Cover-ups

Francis Bloch and Rachel Kranton*

November 2019

On-line Appendix

This Appendix considers traditional comparative statics, which correspond to the effect of *anticipated* changes in the parameters of the model in the paper. These effects are not studied in the paper.

We focus on the effect of a change in the fee structure f_t.

For any period $s < t$, there is a direct negative effect on the continuation values so that the executive has less incentive to cover up. However, there is also an indirect effect, since, as a consequence, the prosecutor is less likely to investigate. We show that for any period $s < t$, the direct effect dominates the indirect effect so that an increase in the penalty f_t reduces V_s in all periods $s < t$. Hence, k_s^* is always decreasing in the fee f_t. The effect of a change in the penalty on w_s^* is ambiguous, however. The threshold of the executive is subject to two opposite effects: on the one hand, as the continuation value goes down, the executive has a higher incentive to report ; on the other hand, as the belief of the prosecutor goes down, the executive has a smaller incentive to report.

In period t, these two forces are also at play. On the one hand, as the executive is more likely to report before t, the prosecutor holds lower beliefs that the employee is a perpetrator and is less likely to investigate. On the other hand, an increase in f_t^a directly results in an increase in k_t^* at period t whereas an increase in f_t directly results in a decrease in k_t^*. On balance, the effect of a change in f_t^a on the equilibrium threshold k_t^* is thus ambiguous. The effect of an increase in f_t however is clear: it reduces the threshold of the prosecutor at period t, k_t^*. However, the effect on w_t^* cannot be ascertained, as it results from two opposite forces: a change in f_t and in k_t^*.

As in the case of unanticipated changes in the environment, the decrease in k_t^* due to the increase in f_t reduces the beliefs of the prosecutor γ_s for all periods $s > t$. This decrease implies a decrease in k_s^* and an in w_s^*.

We summarize in the following Proposition:

Proposition Comparative Statics. *An anticipated increase in the penalty f_t results in a decrease in k_s in all periods. It results in a decrease in w_s for all periods $s > t$, an increase in w_1 and has an ambiguous effect on w_s for $1 < s \leq t$.*

Proof of Proposition Comparative Statics.

We first consider the effect of a change in f_t on the behavior of the executive and the prosecutor before period t. Pick any $s \leq t$. We will show that, for any γ, $V_s(\gamma)$ is lower when f_t increases, or that $\frac{\partial V_s}{\partial f_t} < 0$. The proof is by induction on the periods.

Consider first period t. recall that the continuation value at period t is given by

$$\begin{aligned} V_t(\gamma_t) &= \int_{-\infty}^{w_{t+1}} -f_{t+1}^a dR(\omega) + \int_{w_{t+1}}^{\infty} [\omega + \Omega_{t+1} + H(k_{t+1})\sigma f_{t+1} \\ & \quad +H(k_{t+1})(1-\sigma)\delta V_{t+1}\left(\frac{k_{t+1}(1-\sigma)}{\sigma(1-k_{t+1})}\right) + (1-H(k_{t+1}))\delta V_{t+1}\left(\frac{k_{t+1}}{\sigma}\right)]dR(\omega). \end{aligned}$$

Differentiating V_{t-1} gives

$$\begin{aligned} \frac{\partial V_{t-1}}{\partial f_t} &= -(1-R(w_t))[\sigma H(k_t) + \frac{\partial k_t}{\partial f_t}[-H'(k_t)[f_t\sigma + \delta V_t(\frac{k_t}{\sigma}) - \delta(1-\sigma)V_t(\frac{k_t(1-\sigma)}{\sigma(1-k_t)})] \\ &+ (1-\sigma H(k_t))\delta V_t'(\frac{k_t(1-\sigma)}{\sigma(1-k_t)})\frac{1-\sigma}{\sigma(1-k_t)^2}) + (1-H(k_t))\delta V_t'(k_t)]]. \end{aligned}$$

An increase in f_t has a direct negative effect of $-(1-R(w_t))H(k_t)\sigma$ and an indirect positive effect, due to the fact that an increase in f_t reduces the need to investigate, and thereby lowers the equilibrium value of k_t^*. We show that the indirect effect is always dominated by the direct effect.

To this end, we compute

$$\begin{aligned}
\frac{\partial k_t^*}{\partial f_t} &= -\frac{\frac{\partial \psi_t}{\partial f_t}}{\Psi_t'(k_t)} \\
&= \frac{-R'(w_t)\sigma\gamma_{t-1}(1-\gamma_{t-1})H(k_t)}{R'(w_t)\gamma_{t-1}(1-\gamma_{t-1})B+(1-\gamma_{t-1}R(w_t))^2}
\end{aligned}$$

where $B = -w_t'(k_t)(1-\gamma_{t-1}R(w_t)^2)$ is equal to

$$\begin{aligned}
B &= H'(k_t)[\sigma f_t + \delta V_t(\frac{k_t}{\sigma}) - \delta(1-\sigma)V_t(\frac{k_t(1-\sigma)}{\sigma(1-k_t)})] \\
&+ (1-\sigma)H(k_t)\delta V_t'(\frac{k_t(1-\sigma)}{\sigma(1-k_t)})\frac{1-\sigma}{\sigma(1-k_t)^2} + (1-H(k_t))\delta V_t'(k_t)
\end{aligned}$$

Replacing we obtain

$$\begin{aligned}
\frac{\partial V_{t-1}}{\partial f_t} &= [(1-R(w_t))\sigma H(k_t)](-1+\frac{R'(w_t)\gamma_{t-1}(1-\gamma_{t-1})B}{R'(w_t)\gamma_{t-1}(1-\gamma_{t-1})B+(1-\gamma_{t-1}R(w_t))^2}), \\
&< 0.
\end{aligned}$$

Hence, an increase in the penalty f_t unambiguously decreases the continuation value at the beginning of period t.

Consider now the inductive step. Suppose that $\frac{\partial V_{u-1}}{\partial f_t} < 0$ for all $u > s$ and consider

$\frac{\partial V_{s-1}}{\partial f_t}$:

$$\begin{aligned}
\frac{\partial V_{s-1}}{\partial f_t} &= (1-R(w_s))[\delta(H(k_s)(1-\sigma)\frac{\partial V_s}{\partial f_t}(\frac{k_t(1-\sigma)}{\sigma(1-k_t)})+(1-H(k_s))\frac{\partial V_s}{\partial f_t}(\frac{k_s}{\sigma})) \\
&+ \frac{\partial k_s^*}{\partial f_t}[-H'(k_s)[f_s\sigma+\delta V_s(\frac{k_s}{\sigma})-\delta(1-\sigma)V_s(\frac{k_s(1-\sigma)}{\sigma(1-k_s)}]] \\
&+ (1-\sigma)H(k_s)\delta V_s'(\frac{k_s(1-\sigma)}{\sigma(1-k_s)})\frac{1-\sigma}{\sigma(1-k_s)^2}+(1-H(k_s))\delta V_s'(k_s)].
\end{aligned}$$

An increase in f_t has a direct negative effect on V_{s-1} through the derivative $\frac{\partial V_s}{\partial f_t}$, which is negative by the induction hypothesis. There is also an indirect positive effect through an increase in k_s^*. However, the indirect effect is dominated by the direct effect, as shown by the following argument.

By implicit differentiation, we again compute

$$\frac{\partial k_s^*}{\partial f_t}=\frac{\gamma_{s-1}(1-\gamma_{s-1})R'(w_s)\delta(H(k_s)(1-\sigma)\frac{\partial V_s}{\partial f_t}\left(\frac{k_s(1-\sigma)}{\sigma(1-k_s)}\right)+(1-H(k_s))\frac{\partial V_s}{\partial f_t}\left(\frac{k_s}{\sigma}\right))}{R'(w_s)\gamma_{s-1}(1-\gamma_{s-1})B+(1-\gamma_{s-1}R(w_s))^2},$$

where

$$\begin{aligned}
B &= H'(k_s)[\sigma f_s+\delta V_s(\frac{k_s}{\sigma})-\delta(1-\sigma)V_s(\frac{k_s(1-\sigma)}{\sigma(1-k_s)})] \\
&+ (1-\sigma)H(k_s)\delta V_t'(\frac{k_s(1-\sigma)}{\sigma(1-k_s)})\frac{1-\sigma}{\sigma(1-k_s)^2}+(1-H(k_s))\delta V_s'(k_s)
\end{aligned}$$

Replacing, we have

$$
\begin{aligned}
\frac{\partial V_{s-1}}{\partial f_t} &= (1-R(w_s))\delta(H(k_s)(1-\sigma)\frac{\partial V_s}{\partial f_t}(\frac{k_t(1-\sigma)}{\sigma(1-k_t)}) + (1-H(k_s))\frac{\partial V_s}{\partial f_t}(\frac{k_s}{\sigma})) \\
&\quad (1 - \frac{R'(w_s)\gamma_{s-1}(1-\gamma_{s-1})B}{R'(w_s)\gamma_{s-1}(1-\gamma_{s-1})B + (1-\gamma_{s-1}R(w_s)^2)}), \\
&< 0.
\end{aligned}
$$

This concludes the proof that for any γ, the derivative of V_{s-1} with respect to f_t is negative.

We now show that for any $s < t$, k_s^* is lower when f_t increases. Notice that an increase in f_t affects k_s^* through two channels: it affects the continuation value V_s as well as the initial belief γ_{s-1}. Hence

$$
\frac{\partial k_s^*}{\partial f_t} = \frac{\partial k_s^*}{\partial \gamma_{s-1}}\frac{\partial \gamma_{s-1}}{\partial f_t} + \frac{\partial k_s^*}{\partial V_s}\frac{\partial V_s}{\partial f_t}.
$$

The proof is again by induction on the periods, starting at $s = 1$. At $s = 1$, the initial belief γ_0 is fixed. Hence,

$$
\frac{\partial k_1^*}{\partial f_t} = \frac{\gamma_0(1-\gamma_0)R'(w_1)\delta(H(k_1)(1-\sigma)\frac{\partial V_1}{\partial f_t}\left(\frac{k_1(1-\sigma)}{\sigma(1-k_1)}\right) + (1-H(k_1))\frac{\partial V_1}{\partial f_t}\left(\frac{k_1}{\sigma}\right))}{R'(w_1)\gamma_0(1-\gamma_0)B + (1-\gamma_0 R(w_1))^2}
$$

Now, as $\frac{\partial V_1}{\partial f_t} < 0$ by the previous argument, we have that $\frac{\partial k_1^*}{\partial f_t} < 0$. Hence an increase in f_t reduces the prosecutor's incentive to investigate in period 1. Next suppose by induction that k_u^* is decreasing in f_t for any $u < s \leq t$, and consider k_s^*. A change in f_t affects k_s^* through two channels: it reduces k_{s-1}^* and hence reduces γ_{s-1} and also reduces V_s.

Recall that $\frac{\partial k_s^*}{\partial \gamma_{s-1}} > 0$, and that $\frac{\partial \gamma_{s-1}}{\partial k_{s-1}^*} > 0$. Using the induction hypothesis,

$$\frac{\partial k_s^*}{\partial \gamma_{s-1}} \frac{\partial \gamma_{s-1}}{\partial f_t} = \frac{\partial k_s^*}{\partial \gamma_{s-1}} \frac{\partial \gamma_{s-1}}{\partial k_{s-1}^*} \frac{\partial k_{s-1}^*}{\partial f_t} < 0.$$

Furthermore, as $\frac{\partial V_s}{\partial f_t} < 0$,

$$\begin{aligned} \frac{\partial k_s^*}{\partial V_s} \frac{\partial V_s}{\partial f_t} &= \frac{\gamma_{s-1}(1-\gamma_{s-1})R'(w_s)\delta(H(k_s)(1-\sigma)\frac{\partial V_s}{\partial f_t}\left(\frac{k_s(1-\sigma)}{\sigma(1-k_s)}\right) + (1-H(k_s))\frac{\partial V_s}{\partial f_t}\left(\frac{k_s}{\sigma}\right))}{R'(w_s)\gamma_{s-1}(1-\gamma_{s-1})B + (1-\gamma_{s-1}R(w_s))^2} \\ &< 0, \end{aligned}$$

showing that $\frac{\partial k_s^*}{\partial f_t} < 0$ for all $s \leq t$.

We now consider the effect of a change in f_t on w_s^* for $s \leq t$. Notice that

$$\frac{\partial w_s^*}{\partial f_t} = \frac{\partial w_s^*}{\partial \gamma_{s-1}} \frac{\partial \gamma_{s-1}}{\partial f_t} + \frac{\partial w_s^*}{\partial V_s} \frac{\partial V_s}{\partial f_t}.$$

As a change in V_s does not affect the best response of the prosecutor, it only affects w_s through a change in the prosecutor's investigation decision. Inspecting the threshold values of the prosecutor and the executive, an increase in k_s^* results in a decrease in w_s^*. Hence

$$\frac{\partial w_s^*}{\partial V_s} = \frac{\partial w_s^*}{\partial k_s^*} \frac{\partial k_s^*}{\partial V_s} < 0.$$

In period $t = 1$, since γ_0 is fixed,

$$\frac{\partial w_1^*}{\partial f_t} = \frac{\partial w_1^*}{\partial k_1^*}\frac{\partial k_1^*}{\partial V_1}\frac{\partial V_1}{\partial f_t} > 0.$$

Hence an increase in f_t increases the executive's incentive to report in period 1. For periods $1 < s \leq t$, an increase in f_t has two effects on w_s^*. First, as k_{s-1}^* goes down, the prosecutor's belief γ_{s-1} goes down. Second, as the continuation value V_s^* goes down, the executive has a greater incentive to report. The two effects work in opposite directions, so the total effect is ambiguous.

Next consider periods $s > t$. In these periods, V_{s-1} is not affected by the change in f_t and the only effect goes through the beliefs. As all values of k_s^* for $s \leq t$ are decreasing in f_t, for any $s > t$

$$\frac{\partial k_s^*}{\partial f_t} = \sum_{u=1}^{t} \frac{\partial k_u^*}{\partial f_t}\frac{\partial k_s^*}{\partial \gamma_s - 1}\frac{\partial k_{s-1}^*}{\partial \gamma_{s-2}} .. \frac{\partial k_{u+1}^*}{\partial \gamma_u} < 0,$$

and

$$\frac{\partial w_s^*}{\partial f_t} = \sum_{u=1}^{t} \frac{\partial k_u^*}{\partial f_t}\frac{\partial w_s^*}{\partial \gamma_{s-1}}\frac{\partial k_{s-1}^*}{\partial \gamma_{s-2}} .. \frac{\partial k_{u+1}^*}{\partial \gamma_u} < 0,$$

concluding the proof.■