Risk Sharing across Communities

By Yann Bramoullé and Rachel Kranton*

This paper studies cross-community risk sharing. There is now a large body of theoretical and empirical work on informal insurance, where people mitigate risk by sharing income. A consistent empirical finding is that risk-sharing is not complete within villages, often the observed sets of individuals.[1] One reason, researchers suspect, is that risk sharing does not take place at the village level but between individuals and families.[2] We build a theoretical model of risk sharing between pairs of agents. There are idiosyncratic shocks to individual income and community-level shocks. We ask how the opportunity to form cross-community links affects the shape and efficiency of risk-sharing arrangements.

We find that when links across villages form, there can be less risk sharing within a village. Welfare is higher for those directly or indirectly connected across villages, but lower for everyone else. Overall, welfare can be higher. Thus, empirical findings are not necessarily evidence of inefficient risk-sharing relations. Rather, they are consistent with patterns involving cross-community links, which can yield higher aggregate welfare despite incomplete insurance within villages.

This paper makes two contributions. First, it demonstrates the importance of a network analysis. By examining pairwise incentives to form relations, we can see new risk-sharing outcomes. Second, we uncover aggregate and distributional effects of risk sharing across communities.

We posit a simple two-stage model. We provide a benchmark model for risk sharing for a given network. Then, working backward, we consider incentives to form risk-sharing links.[3] To illustrate, consider a rural population and ask how people might link by marriage in order to establish risk-sharing relations. (There is evidence in India, for example, that families arrange daughters' marriages to men in different villages to increase gains from risk sharing (Mark R. Rosenzweig and Oded Stark 1989). Establishing this relation is costly, e.g., involving a dowry and ceremony, and the relation commits the parties to future income sharing, due to a social norm or punishment in case of nonsharing. The marriage pattern would then be our network. We ask: when do stable networks span communities? And, we examine the welfare properties of such networks.

I. A Model of Risk Sharing

There are two villages, with m_1 agents in village 1 and m_2 agents in village 2. For agent i in village v, income is $\tilde{y}_i = \bar{y} + \tilde{\varepsilon}_i + \tilde{\mu}_v$, where $\tilde{\varepsilon}_i$ is an idiosyncratic shock, and $\tilde{\mu}_v$ is a shock common to all agents of village v.[4] Village shocks are i.i.d. with mean zero and variance σ_μ^2, idiosyncratic shocks are i.i.d. with mean zero and variance σ_ε^2, and the village and idiosyncratic shocks are independently distributed. Agents are risk averse and all have the same increasing and strictly concave utility function $w(y)$. Pairs of agents can transfer money after shocks are realized. To do so, two agents must, ex ante, establish a "link," which represents a relationship-specific investment allowing them to observe each other's monetary holdings and make transfers. A link between two agents in the same village costs each agent c. A link between two agents in different villages costs each agent C, where $C > c > 0$.[5]

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[1] For seminal contributions, see Robert M. Townsend (1994) and Christopher Udry (1995). More recently, see Joachim De Weerdt and Stefan Dercon (2006).

[2] See Marcel Fafchamps and Susan Lund (2003); Rinku Murgai et al. (2002); and Townsend (1994, 541).

[3] Most previous theoretical literature studies the enforcement of risk-sharing agreements (e.g., Stephen Coate and Martin Ravallion 1993; Ethan Ligon, Jonathan P. Thomas, and Tim Worrall 2002). Francis Bloch, Garance Genicot, and Debraj Ray (2006) study enforcement when income is shared by pairs.

[4] We use the word "village" throughout the paper, but it need not be taken literally. It is the common shock that differentiates agents in the population.

[5] Some costs, e.g., the time to build a relation, are not easy to compensate. Bramoullé and Kranton (2005) discuss link costs as another element of shared income.

We represent links and network as follows: **g** is an $(m_1 + m_2) \times (m_1 + m_2)$ symmetric matrix, where $g_{ii} = 0$, $g_{ij} = 1$ when i and j have a link, and $g_{ij} = 0$ otherwise. There is a *path* between i and j if there is a sequence of agents $i_1, \ldots, i_k$ such that $g_{ii_1} = g_{i_1 i_2} = \cdots = g_{i_k j} = 1$. A subset is *connected* if there is a path between any two agents in the subset. A *component* of a network **g** is a maximal connected subset. Components, then, provide a partition of the population. A graph is *minimally connected* when the removal of any link increases the number of components.

A. *Risk Sharing in a Given Network*

Given a network **g**, how do pairs share realized income? Consider this benchmark model. After income is realized, linked pairs meet (sequentially and randomly) and share their current money holdings equally.[6] With many rounds of such meetings, in the limit an individual's money holdings converge to the mean of realized income in his component, yielding the highest possible aggregate expected utility for this set of agents (Bramoullé and Kranton 2005, Proposition 1).

With this benchmark, risk-sharing benefits depend only on the number of agents from each village in a component. Let n_v denote the number of people from village v in a component. Ex post income of an individual in a component containing n_1 and n_2 agents is then $\bar{y} + \left(\sum_{i=1}^{n_1+n_2} \varepsilon_i\right)/(n_1 + n_2) + (n_1\mu_1 + n_2\mu_2)/(n_1 + n_2)$, where ε_i is the realization of i's idiosyncratic shock and μ_v the realization of village v's shock. Let the expected utility of individual i in a component containing n_1 and n_2 be $u(n_1, n_2) = Ew\left[\bar{y} + \left(\sum_{i=1}^{n_1+n_2} \varepsilon_i\right)/(n_1 + n_2) + (n_1\mu_1 + n_2\mu_2)/(n_1 + n_2)\right]$, where the expectation is taken over all realizations of the shocks to agents in the component.

In the rest of the paper, for simplicity, we consider a quadratic utility function $w(y) \equiv y - \lambda y^2$, where $0 < \lambda < 1/(2y)$ for all y. Expected utility then depends only on the mean and variance of an individual's income. Agents then have expected utility

$$u(n_1, n_2) = u_0 + \left(1 - \frac{1}{n_1 + n_2}\right)\lambda \sigma_\varepsilon^2 + \frac{2n_1 n_2}{(n_1 + n_2)^2}\lambda \sigma_\mu^2,$$

where $u_0 \equiv v(\bar{y}) - \lambda(\sigma_\varepsilon^2 + \sigma_\mu^2)$.

The expected utility function gives important insights into the costs and benefits of risk sharing outside one's own community. The first term, u_0, is the utility an individual obtains when he does not participate in any income sharing. The second term shows the benefits from sharing the idiosyncratic shock. It is always increasing in both n_1 and n_2, and it is always beneficial to share this risk with more people, regardless of their village. The third term shows the benefits of sharing the village-level shock. This term is positive as long as $n_1 > 0$ and $n_2 > 0$. It is better to have links across villages than not.

But the number and proportion of agents from each village matters. The largest gain occurs when $n_1 = n_2 > 0$. Any move away from equality hurts agents, as they are subject to greater income variability. To see this, suppose $\tilde{\mu}_v$ is equal to μ or to $-\mu$ with probability ½ each, and the idiosyncratic shock is always equal to zero. Suppose $n_1 = n_2 = 1$. With probability ½, the villages have different shocks, and, after income sharing, each agent has income $\bar{y}$. Now, suppose $n_2 = 2 > n_1 = 1$. When village 2 has the low shock, individuals have income $\bar{y} - \mu/3$, and when village 2 has the high shock, individuals have income $\bar{y} + \mu/3$.

Thus, agents can have a disincentive to link to more agents in their own village. The effect above is not special to our utility function. Indeed, general utility functions could have an additional effect. Increases in expected utility from sharing the village-level shock could reduce the marginal utility from sharing the idiosyncratic shock with yet another agent within a village.[7]

[6] For example, consider an agricultural setting, and at the end of the growing season farmers' incomes are realized. They then visit with their linked partners and share incomes, then visit linked relations again, and so on, until the beginning of the following season.

[7] In our specification, expected utility is additively separable in gains from sharing the idiosyncratic shock and the village shock.

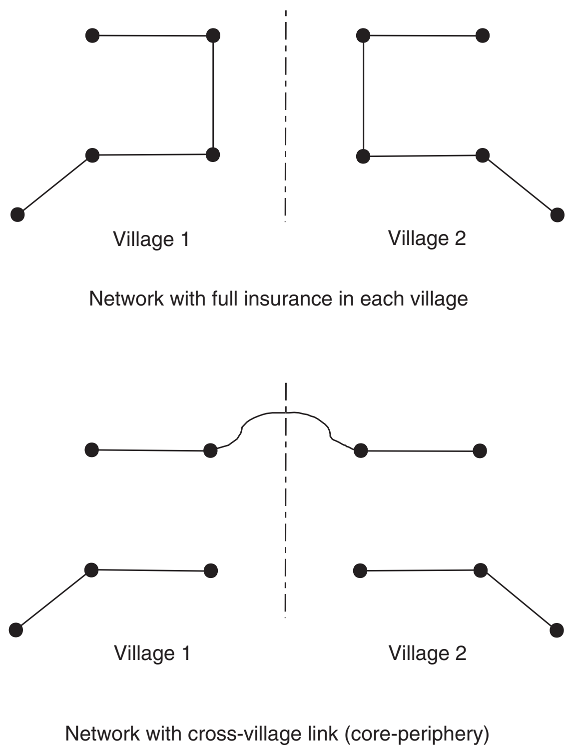

FIGURE 1

B. *Pairwise Stable Networks*

We now consider link formation. In a network $\mathbf{g}$, each agent i earns

$$U_i(\mathbf{g}) = u[n_1, n_2; s_i(\mathbf{g})] - \left[c \sum_{j \text{ s.t. } ij \in V} g_{ij}\right] - \left[C \sum_{j \text{ s.t. } ij \notin V} g_{ij}\right],$$

where $s_i(\mathbf{g})$ denotes the component containing i which determines n_1 and n_2 for i, and the notation $ij \in V$ indicates i and j are in the same village.

We consider *pairwise stable networks* (Matthew O. Jackson and Asher Wolinsky 1996). No agent can improve her payoffs by breaking one of her links, and, for any pair of unlinked agents, if one agent's payoffs increase from a link, the second agent's does not. Let $\mathbf{g} + ij$ denote $\mathbf{g}$ with the addition of a link between i and j, and let $\mathbf{g} - ij$ denote $\mathbf{g}$ subtracting any link between i and j. A network $\mathbf{g}$ is pairwise stable if and only if $\forall ij$ s.t. $g_{ij} = 1$, $U_i(\mathbf{g}) \geq U_i(\mathbf{g} - ij)$, and $\forall ij$ s.t. $g_{ij} = 0$; if $U_i(\mathbf{g} + ij) > U_i(\mathbf{g})$ then $U_j(\mathbf{g} + ij) < Uj(\mathbf{g})$. Observe, first, that all pairwise stable graphs have minimally connected components. If a component is not minimally connected, an agent can sever a link without affecting n_1 and n_2. Second, if C is sufficiently large, pairwise stable networks have only within-village links.

C. *Within-Village Risk-Sharing Networks*

Suppose the villages are the same size, $m_1 = m_2 = m$; (we discuss $m_1 \neq m_2$ below.) Consider a network that yields complete insurance within a village but provides no insurance against village shocks. It has two components. All agents in one village are in one component, and all agents in the other village are in the other component. We call this network a "full-insurance-segregated network." Figure 1 illustrates.

To be stable, within-village link costs must be small enough so that it is beneficial for an individual to maintain a link to the last agent in the village: $[1/(m-1) - 1/m]\lambda\sigma_\varepsilon^2 \equiv \bar{c} \geq c$.[8] This network is pairwise stable if and only if $c \leq \bar{c}$, and no two agents in different villages want to form a link to insure against the village-level shock, $(\lambda/2)\sigma_\mu^2 + (\lambda/2m)\sigma_\varepsilon^2 < C$.

We compare this network to one where some agents insure against village-level shocks.

D. *Cross-Village Risk-Sharing Networks*

Consider in each village a subgroup of size $k \leq m/2$. Let these k be in the same component with k people in the other village, where there is one cross-village link, and call these agents the "core."[9] Let the remaining $m - k$ agents be in a single component in each village. We say these agents are in the "periphery." Figure 1 illustrates this. This network is pairwise stable if and only if: first, no pair within a village periphery wants to cut a link, and no pair within a village core

[8] A minimally connected component always contains an agent that has only one link. The neighbor of this agent has the lowest incentive to keep this link. Hence, this agent must have the incentive to maintain the link in order for the graph to be stable.

[9] Our results also hold when the cores have different sizes, but the difference is not too large.

wants to cut a link. Both of these conditions are satisfied for $c \leq \bar{c}$. Second, (a) no two agents in the periphery have an incentive to form a cross-village link:

$$(1)\qquad \left(\frac{1}{m-k}\right)\frac{\lambda}{2}\sigma_{\varepsilon}^2 + \frac{\lambda}{2}\sigma_{\mu}^2 < C;$$

(b) the pair in the core with the cross-village link does not want to cut their link:

$$(2)\qquad \left(\frac{1}{k}\right)\frac{\lambda}{2}\sigma_{\varepsilon}^2 + \frac{\lambda}{2}\sigma_{\mu}^2 \geq C;\ \text{and}$$

(c) no core agent wants to form a link with a peripheral agent in his own village:[10]

$$(3)\qquad \left(\frac{1}{k} - \frac{2}{m+k}\right)\frac{\lambda}{2}\sigma_{\varepsilon}^2 - \left(\frac{m-k}{m+k}\right)^2\frac{\lambda}{2}\sigma_{\mu}^2 < c.$$

There are two forces at play in this last, key, condition. A core agent would like to connect to an agent in his village to improve risk sharing on his idiosyncratic shock, seen in the positive term $[(1/k) - 2/(m+k)](\lambda/2)\sigma_{\varepsilon}^2$. But this link would worsen risk sharing on the village shock seen in the negative term $-[(m-k)/(m+k)]^2 (\lambda/2)\sigma_{\mu}^2$. Clearly, if $(\lambda/2)\sigma_{\mu}^2$ is large enough, the latter dominates.

There are ranges of link costs where both types of networks are stable. For C, if no two agents in a periphery want to form a cross-village link, then no two agents in a fully connected village would want to do so either.[11] For c, combining $\bar{c} \geq c$ and (3), we have $\{1/[m(m-1)]\}\lambda\sigma_{\varepsilon}^2 \geq c > [(m-k)/2k(m+k)]\lambda\sigma_{\varepsilon}^2 - [(m-k)/(m+k)]^2 (\lambda/2)\sigma_{\mu}^2$, which is satisfied when the village shock variance, σ_{μ}^2, is sufficiently large.[12]

[10] Linking to an agent in the periphery in the other village is more costly. An agent in the periphery would gain more from connecting to the core than an agent in the core gains from connecting to an agent in the periphery when σ_{μ}^2 is not too low: $\frac{1}{2}\sigma_{\mu}^2 \geq [1/(2k) - 1/(m-k)]\sigma_{\varepsilon}^2$.

[11] If $C > (\lambda/2)\sigma_{\mu}^2 + \{1/[2(m-k)]\}\lambda\sigma_{\varepsilon}^2$, then $C > (\lambda/2)\sigma_{\mu}^2 + [1/(2m)]\lambda\sigma_{\varepsilon}^2$.

[12] When this is true, there are also regions where the core-periphery network is stable, but the full-insurance-segregated networks are not for higher ranges of c.

E. *Welfare Comparison: Does Cross-Village Risk Sharing Improve Outcomes?*

Is welfare higher when (some) agents share village shocks? Who benefits and who loses?

Let the welfare of a graph be the sum of all individuals' payoffs. The welfare of a core-periphery network, W_{CP}, is the sum of expected utility of core agents, $2k\{u_0 + [1 - 1/(2k)]\lambda\sigma_{\varepsilon}^2 + (\lambda/2)\sigma_{\mu}^2\}$ and expected utility of peripheral agents, $2(m-k)\{u_0 + [1 - 1/(m-k)]\lambda\sigma_{\varepsilon}^2\}$, minus within- and cross-village link costs, $2C + 2c[2(k-1) + 2(m-k-1)]$. The welfare of a full-insurance-segregated network, W_{FIS}, is simply the expected utility from sharing idiosyncratic shocks within-village, $2m\{u_0 + [1 - (1/m)]\lambda\sigma_{\varepsilon}^2\}$, minus link costs $2c[2(m-1)]$. Comparing, we have $W_{CP} \geq W_{FIS}$ if and only if $k\lambda\sigma_{\mu}^2 - \lambda\sigma_{\varepsilon}^2 \geq 2C - 4c$.

Thus, when both graphs are stable, the core-periphery graph yields higher welfare, if the village shock has high enough variance, $\sigma_{\mu}^2 \geq (3/2)\sigma_{\varepsilon}^2$, and the core is big enough, $k \geq 2$. But not all agents enjoy a gain. Agents in the periphery have lower expected utility. They share idiosyncratic shocks with fewer people. Core agents are better off, and their gains outweigh these losses.

F. *Different Size Villages*

Asymmetric village sizes can strengthen the stability of networks with cross-village links. Consider $m_1 > m_2$ and the following extension of core-periphery graphs. The larger village is divided into two subgroups of sizes m_2 and $m_1 - m_2$. The m_2 agents are in the same component with the m_2 agents in the smaller village, with one link connecting the villages. The $m_1 - m_2$ agents in the other subgroup form a component. This network is pairwise stable when (a) the pair with the cross-village link does not want to cut its link, $(1/m_2)(\lambda/2)\sigma_{\varepsilon}^2 + (\lambda/2)\sigma_{\mu}^2 \geq C$, and (b) in the larger village, core agents do not want to link with agents in the periphery, $[1/(2m_2) - 2/(m_1 + m_2)](\lambda/2)\sigma_{\varepsilon}^2 - [(m_1 - m_2)/(m_1 + m_2)]^2(\lambda/2)\sigma_{\mu}^2 < c$. The novel feature is that there is no counterpart to (1). Since the smaller village does not have a periphery, no lower bound has to be placed on C to guarantee stability. Hence, this network can be stable for values of C for which full insurance segregated networks are not stable.

II. Conclusion

This paper studies informal risk-sharing networks across communities. We build a theoretical model where agents face idiosyncratic and community-level shocks. We find that networks that span communities can yield higher welfare than networks that connect all agents within a village. The gains to agents with a path to another community can outweigh the losses to those with no such path.

While we derive our results with very simple specifications of risk sharing and utility, the trade-offs would hold in a general model. Pairwise stable networks would not necessarily involve minimally connected components. But there would still be an effect of cross-village links on the benefits of within-village links. Thus, this paper opens new questions on the nature of informal risk sharing, and we encourage further theoretical and empirical work to consider cross-community relations.

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