Theorem 5. Let $L$ and $L'$ be any two Lagrangian submanifolds of $P$ intersecting transversely. Then there exists a dense set $J_{\text{reg}}(L, L') \subset J$ such that if $\xi \in J_{\text{reg}}(L, L')$, the zero set of the section $\tilde{\tau}_x$ is regular on all Banach manifolds $\mathcal{B}_{p, \xi}(x, y)$ with $\xi = (\xi^-, \xi^+)$ and $\xi^+ \in [0, \infty) - \delta(A_x)$ and $\xi^- \in (-\infty, 0] - \delta(A_x)$, and such that the zero sets of all maps given in Theorem 4a are regular. In fact, we find $J \in J_{\text{reg}}$ for any prescribed value of $J$ on any neighborhood $U$ of $L \cup L'$ as long as no two different intersections are connected in $U$.

This theorem is from Flor's paper "The Unregularized Gradient Flow of the Symplectic Action.

Here $J$ is the set of time-dependent a.c. structures, i.e.,

For technical reasons, it is sometimes convenient to define the Cauchy-Riemann equation (1.6) with respect to a "time dependent" almost complex structure, i.e., an element of the space

\[(2.1) \quad J = \mathcal{C}^\infty([0, 1] \times \mathcal{S}),\]

In an attempt to unpack thm. 5, here's what I think is going on:

What's going on:

$\mathbb{R} \times [0, 1] = (\xi^-, \xi^+)$

Already transversely intersecting!
We have a bundle

\[ S \xrightarrow{\pi} M \]

\[ S_x := \{ J \in \text{End}(T_x M) : J^2 = -\text{id} \land \omega_x(\cdot, J \cdot) \text{ is a metric on } T_x M \} \]

A smooth section of this is an a.c structure on M.

Transversality (i.e. surjective \& \( D(\overline{\Theta_J}) \))

Problem: We want \( D(\overline{\Theta_J})_u \) to be surjective

for all \( u \in J \)-hol., i.e. we want the \( \overline{\Theta_J} \) to intersect transversely

\( E \)

\( \overline{\Theta_J} \cap E \)

Why? This would ensure

\[ \overline{\Theta_J}^{-1}(0) = M \]

is a manifold by Sard-Smale.

But it's not! In general, \( \rho(\overline{\Theta_J} \) is not surjective. In fact, we suspect the following (may not be true, can't find anywhere)

Proposed claim: (once again, may not be true) the set \( \{ J : D(\overline{\Theta_J})_u \text{ is surjective} \} \) (call these "Good" J's)
is not dense

This is an issue because we want to be able to perturb $J$ to a good $J$, but if the above set is not dense, we have no chance of making the process independent of our choice of perturbation.

So, we really want some sort of dense condition. How?

Extend our bundle trivially to $M \times [0, 1]$.

\[
S \quad \quad S_{(x, t)} := \{ J \in \text{End}(T_x M) : J^2 = \text{id} \text{ and } w(\cdot, J) \text{ is smooth} \}
\]

Now, for each $x$, we have an interval's worth of G.C. structures to choose from. A section of this bundle is precisely a time-dependent G.C. structure.

Let $J_\epsilon$ be a smooth section of this new bundle. Now, our equation becomes

\[
\overline{\partial} J_\epsilon(u)(x, t) = \frac{\partial u}{\partial s} (x, s) + J_\epsilon \frac{\partial u}{\partial \epsilon} (x, s)
\]
Notice just how much more flexibility we've gained — rather than needing just one $J$ to be good at all times, now we just need $J_t$ to be good at the past $(t,s)$.

Because of this flexibility, it's now believable that the set of time-dependent good $a.c.$ schemes, i.e.

$$\{ J_t : D(\overline{J_t}) \text{ is surjective} \}$$

is dense. And indeed, by the thm. from Floer's paper, this is true.