Improving CT Geometry Estimation with Optimization and New Phantom Designs

Sarah Glomski

August 9, 2023

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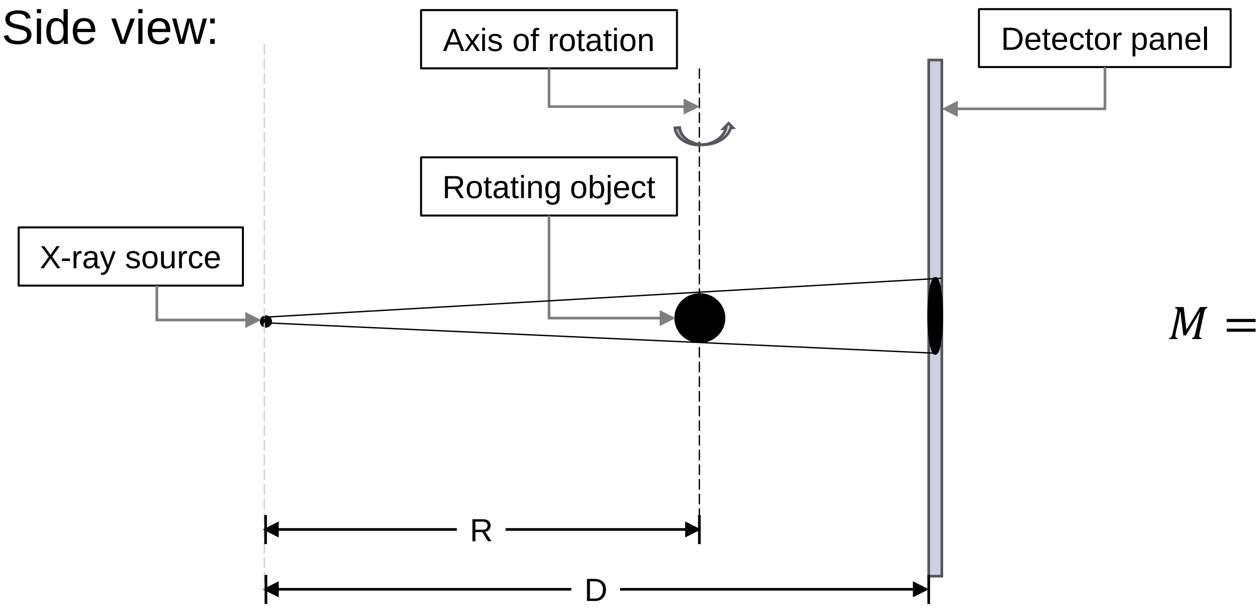
CT Geometry

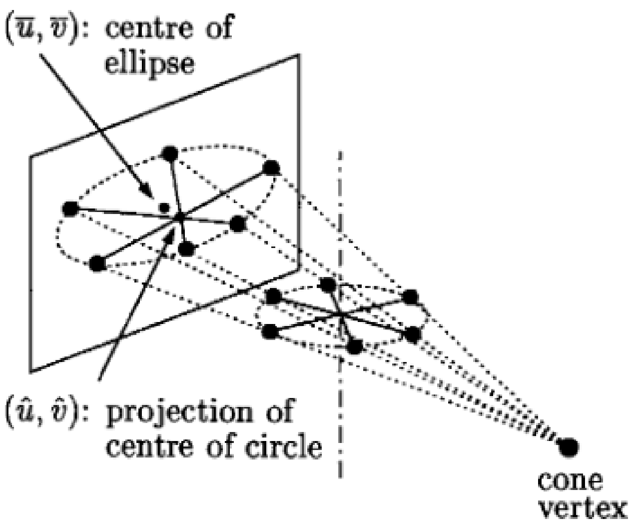
Side view:



$$M = \frac{D}{R}$$

Geometry Estimation

- Noo et al., 2000



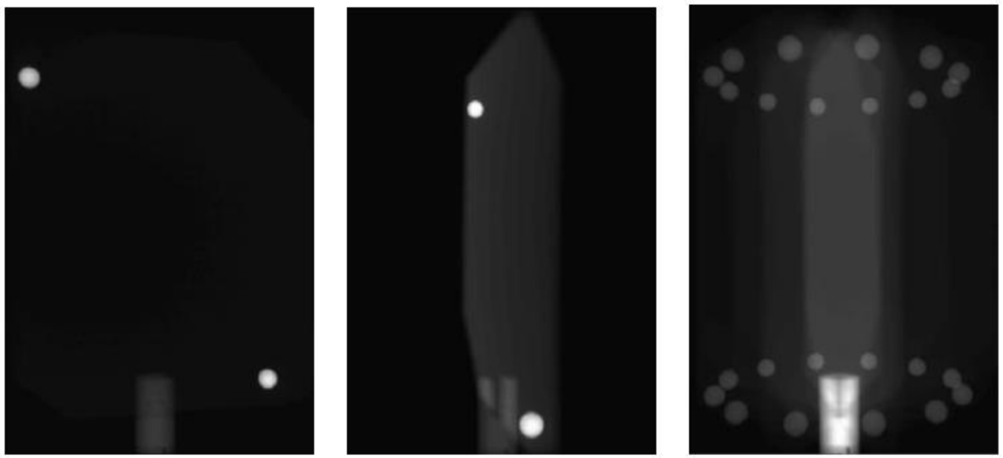


Figure 8. (Left and middle) Two typical projections of the calibration object. (Right) Superposition of 12 CB projections evenly spaced over 360°.

Figure 5. Definition of $(\hat{u}, \hat{v})$ and $(\overline{u}, \overline{v})$ for one point object. Point $(\hat{u}, \hat{v})$ is the projection of the centre of a circle traced by the point object in the field of view. This point lies at the intersection of all lines connecting projections of the point object for two angular positions 180° apart. Point $(\overline{u}, \overline{v})$ is the centre of the ellipse traced by the point object in the detector plane. Geometrically, $(\hat{u}, \hat{v})$ is different from $(\overline{u}, \overline{v})$.

Geometry Estimation

- Noo et al., 2000

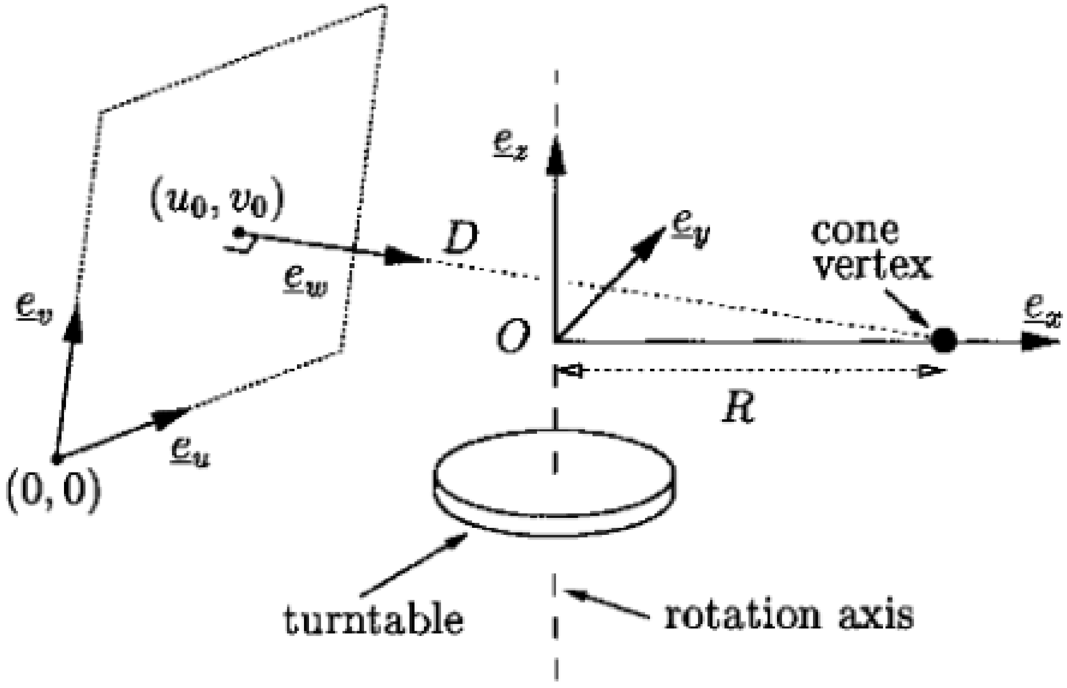
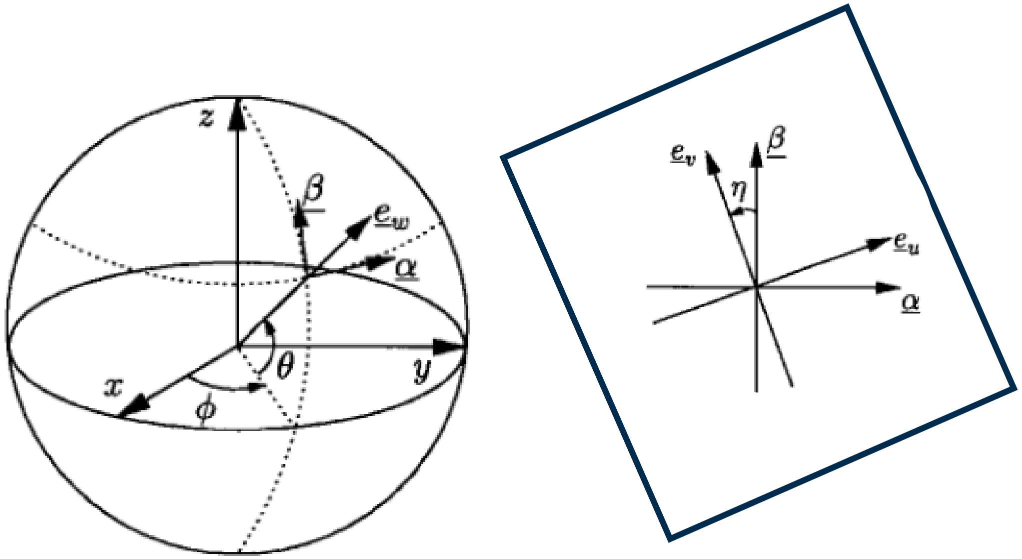


Figure 2. Scanner geometry. See detailed comments in section 2.1.

Geometry Estimation

- Noo et al., 2000

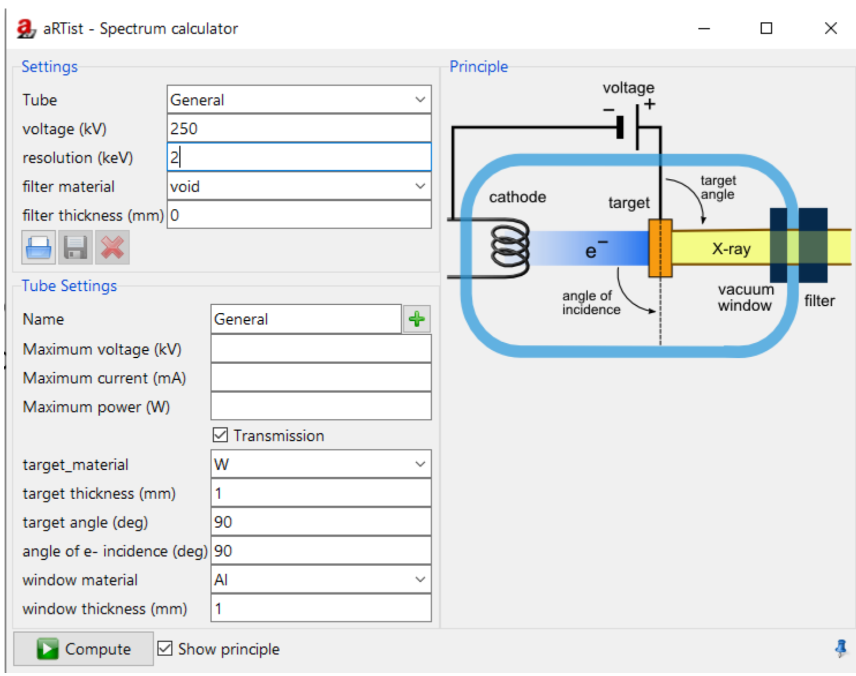


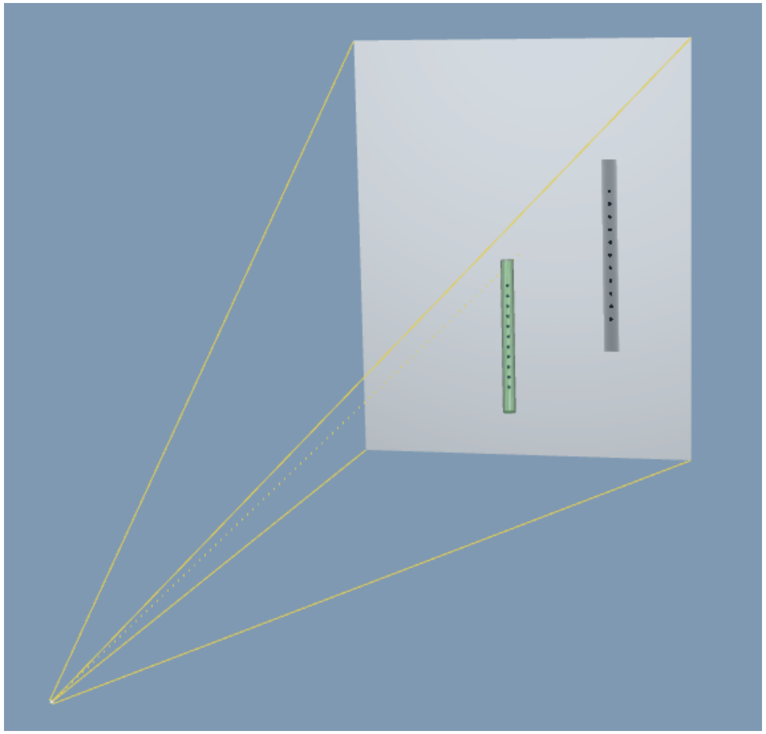
$$\underline{e}_u = \cos\eta\,\underline{\alpha} + \sin\eta\,\underline{\beta}$$
$$\underline{e}_v = \cos\eta\,\underline{\beta} - \sin\eta\,\underline{\alpha}.$$

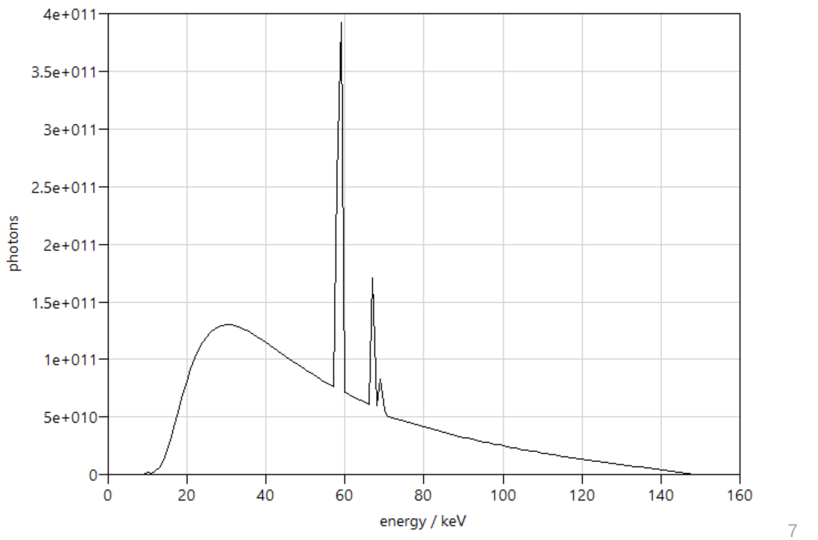
Figure 3. Orientation of detector pixels. Angles θ and ϕ specify the direction of vector $\underline{e}_w$ orthogonal to the detector plane. Vectors $\underline{\alpha}$ and $\underline{\beta}$ are perpendicular to $\underline{e}_w$; $\underline{e}_v$ is an angle η from $\underline{\beta}$.

aRTist Simulations

- Hamamatsu source at 250 kV
- Tungsten target, Aluminum window
- Varex 2520DX-I flat panel detector
- North Star Imaging phantom (medium)







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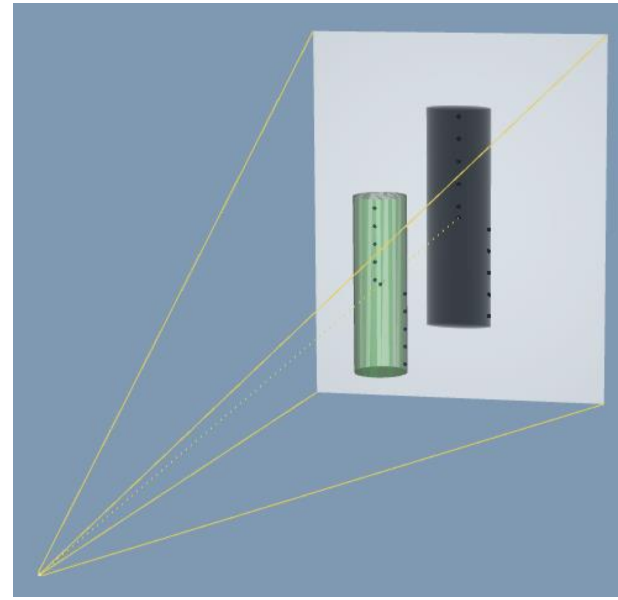
aRTist Simulations



- Performed 110 scans with varying:
  - D, R, and M
  - Horizontal and vertical beam offsets
  - In-plane and out-of-place detector tilts
  - BB size, number, spacing, and material
  - Source voltage, current, focal spot, detector exposure
  - \\e6vault\Students\2023\aRTistTraining\aRTistCTScansSarah

- Used to test current geometry estimation method
  - Broke the algorithm (good)

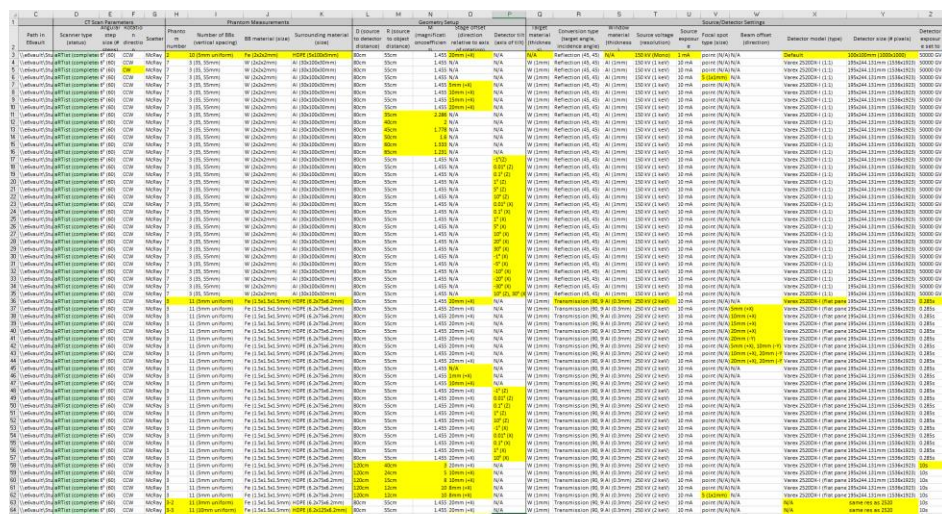


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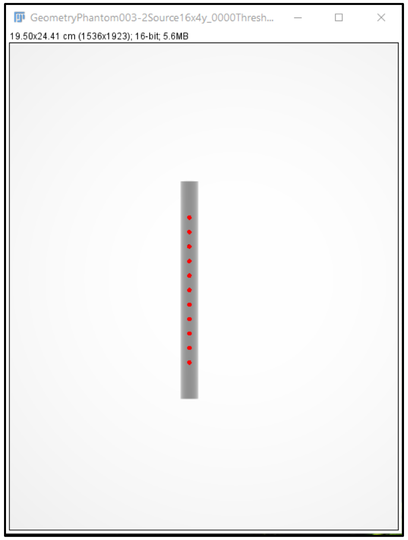
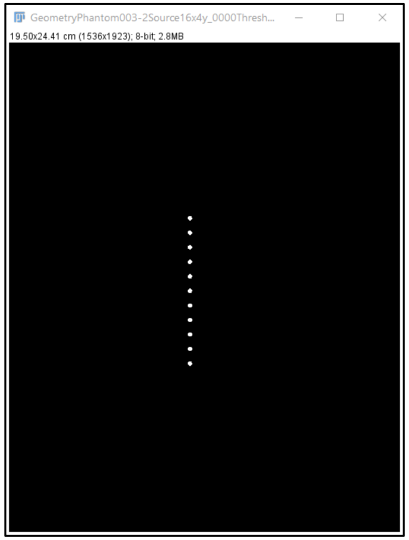
Image Processing

- FIJI macro
  - [\\e6vault\Students\2023\aRTistTraining\aRTistCTScansSarah]
- Threshold set to [0, 6000]
  - Poor performance for large out-of-plane detector tilts
- Analyze particles (sorted top to bottom)
  - Poor performance for large in-plane detector tilts
- Load into Python for analysis





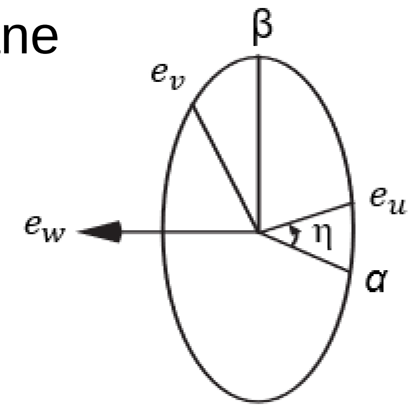
Results

File Edit Font Results

	Area	X	Y
1	0.037069246	9.022604716	8.752139634
2	0.037069246	9.021887155	9.478917945
3	0.036585734	9.021856031	10.205874492
4	0.037069246	9.021721564	10.934296067
5	0.037230417	9.023198023	11.660632803
6	0.036746905	9.021847936	12.387438712
7	0.036746905	9.021959299	13.116527549
8	0.036908075	9.022782358	13.843392786
9	0.037069246	9.021887155	14.569733137
10	0.036585734	9.021856031	15.296689684
11	0.036908075	9.022006226	16.025487514

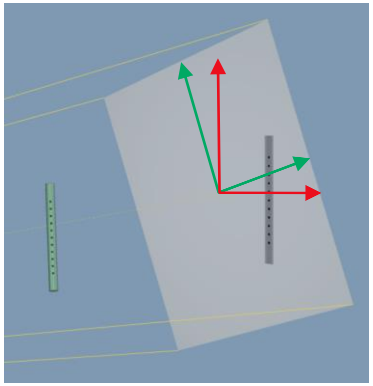
Detector Tilt – Euler Angles

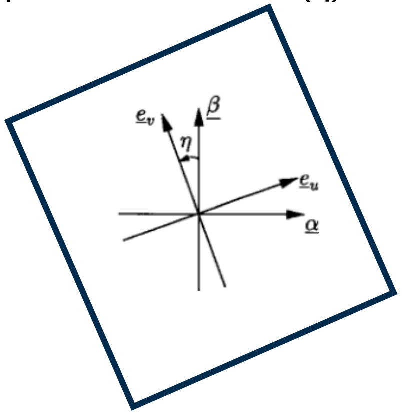
1) In-plane



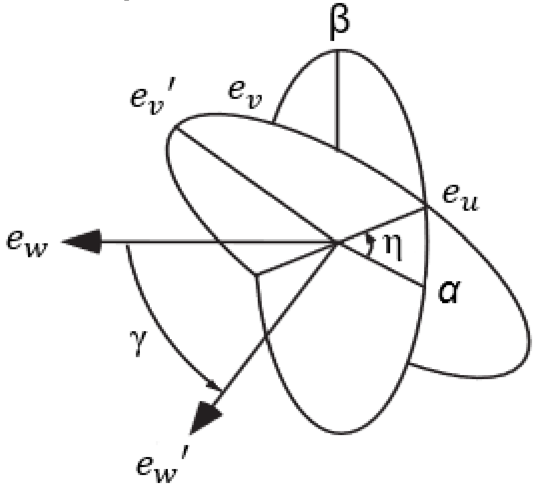
Rotation about e_w

- Noo et al., 2000 considers in-plane detector tilt (η)



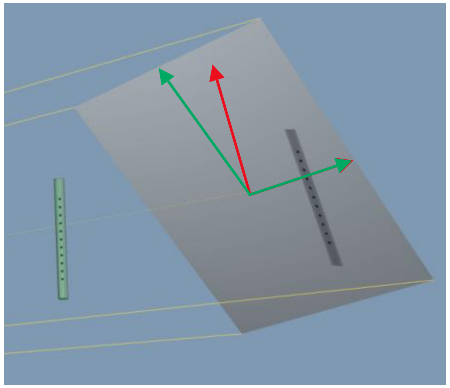


2) Out-of-plane



Rotation about e_w, then rotation about e_u

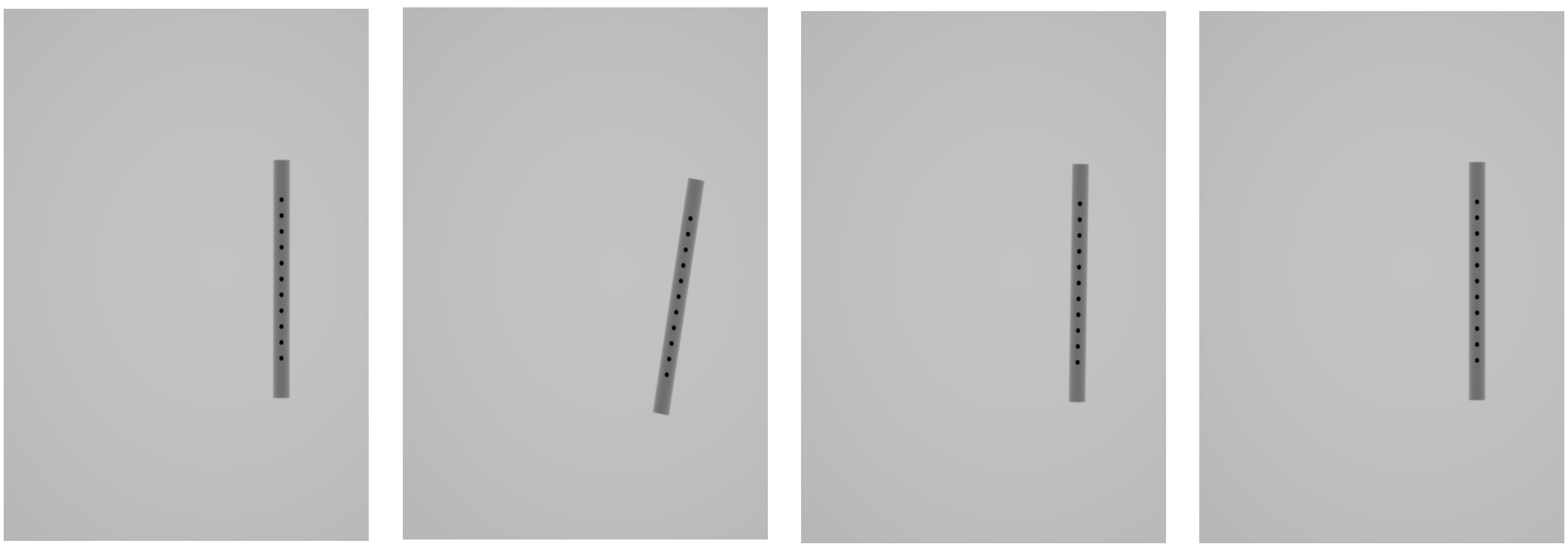
- But ignores out-of-plane detector tilt (γ)
- New method accounts for this tilt



https://mathworld.wolfram.com/EulerAngles.html

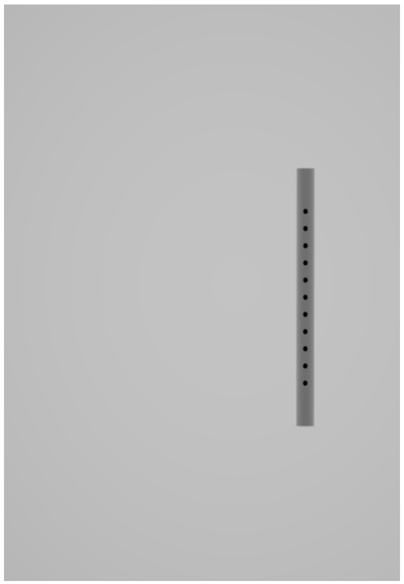
In-plane Detector Tilt

- Try to visually identify which in-plane tilts belong to the scans below
  - $\eta = 0°$
  - $\eta = 0.1°$
  - $\eta = 1°$
  - $\eta = 10°$

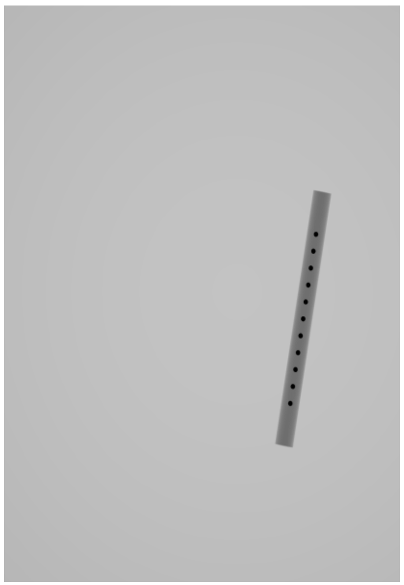


In-plane Detector Tilt

η = 0.1°

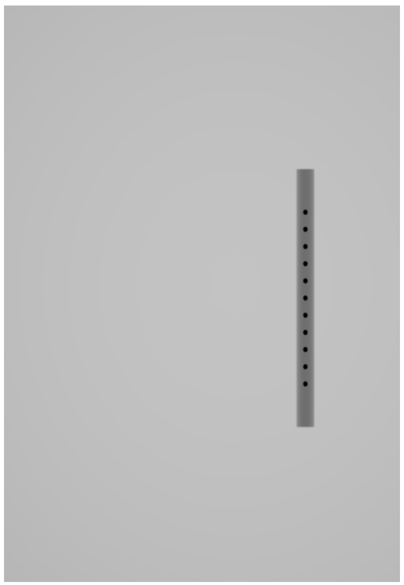


η = 10°



η = 1°

η = 0°



- Large in-plane tilts are easy to visually identify
- Small in-plane tilts are more difficult
- Can all be corrected for using Noo et al. method (IFF γ = 0°)

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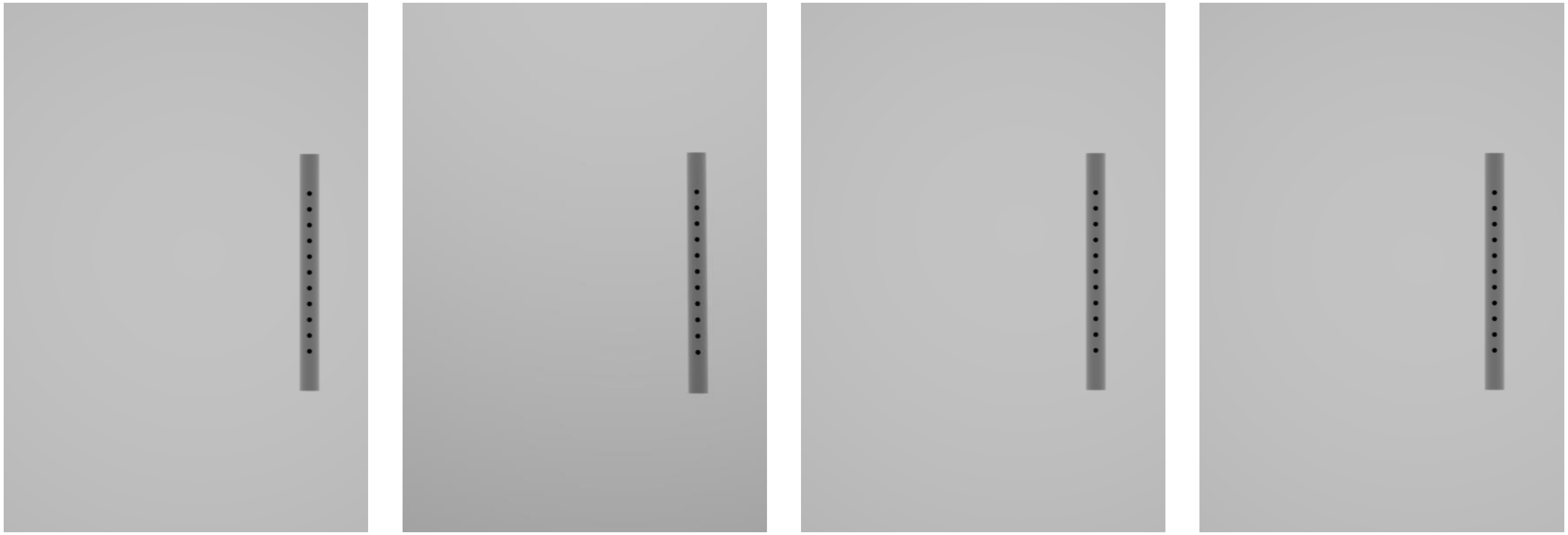
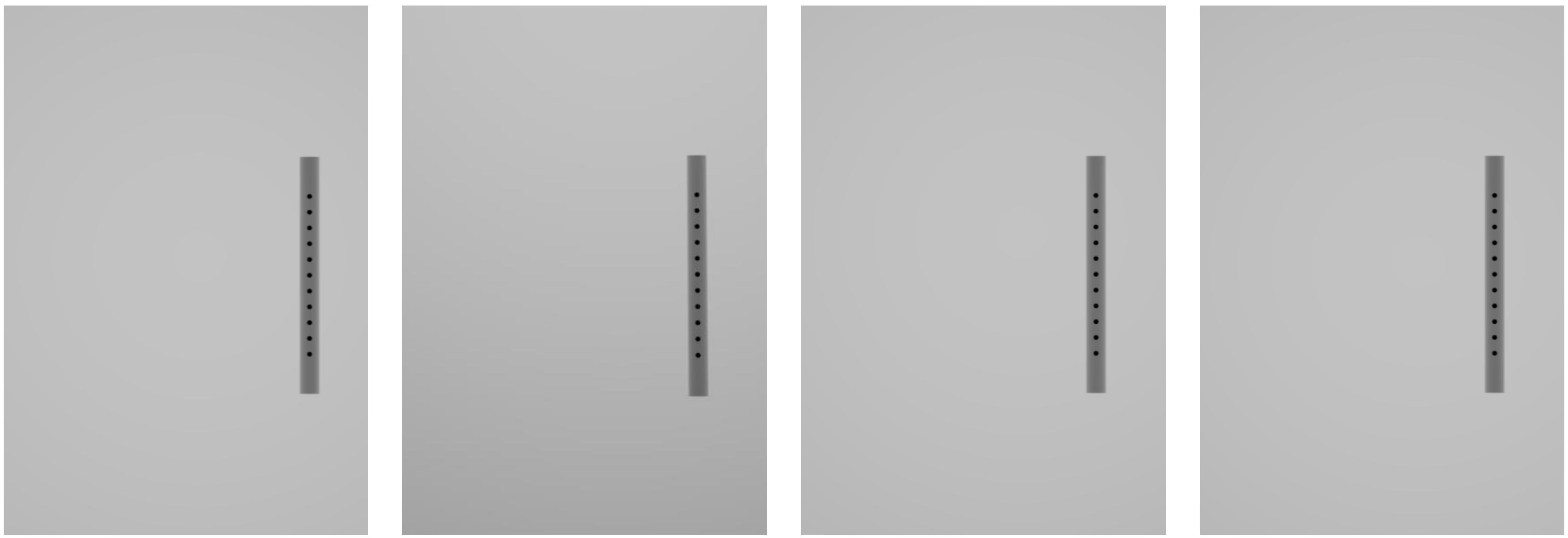
Out-of-plane Detector Tilt

- Try to visually identify which out-of-plane tilts belong to the scans below
  - $\gamma = 0°$
  - $\gamma = 0.1°$
  - $\gamma = 1°$
  - $\gamma = 10°$



Out-of-plane Detector Tilt

γ = 0.1° **γ = 10°** **γ = 1°** **γ = 0°**



- Out-of-plane tilts are more difficult to visually identify
- Cannot be corrected for using Noo et al. method
- Ignoring them adds error to geometry estimation

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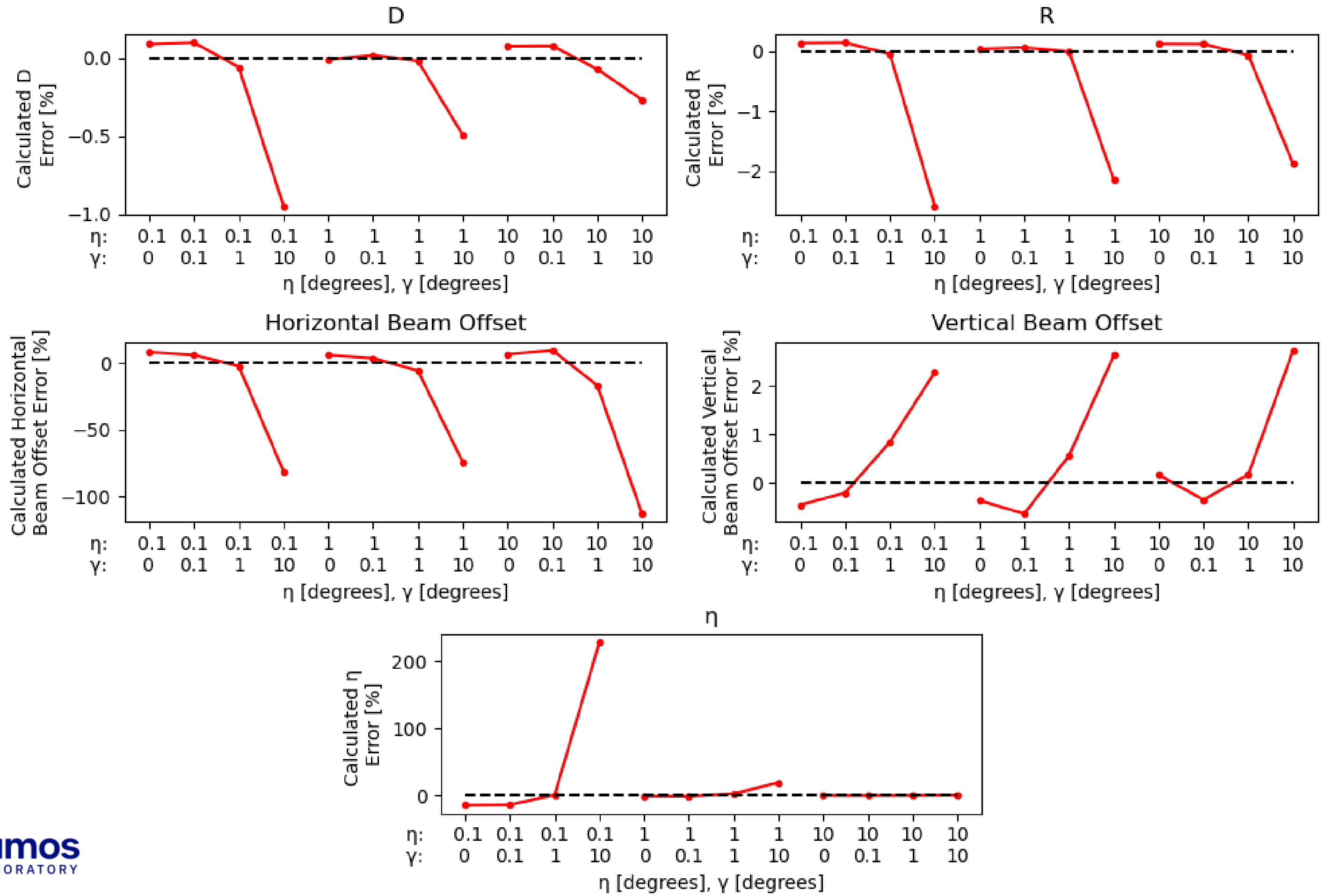
Combined Detector Tilt

- Scans taken at various η (in-plane) and γ (out-of-plane) tilts

Scan Number	D [cm]	R [cm]	Horizontal Beam Offset [cm]	Vertical Beam Offset [cm]	η [degrees]	γ [degrees]
1	80	55	1.6	0.4	0.1	0
2	80	55	1.6	0.4	0.1	0.1
3	80	55	1.6	0.4	0.1	1
4	80	55	1.6	0.4	0.1	10
5	80	55	1.6	0.4	1	0
6	80	55	1.6	0.4	1	0.1
7	80	55	1.6	0.4	1	1
8	80	55	1.6	0.4	1	10
9	80	55	1.6	0.4	10	0
10	80	55	1.6	0.4	10	0.1
11	80	55	1.6	0.4	10	1
12	80	55	1.6	0.4	10	10

Error from Combined Detector Tilt

- Noo et al., 2000 method applied to calculate geometry
  - Error increases as γ increases
  - Interaction between η and γ



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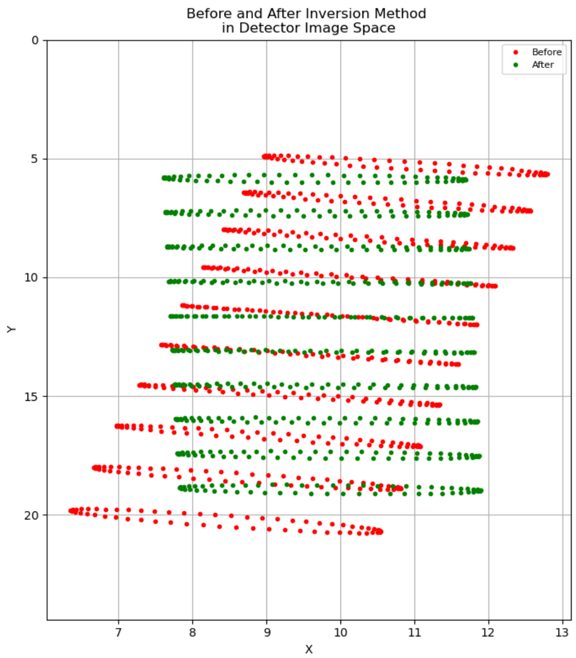
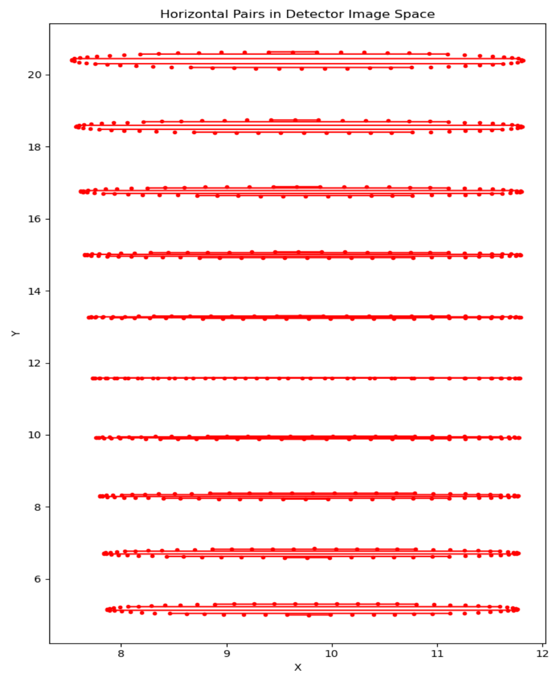
3 New Geometry Estimation Methods

Method 1: Inversion



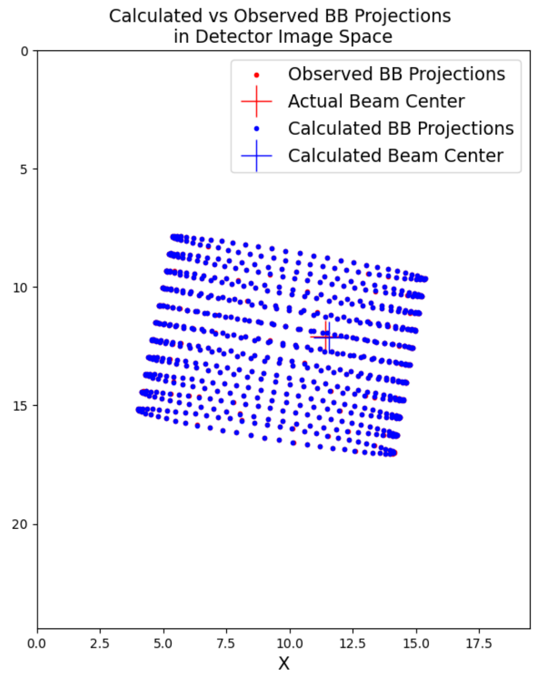
1. Undo η and γ transforms
2. Apply Noo et al., 2000 method to solve for unknowns

Method 2: Magnification



1. Measure magnification factor
2. Use linear algebra to solve for unknowns

Method 3: Optimization



1. Simulate BB projections
2. Use optimization to solve for unknowns

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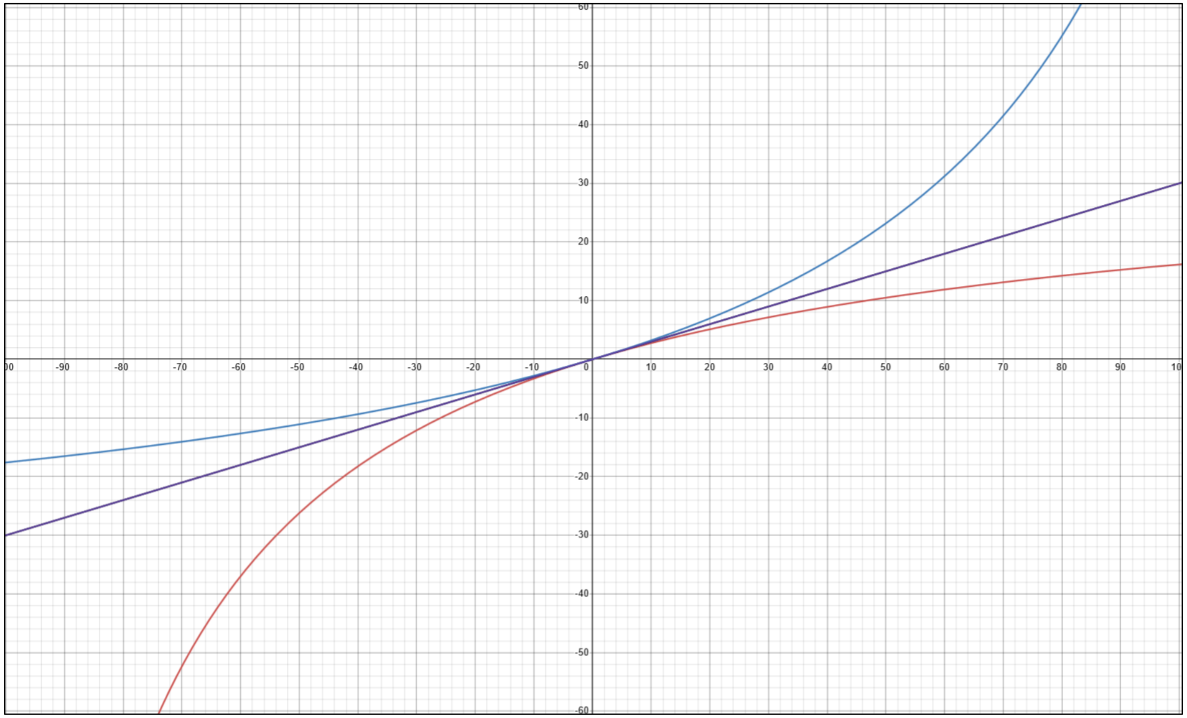
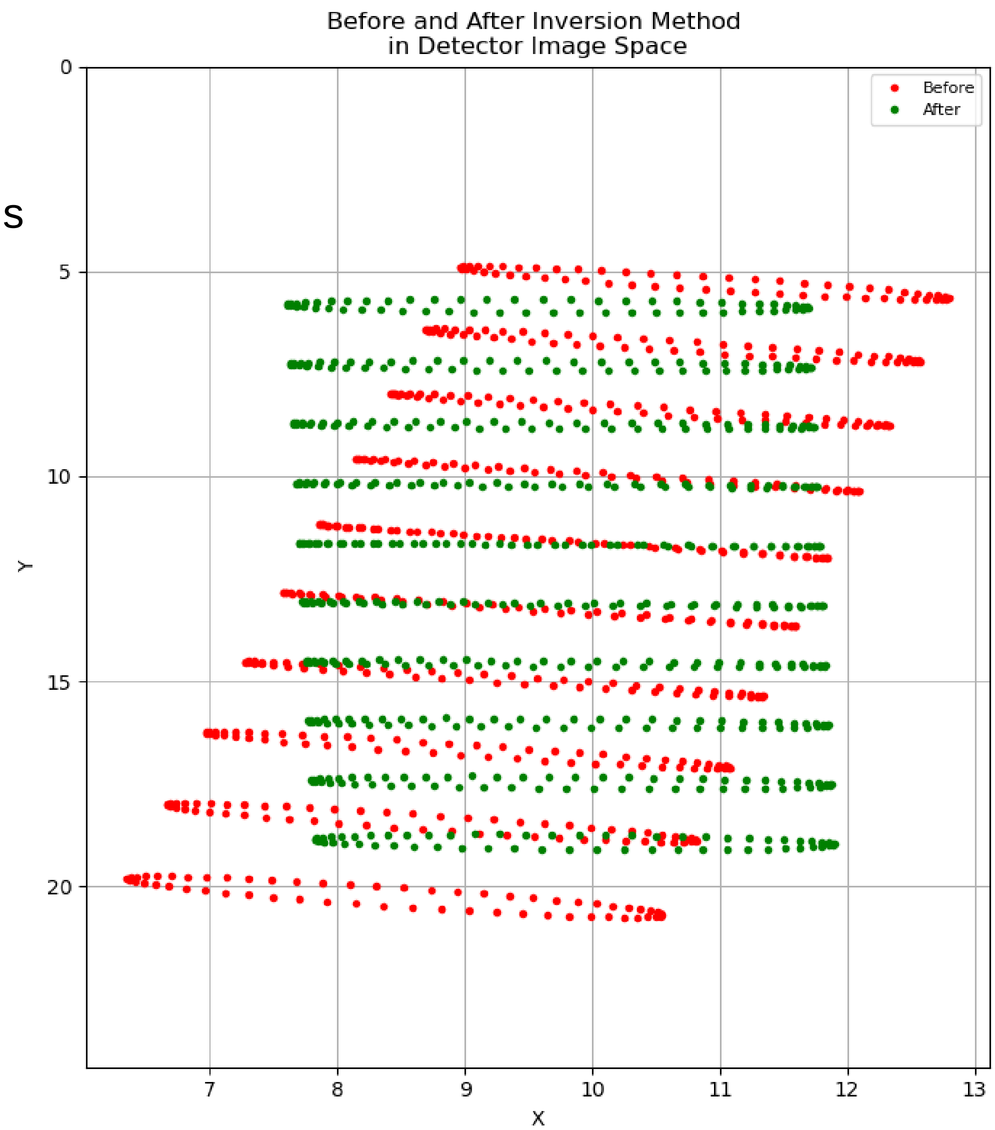
Method 1: Inversion

- Perform the inverse transform of η and γ, then perform Noo et al. method on corrected ellipses
  - η is linear transform, γ is nonlinear
  - https://www.desmos.com/calculator/73cyfcsdtw



Method 1: Inversion

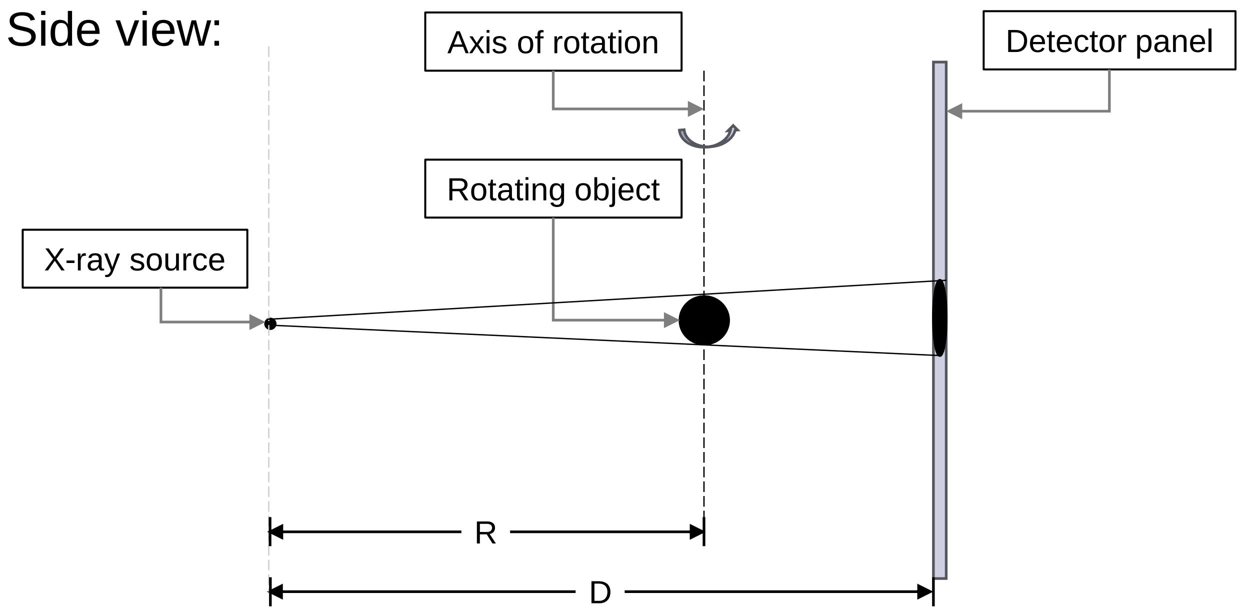
- Pros:
  - Could be used to correct scans retroactively
- Cons:
  - Doesn't fully correct ellipses
  - Requires knowledge of η and γ beforehand
  - Beam center asymmetry
  - aRTist uses Euler angles
  - Order of operations matters
  - Interaction between η and γ



Method 2: Magnification

- Measure observed magnification, then solve for unknowns using linear algebra
- Recall magnification as a function of D and R*:*

$$M_{theoretical} = \frac{D}{R}$$

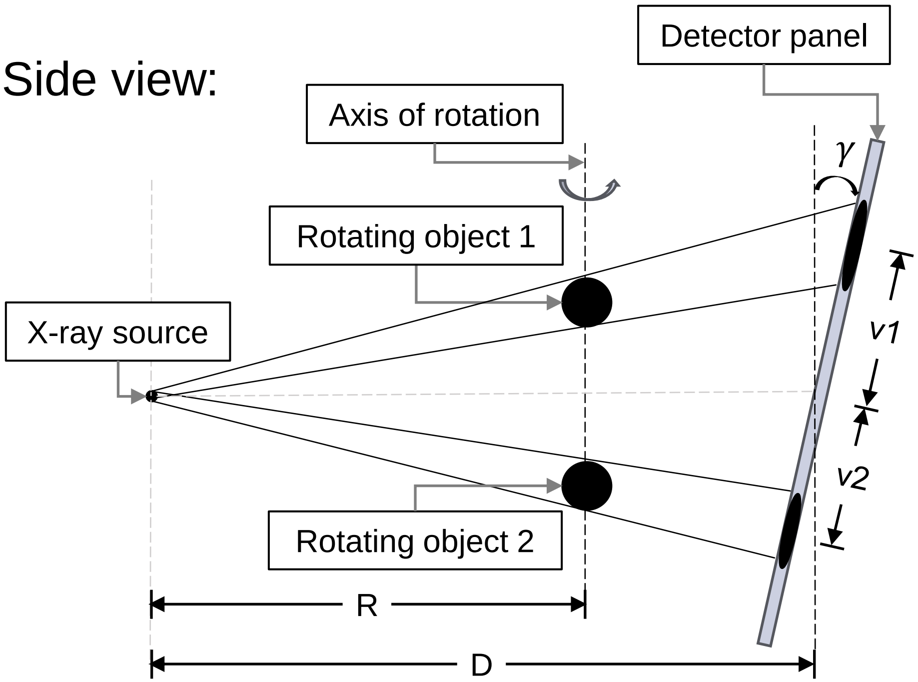


Method 2: Magnification

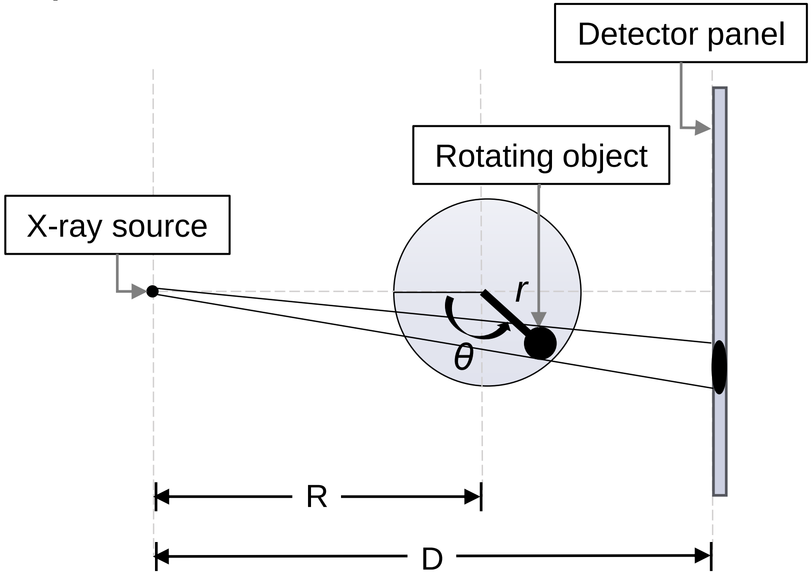
- Magnification as a function of D, out-of-plane detector tilt (γ), projection height in detector space (v), R, rotation radius (r), and rotation angle (θ):

$$M_{theoretical} = \frac{D}{R} \quad \rightarrow \quad M_{theoretical} = \frac{D - v * \sin(\gamma)}{R - r * \cos(\theta + \theta_0)}$$

Side view:

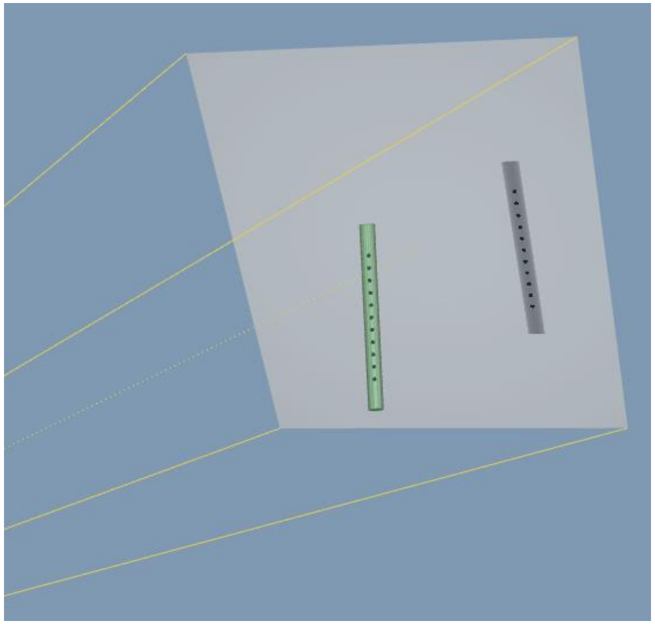


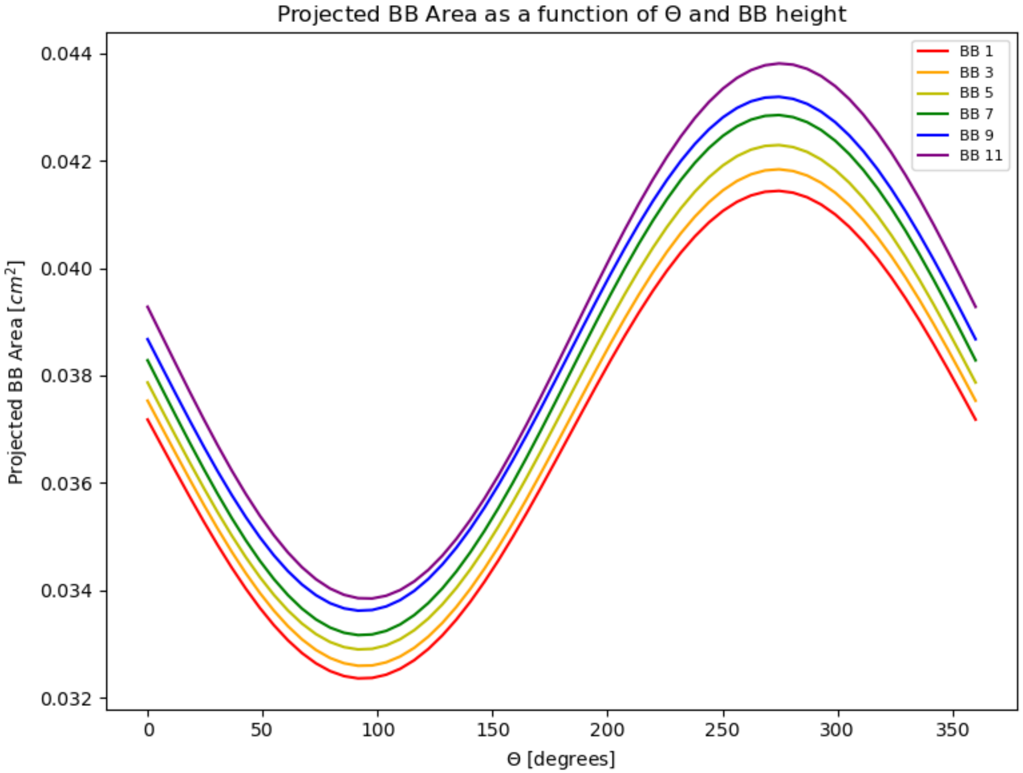
Top view:



Method 2: Magnification

$$M_{theoretical} = \frac{D - v * \sin(\gamma)}{R - r * \cos(\theta + \theta_0)}$$





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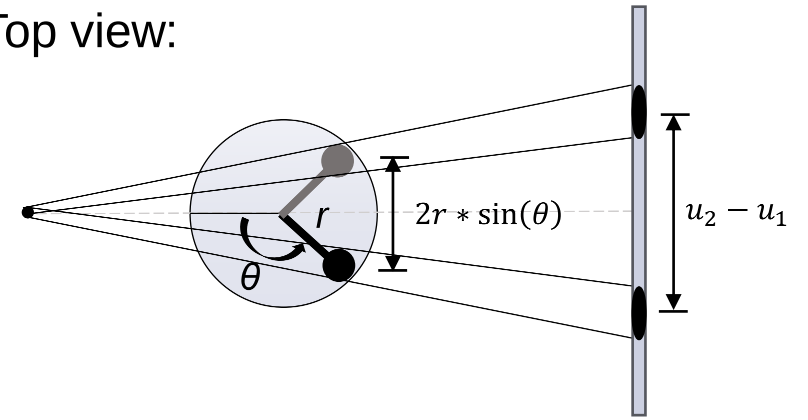
Method 2: Magnification

Top view:



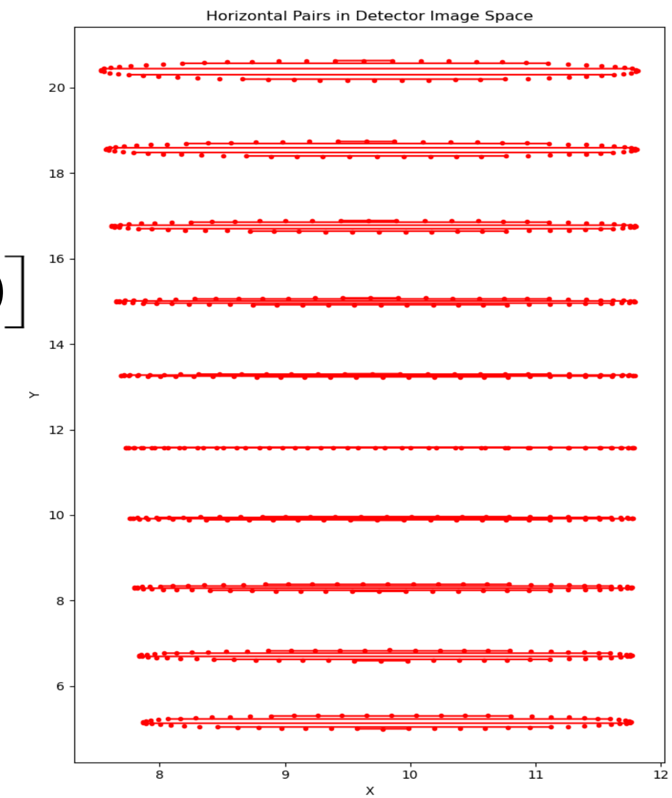
- Use horizontal pairs of BB projections to measure the observed magnification at different heights in detector space:

$$M_{observed} = \frac{u_2 - u_1}{2r * \sin(\theta + \theta_0)}$$

- Use np.linalg.lstsq to solve for D and sin(γ)
  - 11 BBs * 29 horizontal pairs/BB = 319 rows in matrix

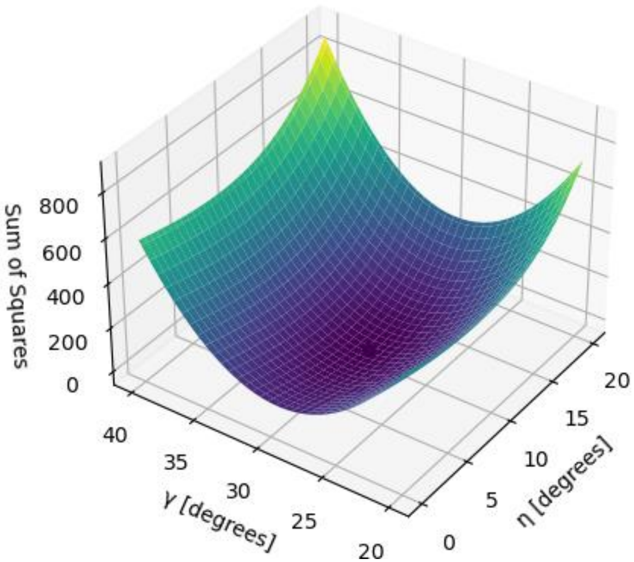
$$\begin{bmatrix} 1 & -\frac{(v_1+v_2)}{2} \end{bmatrix} \begin{bmatrix} D \\ \sin(\gamma) \end{bmatrix} = \begin{bmatrix} M_{observed}(R - r * \cos(\theta + \theta_0)) \end{bmatrix}$$

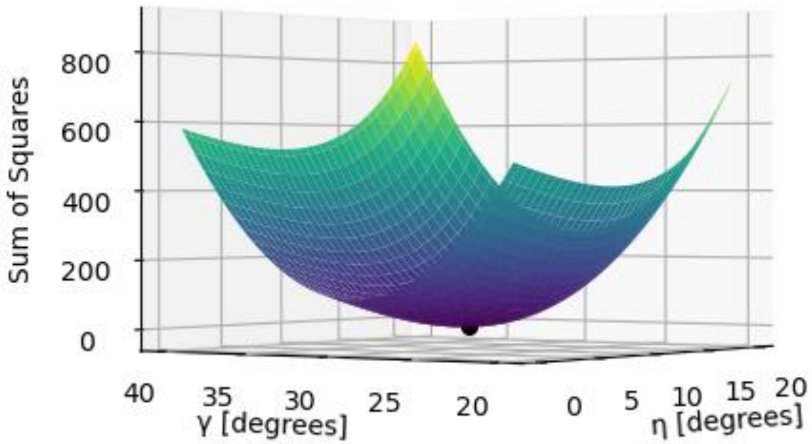
- Pros:
  - Recovers D and γ accurately
- Cons:
  - Requires knowledge of R, r, *and* θ_0 beforehand
  - Becomes more complicated when $\eta \neq 0$, $\theta_0 \neq n * \theta_{step}$



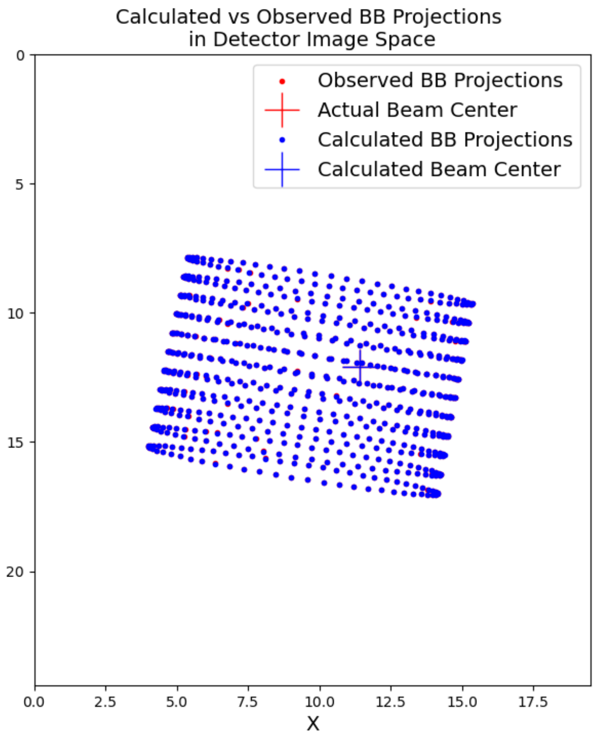
Method 3: Optimization

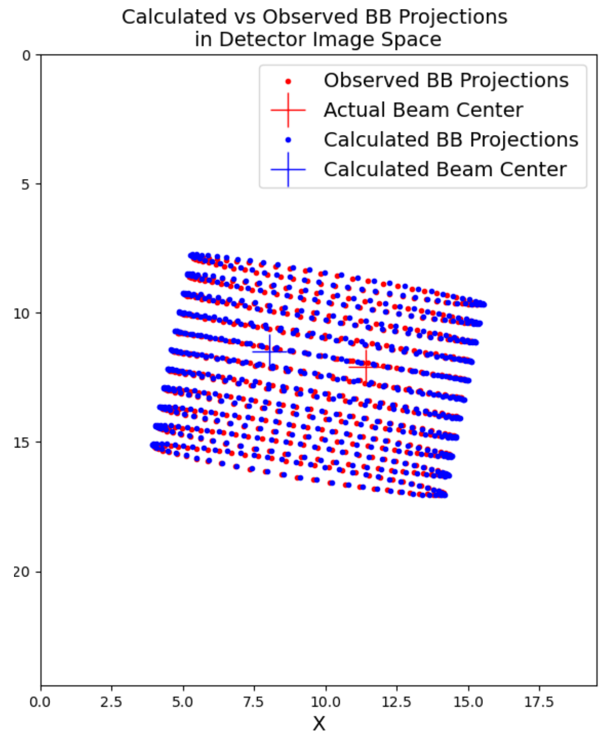
- Simulate projection of BBs onto detector
  - Ray tracing using known BB spacing
- Then use scipy.optimize.minimize to minimize sum of squares between BB projections
  - 9 free variables: D, R, horizontal beam offset, vertical beam offset, η, γ, r, θ_0, h_0
  - All free variables given generic seeds and bounds
  - 11 BBs * 60 projections/BB = 660 Euclidian distances to minimize

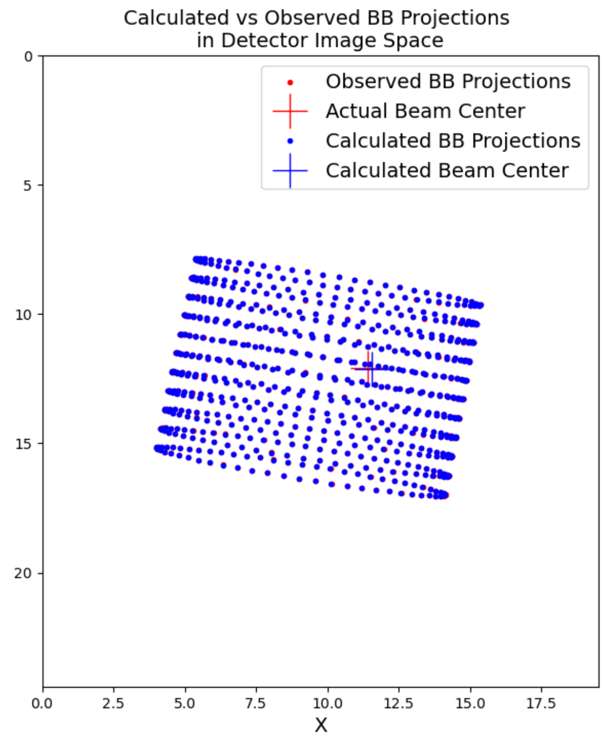




Results: Optimization vs. Noo et al., 2000







Theoretical Simulation

- Horizontal Beam Offset = 1.60 cm
- Vertical Beam Offset = 0.40 cm
- η = 10.00°
- γ = 10.00°

Noo et al., 2000 Simulation

- Horizontal Beam Offset = -1.81 cm
- Vertical Beam Offset = 0.37 cm
- η = 10.54°
- γ = 0.00°

Optimization Simulation

- Horizontal Beam Offset = 1.77 cm
- Vertical Beam Offset = 0.41 cm
- η = 10.00°
- γ = 9.69°

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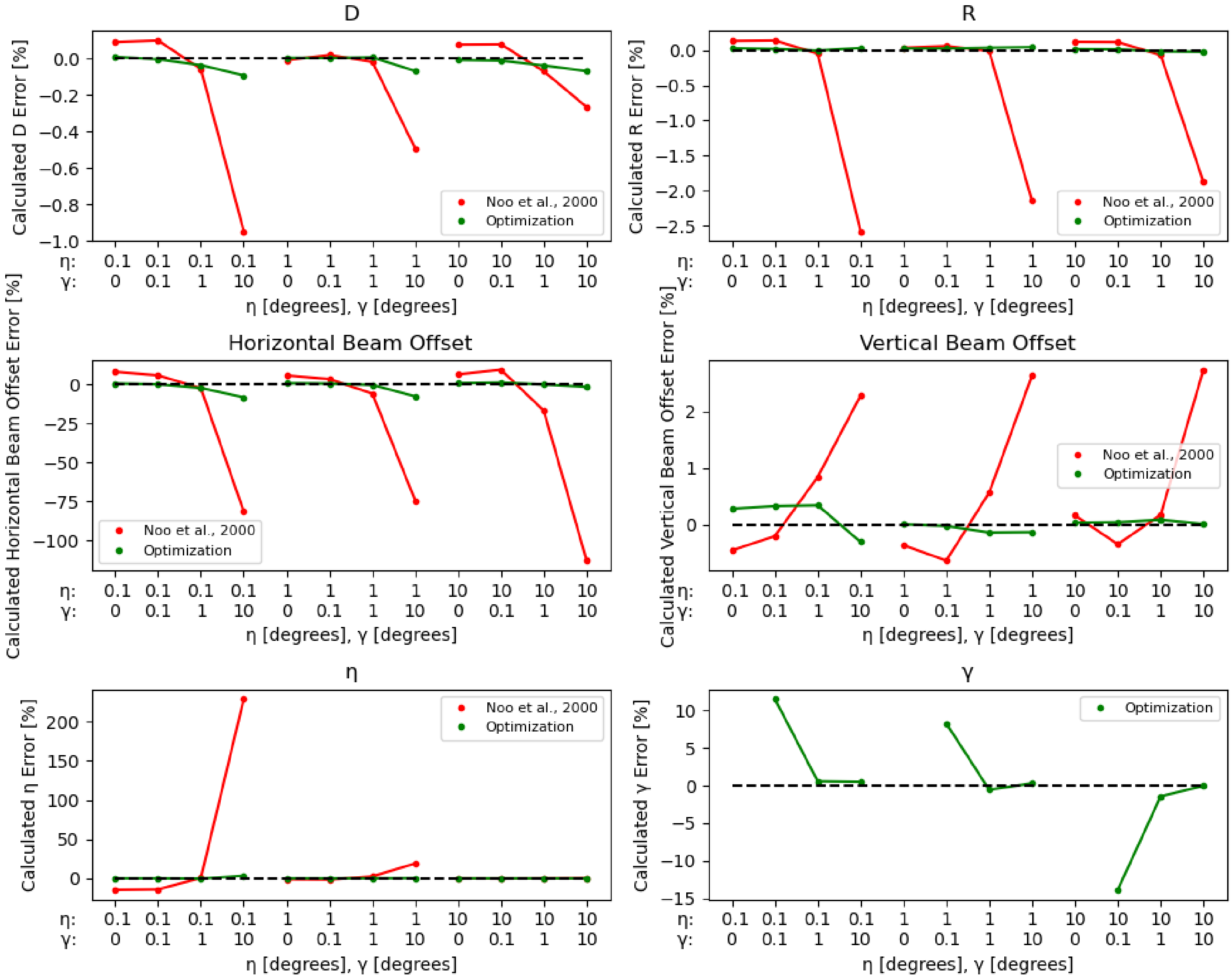
Results: Optimization vs. Noo et al., 2000



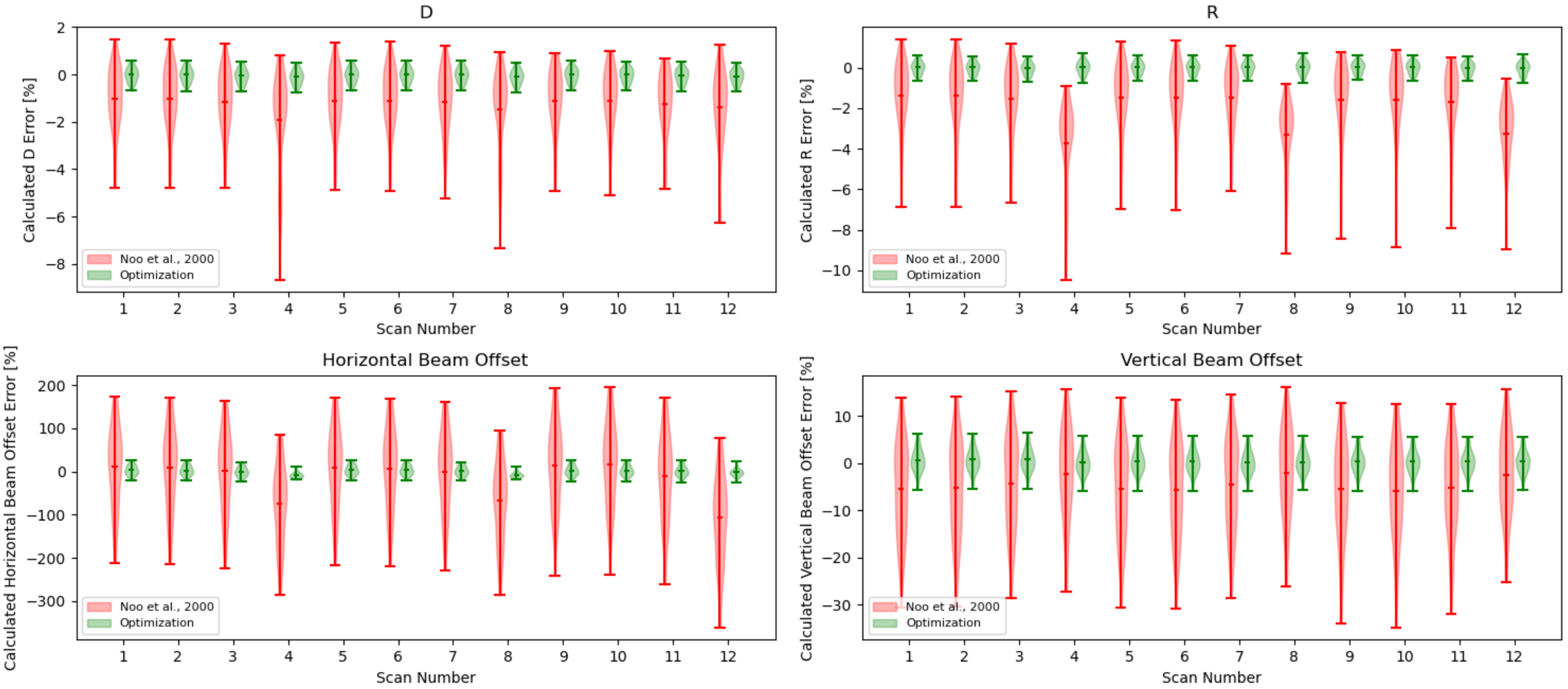
aRTist Test Parameters

- Gaussian noise added to each scan (σ = 1 pixel, n = 50)
  - Both methods given the same random sample for pairwise comparison

Scan Number	D [cm]	R [cm]	Horizontal Beam Offset [cm]	Vertical Beam Offset [cm]	η [degrees]	γ [degrees]
1	80	55	1.6	0.4	0.1	0
2	80	55	1.6	0.4	0.1	0.1
3	80	55	1.6	0.4	0.1	1
4	80	55	1.6	0.4	0.1	10
5	80	55	1.6	0.4	1	0
6	80	55	1.6	0.4	1	0.1
7	80	55	1.6	0.4	1	1
8	80	55	1.6	0.4	1	10
9	80	55	1.6	0.4	10	0
10	80	55	1.6	0.4	10	0.1
11	80	55	1.6	0.4	10	1
12	80	55	1.6	0.4	10	10

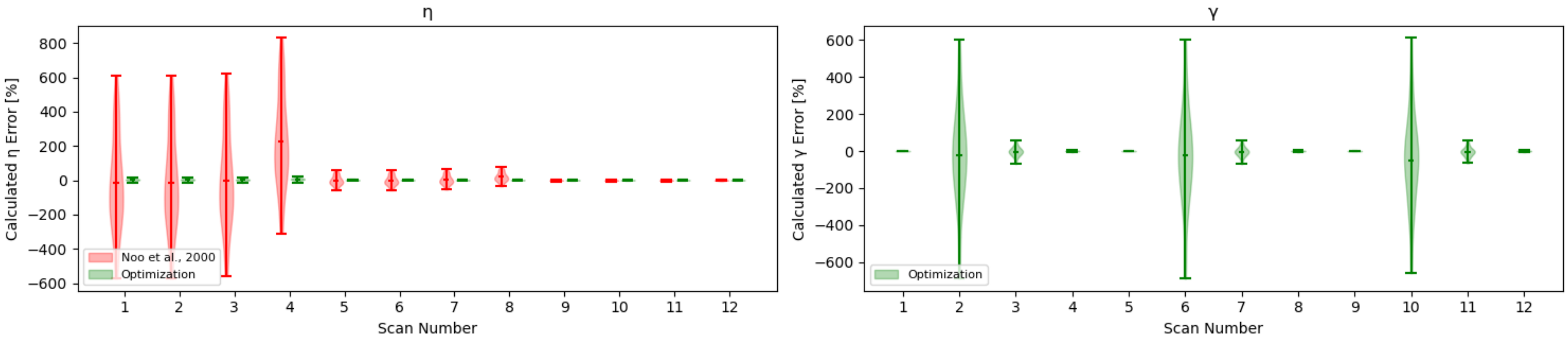
Results: Optimization vs. Noo et al., 2000

- Optimization method is more accurate and less sensitive to noise than Noo et al., 2000 for D, R, and both beam offsets



Results: Recovering Detector Tilt

- Optimization method is more accurate and less sensitive to noise than Noo et al., 2000 for recovering η
  - Noo et al., 2000 does not calculate γ
- Optimization method is better at recovering η than γ
  - % error calculation blows up near 0



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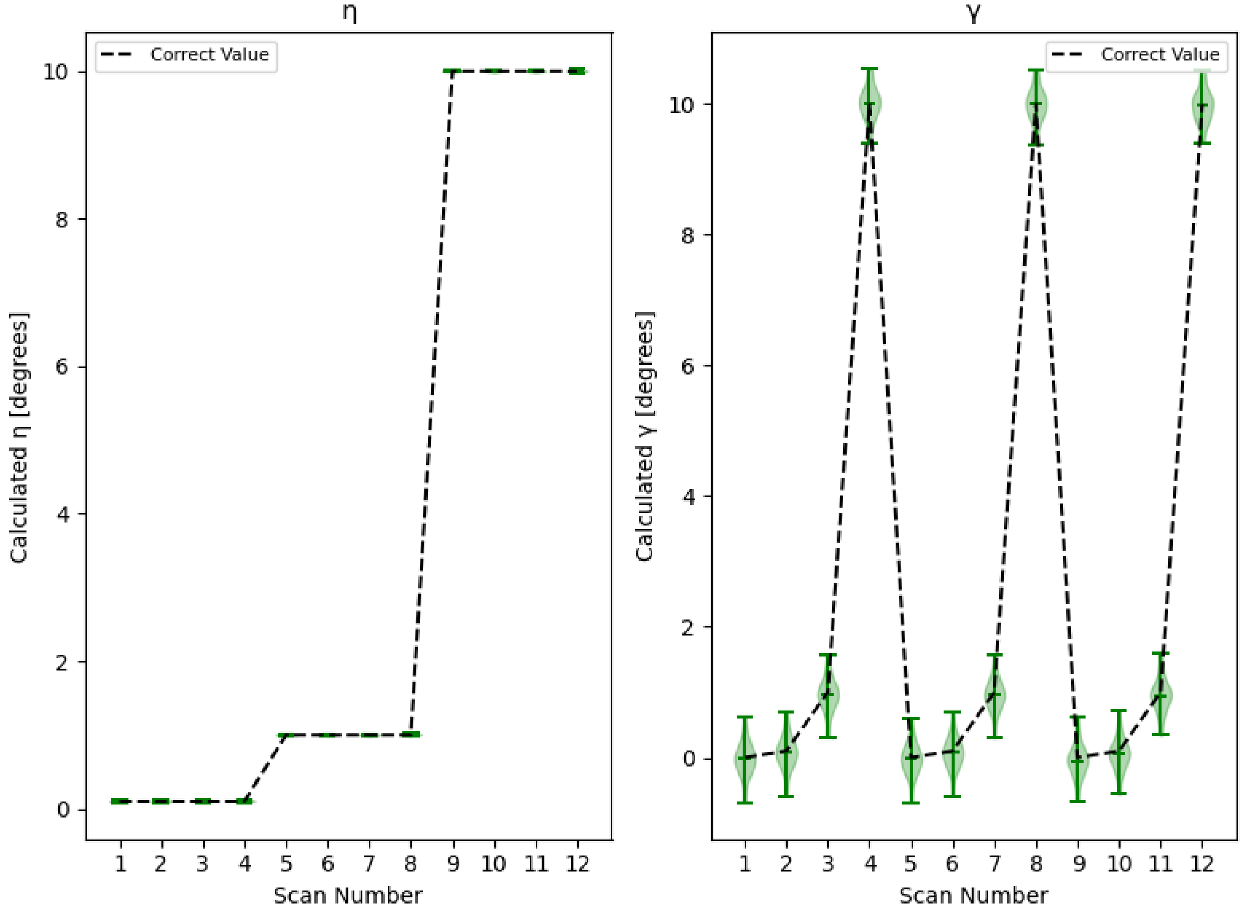
Results: Recovering Detector Tilt with Optimization

- Average η off by ~0.001°
- Average γ off by ~0.015°
  - BB centroid error due to tilt



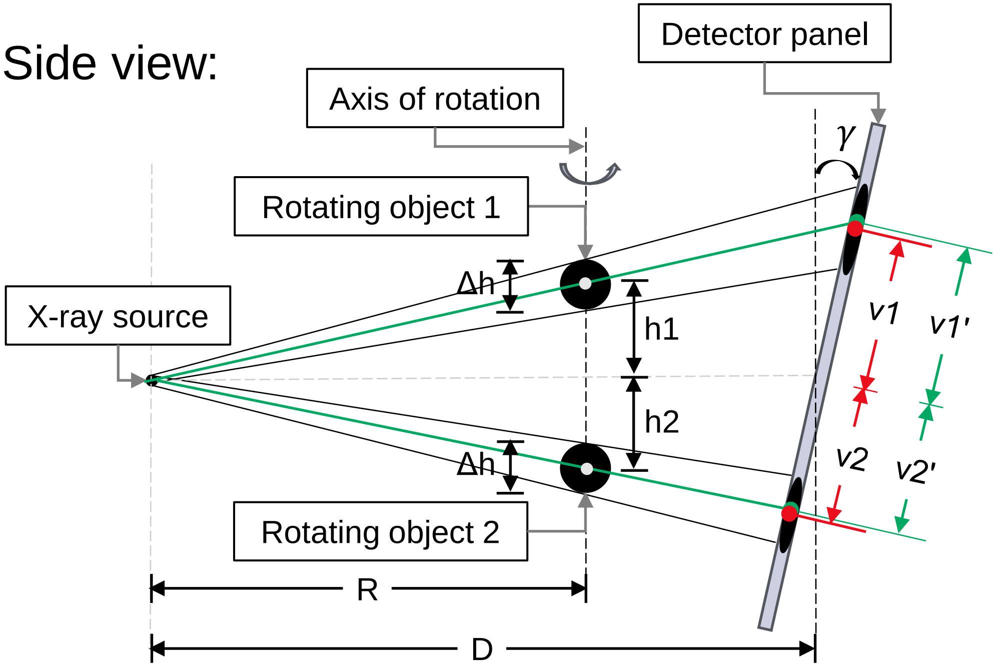
BB Centroid Error

- Out-of-plane tilt stretches BB projection
  - BB is imperfect “point” object
  - Oblique circular cone geometry
  - Circular cross section → elliptical
  - Centroid of ellipse ≠ ray trace of center
  - Seemingly negligible error (<1 pixel for most real cases), but may propagate

$$v' = v\left(1 - \left(\frac{a}{c+R\frac{|v|}{v}}\right)^2\right) + \frac{ab\sqrt{\tan^2(\gamma)+1}}{(c+R\frac{|v|}{v})^2},$$

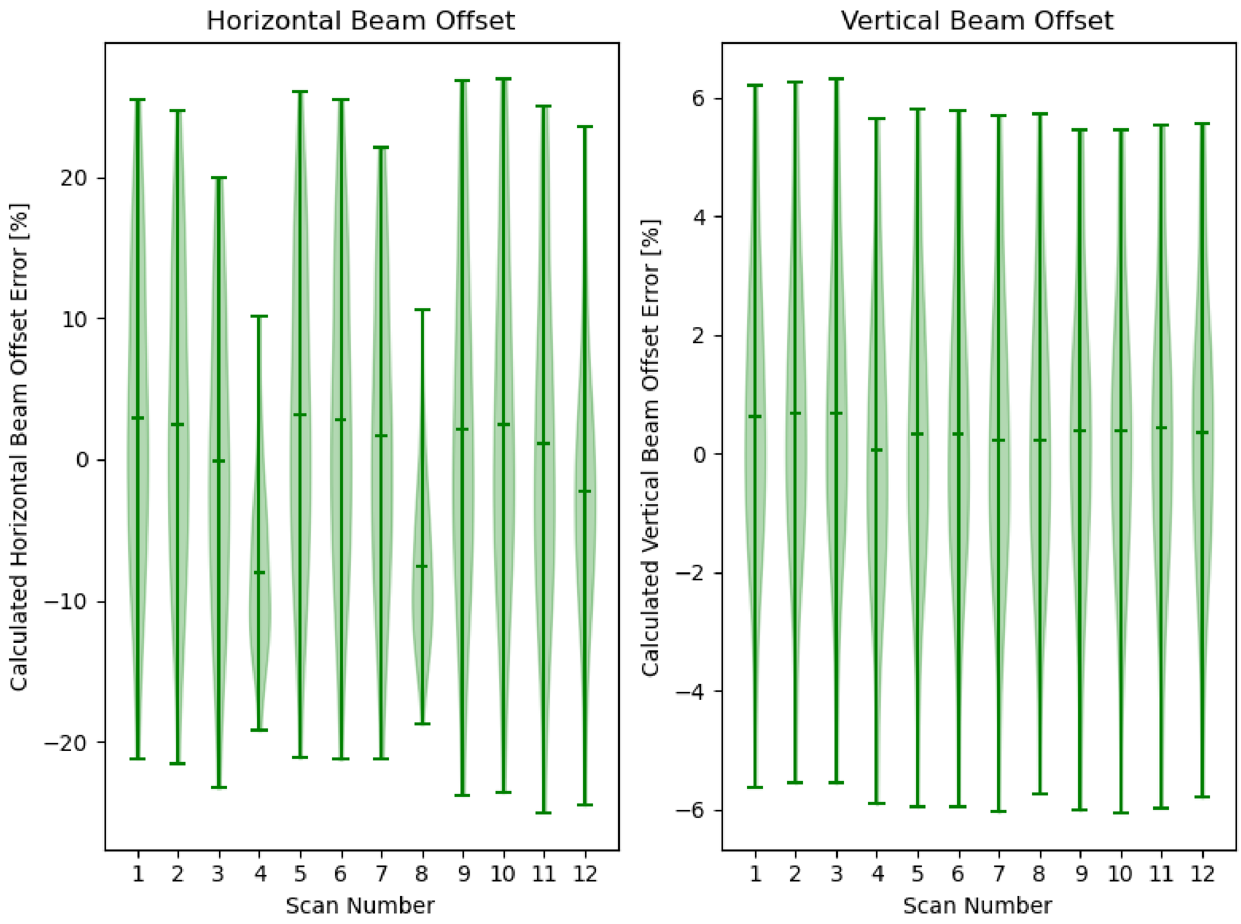
where $a = \frac{\Delta h}{2}\tan(\gamma)$, $b = D\frac{\Delta h}{2}$, $c = \left(h + \frac{\Delta h}{2}\right)\tan(\gamma)$

Side view:



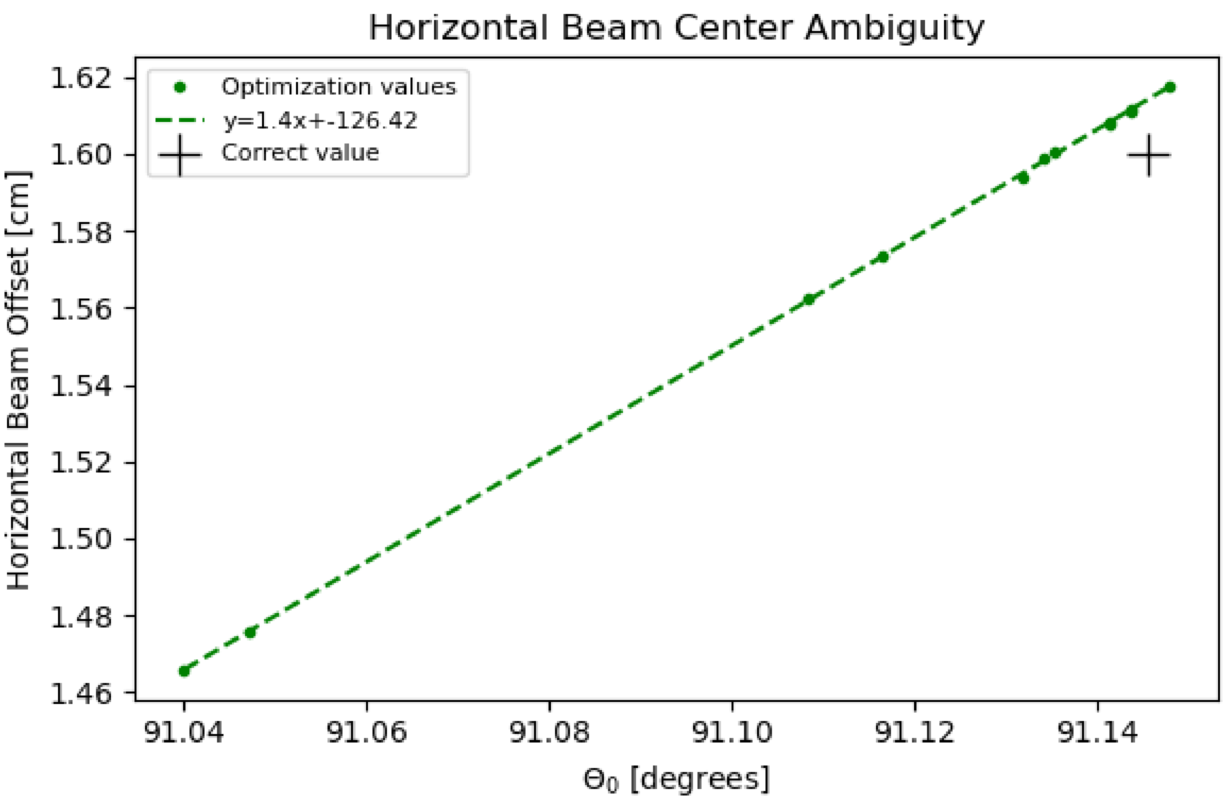
Results: Beam Offset Calculations

- Optimization method is better at recovering vertical beam offset than horizontal
  - Due to horizontal shift ambiguity



Horizontal Shift Ambiguity

- Optimization doesn't converge on correct values
  - Small angle approximation
  - https://www.desmos.com/calculator/fmvqlhyhgw



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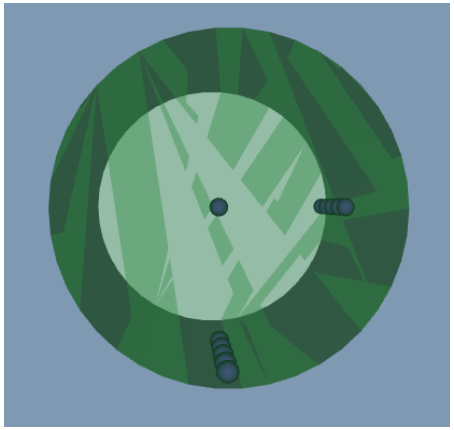
Method 3: Optimization Method

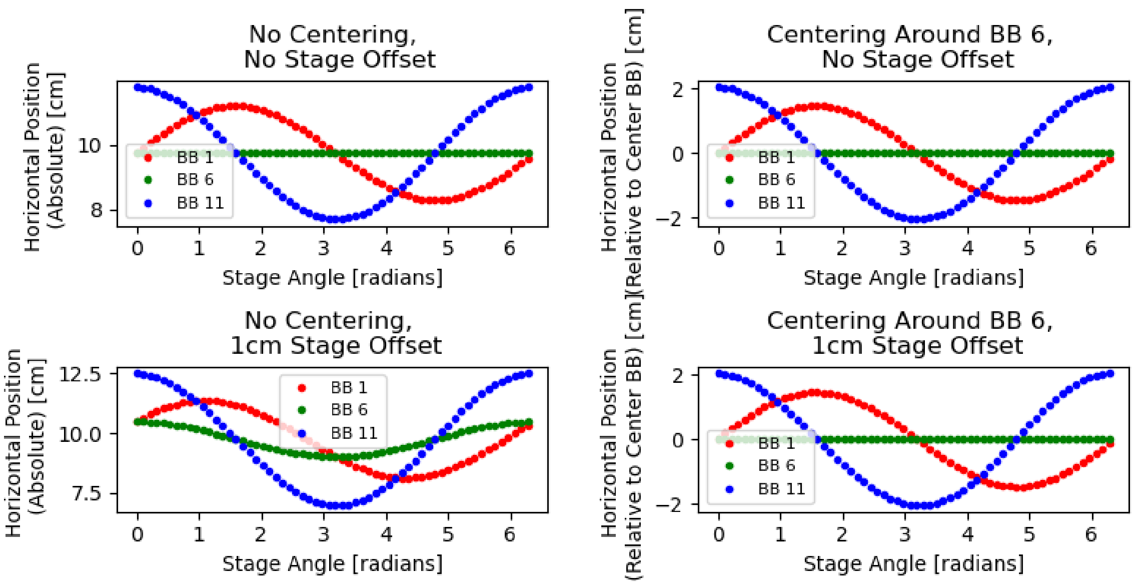
- Pros:
  - Accurately recovers D, R, and both beam offsets
  - Recovers η and γ for detector correction (alternative: inversion method)
  - Highly resistant to random noise
  - Only required knowledge is BB spacing (NSI: 0.762 mm (S), 5 mm (M), 15 mm (L))
  - Also recovers initial placement of phantom in 3D

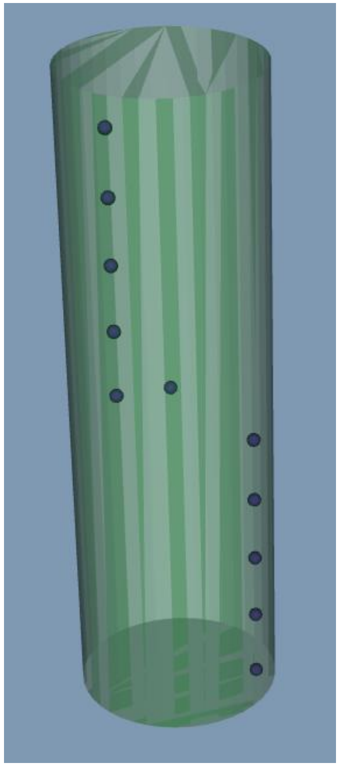
- Cons:
  - Long run time compared to other methods (27.2s vs. 0.001s using Noo et al., 2000)
  - Low accuracy in horizontal beam center due to horizontal shift ambiguity

Rotary Stage Calibration



- Built-in BB phase offset of 90° between top and bottom
- Middle BB is placed in center of phantom (no radius)
- Track horizontal BB positions (relative to center BB) over stage angle
  - Must sample at higher angular frequency to calibrate accurately





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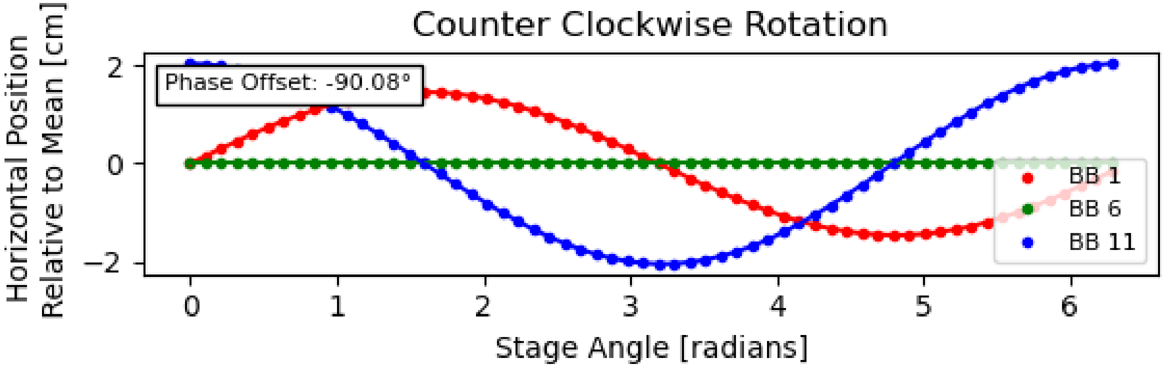
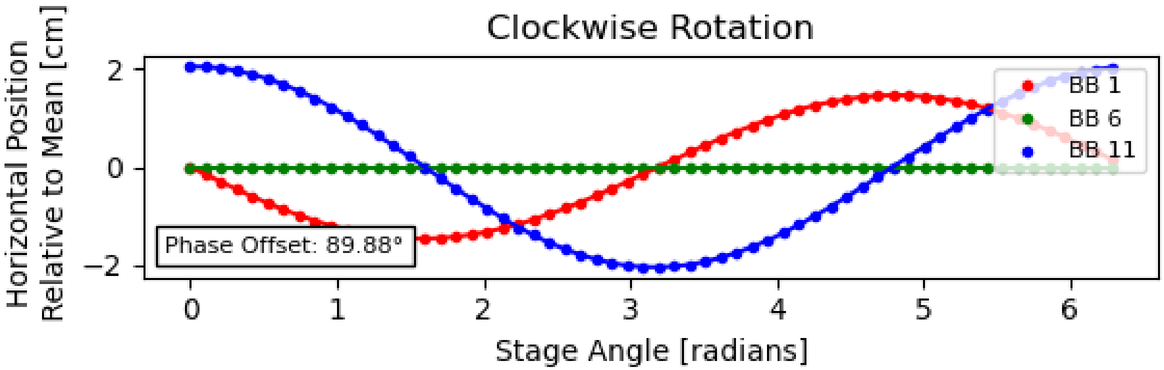
Rotary Stage Calibration

- Fit a sine curve to recover phase offset and rotation direction
  - Works best with no detector tilt, no beam offsets
  - Fit more complex curves for more complex geometry





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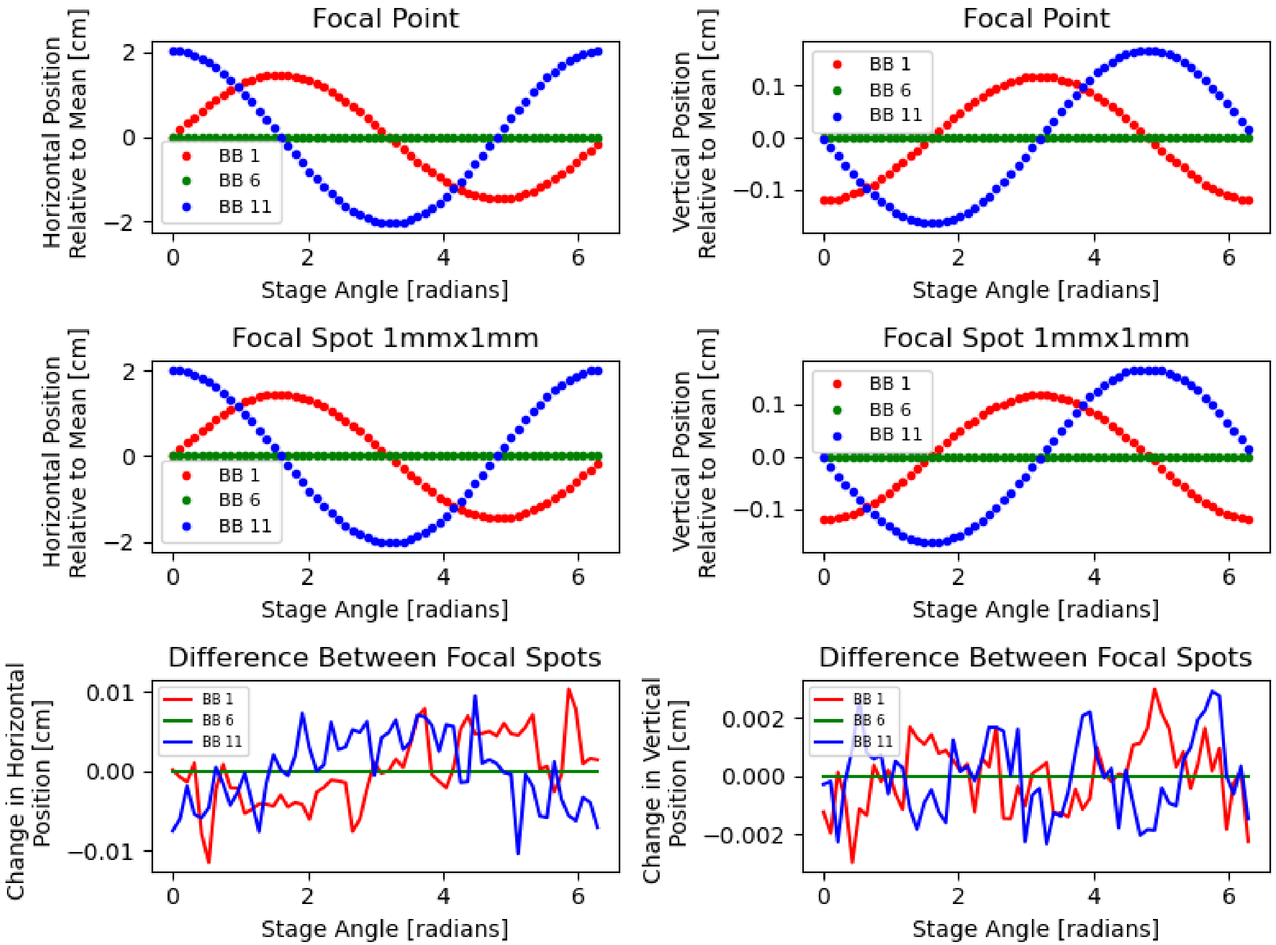
Rotary Stage Calibration

- Focal spot blur causes error in position
  - Higher for horizontal position than vertical position



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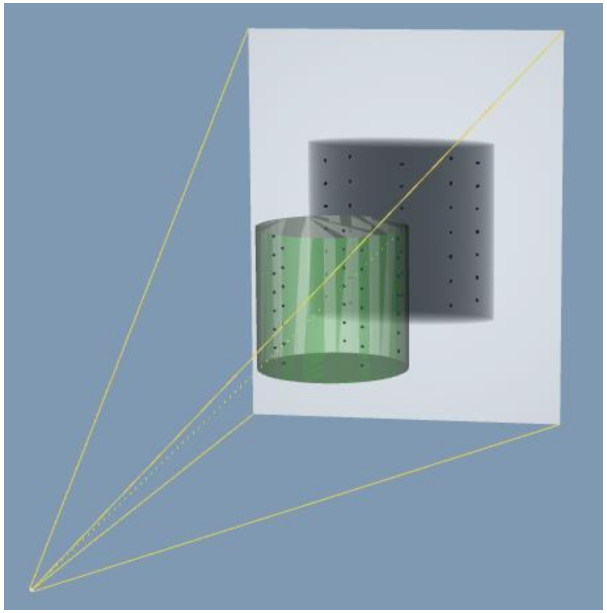
Conclusions

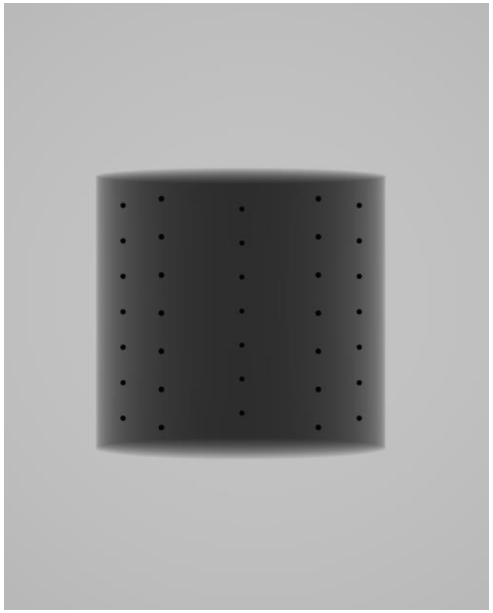
- Summary
  - Performed 110 aRTist simulations for testing current code
  - Wrote a macro for image processing and BB tracking
  - Developed 3 methods to address out-of-plane detector tilt
  - Optimization method showed higher accuracy and resistance to noise than Noo et al., 2000 method
  - Designed a new phantom for rotary stage calibration

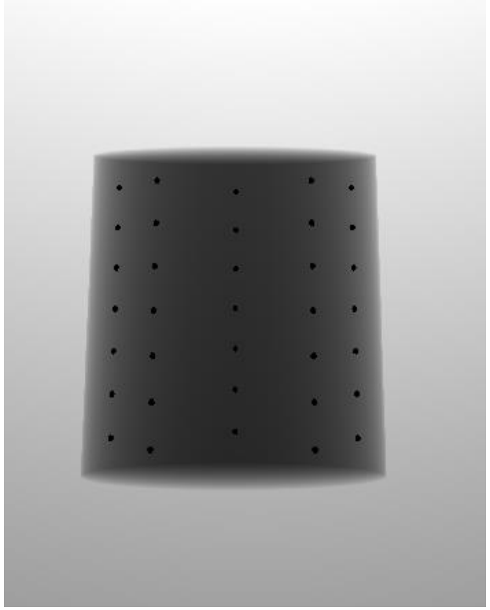
- Implications
  - More accurate, consistent geometry estimation going forward
  - More accurate reconstructions
  - Retroactive diagnoses and/or corrections of detector tilt
  - Exploring the consequences of other broken assumptions

Future Work: Radiography Geometry Phantom

- Adapting code to work with 1 scan as opposed to 60
  - Must avoid overlapping BBs in scan
  - Maybe use some other known geometry pattern

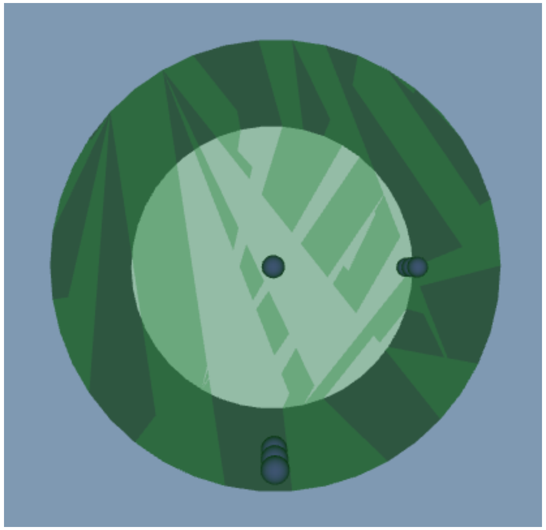


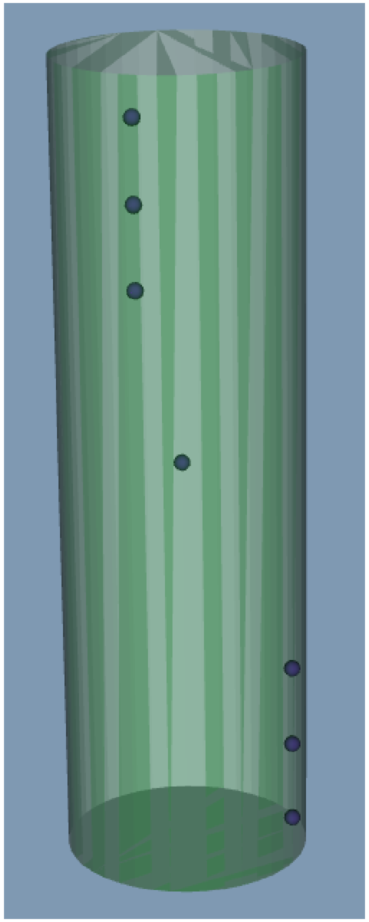




Future Work: Combined Geometry & Metrology Phantom

- Machined from Aluminum with Tungsten BBs adhered in place
- Use coordinate measuring machine to validate
- Space for ASTM E1695 cross sectional reconstruction
  - MTF, PSF, CDF
- Adapt optimization code to work with new BB geometry
- More BBs could be added to top and bottom
  - Maintain space around center BB to prevent index flipping





Resources

- Git repo
  - E-6>Members>Students>SarahGlomski
  - https://git.lanl.gov/e-6/members/students/sarahglomski
- Image processing macro
  - \\e6vault\Students\2023\aRTistTraining\aRTistCTScansSarah
- aRTist scans and test parameter Excel sheet
  - \\e6vault\Students\2023\aRTistTraining\aRTistCTScansSarah
- Desmos graphs
  - https://www.desmos.com/calculator/73cyfcsdtw
  - https://www.desmos.com/calculator/fmvqlhyhgw
- Various math (magnification equations, BB centroid error)
  - Notebooks (good luck)
  - Email: sarah.glomski@duke.edu

References & Acknowledgements

- Noo, Frédéric, et al. “Analytic Method Based on Identification of Ellipse Parameters for Scanner Calibration in Cone-Beam Tomography.” Physics in Medicine and Biology, vol. 45, no. 11, 2000, pp. 3489–3508, https://doi.org/10.1088/0031-9155/45/11/327.

- Mentors
  - Michelle Espy
  - Cort Gautier
  - Shannon Scott
  - Matt Sheats
  - Andre Spears

- Collaborators
  - Galen Brown

aRTist Quirks & Random math

- aRTist uses Euler angles for detector tilt
- aRTist defines the axis of rotation as the vertical axis that intersects the line between detector center and the source at the point closest to the initial position of the part
- Typed up math for BB centroid error:

$$v' = v\left(1 - \left(\frac{a}{c+R\frac{|v|}{v}}\right)^2\right) + \frac{ab\sqrt{\tan^2(\gamma)+1}}{(c+R\frac{|v|}{v})^2},$$

where $a = \frac{\Delta h}{2}\tan(\gamma)$, $b = D\frac{\Delta h}{2}$, $c = \left(h + \frac{\Delta h}{2}\right)\tan(\gamma)$

- Log plot of recovered tilts