Perturbative heterotic duals of M-Theory on G_2 orbifolds

David R. Morrison

University of California, Santa Barbara

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Compactification

The classical physical "supergravity" theories in 10 and 11 spacetime dimensions, and their quantum cousins (the "superstring" theories and "M-theory") form an important component of the string theory approach to quantizing gravity. These theories were constructed in the 1980's (with antecedents dating to the 1960's), and in order to use them to study realistic quantum gravity theories in our world of 4 spacetime dimensions, one must "compactify," that is, study 10- or 11-dimensional spacetimes $\mathcal{M}$ which are fibered over our 4-dimensional spacetime M with compact fibers F.

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If we take the characteristic length scale of the compact fiber to be, say, the Planck length 10^{-33}cm, then one would not expect to directly observe F, but might detect geometric features of F through the physical effects which are produced.

In the simplest version of compactification, one takes $\mathcal{M} = M \times F$, although more complicated scernarios are also possible.

Compactification and special holonomy

The 10- and 11-dimensional theories in question are supersymmetric, and one can correspondingly study supersymmetric theories in 4 spacetime dimensions. (The “experimental jury” is still out on the question of whether the world around us is actually supersymmetric at high enough energies, but supersymmetric models of phyiscs are nonetheless interesting to study.) In order to preserve at least some of the higher-dimensional supersymmetry in the lower-dimensional theory, the compact fiber F must have a metric with a covariantly-constant spinor field in the product scenario (and there should be an appropriate fibration by such in the more complicated scenarios.)

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Now the mathematical question of which compact Riemannian manifolds have covariantly-constant spinor fields is a question about holonomy of Riemannian manifolds, and the answer is known: after a finite unbranched cover, F is a metric product of compact Riemannian manifolds whose holonomy is either $SU(n)$, $Sp(n)$, G_2, Spin(7), or trivial.

Compactification and special holonomy, con.

In fact, to obtain a theory in 4 dimensions from M-theory with only a single 4-dimensional supersymmetry transformation, one takes F to be a Riemannanian 7-manifold with holonomy G_2. The aim is then to construct a dictionary between geometric properties of such fibers F, and physical properties of the corresponding 4-dimensional physical theory (and perhaps even to go backwards: given a set of properties of the physical theory, one might want to "engineer" a fiber F which produces them, and possibly even classify all fibers which could produce them).

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One immediately enounters the problem that two key physical properties – non-abelian gauge symmetry, and chiral matter – cannot be realized if F is a manifold. However, physicists have studied more general compactifications in which the fiber (and the total space) are allowed to have controlled singularities, of types known to produce the desired physical features. Thus, one wants to allow "ADE" singularities in real codimension four in order to produce non-abelian gauge symmetry, and to allow certain further singularities in real codimension seven in order to produce chiral matter.

$\mathbb{C}^2/\Gamma$ $\quad \Gamma \subset SU(2)$

Holonomy G_2

A Riemannian 7-manifold F has *holonomy* G_2 if the parallel transport along any loop $L \subset F$ based at a point $P \in F$ induces an automorphism of $T_P(F)$ contained in the subgroup $G_2 \subset O(7) = O(T_P)$.

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The first compact examples were constructed by Joyce in the 1990's. Joyce started with singular spaces T^7/Γ for appropriate finite groups Γ acting on T^7, and then worked quite hard to show that the singularities could be resolved, leaving one with a manifold of holonomy G_2. Of course, restricting the loops used to measure holonomy to the nonsingular part of T^7/Γ would also produce holonomy (contained in) G_2; moreover, the singularities on T^7/Γ are of "ADE type," so paradoxically, Joyce's starting singular space is actually a more general space of the type relevant to physics.[1]

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Singular forms of Joyce's examples

In this project, we investigated the singular spaces T^7/Γ which were Joyce's starting points, hoping to get a better handle on the physics involved. Our key tool was the use of certain dualities, in order to represent the physical models in a different way: as a compactification of one of the heterotic superstring theories on a Calabi–Yau threefold (a manifold with holonomy $SU(3)$). A heterotic compactification naively requires not just a metric on the space F, but also a bundle on that space – precisely what type of bundle depends on which of the two heterotic theories we are considering.

Our initial expectation was that the data of the singular 7-dimensional space T^7/Γ would determine both the corresponding 4-dimensional physics, and the bundle data required for a heterotic interpretation of that same physics. The final story turned out to be somewhat more complicated.

M-theory/heterotic duality and K3 surfaces

Our starting point is one of the string theory dualities discovered in the 1990's, relating M-theory compactified on a Ricci-flat K3 surface,[2] to the heterotic string compactified on a flat T^3 with appropriate bundle data. These compactified theories are 7-dimensional, and each has a moduli space with a description of the form

$$\mathbb{R}^+ \times (\Lambda \backslash O(3,19)/(O(3) \times O(19)))$$

$\Lambda = \text{Diff}/\text{Diff}^0$

for an appropriate discretely acting group Λ. In the case of M-theory compactified on a K3 surface, the moduli space is that of Ricci-flat metrics; in the case of a heterotic string compactified on a flat T^d, Narain made a general perturbative string calculation showing that the moduli space has the form

$$\mathbb{R}^+ \times (\Lambda_d \backslash O(d, d+16)/(O(d) \times O(d+16)))$$

[2] A K3 surface is a real 4-manifold, unique up to diffeomorphism. The name arises from putting on a compatible complex structure, which gives it two *complex* dimensions.

Nonabelian gauge symmetry

What we referred to as the "moduli space" on the previous slide contains some points corresponding to theories with non-abelian gauge symmetry. In the heterotic case, this is part of Narain's perturbative heterotic string calculation: he found gauge fields in the spectrum of the theory which become massless only at certain moduli values determined by the "Narain lattice" $O(d, d+16; \mathbb{Z})$. The corresponding statement on the K3 side deals with limits which are not Ricci-flat metrics on K3 *per se;* but rather limiting Ricci-flat orbifold metrics (with "ADE" singularities) corresponding to limits in which rational curves on the nearby K3 which have shrunk to zero area. This is the origin of non-abelian gauge symmetry in M-theory compactifications on singular spaces.

M-theory/heterotic duality and K3 surfaces, con.

There is a particular limit in these moduli spaces, known as the "half-K3 limit," which makes the duality apparent. It is a limit in which the K3 surface is equipped with a sequence of Ricci-flat metrics which leave a large open subset of the K3 surface in the form $(a, b) \times T^3$ for an open interval (a, b); each of the complicated geometric ends of this picture carries homology related to the E_8 lattice and can be reinterpreted in terms of E_8 bundle data on T^3. This relates it directly to the "$E_8 \times E_8$" heterotic string.

This duality can also be studied for families of K3 surfaces whose general member is singular, compared with families of bundles on T^3 whose general member has non-abelian struture group. Again, the correspondence between the physical theories is evident.

Fibering the duality

We now wish to invoke the same duality for a compact singular space F (of holonomy G_2) which has a map $F \to B$ whose fibers are K3 surfaces (possibly with ADE singularities). As Sen discusses in hep-th/9604070, there can be complications when applying dualities fiberwise, and indeed we shall find some in this case.

Our basic idea is to take a fiberwise half-K3 limit, in other words, a map $F \to (a, b)$ such that the K3 fibers within F have a T^3 fibration over (a, b). We call this the "half-G_2" limit. The 3-parameter family of T^3's will sweep out a Calabi–Yau threefold (in fact they form an SYZ fibration on that threefold) and that Calabi–Yau is the space we need for compactifying the heterotic string.

An example

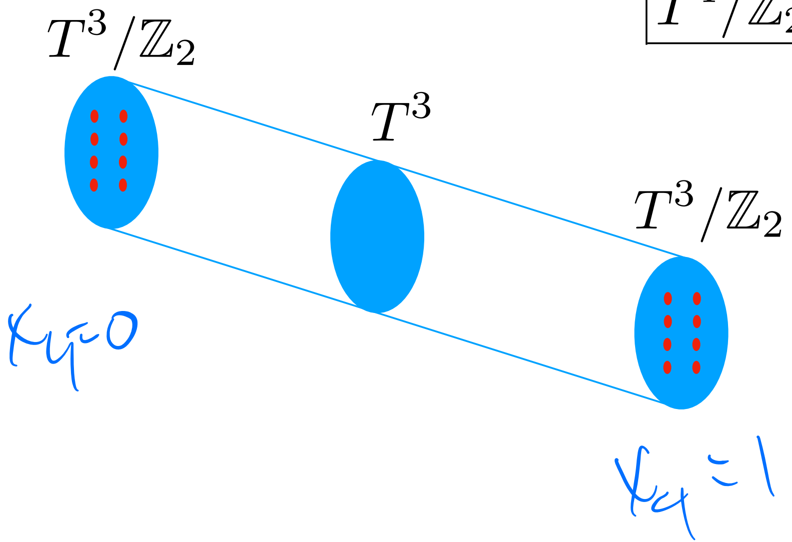
$F = T^7/(\mathbb{Z}_2)^3$ with the group action generated by

$$\alpha : (x_1, \ldots, x_7) \mapsto (-x_1, -x_2, -x_3, -x_4, x_5, x_6, x_7)$$

$$\beta : (x_1, \ldots, x_7) \mapsto (-x_1, \frac{1}{2} - x_2, x_3, x_4, -x_5, -x_6, x_7)$$

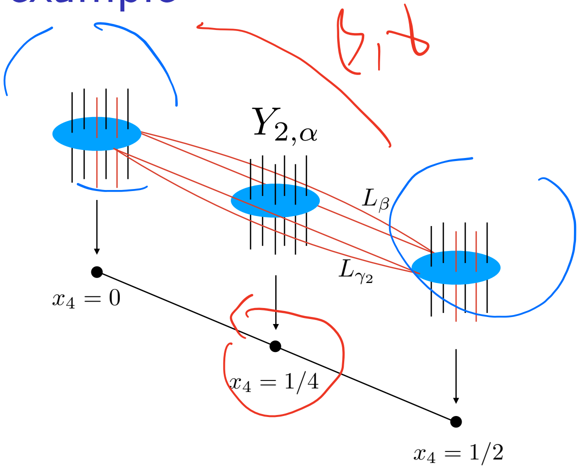
$$\gamma_2 : (x_1, \ldots, x_7) \mapsto (\frac{1}{2} - x_1, x_2, \frac{1}{2} - x_3, x_4, -x_5, x_6, -x_7)\ .$$

An example



$T^4/\mathbb{Z}_2$

An example



A schematic view of the half-G_2 limit of the G_2 orbifold X_2 from example 3.2 with the α-fibration. We have stretched X_2 along the direction of x_4, the throat coordinate. The heterotic dual geometry $Y_{2,\alpha}$ is the inverse image $\pi_4^{-1}\left(\frac{1}{4}\right)$, and is shown with its SYZ fibration of T^3 fibers (black lines) over the 3-orbifold base $Q_{2,\alpha}$ (blue disk). Some of the black lines are singular fibers that do not create singularities in the total space; the singularities in the total space are displayed by red lines.

Example, con.

The α-fixed loci (vertical red lines) are confined to the ends of the x_4 interval, while the β-fixed loci L_β and γ_2-fixed loci L_γ stretch across the interval. These T^3 loci that stretch across the interval intersect $Y_{2,\alpha}$ in a 2-component locus $T^2 \sqcup T^2$. The monodromy action of α on the singular T^2 of $Y_{2,\alpha}$ fixed by β is to travel around a loop in x_4 that begins at $x_4 = \frac{1}{4}$, passes through $x_4 = 0$ or $x_4 = \frac{1}{2}$, and returns to $x_4 = \frac{1}{4}$ along the other leg of L_β, so that the singular T^2 are swapped in pairs.

Heterotic dual

Dual geometry: SYZ fibration

can be calculated

"Borcea-Voisin structure

$(K3 \times T^2)/D$"

Bundle?

E_8 ⟵——⟶ E_8

16 A_1 points on Kummer surface

$SU(2)^{\oplus 16}$ → smaller gauge symmetry in total space

We found a het dual:

involves pert. + non-perturbative $E_8 \times E_8$ heterotic theory

$\mathcal{M} = X^6 \times M^{1,3}$ perturbative: metric + $E_8 \times E_8$ bundle on X^6.

beyond perturbation: other effects "instantons"

M-theory gauge group = $G_1 \times G_2$

heterotic: G_1 ↑ perturb., G_2 ↑ non-perturb.

Conclusion: determining low energy 4D theory from M-theory on G_2 a challenge