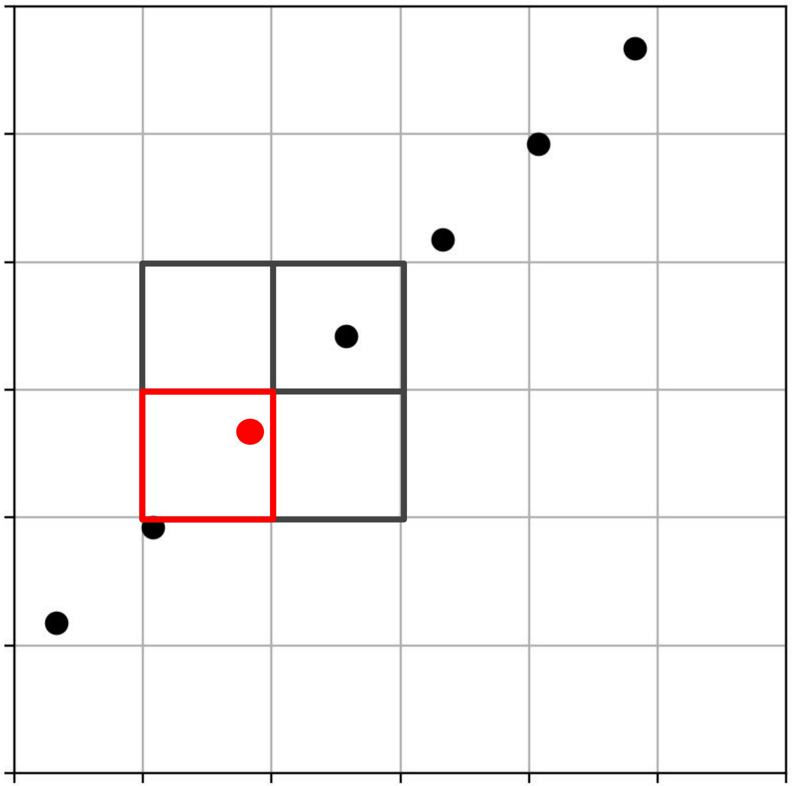
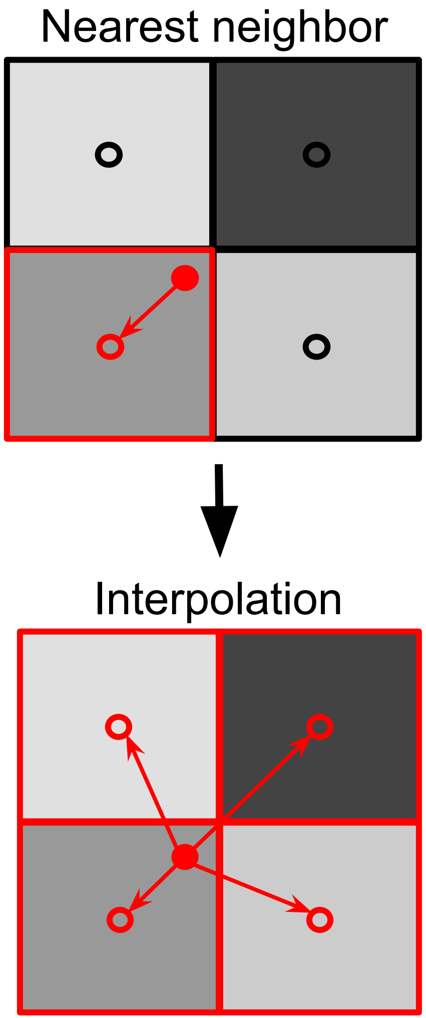
Improving ART Resolution





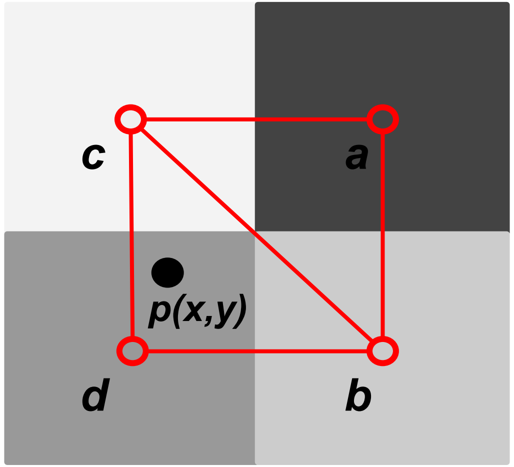
Interpolation

```
# Square interpolation function

# param
#    f00,f10,f01,f11: four corner values
# return: numpy vector (a,b,c,d)

def squareInterp(f00,f10,f01,f11):
  # write system of eq as matrices
  A = np.array([[0,0,0,1],[0,1,0,1],[0,0,1,1],[1,1,1,1]])
  b = np.array([f00,f10,f01,f11])

  # solve Ax = b manually
  d = f00
  b = f10-d
  c = f01-d
  a = f11-b-c-d
  return [a,b,c,d]
```



$$p(x, y) = axy + bx + cy + d$$

Interpolation

```python
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```
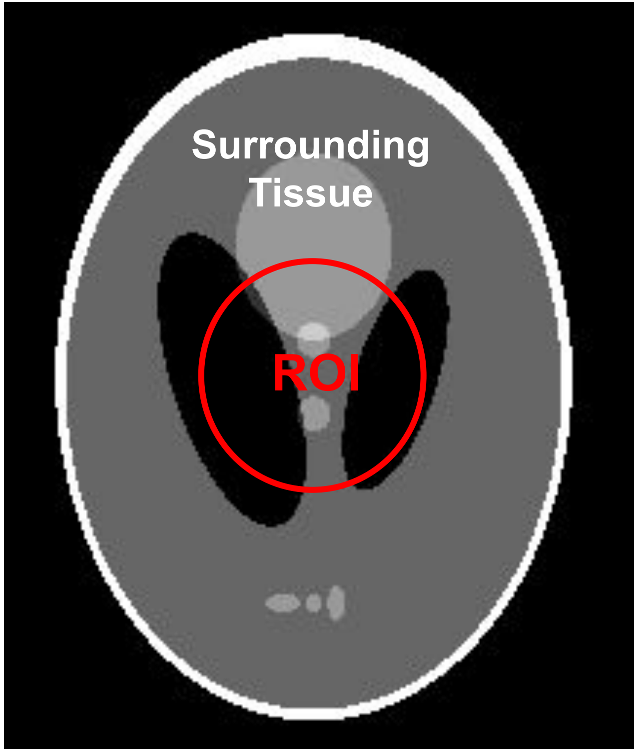

$$p(x, y) = axy + bx + cy + d$$

$$\begin{cases} d = f(0,0) \\ b + d = f(1,0) \\ c + d = f(0,1) \\ a + b + c + d = f(1,1) \end{cases}$$

Region of Interest Isolation



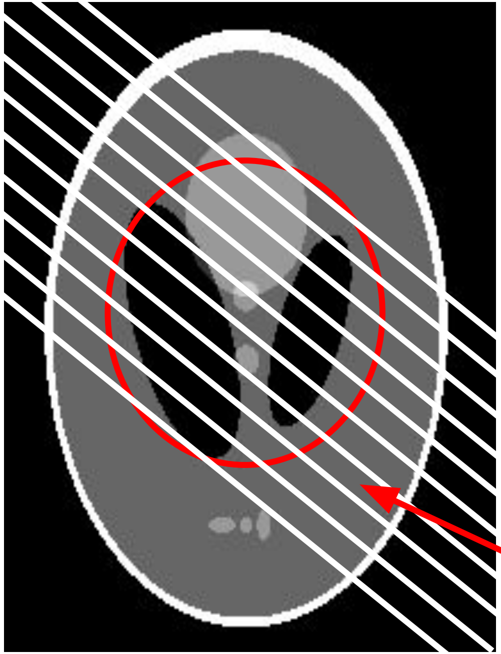
Objective 1: Increase resolution within the ROI

Objective 2: Minimize surrounding tissue damage

This is very challenging

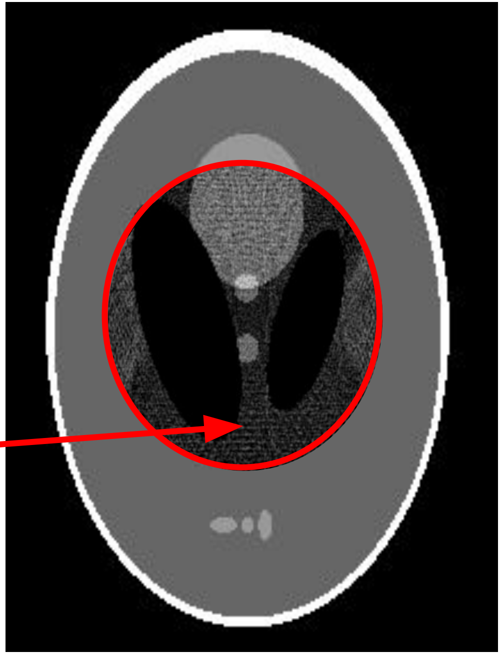
Naive Approach

Restrict beams to only ROI



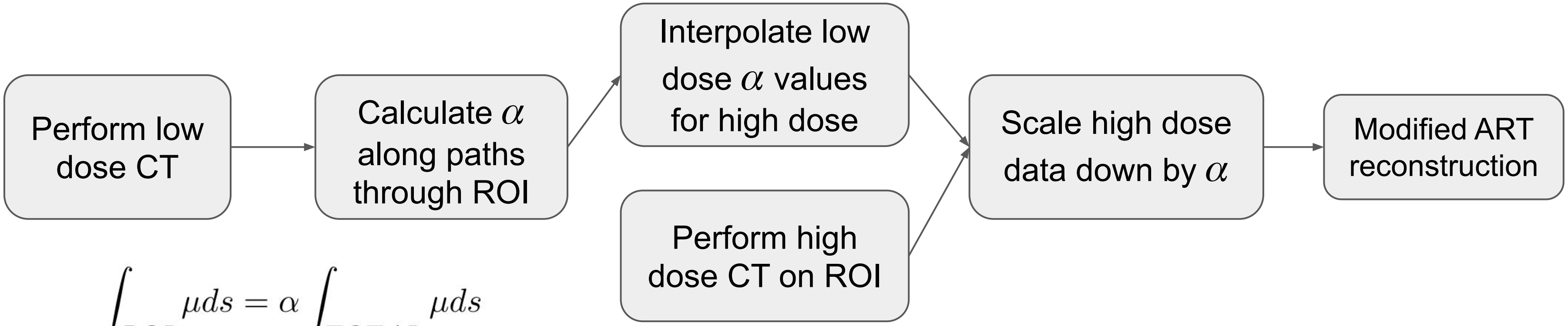
Regular ART

ROI reconstruction (inaccurate)

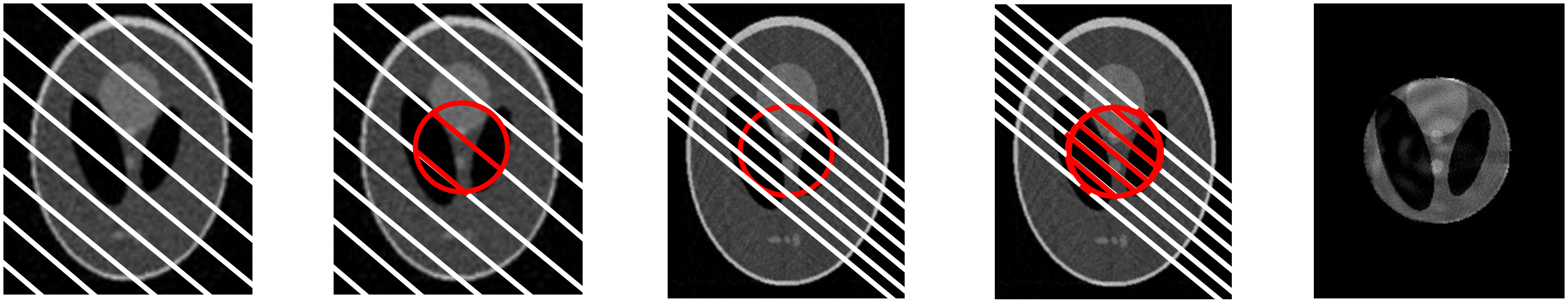


Poor resolution
due to attenuation
interference from
surrounding tissue

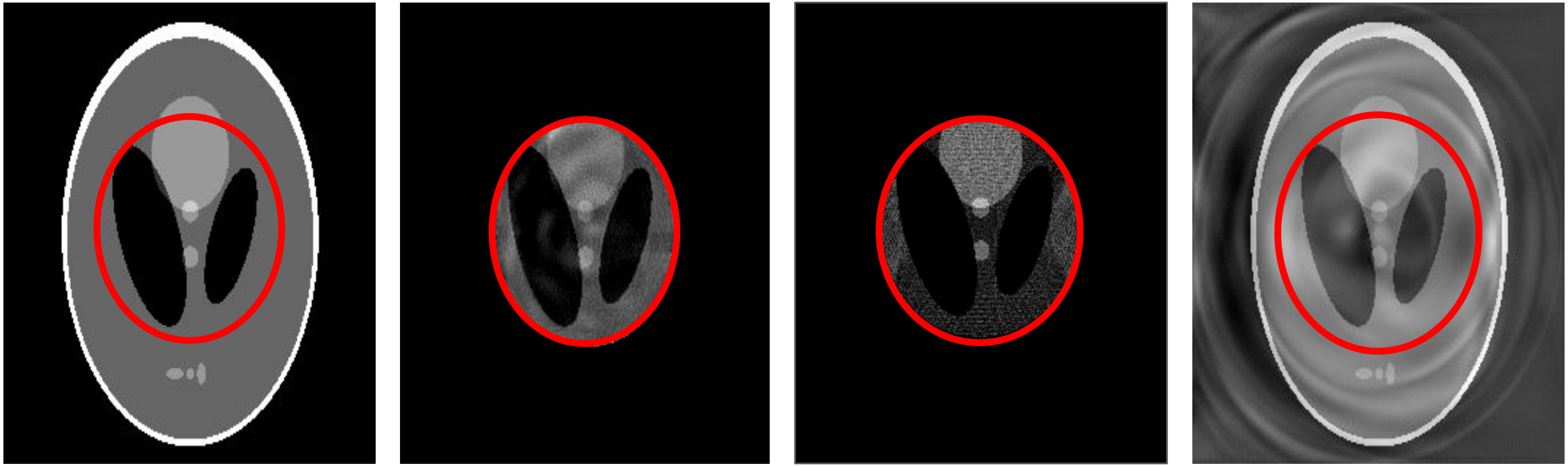
Dual Scan Method



$$\int_{ROI} \mu ds = \alpha \int_{TOTAL} \mu ds$$



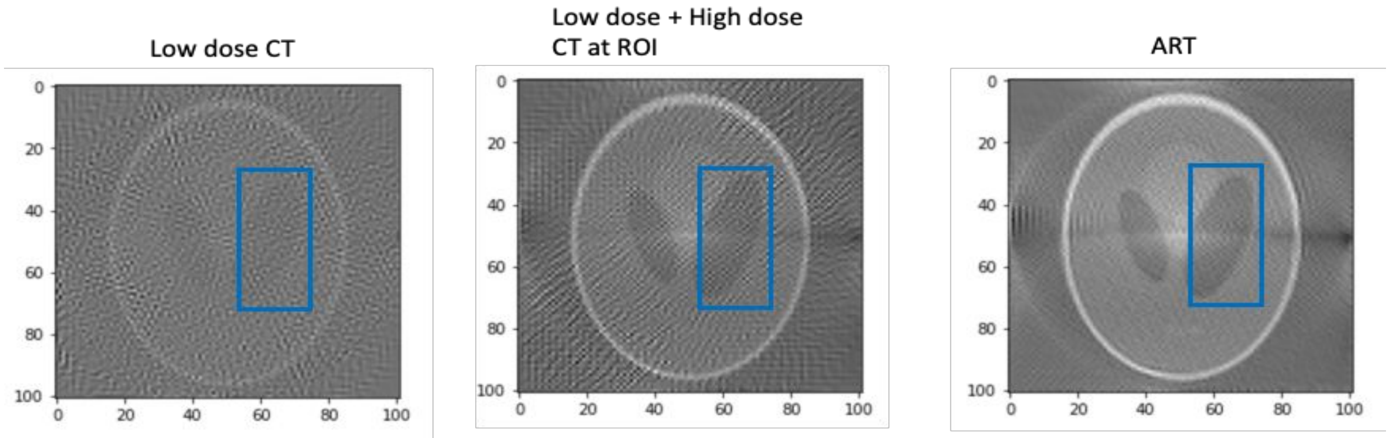
ROI Reconstruction Results



Original | Dual Scan Method | Naive Approach | Full ART Reconstruction

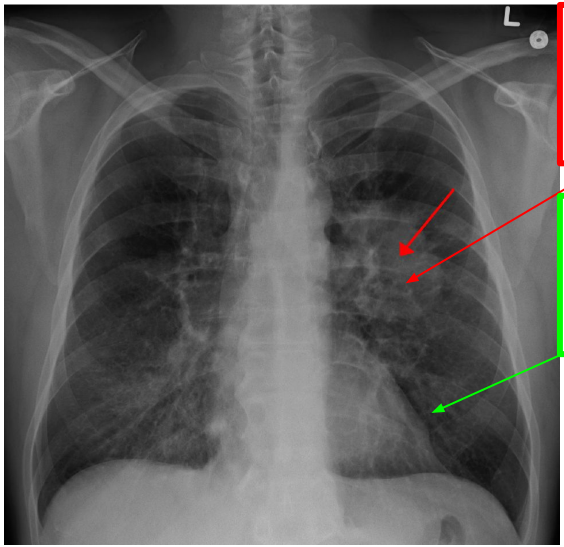
Low Dose Interpolation Method

- Use low dose CT (reduced resolution by ¼, ½ in each direction)
- Isolate region of interest
- Shoot high resolution CT on ROI, creating a smaller system matrix
- However, since X-rays goes through whole body, cannot easily isolate X-rays that pass through just the ROI
- Interpolate to get values at other regions



Inverse Planning

- An optimization algorithm to “optimize” the dose of radiation given to the patient
- Aim is to keep radiation within certain limits in the tumor region and keep radiation below a certain dose outside the non-tumor region



$$R_T^L \leq \sum_{i \in S} \lambda_i B_i x_T \leq R_T^U$$

$$\sum_{i \in S} \lambda_i B_i x_N \leq R_N$$

λ_i: radiation dose, $\lambda_i \geq 0$

B_i: ith row of system matrix B

x_N, x_T: indicator functions in the form of vectors, 1 if in the region of tumor, 0 otherwise and vice versa

R_T^L, R_T^U: lower and upper limits of radiation dose in tumor region

R_N: upper limit of radiation dose in non − tumor region

Inverse Planning

- Goal: Minimize the radiation dose subjected to the 3 constraints using linear programming methods (the simplex method is the most basic and most commonly used)
- However, it is often the case that the equations are not feasible and we have to add penalty terms to make them feasible:

$$R_T^L - \propto \leq \sum_{i \in S} \lambda_i B_i x_T \leq R_T^U + \beta$$

$$\sum_{i \in S} \lambda_i B_i x_N \leq R_N + \gamma$$

$$\lambda_i \geq 0$$