit" and obtain enhanced gauge symmetry of e_n type.

T-duality

The various superstring theories are related by a number of "dualities." The oldest of these, known as T-duality, operates at weak string coupling.
T-duality asserts an equivalence between type IIA string theory compactified on an S^1 of radius R, and type IIB string theory compactified on an S^1 of radius α'/R, once the "momentum" and "winding" modes have been exchanged.

This also works in the presence of BPS D-branes: a Dp-brane wrapped around the S^1 is mapped to a D(p-1)-brane not wrapped around the other S^1.

A variant applies which relates the type I and type I' theories.

Also T-duality relates the two heterotic theories.

Strong coupling limits and M-theory

A superstring has two fundamental parameters: the size of the string (usually measured as the area of the string worldvolume and denoted by α') and the "string coupling" which corresponds to a scalar field in the supergravity theory known as the "dilaton." One question which was answered in the 1990's (using the stability of BPS D-branes) was: what is the strong coupling behavior of each superstring theory? The answer is that type I and the heterotic $so(32)$ theory are each other's strong coupling limit, and the strong coupling limit of type IIB is again type IIB. But the type IIA theory and the heterotic $e_8 \oplus e_8$ theories have a strong coupling limit which is a quantum 10+1 dimensional theory known as *M-theory*. (This is one of the string theory "cousins" in the title.) When this larger theory is compactified on S^1 we get the type IIA string, and when it is compactified on an interval we get the heterotic $e_8 \oplus e_8$ string (with e_8 gauge fields at the endponts).

M-theory branes

There are also supergravity solutions for M-theory branes: an M2-brane and an M5-brane (which couple to the M-theory 3-form field and its dual 6-form field).
When we compactify M-theory on a circle to get type IIA, we can ask how the branes correspond. Now part of the argument for the emergence of M-theory was that at strong coupling, stacks of N D0-branes behave like the N^{th} "Kaluza–Klein mode" for the compactification on a circle.
Beyond that, the type IIA string and the D2-brane both come from the M2-brane (one wrapped on the circle and one not); the D4-brane and the NS5-brane both come from the M5-brane (one wrapped on the circle and one not); the D8-brane does not lift to M-theory unless we also consider a boundary M9-brane (carrying an e_8 gauge field).

Another mechanism

The remaining case of the D6-brane is quite interesting: a stack of N D6-branes lifts to an M-theory geometry of Atiyah–Hitchin type, with an A_{N-1} singularity in real codimension 4. This provides a new mechanism for non-abelian gauge symmetry – through singularities in real codimension 4, and in fact the D_k and E_k singularities can be realized as well (the former involving orientifolding). This phenomenon of ALE singularities in real codimension four giving rise to nonabelian gauge symmetry also works directly in type IIA. The analogue of separating branes to get a massive model is blowing up the singularities, with only a small area for the 2-spheres thereby created.

Analyzing the new mechanism

A complete analysis of this mechanism should involve analyzing a classical physical system consisting of 10+1 dimensional supergravity on a singular background coupled to 6+1 dimensional super-Yang–Mills theory of the appropriate type. This has been done (by Anderson, Barrett, and Lukas) in the A_{N-1} case but not yet in the other cases.

F-theory

The transformation of the type IIB theory mentioned above is part of a larger discrete symmetry group of that theory isomorphic to $SL(2, Z)$. The other "cousin" of string theory is known as *F-theory*, and is a quantum version of the supergravity theory obtained from type IIB by gauging the $SL(2, Z)$ symmetry. A prominent feature of this theory is the D7-branes (and other kinds of 7-branes) which account for non-abelian gauge symmetry in these theories. Because of the "sourcing" behavior of branes, they also account for the mutivaluedness of the scalar fields, which was to be expected since $SL(2, Z)$ acts on the scalar fields through fractional linear trasformations.

F-theory/M-theory duality

The real scalar fields in type IIB supergravity (one of which is constrained to be positive) can be combined into a single complex scalar τ in the upper half-plane, on which $SL(2, Z)$ acts by fractional linear transformations. It follows that there is an elliptic curve E_τ at each point in the type IIB spacetime. The statement of F-theory/M-theory duality is that M-theory compactified on the total space of the family of elliptic curves will approach the F-theory vacuum when the area of the elliptic curves (which is constant in the family) approaches zero.

F-theory/M-theory duality con.

To acieve that limit, assuming that the family has a section, one can first shrink all components of fibers not meeting the section to zero area. This will create loci of ALE singularities in real codimension 4. Then, locally in the family one may identify one of the two circles within the elliptic curve as being invariant under monodromy; if that circle is shrunk, the resulting type IIA compactification will have stacks of D6-branes replacing the ALE-singularities. We can then do T-duality on the remaining circle, turning stacks of D6-branes into stacks of D7-branes. (Technically, this only works as stated for A type; it can be modified with orientifolds to work for D type. The E type cases are going to be a challenge.)