

# Risk-sharing networks

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## Abstract

This paper considers the formation of risk-sharing networks. Following empirical findings, we build a model where pairs form links, but a population cannot coordinate links. As a benchmark, individuals commit to share monetary holdings equally with linked partners. We find efficient networks can (indirectly) connect all individuals and involve full insurance. But equilibrium networks connect fewer individuals. When breaking links, individuals do not consider negative externalities on others in the network. Thus identical individuals can end up in different positions in a network and have different outcomes. These results may help to explain empirical findings that risk-sharing is often asymmetric.

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## 1. Introduction

In this paper, we study the formation of risk-sharing networks. In many settings, formal insurance mechanisms are not available. People often mitigate risk by making insurance arrangements among themselves. We see people sharing income and helping each other in many different countries and settings. There is now a large body of empirical work on risk-sharing arrangements, and one major finding of this research is that informal risk-sharing is often not complete within the observed set of individuals.<sup>2</sup> That is, within a village, people do not enjoy the benefits of

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<sup>2</sup> For seminal contributions see Townsend (1994) and Udry (1995).

complete risk-sharing across individuals in the village. One reason, researchers suspect, is that risk-sharing does not take place at the village level, but within families and between individuals. In their study of the rural Philippines, [Fafchamps and Lund \(2003, p. 216\)](#) find, for example, that “mutual insurance does not appear to take place at the village level; rather households receive help primarily through networks of friends and relatives.”<sup>3</sup>

In line with these empirical findings, we build a model where risk-sharing takes place between pairs of individuals. Most theoretical work on informal insurance has assumed that risk-sharing takes place within groups, where the group can be as large as the village or as small as two people.<sup>4</sup> A notable recent exception is [Bloch et al. \(2005\)](#), which we discuss below. We follow the empirical research and suppose that within a given population, there is no cohesive risk-sharing group, per se. Rather, individuals can form bilateral risk-sharing relations, and while individuals can have many such bilateral relationships, there is no requirement that if one person shares income with a second and the second shares with a third that the first and the third automatically have a relationship with each other as well. That is, there is a network of sharing relationships. In our model, we assume that people make bilateral transfers and can make transfers to each other *only if* they have previously established a relationship that allows them to observe income levels and commit to a sharing agreement. It is costly to form such a relationship, as there may be investments that allow monitoring or the ability to enforce a risk-sharing agreement.<sup>5</sup> Our key assumptions, then, are that relationships are bilateral and establishing such a relation is costly.

We study and contrast the equilibrium and the efficient patterns of risk-sharing relations when individuals form bilateral agreements. We consider the formation of risk-sharing links. We ask what structures will emerge when pairs can agree to form links but agents cannot coordinate link formation across the whole population. Agents form links, then agents earn utility from sharing with their bilateral relations.

We consider a benchmark model where identical individuals commit to share their monetary holdings equally with their linked partners. We show that if individuals are committed to share income equally within pairs and interact repeatedly with their neighbors, they can end up sharing income equally within components of the network. The process is useful for our analysis for two reasons. This outcome corresponds to the highest level of insurance that can be secured in a network. Second, it matches the efficient level of insurance. Thus any divergence between efficient networks and equilibrium networks would come from the network formation process. That is, we hold the risk-sharing level as constant and ask how the necessity of forming relations bilaterally affects the configuration of the risk-sharing arrangements.

In this setting, we have several findings.

First, efficient risk-sharing networks can (indirectly) connect all individuals within a society and involve full insurance. That is, efficient networks can result in the equivalent of full-income pooling with a population, despite bilateral relations and commitment costs.

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<sup>3</sup> See also [Murgai et al. \(2002\)](#). These findings support suspicions of earlier work that finds incomplete insurance at the village level. For example, [Townsend \(p. 541\)](#) writes that “kinship groups or networks among family and friends might provide a good, if not better, basis for testing the risk-sharing theory.” In recent work, [De Weerdt \(2004\)](#) and [Dercon and De Weerdt \(2006\)](#) analyze a survey of villagers in Tanzania asking, “Can you give a list of people from inside or outside of Nyakatoke, who you can personally rely on for help and/or that can rely on you for help in cash, kind, or labour?” They notably find that, on average, households are linked to relatively few other households.

<sup>4</sup> See, for example, [Kimball \(1988\)](#), [Coate and Ravallion \(1993\)](#), and [Ligon et al. \(2002\)](#).

<sup>5</sup> For example, an investment could be arranging a marriage with a member of another household. This marriage would allow both greater observability and greater ability to punish (say, socially) the partner for not sharing income. We discuss this further below.

Second, equilibrium risk-sharing networks, in general, connect fewer individuals than efficient risk-sharing networks. This inefficiency arises because of a basic externality. While income transfers are bilateral, individuals can benefit from their partners' risk-sharing relations with others, and those parties relations with yet others, and so on. When forming a single relation (or, critically, breaking a relation) individuals do not take into account the effect on others distant in the network. Hence, models of risk-sharing with commitment in groups would overstate the benefits that arise from informal risk-sharing.

Third, we find that equilibrium networks divide the population into sets of different sizes, and hence, individuals are in asymmetric positions. That is, individuals have different risk-sharing outcomes, despite that individuals have identical preferences and their incomes are identically distributed. In general, we show that equilibrium risk-sharing networks involve separate components, with one smaller than the others. A single component cannot be too large because the benefits of linking to an individual in a larger and larger component eventually will not exceed the cost. Hence, the largest component is of bounded size. A second component is smaller. If the second component is too large, then there is an incentive for a pair to make a link between components, but such a super-sized component is itself not stable. Agents on the edges would be cut out. This finding of asymmetric outcomes suggests a more precise reason for the empirical finding of incomplete risk-sharing. The process of network formation can lead similar individuals to be in different positions, and thus have different outcomes.

Fourth, as hinted at above, there is an inherent instability in risk-sharing networks. Components must be neither too large nor too small to prevent incentives to cut links or form links between components. Often, the population cannot be divided into such components, and there is no pairwise stable network.

Finally, in an environment where agents continually break and make links, agents who find themselves on the edge of a network are the most vulnerable to being cut out completely. They bring little benefit because they have connections to fewer other agents. In a dynamic model of network formation we show that if pairwise stable networks exists, there is eventual convergence to a pairwise stable network. When there is no pairwise stable network, we see cycles, where the size of connected components grow and shrink over time, and agents at the edge of networks are cut off. This finding leads us to question the idea of stable risk-sharing networks. In a snapshot, the risk-sharing relations we observe may just be part of a long cycle that changes over time.

Our paper focuses on the formation of risk-sharing relationships. Previous theoretical literature on informal insurance focuses, for the most part, on the enforcement of risk-sharing agreements. Typically, there is a repeated game where a given set of agents receives income shocks in each period. They are then supposed to share income with another given set of agents. If an agent does not share income, he is punished in the future by exclusion from the income-sharing group.<sup>6</sup> The question then becomes how the severity of the punishment determines the level of risk-sharing that can be sustained in equilibrium. Bloch et al.'s recent paper considers this enforcement problem when income is shared in pairs. Income is transferred only between pairs of agents, where many pairs can transfer income to one another. The question then becomes what pattern of transfers is sustainable in an equilibrium of a repeated game. That is, they define a risk-sharing network as the pattern of equilibrium transfers. We define a risk-sharing network differently in our present

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<sup>6</sup> In a bilateral situation, this "group" would consist of the one other person in the pair. In a larger population, the deviating agent could be punished by all other agents or some subset of agents (see [Genicot and Ray, 2003](#)).

effort. A risk-sharing network is a pattern of existing relations where agents can commit to sharing income. There is no enforcement problem per se.

An illustration might clarify the distinction and our contribution. Consider a population and suppose that risk-sharing takes place within extended families. Our paper asks how a family might form a relation with another family (by marriage say) in order to establish a risk-sharing relation. There is evidence in India, for example, that marriages of daughters are arranged to maximize gains from risk-sharing (see Rosenzweig and Stark, 1989). Establishing this relation is costly, involving a dowry and marriage ceremony and so on, and the relation commits the parties to future income sharing, say, due to a social norm or social punishment in case of non-sharing. The marriage pattern would then be our network. In contrast, for Bloch et al. (2005), there is no ex ante formation of relations and transfers are not restricted by marriage. Rather any individual can share with any other individual in the population, but sharing must be enforced through the threat of withdrawing future interaction. They determine equilibrium patterns of bilateral transfers, and use the term network to describe these patterns.

Our paper, then, contributes to the growing theoretical literature on the formation of social networks. We develop the first model of ex ante formation of links that are later used for risk pooling. We apply the equilibrium notion of pairwise stability, introduced by Jackson and Wolinsky (1996), to a context of risk-sharing, and we characterize pairwise stable and efficient networks. Our model is representative of a general setting: links have positive externalities, individual benefits depend only on the size of components, and individuals always benefit (at a decreasing rate) from an increase in the number of people in their component. Our results would hold in any economic environment that yields these network characteristics.

More generally, our paper considers the relationship between individual interactions and aggregate outcomes.<sup>7</sup> In the present paper, individual interactions determine the risk-sharing networks, which in turn determine the extent of risk-sharing within and across the population. As discussed above we have a striking finding. Ex ante symmetric agents can end up with very different risk-sharing outcomes. That is, different outcomes across individuals may not be the result of some underlying heterogeneity but the result of the interactions among agents.

The rest of the paper is organized as follows. In Section 2 we introduce concepts from the theory of networks to describe the pattern of risk-sharing relations and develop our model of bilateral risk-sharing. In Section 3 we solve for efficient networks. In Section 4 we study network formation: agents form bilateral relations, then share incomes with these relations. We solve for the pairwise stable networks, networks where no more pairs have an incentive to form a relation, and no individual has incentive to break a relation. We then compare pairwise stable networks to efficient networks. In Section 5, we look at a dynamic model of network formation where agents may form and break relations over time. In Section 6, we examine alternative specifications of the model where individuals can share some of their link costs. We conclude in Section 7.

## 2. The model

Consider a society of  $n$  individuals. Individuals are risk-averse and face shocks to their incomes. Each individual's income,  $y_i$ , is a random variable, and incomes are independent and identically distributed with mean  $\bar{y}$  and variance  $\sigma^2$ . People have identical preferences, and we represent their utility by a utility function  $v$ , which is increasing and strictly concave. Formal insurance

<sup>7</sup> We see this theme in, for example, Kirman (1993) and the volume Kirman and Zimmermann (2001).

mechanisms are not available. The only way for people to mitigate risk is to make insurance arrangements among themselves. We assume that two individuals can make transfers to each other *only if* they had previously established a risk-sharing relation. Our key assumptions are that individuals can only make bilateral transfer and that establishing a bilateral relation is costly.

### 2.1. Definition of links and networks

To model bilateral risk-sharing in a large population, we use tools from the theory of networks. An individual  $i$  and  $j$  can form a risk-sharing relationship by each incurring a cost  $c$ .<sup>8</sup> If they incur this cost, we say they have a *link*. We think of the link cost as a fixed cost that must be incurred by each of the agents. That is, one agent cannot compensate another agent for the expense of building a link. This assumption reflects the idea that some costs, such as the time incurred to build a relation are not easy to compensate or transfer. In the case of marriage, this cost would be, for example, the time and money involved in courtship and providing a dowry. After the link is formed, the cost is sunk.

In an alternative model, link costs could be seen as another element of income to be shared between agents in a risk-sharing relation. Such a model would still lead to a divergence between individual, or pairwise, incentives to form links and the criteria of a social planner. We discuss this and another alternative specification in Section 6.

We represent links and a network of links with the following notation:  $g$  is an  $n \times n$  matrix, where  $g_{ij} = 1$  when  $i$  and  $j$  have a link (i.e., have established a risk-sharing relation) and  $g_{ij} = 0$  otherwise. We assume that risk-sharing relations are mutual, so that  $g_{ij} = g_{ji}$ . By convention,  $g_{ii} = 0$ . We say there is a *path* between two individuals  $i$  and  $j$  in the graph  $g$  if there exists a sequence of individuals  $i_1, \dots, i_k$  such that  $g_{ii_1} = g_{i_1i_2} = \dots = g_{i_kj} = 1$ . A subset of individuals is *connected* if there is a path between any two individuals in the subset. A *component* of the graph  $g$  is a maximal connected subset. Components provide a partition of the population. A graph is *minimally connected* when the removal of any link increases the number of components.

In what follows, we first model, for a given network  $g$ , how much each individual gains in risk reduction. This process results in individual benefit functions that depends on the network. We then use these benefit functions to solve for efficient networks and specify and analyze network formation.

### 2.2. Risk-sharing in networks

Given a network  $g$ , how do people share risk?

We consider a benchmark model where individuals commit to share their monetary holdings equally with their linked partners. Monetary holdings are individual incomes net any transfers. Specifically, suppose that people meet repeatedly over time after the income shock. For any two linked individuals, they are committed to share their monetary holdings equally *every time they meet*. For example, suppose we are studying an agricultural setting and at the end of the growing season, farmers' incomes are realized. They then visit with their linked partners and share incomes. Then they visit linked relations again, and so on, until the beginning of the following season. This process is a benchmark for income sharing, since (as we show below) when pairs meet often enough it achieves complete risk-sharing within a component of a network. It is as if individuals

<sup>8</sup> We can think of this cost in utility terms. Agents then have additively separable utility functions, as described below.

in the component completely pooled their income. By sharing net monetary holdings in bilateral relations, individuals mitigate risk throughout the connected network. Complete income pooling within a set of individuals yields the highest possible aggregate utility.

To illustrate consider a population of three people,  $n = 3$ . Agent 1 is linked to agent 2, and agent 2 is linked to agent 3, but agent 1 is not linked to agent 3. That is, the network is a star. Agents' receive incomes  $y_1$ ,  $y_2$ , and  $y_3$ . The agents then interact as follows. Agents randomly meet their linked partners such that each agent meets each of his linked partners at least once. Whenever agents meet they share equally their current monetary holdings. For example, first agent 1 meets agent 2, and they share incomes. Each then has  $(y_1 + y_2)/2$ . Agent 3 then meets agent 2, and they share their current monetary holdings. Agents 2 and 3 then each have  $((y_1 + y_2)/2 + y_3)/2$ . Agent 2 then meets agent 1. They share monetary holdings equally, and agents 1 and 2 then have  $((y_1 + y_2)/2 + ((y_1 + y_2)/2 + y_3)/2)/2 = ((y_1 + y_2 + y_3)/4) + ((y_1 + y_2)/8)$  each. Agent 3 still has  $((y_1 + y_2)/2 + y_3)/2 = ((y_1 + y_2)/4) + (y_3/2)$ , and so on.

We formalize such random meetings as follows. There are  $T$  rounds of interactions. (1) Every pair of linked individuals meets at least once in each round. (2) Pairs meet sequentially, and the sequence of matching is random. This interaction process represents risk-sharing following a realization of income. Again, we emphasize that it is a benchmark. It implicitly assumes that agent's cannot hide transfers from others nor exaggerate transfers to others.

With this interaction process we can show the following result. As the number of rounds of interaction approaches infinity, monetary holdings will be equalized for all individuals belonging to the same component of the network. All formal proofs are given in appendix of the supplementary material.<sup>9</sup>

**Proposition 1.** *As  $T$  tends to infinity, the distribution of incomes in a component converges in probability to the equal distribution. As the number of rounds increases, an individual's money holdings converge to the mean level of the income shocks in his risk-sharing component.*

To prove this result, we first show that the dispersion of monetary holdings within components must decrease weakly after each round. We then show that the decrease is strict when all individual monetary holdings are not equal. We finally show that dispersion must converge to zero as the number of rounds becomes arbitrarily large.

This result demonstrates that if individuals are committed to share income equally within pairs and interact repeatedly with their neighbors, they end up sharing income equally within components. The process has two noteworthy features. First, individuals do not need to have information on the past transfers between people. They only need to know the current level of monetary holdings of their neighbors when they meet them. Second, modifying the interaction process could provide one way to introduce frictions in the way transfers flow through links. For instance, if the process takes place over a finite number of rounds, monetary holdings within the component will, in general, not equalize within the component, but depend on an agent's position within the component, or if a critical pair is left out of the interaction process, individual incomes may converge to different values.<sup>10</sup>

<sup>9</sup> Appendix of the supplementary material is available on the website of the *Journal of Economic Behavior and Organization*.

<sup>10</sup> There are other ways to understand equal sharing. Bloch et al. show that equal sharing within a pair is one fixed point of bilateral transfers and focus on equal sharing because it is a "social norm".

This outcome corresponds to the highest level of insurance that can be secured, given the constraint that income-sharing occurs between pairs (e.g., not in groups).<sup>11</sup> It thus allows us to study how the formation of a network affects outcomes, where the best level of risk-sharing in a network is the same as the best level possible. That is, we hold the risk-sharing level as constant and ask how the necessity of forming relations bilaterally affects the configuration of the risk-sharing arrangements.

With this interaction process, risk-sharing benefits only depend on the number of individuals in the risk-sharing component. If  $s$  denotes the size of this component, the expected utility  $u(s)$  is given by

$$u(s) = \text{Ev} \left( \frac{y_1 + \cdots + y_s}{s} \right),$$

where the expectation is taken over all realizations of incomes for all individuals  $(1, \dots, s)$  in the component. Thus,  $u(1) = \text{Ev}(y_1)$  is the expected utility for an individual who has no links. Let  $s_i(g)$  denote the size of  $i$ 's component in  $g$ . The expected utility of individual  $i$  from belonging to network  $g$  is  $u(s_i(g))$ . This reduced-form expected utility function  $u$  captures all the properties of the distribution of income shocks, the primitive utility function  $v$ , and the graph that matter for our analysis of efficient networks and network formation.

In general, the expected utility function  $u$  satisfies two properties. With  $v$  strictly increasing and concave, expected utility  $u(s) = \text{Ev}((y_1 + \cdots + y_s)/s)$  is non-decreasing and bounded from above (i.e.,  $u(s) = \text{Ev}((y_1 + \cdots + y_s)/s) \leq v(\bar{y})$ ; see [Rothschild and Stiglitz, 1971](#)). We make the following additional assumption: we assume that  $u(s)$  is increasing in the size of the component at a decreasing rate.

**Assumption 1.**  $\forall s, u(s+1) > u(s)$  and  $u(s+2) - u(s+1) < u(s+1) - u(s)$ .

To our knowledge, there is no result in the literature on uncertainty giving the conditions under which this property holds in general. It does embody an economic intuition: the more and more people share risk, the lower the benefit of pooling income with an additional individual.<sup>12</sup> We show in the examples below that this assumption is indeed satisfied in several important cases.<sup>13</sup>

**Example 1.** Consider a quadratic utility function  $v(y) = y - \lambda y^2$ , where  $\lambda$  is a positive parameter such that  $\lambda < 1/2y$  for all values of  $y$ . Observe that a larger  $\lambda$  corresponds to a more risk-averse individual. By the law of large numbers,  $u(s)_{s \rightarrow \infty} = v(\bar{y})$ . It is then easy to see that  $u(s) = v(\bar{y}) - (\lambda\sigma^2)/s$ , where recall  $\bar{y}$  is the mean of a distribution of income and  $\sigma^2$  is the variance. With this utility function,  $u(s)$  is greater when risk-aversion, measured by  $\lambda$ , is lower. We can see that  $u(s)$  is increasing in  $s$ , and  $u(s)$  satisfies our “concavity” assumption.

**Example 2.** Consider a CARA utility function  $v(y) = v_0 - e^{-\mu y}$ , where  $\mu > 0$  denotes the level of absolute risk-aversion. If income is normally distributed, we can see that  $u(s) = v_0 - e^{-\mu\bar{y} + (\mu^2\sigma^2)/2s}$ . In contrast, if income is exponentially distributed according to  $v e^{-\nu y}$ , then

<sup>11</sup> More precisely, under the constraint that only linked individuals can make transfers, complete insurance within components maximizes the sum of ex ante utilities over all possible transfers.

<sup>12</sup> However, we are also aware that in expected utility theory, “intuitive” properties often do not hold for all risk-averse preferences and all income distributions.

<sup>13</sup> Also, observe that since  $u$  is non-decreasing and bounded, the number of values of  $s$  such that  $u(s+2) - u(s+1) \leq u(s+1) - u(s)$  must be infinite.

$u(s) = v_0 - (1 - (\mu/\mu + sv))^s$ . In both cases, the utility  $u(s)$  is increasing and concave in  $s$  (see appendix of the supplementary material).

### 3. Efficient networks

We now characterize efficient networks. We find that two cases emerge depending on the properties of aggregate expected utility. In one case, where aggregate expected utility function is convex, the benefits of adding more people to a component gets higher and higher. The efficient network then either contains one component connecting the whole population or involves no links. In the other case, where aggregate expected utility is concave, the benefits of adding more people to a component gets smaller and smaller. The efficient network involves intermediate size components that balance benefits and costs. We later compare pairwise stable networks to efficient networks. We find that pairwise stable networks are always underconnected from a social welfare perspective. This result expresses a positive externality inherent to risk-sharing in networks.

In order to study efficiency issues, we introduce a simple welfare measure. Let the welfare of graph  $g$ ,  $W(g)$ , equal the sum of the net utility of the agents:

$$W(g) = \sum_i u(s_i(g)) - c \sum_i \left( \sum_j g_{ij} \right).$$

That is, welfare is the difference between the total expected utility of agents in the network and the total link costs. We say a risk-sharing network  $g$  is *efficient* if it yields the highest welfare of all possible graphs. Formally, a network  $g$  is efficient if and only if there does not exist a network  $g'$  such that  $W(g') > W(g)$ .

We characterize efficient networks here and compare them to pairwise stable networks below. Let  $k$  denote the number of components of  $g$ , and let  $s_1, \dots, s_k$  denote the sizes of these components. Observe, first, that components in efficient networks must be minimally connected. Removing any “redundant” link reduces costs by  $2c$  without affecting benefits. We can then see that total number of links in a network  $g$  are a simple linear function of the number of components of the network and do not depend on their sizes or shapes. Let  $|g|$  denote the number of links in network  $g$ . Since the number of links in a minimally connected component of size  $s$  is always equal to  $s - 1$ , we have  $|g| = \sum_{i=1}^k (s_i - 1) = n - k$ . In other words, if there are  $n$  components,  $k = n$ , no individual has links, and  $n - k = 0$ . On the other extreme, if there is a single component,  $k = 1$ , the number of links is  $n - 1$ , the minimal links necessary to connect the population in a single component. The total costs of links are then equal to  $2c(n - k)$ . Since every individual in the same component obtains the same risk-reducing benefits, aggregate benefits on a component of size  $s$  are equal to  $su(s)$ . We summarize these observations in the following lemma.

**Lemma 1.** *In an efficient network,  $g^*$ , components are minimally connected and welfare can be written as follows*

$$W(g^*) = \sum_{i=1}^k s_i u(s_i) - 2c(n - k).$$

We can then see that the shape of an efficient network (whether it should have none, few, or many components) depends on how component size affects the aggregate expected utility of agents in the component. Let  $U = su(s)$  be this aggregated expected utility.

The functions  $u(s)$  and  $su(s)$  are defined over integers. To illustrate our results, we find it helpful to use extensions of these functions that are continuous and twice-differentiable. We also use these extensions for intermediate steps in proving our results. Below we will use derivatives, and the reader should understand that any derivative refers to a derivative of the continuous, twice-differentiable extensions of our basic functions.

Let us consider the shape of networks that maximize aggregate expected utility  $U = su(s)$ . As component size  $s$  increases, there is a direct effect on aggregate utility, as an additional agent's utility is added to the sum, and there is an indirect effect as the increase changes other agents' utility. To illustrate, consider  $u(s)$  that is continuous and twice-differentiable. We would then have

$$\frac{dU}{ds} = u(s) + su'(s).$$

Both of these effects are positive, as expected utility is always increasing in the size of the component ( $u' > 0$ ). The question is then whether or not these positive effects are increasing enough to justify the cost of adding more agents to a component. That is, we must examine the derivative,  $(d^2U)/(ds^2)$ , how marginal benefits change as component size increases:

$$\frac{d^2U}{ds^2} = 2u'(s) + su''(s).$$

Since the expected utility function  $u(s)$  is concave,  $(d^2U)/(ds^2)$  could be positive or negative. When  $-s((u''(s))/(u'(s))) < 2$ , the aggregate expected utility function  $U$  is convex, and when  $-s(u''(s)/u'(s)) > 2$  the aggregate expected utility function  $U$  is concave.

Consider first a convex aggregate expected utility. Marginal benefits are increasing in component size, and we have corner solutions. The efficient network will either involve one large component connecting all agents in the population or contain  $n$  components so that no agents have links. A network where all  $n$  agents are in a component yields net payoffs  $nu(n) - 2(n - 1)c$ . If these payoffs are greater than the payoffs earned by individuals when they are all isolated,  $nu(1)$ , then the efficient network involves all  $n$  agents in a single component. That is, for  $((d^2U)/(ds^2)) > 0$ , if  $nu(n) - 2(n - 1)c > nu(1)$ , the efficient network will involve all  $n$  agents in the same component. Otherwise, no agents are connected in the efficient network.

Next consider concave aggregate expected utility. Marginal benefits are decreasing in component size and the efficient network will involve smaller, intermediate size, components. An efficient network is composed, as much as possible, of components of size  $s$  such that  $s^2u'(s) = 2c$ .

The next result formally characterizes efficient risk-sharing networks. Let  $c^* = (n[u(n) - u(1)])/(2(n - 1))$  denote the cost level for which an empty network and a component connecting all  $n$  agents yield the same welfare.

**Proposition 2.** *If  $su(s)$  is linear or strictly convex, an efficient network contains one component connecting all  $n$  agents if  $c < c^*$  and contains no links if  $c > c^*$ .*

*If  $su(s)$  is strictly concave, the efficient network contains components of sizes that differ by at most one agent. For any  $\varepsilon > 0$ , if  $n$  is large enough, the average size component in an efficient network lies between  $\hat{s} - \varepsilon$  and  $\hat{s} + 1 + \varepsilon$  where  $\hat{s}$  is the highest integer smaller than or equal to the solution of the equation  $s^2u'(s) = 2c$ .*

This result tells us that, despite costly link formation and bilateral risk-sharing, there are circumstances where the efficient network yields complete income pooling in the population.

Hence any divergence from complete income pooling would come from a decentralized formation process. A quadratic primitive utility function yields this case, and we provide an example below. When people are risk averse enough, the gains from connecting all individuals exceed the link costs. Otherwise, no-one should be connected.

**Example 3.** When the primitive function is quadratic,  $su(s) = sv(\bar{y}) - \lambda\sigma^2$ , which is linear in  $s$ . Hence, the efficient network involves either everyone in a one component, or no risk-sharing. The condition  $c < c^*$  (all agents should be in a single component) simply reduces to  $2c < \lambda\sigma^2$ . Recall the parameter  $\lambda$  represents the level of risk aversion. When  $\lambda$  is high enough, all agents should be in a single component. Similarly, when the variance of incomes is high enough, all agents should be in a single component.

When  $su(s)$  is concave, there should be an intermediate level of risk-sharing: the costs of link formation impose a limit on the optimal size of connected components. Incomplete income pooling is a socially optimal outcome.

The curvature of  $su(s)$  thus critically determines the shape of efficient networks. Ideally, we would like to know what properties of  $v$  and  $y$  determine the curvature of  $su(s)$ . Unfortunately, as with the concavity of  $u(s)$ , no such result currently exists in the literature on uncertainty. Therefore, we examine properties of  $su(s)$  in the particular cases from Example 2. When the primitive utility function is CARA and income is normally distributed, we can see that  $su(s)$  is always concave (see appendix of the supplementary material). This is not a general property of CARA utility functions, however. If income is exponentially distributed,  $su(s)$  may not have a fixed curvature.<sup>14</sup>

#### 4. Link formation and stable networks

We now consider the formation of risk-sharing networks. We ask what structures will emerge when links are formed by pairs of agents, but agents cannot coordinate link formation across the whole population. We solve for *pairwise stable* networks, a concept developed by Jackson and Wolinsky: each agent  $i$  earns some payoffs  $Y_i(g)$  that depend on the graphs. A pairwise stable network is a network where no agent has an incentive to make a change in its links. No single agent can improve its situation by breaking a link, and for any pair of agents, if one agent could benefit from a new link, the second agent would not and hence the link is not formed. That is, we do not specify a link formation game, per se, or a particular protocol for link formation. Rather, pairwise stability “identifies networks that are the only ones that could emerge at the end of any well defined game where the process does not artificially end, but only ends when no player(s) wish to make further changes to the network” (Jackson (2003, p. 116)).

In our setting, each agent earns

$$u(s_i(g)) - c \sum_j g_{ij},$$

which is the individual utility from risk-sharing minus the sum of an individual’s link costs. To describe pairwise stable networks, formally let  $g + ij$  denote the graph  $g$  with the addition of a link between agents  $i$  and  $j$ , and let  $g - ij$  denote a graph  $g$  subtracting any link between agents  $i$  and  $j$ .

<sup>14</sup> Thus, when  $\mu = 3$  and  $\nu = 1$ ,  $su(s)$  is convex if  $s \leq 9$  and concave if  $s \geq 10$ .

**Definition 1.** A risk-sharing network  $g$  is pairwise stable iff

- (1)  $\forall ij$  s.t.  $g_{ij} = 0$ , if  $u(s_i(g + ij)) - c > u(s_i(g))$  then  $u(s_j(g + ij)) - c < u(s_j(g))$ .
- (2)  $\forall ij$  s.t.  $g_{ij} = 1$ ,  $u(s_i(g)) - c \geq u(s_i(g - ij))$ .

The first condition says that, given others' links, for a pair of agents  $ij$ , if  $i$  could benefit from the link,  $j$  could not (and hence the link is not formed). The second condition says that, given the set of links, no individual wants to sever one of his links unilaterally.

#### 4.1. Pairwise stable networks

We now characterize pairwise stable networks. We find that the basic structure of pairwise stable networks is unique. For any population size  $n$  and link cost  $c$ , pairwise stable networks have a well-defined shape. First, there are no “extra” links in a pairwise stable network. If a link does not increase the size of a component, then it brings no benefits and an agent would have an incentive to cut the link. Second, pairwise stable networks divide agents into distinct components. Third, the size of the components is bounded. A component cannot be too large because the benefits of linking to an individual in a larger and larger component eventually will not exceed the cost. A second component is smaller. If the second component is too large, then there is an incentive for a pair to make a link between components, but such a super-sized component is itself not stable. Hence, stable components will not generally include all members of the population. Finally, combining the second and third findings implies that pairwise stable networks do not always exist. In such a setting, we might expect to see cycles of networks.

We proceed by deriving successive restrictions on the shape of pairwise stable networks.

First, pairwise stable networks must have minimally connected components. This is due to the fact that benefits generated by a risk-sharing network depend only on the size of the components of the network. Therefore, individuals will sever any link that does not affect the size of the components.

**Lemma 2.** In a pairwise stable network  $g$ , any component is minimally connected.

Clearly, this property is shared by any network model where the benefits depend only on the size of the network components.

Second, components of a pairwise stable network cannot be too large. This result follows from the concavity of the expected utility function. As the size of a component increases, benefits increase but at a decreasing rate. Hence, there exists a threshold size  $s^*$  where the benefits of an additional agent do not exceed the cost:<sup>15</sup>

$$s^* = \max\{s : u(s) - u(s - 1) \geq c\}.$$

**Lemma 3.** In a pairwise stable network  $g$ , all components have a size lower than or equal to  $s^*$ .

This result follows from the second condition of pairwise stability: no agent can have an incentive to cut a link. In general to check that no individual wants to cut a link, it is necessary and sufficient to find the individual earning the least from a link and to check that he does not want to

<sup>15</sup> Since  $u$  is concave,  $u(s) - u(s - 1)$  is decreasing and  $s^*$  is well-defined as soon as  $u(2) - u(1) \geq c$ . When  $u(2) - u(1) < c$ , only the empty network is pairwise stable.

cut this link. In our setting, we can define this individual precisely. He is connected to a *peripheral* agent, which we define as an agent with a single link (a minimally connected graph always has at least two peripheral agents). Cutting a link to a peripheral agent reduces the component size by one, while cutting a link to a non-peripheral agent reduces the component size by more than one. Hence, the stability condition for cutting a link is derived from incentives to cut links to peripheral agents. In a pairwise stable graph, individuals must earn more with the link to a peripheral agent than without it.<sup>16</sup>

Third, we show that in pairwise stable networks, the size of the larger of two components must be exactly equal to  $s^*$ . This outcome arises from the first condition of pairwise stability (no two agents can want to form a link) together with the concavity of  $u$ .

**Lemma 4.** *For any two components of a pairwise stable network  $g$ , the size of the largest component is equal to  $s^*$ .*

The proof follows from the requirement that, in a pairwise stable network, no two individuals in the different components can want to form a link. Individuals in the largest component have a lowest incentive to do so; hence it is their payoffs that give us the stability condition. If the size of the largest component,  $s'$ , is lower than  $s^*$ , individuals in the largest component would benefit from connecting to an individual in the smaller component. [At a component of size  $s' < s^*$ , individuals would gain from connecting to one individual; hence they would also gain from connecting to an individual connected to others.] Hence, along with Lemma 3, we have that the largest component must be equal to  $s^*$ .

To obtain the final characterization, we find the largest possible size of the smaller component. We define the threshold  $s^{**}$  as follows:

$$s^{**} = \max\{s \leq s^* : u(s^* + s) - u(s^*) < c\}.$$

An agent in a component of size  $s^*$  would want to form a link to an agent in a component larger than  $s^{**}$ . Hence, any components larger than  $s^{**}$  cannot be part of a pairwise stable network alongside the component of size  $s^*$ . This threshold size  $s^{**}$  is well-defined and greater than or equal to 1 since by definition of  $s^*$ ,  $u(s^* + 1) - u(s^*) < c$ . Also, observe that by definition  $s^{**} \leq s^*$ .

We then have our result that completely characterizes pairwise stable networks. For any population size  $n$  and link cost  $c$ , pairwise stable networks have a unique shape.

**Proposition 3.** *When  $s^{**} < s^*$  (unequal components case) a risk-sharing network  $g$  is pairwise stable if and only if (1) if  $n \leq s^*$ , it is minimally connected, and (2) if  $n > s^*$ , it is composed of two minimally connected components of sizes  $s^*$  and  $s \leq s^{**}$ . When  $s^{**} = s^*$  (equal components case) a risk-sharing network is pairwise stable iff (1) all its components are minimally connected, (2) all its components except 1 have size  $s^*$ , and (3) the size of the remaining component is lower than or equal to  $s^*$ .*

While the basic structure of pairwise stable networks is unique, they might not always exist. Existence depends on the relationship between the necessary component sizes and the population size. The equal components case appears if the utility function  $u$  is sufficiently flat above  $s^*$ . In this case, we can have  $u(s^*) - u(s^* - 1) \geq c$  and  $u(2s^*) - u(s^*) < c$ , and pairwise stable networks exist for all values of  $n$ . In other cases, the existence of pairwise stable networks

<sup>16</sup> More generally, individual  $i$  earns less from a link with  $j$  if the number of indirect neighbors of  $j$  in the graph without the link  $ij$  is lower.

may be *non-monotonic* in the link formation cost  $c$ . A decrease in  $c$  has two effects. First, it may lead to a increase in  $s^*$ ; the largest component is higher since it is less costly to maintain a link to a peripheral agent. Second, it can decrease  $s^{**}$ , since building links between components is less costly. Thus, for a certain population level  $n$ , there could exist three values  $c_1 > c_2 > c_3$ , such that a pairwise stable network exists for  $c_1$  and  $c_3$ , but not for  $c_2$ . For the high cost level, there is a small difference between the size of the components, and for the low cost level there is a higher difference. Thus higher link costs can equalize risk-sharing outcomes. For the unequal components case, the existence of a pairwise stable network depends on the relative magnitude of the components and the population. Since there can be no more than two components in a pairwise stable network, the population size cannot exceed the sum  $s^{**} + s^*$ . This is obviously a very restrictive case as the following example, using a quadratic utility function, illustrates. In Section 5, we discuss what happens when pairwise stable networks do not exist.

**Example 4.** Consider, again, a quadratic primitive utility function that yields the expected utility function  $u(s) = v(\bar{y}) - ((\lambda\sigma^2)/s)$ , and set  $\lambda\sigma^2 = 1$ . Table 1 below gives the threshold values  $s^*$  and  $s^{**}$  as functions of the cost of link formation  $c$ . The third row gives the maximum population size,  $\bar{n}$ , for which there exists a pairwise stable graph. We have  $\bar{n} = \infty$  when  $s^* = s^{**}$ , and  $\bar{n} = s^* + s^{**}$  when  $s^* < s^{**}$ . When  $c > (1/2)$ , the link cost exceeds the benefit of any risk-sharing; hence  $s^* = s^{**} = 1$ , and the empty network is always stable. We see that as  $c$  decreases, the critical size of the largest component increases, and as  $c$  increases, there is greater gap between the size of the smallest and largest component  $c$  values.

4.2. Pairwise stable versus efficient network

Finally, we compare pairwise stable and efficient networks. In general, there are two sources of divergence between pairwise stable and efficient networks. First, there could be a pairwise inefficiency: a link between agents  $i$  and  $j$  could increase their joint payoffs by  $2c$ , but since the benefits are not equal,  $i$  is willing to pay  $c$ , but  $j$  is not. In our model, a link between  $i$  and  $j$

Table 1  
Population and Component Sizes in Pairwise Stable Networks

Values of $c$	$(s^*, s^{**})$	Max $n$ with pws graph
$c > \frac{1}{2}$	(1,1)	$\infty$
$\frac{1}{2} \geq c \geq \frac{1}{4}$	(2,2)	$\infty$
$\frac{1}{4} > c > \frac{1}{6}$	(2,1)	3
$c = \frac{1}{6}$	(3,3)	$\infty$
$\frac{1}{6} > c \geq \frac{2}{15}$	(3,2)	5
$\frac{2}{15} > c > \frac{1}{12}$	(3,1)	4
$c = \frac{1}{12}$	(4,2)	6
$\frac{1}{12} > c > \frac{1}{20}$	(4,1)	5
$\frac{1}{20} \geq c > \frac{1}{30}$	(5,1)	6
$\frac{1}{30} \geq c > \frac{1}{42}$	(6,1)	7

increases the sizes of their components. For  $i$  the increase is from  $s_i$  to  $s_i + s_j$ ; for  $j$  the increase is from  $s_j$  to  $s_i + s_j$ . Since  $i$  and  $j$  can start in different size components, they do not have the same benefit from the link: it is then possible that the link is worth more than  $2c$  to both agents, but less than  $c$  to agent  $j$ .

$$u(s_i + s_j) - u(s_j) - c < 0 < 2u(s_i + s_j) - u(s_i) - u(s_j) - 2c.$$

This case can arise when agent  $i$  is a peripheral agent. From a social welfare point of view the link should be formed, but  $j$  does not have the incentive to do so. Second, there could be a global inefficiency: a link between agent  $i$  and agent  $j$  can benefit others, but  $i$  and  $j$  do not benefit enough for both to pay the link cost. In our model, the increase in utility for  $i$  and  $j$  from being in larger components does not exceed the link costs, but the link increases overall welfare:

$$2u(s_i + s_j) - u(s_i) - u(s_j) - 2c < 0 < W(g + ij) - W(g).$$

This case can arise when a link between components would increase overall welfare, but not the joint payoff for the pair. As both types of inefficiency arise in our model, pairwise stable networks in general involve (weakly) smaller components than efficient networks. We can show this result directly, by comparing [Propositions 3 and 2](#).

**Proposition 4.** *If  $su(s)$  is strictly convex, or if  $su(s)$  is strictly concave and  $n$  is high enough, the size of components in efficient networks is always greater than the size of components in pairwise stable networks.*

This inefficiency is severe in the case where the efficient network involves a single component connecting all the agents. Recall that when aggregate expected utility is convex, the efficient network connects all individuals when  $nu(n) - 2(n - 1)c \geq nu(1)$ , which can be written  $((u(n) - u(1))/(n - 1)) \geq (2c/n)$ . That is, a social planner would want to increase the component size to  $n$  as long as the average increase in benefits exceeds the average cost. In this case, the efficient network would yield risk-sharing that mimics complete income pooling within the population. However, the pairwise stable network will include components only of size  $s^*$  and of smaller sizes, where, recall,  $s^* = \max\{s : u(s) - u(s - 1) \geq c\}$ . That is, an individual would maintain a link only when the marginal benefits exceed the cost. Hence, due to the decentralized link formation process, risk-sharing networks would yield outcomes that look like incomplete income pooling within a population, as well as inequalities in risk-sharing outcomes.

When the aggregate expected utility function is concave, the efficient network involves intermediate size components. A social planner again would choose a network considering the change in average benefits and average costs, but since individuals only consider their own benefits, the components in a pairwise stable network are generally smaller than in efficient networks. And in pairwise stable networks, individuals can be in different size networks. Individuals do not have symmetric outcomes.

However this argument may be affected by the discrete nature of networks. Consider the extension of the welfare maximization program obtained by allowing component sizes to be real numbers. The solution to this extended problem is always higher than  $s^*$ . Then, when  $n$  is high enough, actual efficient size is close enough to this “real-valued” efficient size to also be greater than  $s^*$ . In contrast, when  $n$  is low, the efficient size could be too low with respect to the real-valued efficient size. This could, in principle, lead the sizes of components in efficient networks to be lower than  $s^*$ . To investigate the likelihood of this outcome, we ran simulations with  $v$

CARA,  $y$  normal, and  $n = 6$ . For 500,000 different values of the parameters, picked at random, this possibility was realized in only four cases.<sup>17</sup>

We illustrate [Proposition 4](#) with an example using a quadratic utility function.

**Example 5.** Consider, again, a quadratic primitive utility function that yields the expected utility function  $u(s) = v(\bar{y}) - ((\lambda\sigma^2)/(s))$  and set  $\lambda\sigma^2 = 1$ . [Example 4](#) solves for the pairwise stable networks for different cost levels. Let us consider  $c = 1/4$ . As calculated above, for any size population, a pairwise stable network exists and consists of components of size 2 (subject to integer constraints). The efficient network, in contrast, places all individuals in a single component. [Example 3](#) showed that when  $2c < \lambda\sigma^2$ , this network form is efficient. Here we have  $2(1/4) < 1$ , which satisfies this condition. Thus, the decentralized network formation leads to very small connected components and limited income pooling relative to the efficient outcome.

## 5. A dynamic link formation model

In this section, we consider a dynamic model of network formation. We ask what networks emerge when agents can continually make and break links. We do so in order to gain insights into the evolution of risk-sharing networks.

Starting from an initial set of risk-sharing relations, what type of network emerges? Consider the following stochastic process from [Jackson and Watts \(2002\)](#).<sup>18</sup> Start with an initial network  $g^0$ . At each time  $t$ , a pair  $(i, j)$  is picked with probability  $p_{ij}$  where  $\forall(i, j), p_{ij} > 0$  and  $\sum_{(i, j)} p_{ij} = 1$ . When the pair is picked, they decide either to make a link if they are not already connected or any one of the pair can break the link if they are connected. That is, if  $g_{ij}^t = 1$ , each considers whether she wants to keep the link. If  $u(s_i(g^t - ij)) > u(s_i(g^t)) - c$ ,  $i$  would be better off without the link and the link is cut. Similarly for  $j$ . The network evolves, and we have  $g^{t+1} = g^t - ij$  when the link is cut. Otherwise, the network remains unchanged. If  $g_{ij}^t = 0$ , the agents both consider whether they want to form the link. For instance, if  $u(s_i(g^t + ij)) - c > u(s_i(g^t))$  and  $u(s_j(g^t + ij)) - c \geq u(s_j(g^t))$ , the link is formed. Similarly if  $u(s_j(g^t + ij)) - c > u(s_j(g^t))$  and  $u(s_i(g^t + ij)) - c \geq u(s_i(g^t))$ . In this case, the network evolves and  $g^{t+1} = g^t + ij$ .

This defines a stochastic, dynamic process of network formation. Links are formed or cut one at a time, in a myopic fashion. We ask if there is any prediction as to the outcome of this process.

We use Jackson and Watts' notions of improving paths and closed cycles to characterize the outcomes. There is an *improving path* from  $g$  to  $g'$  if and only if there is a strictly positive probability that starting at  $g$ , the dynamic process leads to  $g'$ . A *closed cycle*  $C$  is a set of networks such that for any two networks  $g, g' \in C$ , there is an improving path from  $g$  to  $g'$ , and improving paths from a network in  $C$  can only lead to another network in  $C$ . If  $g$  is pairwise stable,  $\{g\}$  is a closed cycle.

The results confirm the insights from the static setting. The dynamic process yields pairwise stable networks, when pairwise stable networks exist. When pairwise stable networks do not exist, we see cycles among different networks. The cycles, though, include only certain kinds of

<sup>17</sup> For instance, when  $\mu = 8.45194$ ,  $\bar{y} = 2.20984$ ,  $\sigma^2 = 1.52213$  and  $c = 0.22373$ , then  $s^* = 4$  while efficient networks are composed of two components of size 3.

<sup>18</sup> Here, income shocks are realized and agents share risk *between* any two periods where the network evolves. The many rounds of income-sharing (as considered in Section 2) take place when the links are fixed (that is, between the periods when agents can change links).

networks. In particular, the networks are minimally connected and always include networks with small components, of size less than or equal to  $s^*$ . Hence, we see that the networks that emerge from this process are often not socially efficient. They connect too few people, and agents on the periphery of networks are vulnerable; links to them are cut more often.

Our formal results follow.

First, if there is a pairwise stable network, the dynamic process eventually yields a pairwise stable network. No other outcome is possible.

**Proposition 5.** *If a pairwise stable network exists, there is no other closed cycles.*

**Corollary 1.** *If a pairwise stable network exists, starting from any network, the dynamic process converges to a pairwise stable network with probability 1.*

Second, when no pairwise stable network exists, the networks that emerge from this dynamic process are well-defined. They are all minimally connected and may have small size components, less than or equal to  $s^*$ .

**Proposition 6.** *If a pairwise stable network does not exist, there is a unique closed cycle  $C$ . Any network in  $C$  has minimally connected components and has one component of size greater than or equal to  $s^*$ . In addition, all networks with minimally connected components of sizes  $(s^*, \dots, s^*, s')$  where  $s' \leq s^*$  belong to  $C$ .*

In this case, there are cycles where individuals make and break links forming new networks.

**Corollary 2.** *If a pairwise stable network does not exist, starting from any network, the dynamic link formation process leads to a network in  $C$  with probability 1. As soon as a network in  $C$  is reached, all subsequent networks are also in  $C$ .*

Third, we see a further illustration of the vulnerability of peripheral agents. In this dynamic process, individuals with fewer links are more likely to be cut out of a component. They bring fewer benefits than other agents, and links to them are less valuable.

**Proposition 7.** *Within  $C$ , as soon as a link is deleted, it happens in a component of size strictly greater than  $s^*$  and to an individual who has less than the average number of indirect neighbors.*

To summarize, suppose that we begin in a situation where all individuals are isolated. At first, individuals connect with each other and the size of the networks' components grows. As soon as the size is greater than  $s^*$ , the dynamics results from two countervailing forces. First, neighbors of individuals situated at the periphery would want to sever their links to them. Second, individuals want to form links to reap risk-sharing benefits. The size of connected components will then grow and shrink over time.

## 6. Alternative specification of link costs

In this section we consider alternative specifications of the model, where agents can share some of their link costs with others. We discuss how these different specifications still lead to a divergence between pairwise stable networks and efficient networks.

### 6.1. A linked agent can compensate partner for link cost

Our first alternative is just a slight divergence from our current model. We suppose that link costs are transferable between a pair of agents, but are not viewed as income. That is, for a pair

$ij$  an agent  $i$  can pay some or all of  $j$ 's cost of forming the link  $ij$ . This specification changes the pairwise stability conditions. The conditions would be as follows.

A risk-sharing network  $g$  is pairwise stable iff

**Definition 2.**

- (1)  $\forall ij \text{ s.t. } g_{ij} = 0, u(s_i(g + ij)) + u(s_j(g + ij)) - 2c \leq u(s_i(g)) + u(s_j(g)),$
- (2)  $\forall ij \text{ s.t. } g_{ij} = 1, u(s_i(g)) + u(s_j(g)) \geq u(s_i(g - ij)) + u(s_j(g - ij)) + 2c.$

That is, a pairwise stable network involves all links that improve the sum of two agents' utility from risk-sharing. It is easy to see that a pairwise stable networks will now have more links than in our current specification. However, there will remain a divergence between pairwise stable and efficient networks. While agents now consider the benefit of a link to their linked partner, they still do not consider the benefits of a link to other agents in their component. In general, pairwise stable networks will be underlinked.

6.2. Agents treat link costs as income

In our second alternative, we suppose that the cost of link formation  $c$  is a monetary cost. Agents share this cost just as they share income. In this specification, an individual's link costs are ultimately borne by all the agents in an individual's component. An agent in a component of size  $s$  then receives utility

$$Ev \left( \frac{y_1 + \dots + y_s}{s} - \frac{2c|g|}{s} \right).$$

Clearly in this case, as in the base case in the paper, only minimally connected networks can be pairwise stable, or efficient. We can then write an agent's utility simply as a function of component size:

$$u(s) = Ev \left( \frac{y_1 + \dots + y_s}{s} - \frac{2c(s - 1)}{s} \right).$$

Note that agents now share link costs with all agents in the component, as well as share income shocks. Denote by  $S$  the set of component sizes in a minimally connected network. If  $s \in S$ , denote by  $N_s$  the set of sizes of the subcomponents obtained after severing one link in a component of size  $s$ . A network is efficient if and only if it solves the following maximization problem:

$$\max_k \max_{s_1 + \dots + s_k = n} \sum_{i=1}^k s_i u(s_i),$$

and a network is pairwise stable if and only if

- (1)  $\forall s \in S, \forall k \in N_s, u(k) \leq u(s),$  and
- (2)  $\forall s_1, s_2 \in S, u(s_1 + s_2) > u(s_1) \Rightarrow u(s_1 + s_2) < u(s_2).$

It is easy to see that there is divergence between pairwise stable and efficient networks. In our original model, pairwise stable networks in general involve fewer links than efficient networks. This outcome arises because agents do not consider the benefits of their links to the risk-sharing utility of agents in their component. In this alternative, pairwise stable networks could involve

greater or fewer links than the efficient network. This outcome arises because the agent does not consider the benefits to others of adding a link but does consider the costs as the link costs are ultimately borne by all agents in the component. For example, if  $u(s)$  is increasing, the only pairwise stable network includes all the agents in a single component, but if  $su(s)$  is concave, the only efficient network is the empty network. No links should be formed. Hence, individual agents form too many links relative to the efficient network. It is easy to see there are also examples of the opposite outcome. In general, the divergence between pairwise stable and efficient networks will depend on the precise nature of the uncertainty and the tradeoff of additional shared linked costs.

## 7. Conclusion

This paper develops the first model of network formation where links are used to share risk. We model a benchmark case where individuals share income equally with their linked partners, and hence, complete insurance can be attained in a component of the network. That is, individuals earn the average income of agents in the component, and an agent would prefer to be in larger components since there is lower variability of the mean income.

We ask what structures will emerge when pairs can agree to form links but agents cannot coordinate link formation across the whole population. We compare efficient networks, which maximize aggregate expected utility, and pairwise stable networks, which are formed by individuals acting to maximize their own expected utility.

We find, first, that efficient networks can (indirectly) connect all individuals and involve full insurance, despite the cost of links. Pairwise stable networks, however, connect fewer individuals. There is an externality: when breaking a link individuals do not take into account the negative effect on others distant in the network. Peripheral agents (agents with no links to others) are especially vulnerable to being cut out of a component.

Second, the network formation process can lead ex ante identical individuals to be in different positions ex post and thus have different risk-sharing outcomes. These results may help to explain empirical findings that risk-sharing within a similar population is often not symmetric or complete.

In our static model, stable networks may not exist. If components are too large, links to peripheral agents are cut, but if components are too small or agents are isolated, they have an incentive to form links to others. What happens in this case? In our study of the dynamic model, we show that cycles may emerge. Especially, during the dynamic making and breaking of links, peripheral agents are vulnerable. They are likely to be cut out of a connected component. This occurs because network components cannot stay large for a long time. In large components, agents have an incentive to cut links when they have the chance to do so. Hence, even in the long term, we do not expect that agents, acting on their own to establish risk-sharing relations, will form efficient risk-sharing networks.

Frictions in the income sharing process is a direction for future research. Our current model has no frictions: we show that if individuals commit to share monetary holdings equally within pairs and pairs interact repeatedly, in the limit individuals' incomes are shared equally within components. In this process, individuals do not need to have information on the past transfers between people. They only need to know the current level of monetary holdings of their neighbors when they meet them. There is also no discounting or other income losses that occur as these interactions proceed.

This benchmark model then indicates several ways to introduce frictions into the risk-sharing process. If the process takes place over a finite number of rounds, monetary holdings within

the component will, in general, not equalize within the component, but depend on an agent's position within the component. A more connected person might meet more often with neighbors and have greater risk-sharing. If individuals can only partially observe net monetary holdings, then only some parts of income and transfers can be shared. Finally, if some income is consumed before all transfers are complete, expected utility will not be equalized within a component. With these frictions, agents' utility would depend not only on the size of their components, but their positions in the component. We expect that pairwise stable networks are no longer minimally connected, as people gain more from direct connections than from indirect connections. Yet, positive externalities will remain.

Thus, we expect the central effects that we identify in this paper will hold. Peripheral individuals will be vulnerable because links that connect them to others are less valuable than links to better connected agents. Pairwise stable networks will connect fewer individuals than efficient networks, and the decentralized link formation process can lead to asymmetric outcomes for otherwise similar individuals.

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### Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at [doi:10.1016/j.jebo.2006.10.004](https://doi.org/10.1016/j.jebo.2006.10.004).

### References

- Bloch, F., Genicot, G., Ray, D., 2005. Informal insurance in social networks. Working paper, available at <http://www.vcharite.univ-mrs.fr/PP/bloch/papers/>.
- Coate, S., Ravallion, M., 1993. Reciprocity without commitment: characterization and performance of informal insurance arrangements. *Journal of Development Economics* 40, 1–24.
- Dercon, S., De Weerd, J., 2006. Risk-sharing networks and insurance against illness. *Journal of Development Economics* 81, 337–356.
- De Weerd, J., 2004. Risk-sharing and endogenous network formation. In: Dercon, S. (Ed.), *Insurance Against Poverty*. Oxford University Press, Oxford.
- Fafchamps, M., Lund, S., 2003. Risk sharing networks in rural Philippines. *Journal of Development Economics* 71, 261–287.
- Genicot, G., Ray, D., 2003. Endogenous group formation in risk sharing arrangements. *Review of Economic Studies* 70 (1), 87–113.
- Jackson, M., 2003. The stability and efficiency of economic and social networks. In: Jackson, M.O., Dutta, B. (Eds.), *Networks and Groups: Models of Strategic Formation*. Springer-Verlag, Heidelberg, pp. 99–140.
- Jackson, M., Watts, A., 2002. The evolution of social and economic networks. *Journal of Economic Theory* 106, 265–295.
- Jackson, M., Wolinsky, A., 1996. A strategic model of social and economic networks. *Journal of Economic Theory* 71, 44–74.
- Kimball, M., 1988. Farmer cooperatives as behavior towards risk. *American Economic Review* 78, 224–232.
- Kirman, A., 1993. Ants, rationality and recruitment. *Quarterly Journal of Economics* 108 (1), 137–156.

- Kirman, A., Zimmermann, J.B. (Eds.), 2001. Economics with heterogeneous interacting agents. Springer-Verlag, Berlin.
- Ligon, E., Thomas, J., Worrall, T., 2002. Mutual insurance and limited commitment: theory and evidence in village economies. *Review of Economic Studies* 69, 115–139.
- Murgai, R., Winters, P., Sadoulet, E., de Janvry, A., 2002. Localized and incomplete mutual insurance. *Journal of Development Economics* 67 (2), 245–274.
- Rosenzweig, M.R., Stark, O., 1989. Consumption smoothing, migration, and marriage: evidence from rural India. *Journal of Political Economy* 97 (4), 905–926.
- Rothschild, M., Stiglitz, J., 1971. Increasing risk II: its economic consequences. *Journal of Economic Theory* 3, 66–84.
- Townsend, R., 1994. Risk and insurance in village India. *Econometrica* 62 (3), 539–591.
- Udry, C., 1995. Risk and saving in Northern Nigeria. *American Economic Review* 85 (5), 1287–1300.