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Optimizing formation flight via the Constructal Law

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Abstract

Flyers do not often fly alone. Fighter jets have a "wingman" to assist in their mission and provide aerodynamic benefits. Birds have a natural advantage: they can "feel" what is easiest as they fly. They thus will naturally tend to fly in the optimal configuration (that is, the configuration that uses the least energy). Birds naturally fly in a V-formation (figure 1), and fighter jets use a nearly identical formation in practice called "fingertip" [1, 2].

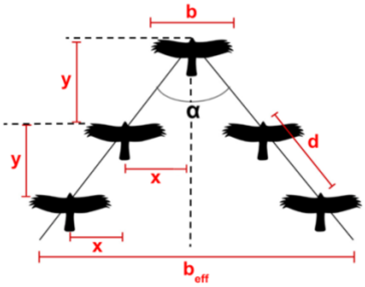
Figure 1: Flock of geese flying in a V-formation, credit: Birding World [3]

The V-formation employs a concept known as "vortex surfing" in which trailing flyers benefit from the updraft of wingtip vortices trailing from the preceding flyers – boosting lift, reducing drag, and making flight easier [4, 5]. Wieselsberger first suggested that the ideal V-formation among birds would provide an equal distribution of drag to maximize collective energy savings of the flock [6]. For a fixed-wing flyer to maximize these benefits, it must be positioned at the ideal angle behind the leader. This paper models the V-formation design, defined by the V-angle α as shown in figure 2, to maximize energy savings and provide insight into predicting the natural optimal design for formation flyers.

This paper applies fundamental principles of physics to this question, specifically the Constructal Law which states that a flow system persists when it evolves to provide easier flow access to its currents [7]. The Constructal Law suits the formation flight problem well as its application can lead to the prediction of nature and can be employed to predict the V-formation design at which birds fly. In this case, the formation flight configuration is a flow system of aerodynamic lift in which the induced drag is imperfection. The Constructal Law, then, predicts that the optimal design is the configuration for which the imperfection is most evenly distributed across the system, or the induced drag is equally partitioned among flyers in formation.

The "optimal configuration" or "design" for formation flight maximizes energy savings by maximizing the boost in lift and corresponding reduction of induced drag. To determine the relationship between the formation flight configuration and the resulting energy savings, a theoretical model is developed for a flat-plane system of N fixed wing flyers (in straight and level flight) with individual wingspan b, individual weight W, air density ρ, and formation velocity V_∞ in a V-formation configuration. The V-formation is parameterized with fixed lateral and longitudinal spacing coordinates x and y, respectively, as shown in figure 2 below.

Figure 2: Design schematic parameterized of V-formation



Optimizing this design for energy savings yields the following model:

$$y_{opt} = \frac{2\pi\rho V_\infty^2}{NW}\left[(N-1)x_{opt}^3 + bx_{opt}^2\right] \tag{1}$$

Since the relationship is nonlinear, there is no constant configuration that optimizes the V-formation. Rather, the optimal configuration adapts to the variation of the formation system parameters, and the optimal flyers will evolve their formation to reflect this. To employ the model in equation 1 to find the optimal V-formation angle αopt, each (x_{opt}, y_{opt}) coordinate corresponds to a scenario-dependent nose-to-nose distance d, held constant in the V-formation. Given d, the corresponding (x_{opt}, y_{opt}) is determined from the model in equation 1, and then α_{opt} can be calculated geometrically.

Analyzing the model in terms of the fundamental formation parameters provides greater insight into the model's accuracy and predictive applications to the real world. To determine trends between each parameter and the resulting optimal formation flight design, parameters d, V_∞, and N are computationally varied for an arbitrary aircraft and bird formation flight system. Beginning with nose-to-nose separation distance, as the flyers fly further apart, the optimal V-formation becomes narrower. This trend aligns with experimental bird data from Williams, observing that Canada Geese tend to fly narrower V-formations as the overall length of the V-formation increases [8]. This is likely due to an effort of the birds to distribute the induced drag as evenly as possible along the extended formation by remaining closer laterally, in line with the predictions of the Constructal Law.

Next, as V_∞ increases, the optimal configuration gets narrower. Therefore, the model suggests faster flyers fly narrower formations to maximize energy savings. Birds have also been observed to follow this trend, as bigger, faster birds tend to fly in narrower V-formations [8]. Boats display a similar phenomenon as their V-formation wakes become narrower with increasing speed.

Lastly, the optimal configuration model is evaluated for varying N flyers. This trend reveals an asymptotic relationship between optimal configuration and flyers. When there are few flyers ($N < 10$), the flyers should fly in a slightly wider formation as N increases to account for the wider divergence of the trailing wakes. However, for many flyers ($N > 10$), adding additional flyers has negligible impact on the wake geometry leading to a constant optimal configuration angle.

To test the prediction of the Constructal Law of uniform distribution of drag (imperfection) in the optimal configuration, induced drag for each flyer in the formation is calculated. Results show that the induced drag force felt by the individual flyers varies very little (<6% body weight). Thus, in both cases, the flyers are flying in the configuration that distributes the induced drag as evenly as possible among flyers – supporting Constructal Law predictions. The effectiveness of this distribution of drag is also largely determined by the longitudinal spacing of flyers. However, in practice, the formation flight system can never achieve this perfect drag distribution because the induced drag consistently decreases along the V-formation, meaning the leading flyers will always be working harder than the trailing flyers to maintain the V-formation. The Constructal Law also describes this inherent diversity in nature: No optimal system can eliminate diversity and imperfection. Rather, it will constantly evolve towards a state that provides better access for its currents, as does the formation flight model.

Keywords: formation flight, birds, aircraft, aerodynamic optimization, Constructal Law

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