Expert Report on the North Carolina State Legislature

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1 Introduction

I am Professor of Mathematics and Statistical Science at Duke University. My degrees are from the North Carolina School of Science and in Math (High School Diploma), Yale University (B.S.), and Princeton University (Ph.D.). I am presently the Chair of the Mathematics Department at Duke University. I grew up in Charlotte, North Carolina and currently live in Durham, North Carolina.

I lead a group at Duke University which conducts non-partisan research to understand and quantify gerrymandering. This report grows out of aspects of our group's work around the current North Carolina legislative districts which are relevant to the case being filed.

I previously submitted an expert report in Common Cause v. Rucho, No. 18-CV-1026 (M.D.N.C.) and Diamond v. Torres, No. 17-CV-5054 (E.D. Pa.), and was an expert witness for the plantiff in Common Cause. I am being paid at a rate of $400/per hour for the work on this case. Much of the work derives from an independent research effort, unrelated to this lawsuit, to understand gerrymandering nationally and in North Carolina specifically.

2 Overview

2.1 Overview of Findings

Using historic voting data, we compare election results under the enacted districting plans for the North Carolina House and North Carolina Senate with election results under a collection of non-partisan maps. One strength of this method is that it make no assumptions in advance about what structure an election should have such as a relation to proportional representation or some type of symmetry considerations. We examine both the number of seats that would have been won under these vote counts, along with the expected margins of victory. We see that the enacted plans are extreme outliers. Both the House and Senate plans systematically favor the Republican Party to an extent which is rarely, if ever, seen in the non-partisan collection of maps. Under many historic elections considered, the enacted maps in both the North Carolina Senate and House elect significantly fewer Democrats than the typical number of Democrats found in the collection of maps. At times the Democratic Party is denied a majority of seats when the overwhelming majority of maps in our collection would have resulted in a Democratic majority. In the North Carolina Senate, we find instances in which the Republicans would have gained a supermajority under the enacted plan, but would have lost a supermajority in nearly every map in our collection. In the North Carolina House, we find instances in which the Republicans won the supermajority of seats under the enacted plan but they would have not won the supermajority in the majority of maps in our collection. Again in the North Carolina House, we found an instance where the enacted map resulted in the chamber's seats being equally divided but a number of maps in our ensemble gave the Democratic Party a supermajority.

The extreme statewide tilt towards the Republican Party is the result of a significant number of truly independent choices at the level of the county-clusters into which the state is divided. The chance of making so many independent choices which bias the results towards the Republican Party unintentionally is astronomically small.

In addition to this systematic bias towards the Republican Party which when aggregated produces highly atypical results, the enacted plan also has highly atypical results in a number of county clusters even when viewed alone. Beyond often creating atypical results in terms of the number of seats won in a given cluster, our results also show a durability in the results in certain clusters under the enacted plans. By durable, we mean that the results remain atypically unchanged over a wide range of elections. This unresponsiveness to changes in vote counts is another problematic feature revealed by our analysis of the enacted plan.

2.2 Overview of Method

We generate a collection of alternative restricting maps using Markov Chain Monte Carlo methods, and use this collection to characterize what would be expected if only non-partisan redistricting criteria were used (see also [1, 2, 3]). No partisan information is used to construct this ensemble of maps; only the generally accepted districting criteria of approximately equal population per district, contiguous and relatively compact districts, reducing traversals, and keeping counties, precincts, and municipalities whole.

At the request of the plaintiffs, we used an ensemble of maps which did not modify districts drawn by the Special Master. In particular, the following districts were left unchanged: Senate districts 19, 21, 24, and 28, and House districts 21, 22, 57, 61, and 62.

We use the term *compliant maps* to refer to maps which satisfy certain minimal design criteria such as the number of counties split or the total population deviation being below accepted thresholds which is based on the values of the enacted plan. We will generally refer to our collection of compliant maps as the *ensemble of maps*.

To generate the ensemble of alternative maps, we define a distribution on all of the redistricting maps and then sample this distribution using a Markov Chain Monte Carlo algorithm; such algorithms are widely accepted for sampling high-dimensional distributions. The distribution is defined to be concentrated on districting plans that contain districts near the ideal district population based on one-person-one-vote. It is also designed to produce contiguous districts that are relatively compact and to reduce the number of counties and municipalities which are split. The distribution on redistricting plans is tuned so that these non-partisan quantities are similar to the number of counties, municipalities, and precincts which are split in the enacted plan.

2.3 County Clusters

In Stephenson v. Bartlett, 562 S.E.2d 377 (N.C. 2002), the North Carolina Supreme Court ruled that North Carolina's state legislative districts should be clustered into groups of counties and that no district should cross between two of the "county clusters." For purposes of this case, we accept the county clusters created by the General Assembly in 2017 and will produce maps which strictly respect them. In addition, we apply the Whole County Rule within each cluster, meaning that none of our maps split counties that are kept whole under the 2017 enacted plans. We also keep the number of county traversals to be the same as the number of county traversals in the enacted plan with the exception of two county clusters in the House.[1]

Because of the county cluster structure, we produce an ensemble of maps within each county cluster. To produce a statewide ensemble of maps, we then combine maps from each cluster. The total number of maps in each county cluster ensemble are shown in Appendix F in Table 13 for the Senate and Table 14 for the House. We combine the county cluster-wide ensembles to produce a statewide ensemble. Because the combinations for different county clusters are independent, our state-wide ensemble has a total of approximately 1.1×10^{108} plans in the House and approximately 3.7×10^{93} plans in the Senate. Of these plans, 6.6×10^{86} of the plans in the House and 5.3×10^{30} of the plans in the Senate are unique.[2] The redundancy captures the fact that plans which better satisfy the non-partisan criteria are preferred in the distribution of redistricting plans. In Section E.25, we show that our qualitative conclusions do not change when we only examine the space of unique plans, as opposed to allowing for duplicate plans to be sampled; we also show that our results are insensitive when considering the effects of incumbency or when narrowing the scope of which plans are compliant.

The combined plans from each of the cluster ensembles define a statewide ensemble of maps that are drawn without partisan consideration. To compare the statewide ensemble of maps with the enacted plan, we examine the partisan results of each map when considering a range of historic elections. For each election and each district within a plan we aggregate the partisan votes cast for each party to determine the given district's partisan makeup under those historic votes. By considering the partisan makeup of all districts within a redistricting plan, we can compare the enacted plan with the plans in the ensemble.

2.4 Election Data Used in Analysis

The historic elections we consider are the Attorney General race in 2008 and 2016, the Commissioner of Insurance race in 2008 and 2012, the Governor race in 2008, 2012, and 2016, the Lieutenant Governor race in 2008, 2012, and 2016, the Unites States President race in 2008, 2012, and 2016, and the United States Senate race in 2008, 2010, 2014 and 2016. We use the census level vote counts provided by the North Carolina Legislature in the Redistricting Archives (See [4, 5]) along with more recent voting data provided by Blake Esselstyn through a public records request (see [6, 7]). Below we will use the following abbreviations: AG for Attorney General, USS for United States Senate, CI for Commissioner of Insurance, LG for Lieutenant Governor, GV for Governor, and PR for United States President. We add to these abbreviations the last two digits of the year of the election. Hence CI08 is the vote data from the Commissioner of Insurance election in 2008.

3 Statewide Analysis of Partisan Gerrymandering

We now use our collection of ensembles at the county-cluster level as well as global ensembles built from them to investigate the typical structure of elections from a statewide perspective. We will first consider the North Carolina Senate and then turn our attention to the North Carolina House.

3.1 North Carolina Senate

Using the precinct-by-precinct historical vote counts, we determine the number of Democrats elected by each plan in the ensemble based on each historical election. The number of Democrats elected will change under different plans and some

[1]In the Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly county cluster and the Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander county cluster the ensemble of plans within these clusters occasionally introduces one more county traversal than appears in the enacted plan.

[2]For information on how the ensembles were produced, see Section C; for parameters used to generate the ensemble examined in this section, see Table 7 and Table 6; for cluster-by-cluster validations of each cluster ensemble, see Sections E and F.1; for details on the number of sampled and unique plans for each cluster see Table 14 and Table 13 in Appendix F

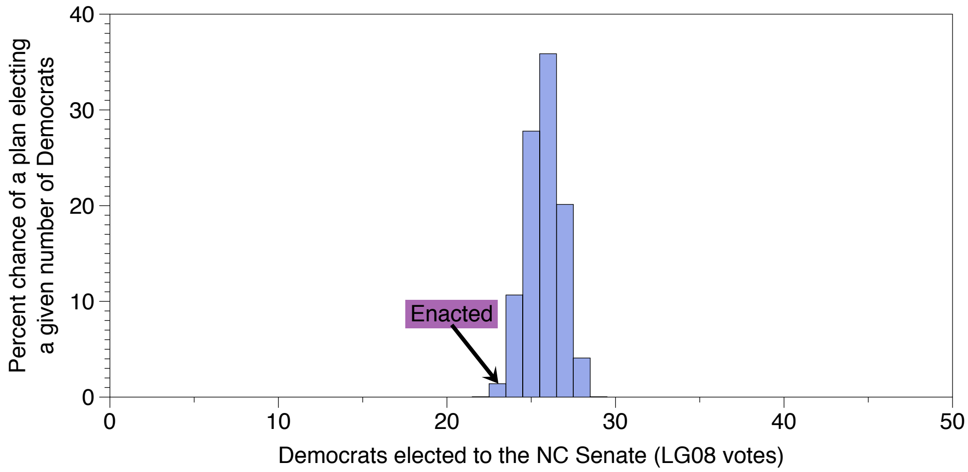


Figure 1: We plot the chance that a certain number of Democrats are elected to the North Carolina Senate using the partisan vote counts from the 2008 Lieutenant Governor. Less than 1.5% of the plans in the ensemble lead to 23 elected Democrats or fewer; in contrast the enacted plan would elect 23 Democrats out of 50 available seats.

results occur more often than others. When using the 2008 Lieutenant Governor votes, we find that plans in the ensemble lead to 23 Democrats out of 50 available seats in less than 1.46% of the plans in the ensemble; similarly there would be 24 Democrats elected in 10.37% of the plans in the ensemble, 25 Democrats in 28.25% of the plans in the ensemble, and so on. We plot these results as a histogram in Figure 1. Using the same 2008 Lieutenant Governor votes under the enacted plan would lead to the election of 23 Democrats out of 50 available seats. The enacted plan is an outlier in the context of the ensemble of plans. In fact, the histogram reveals that Democrats would win 25 or more seats in over 88% of the plans, and would win a majority of the seats in over 59% of the plans.

Comparing the enacting plan to the ensemble plans using one historical election can be probative. However, in doing so, we don't have a sense if the atypical behavior found in the elections will persist across other elections or shifts in the vote. We therefore turn to consider a larger set of historical votes; this will reveal how each map responds to a range of possible variations in the partisan vote fraction and in spatial variations within the statewide vote. By accounting for a collection of historic elections, we also minimize the peculiarities specific to any given race.

To summarize the results from a range of historic elections, we first note that as the statewide vote totals shift from favoring one party to another, we expect the statewide number of seats to change. To capture the change in the statewide vote and understand its impact on both the enacted plan and the ensemble, we plot histograms for a number of elections in Figure 2 and position their relative height to represent the Democratic statewide vote fraction. For a detailed break down over all elections, see Tables 1 and 3.

The various plans in the ensemble elect a range of Democrats for any given historical election; we view the median of this range as a baseline for each election. When comparing the medians of the ensemble with the results of the enacted plan over the 17 considered elections, we find that the enacted plan nearly always elects fewer Democrats than the median number of Democrats elected in the ensemble (see the table of Figure 3). Theoretically it is possible that some plans in the ensemble may consistently elect fewer Democrats than the median number of elected Democrats in the ensemble; it is also possible that typical plans will alternate between electing more Democrats than the median in some elections, and fewer Democrats than the median in other elections. To explore this, we test if the enacted plan is atypical of the ensemble by determining the net number of elections for which a plan skews in favor of the Democratic or Republican Party.

To calculate this number, we select a plan in the ensemble and count the number of elections (out of the 17 historical elections considered) in which that plan elects more Democrats than the median number of elected Democrats in the ensemble; we then subtract the number of elections in which that particular plan elects fewer Democrats than the median number of elected Democrats in the ensemble. We then cycle through all of the elections under consideration. For example, in the enacted plan, 15 of the 17 elections lead to fewer elected Democrats than the median number of elected Democrats over the ensemble, and 0 of the 17 elections lead to more elected Democrats than the median number of elected Democrats: this would provide a value of $0 - 15 = -15$ (see the 'seat shift' column in the table of Figure 3). We repeat this procedure for each plan in the ensemble. By contextualizing the net median skew within the ensemble of plans, we measure partisan bias in a way that is adapted to the geographic structure in the votes across the state. The baseline for what is typical is set by

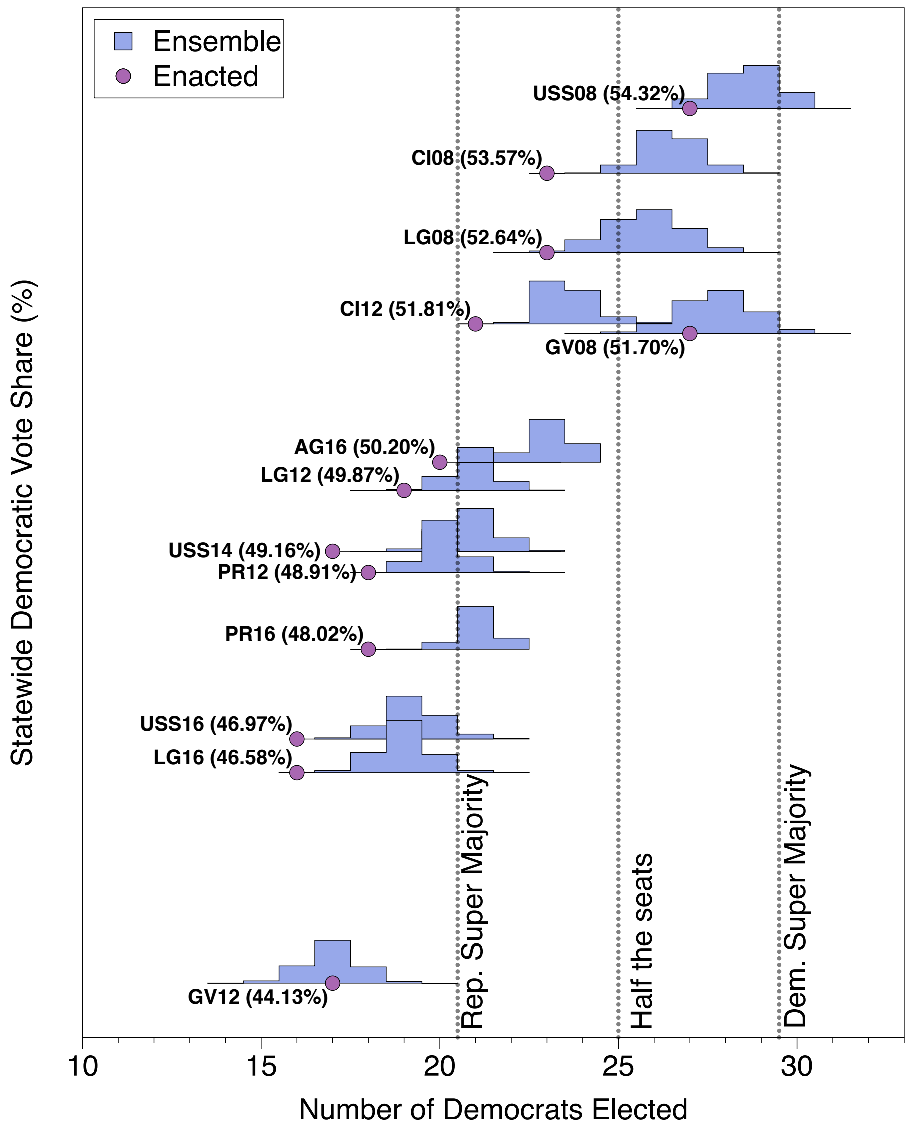


Figure 2: Each blue distribution represents the range of Democratic seats won in the ensemble of plans, given a set of historic votes; the height is the relative probability of observing the result. We only include a selection of the historic vote counts for clarity. Other vote counts either lead to qualitatively similar conclusions (e.g. USS10), or are well above the displayed range of statewide vote fractions (AG08). We label each distribution with an abbreviation for the votes used along with the Democratic statewide vote percentage. Abbreviations contain the year of the last two characters, and AG for Attorney General, USS for United States Senate, CI for Commissioner of Insurance, GV for Governor, LG for Lieutenant Governor, and PR for United States President.

the emergent ensemble of plans. To display this baseline, we plot the probability of seeing a certain net number of elections favoring each party and contextualize the enacted plan in this distribution (see the upper right figure in Figure 3).

We find that less than 0.03% of plans in the ensemble have a net median skew of -15 or less, meaning that it is highly atypical to find a neutrally selected plan from the ensemble which favors the Republican Party as consistently as does the enacted plan.

Although the enacted plan favors the Republicans in an atypically large number of elections, it may be the case that degree of skew within any particular election is typical of the ensemble. To test this we ask how often plans in the ensemble are as far, or farther, away from the median number of elected Democrats in the ensemble of plans. Under many historic vote counts, the number of Democrats elected in the enacted plan is abnormally low with respect to the ensemble. In 12 of the 17 examined elections, less than 2% of the plans in the ensemble give a skew from the median that is the same or greater than is seen in the enacted plan. In 9 of the 17 examined elections (including all 5 elections in 2016), less than 0.04% of the plans in the ensemble give a skew from the median that is the same or greater than is seen in the enacted plan (see the '%' column in the table in Figure 3). In short, we find that the number of seats elected by the enacted plan is an extreme outlier with respect to the ensemble in the majority of considered elections.

Besides just noting the extremity of the enacted plan in the context of the ensemble, we also quantify the *number* of seats by which the enacted plan typically differs from the results of the ensemble. To do so, we calculate the median number of seats elected in the ensemble of maps for each set of votes. For each election, we tabulate the difference between the number of Democrats elected by each ensemble map, and the median number of Democrats elected over the ensemble. We then average this difference in seats over all elections. We repeat this procedure for the enacted plan and, again, contextualize the result in the ensemble of results.

Before turning to the enacted plan, we make a few observations about the ensemble of plans: In the ensemble, there are roughly the same number of maps biased (relative to the median) toward the Republican Party as the Democratic Party. The average seat bias across the elections considered is typically less than one in magnitude with nearly all of the ensemble plans having an average seat bias less than two.

The enacted plan, in contrast, deviates from the median number of elected Democrats, on average, by just over two seats in favor of the Republicans (see the 'seat shift' column and last row in the table of Figure 3). We find that this difference is greater than all plans in the ensemble (see the bottom right figure of Figure 3). The ensemble shows a typical average deviation of plus or minus half of one seat, and can range as high as plus or minus one seat, but we never see any plan that favors the Republican Party to the same extent with which the enacted plan does according to this metric. We see that in all five 2016 elections, the shift is 3 seats in favor of the Republican Party.

To understand the above results, it is not enough to understand *who* won the election, but we must also understand the *margin* of the partisan victory. We compare the typical margins of victory in the ensemble with the enacted plan by taking a similar approach to that taken previously (see [1, 2, 3]): for a given election and a given plan, we rank the districts from least to most Democratic. By examining this ranking over a number of plans in the ensemble, we examine the typical Democratic/Republican vote fractions in the most Democratic district, the second most Democratic district, etc... and similarly in the most Republican district, the second most Republican district, and so on. We display the resulting ranked-vote marginal distributions for the middle 20 districts in two of the elections in Figure 4; we display the full ranked marginal distributions across all other elections in the appendix (see Section G).

The ranges of each ranked district are represented by box-plots in Figure 4. In these box-plots 50% of all plans have a corresponding ranked district that lies within the central box; the median is indicated by the line through the center of the box; the tick marks denote location of the 1% and 99% outliers; the extent of the lines outside of the boxes represent the full range of results observed in the ensemble (i.e. the maximum and minimums). We compare the ranked-votes curve of the enacted plan with the ranked-votes marginal distributions. There are 50 seats. Any dot (or box) that lies above the 50% line on the vertical axis will elect (or typically elect) a Democrat; any dot (or box) that lies below the 50% line will elect (or typically elect) a Republican.

The enacted plan has significantly fewer democratic votes in the middle districts than is typical of the ensemble. In the 2008 Commissioner of Insurance race, we see that the Democrats would typically win about 26 seats, as the (24th ranked marginal distribution straddles the 50% line), but that the enacted plan leads to 23 elected Democrats, with corresponding ranked districts having far fewer Democratic votes than is typical. When the statewide vote share shifts toward the Republicans, as in the 2016 Attorney General election, the Democrats tend to win fewer seats, both in the ensemble and in the enacted plan, however the depression in the middle districts remains fairly consistent, as does the disparity between the expected number of seats going to the Democrats and the number of Democratic seats gained in the enacted plan.

To test the observation that the middle districts have an abnormally low Democratic vote fraction, we consider the 20th

Election	Median Dems. in Ensemble	Dems. elected in enacted plan	Seat shift	% of plans that are as far or farther from the median
USS10	15	15	0	-
GV12	17	17	0	-
LG16	19	16	R+3	0.03%
USS16	19	16	R+3	2.5e-3%
PR12	20	18	R+2	0.22%
LG12	21	19	R+2	1.28%
USS14	21	17	R+4	6.7e-3%
PR16	21	18	R+3	0.00%
PR08	22	19	R+3	0.01%
GV16	22	19	R+3	0%
CI12	23	21	R+2	0.01%
AG16	23	20	R+3	4.8e-3%
CI08	26	23	R+3	0.00%
LG08	26	23	R+3	1.49%
GV08	28	27	R+1	39.8%
USS08	29	27	R+2	9.32%
AG08	40	39	R+1	40.1%
Average	-	-	R+2.24	-

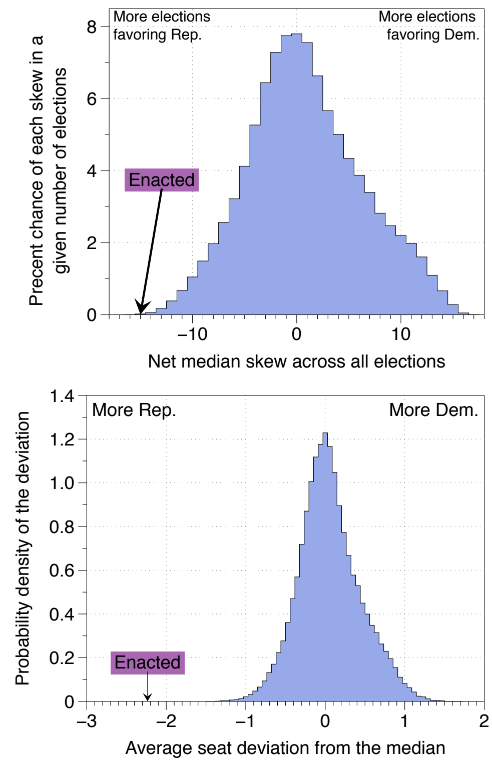


Figure 3: In the table (left), we list the median number of elected Democrats over the plans in the Senate ensemble for each considered election. We then list the number of Democrats that would have been elected by the enacted plan for each election. We calculate the difference between the median number of Democrats and the Democrats that would have been elected by the enacted plan, and then display the chance that a random plan from the ensemble would be as far or farther than the enacted plan is away from the median. We then plot the net median skew (out of 17 elections) that give a favorable result to the Democrats versus those that give a favorable result to the Republicans; a skewed result is one in which a party wins more than the median number of seats (top right; the number of skewed elections for the enacted plan is 15 for the Republicans and 0 for the Democrats over the 17 considered elections). Next, we show the amount the plans in the ensemble deviate from the median number of seats in the ensemble, averaged over the set of historic vote counts. In both cases, we find that the enacted plan is an extreme outlier when compared to the ensemble of plans.

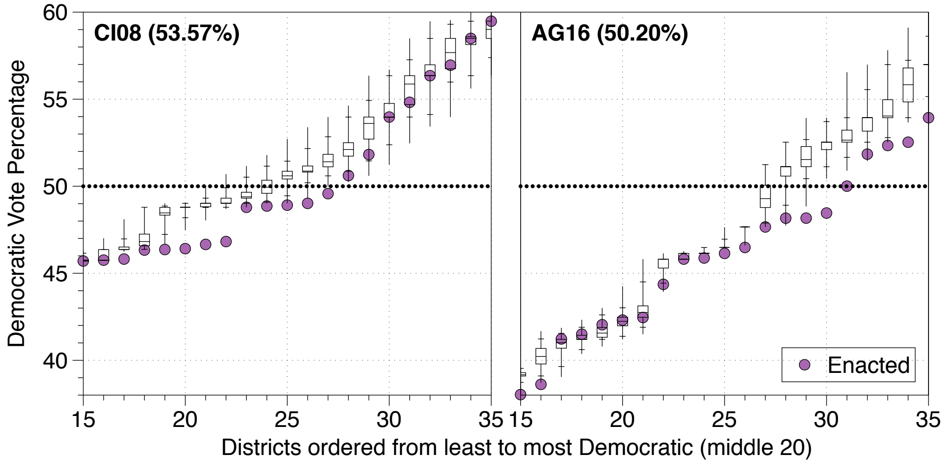


Figure 4: Over two elections, we plot the typical range of the 15th least Democratic district to the 35th least democratic district. The ranges are represented by box-plots: 50% of all plans in the ensemble have a corresponding ranked district that lies within the box; the median is given by the line within the box; the ticks mark the 1% and 99% quartiles; the extent of the lines outside of the boxes represent the range of results observed in the ensemble. We compare the ranked-votes curve of the enacted plan (purple dots) with the ranked-votes marginal distributions (box plot). There are 50 seats; any dot (or box) that lies above the 50% line on the vertical axis will elect (or typically elect) a Democrat; any dot (or box) that lies below the 50% line will elect (or typically elect) a Republican.

through 30th least Democratic districts and consider the average vote share in this range. We then compare average vote share in the ensemble with the average vote share in the enacted plan. We choose the ranges at 20 and 30 because this is where power shifts to give a supermajority for either party. In 14 of the 17 elections, we see that fewer than 0.0005% of the plans in the ensemble have fewer average Democrats in these middle districts than the enacted plan; in the AG08 and LG08 elections, we find that fewer than 0.1% of the plans in the ensemble have fewer Democrats in the middle plans than the enacted plan. There is only one anomalous election —GV08 —in which roughly 80% of all plans in the ensemble have more Democrats in the middle districts than in the enacted plan (see Section G).

The above differences between the enacted plan and the ensemble can affect majority representation. In the 2008 Commissioner of Insurance election the Democrats are expected to gain a majority of the seats in over 90% of the plans in the ensemble; in the 2008 Lieutenant Governor race the Democrats are expected to gain a majority of the seats in over 59% of the plans; yet in both sets of election data, the Republicans win a majority of the seats under the enacted plan. In contrast, we find no elections under which the Republicans would have been expected to receive a majority under the ensemble, but would not receive a majority in the enacted plan.

The differences in the ensemble and the enacted plan also can affect the supermajority. Under two of the elections (LG12, USS14) the ensemble of plans yields a Republican supermajority in less than 30% of the plans, yet the enacted plan provides a Republican supermajority with 19 and 17 Democratic seats, respectively. In PR08, the Republicans do not gain a majority in over 98.7% of plans in the ensemble, whereas they do gain a supermajority under the enacted plan. In three other elections (AG16, GV16, PR16) the Republicans do not gain a supermajority in over 99.5% of the plans in the ensemble, whereas they gain a supermajority in each of these elections under the enacted plan.

3.2 North Carolina House

We continue with our statewide analysis of partisan gerrymandering in the North Carolina House. Having already introduced above the concept of using histograms capturing the distribution of the number of seats won by a given party, we begin by examining the broad range of possible partisan outcomes across multiple historic elections. We again plot histograms for a number of elections taken over the ensemble of plans for the House and compare what the result would have been when using the enacted plan (see Figure 5). We position the relative height of each histogram by Democratic statewide vote fraction. For a detailed break down over all elections, see Tables 2 and 4.

Each plan in the ensemble elects a range of Democrats; we view the median of this range as a baseline for each election. When comparing the medians of the ensemble with the results of the enacted plan over the 17 considered elections, we find that the enacted plan nearly always elects fewer Democrats than the median number of Democrats elected in the ensemble

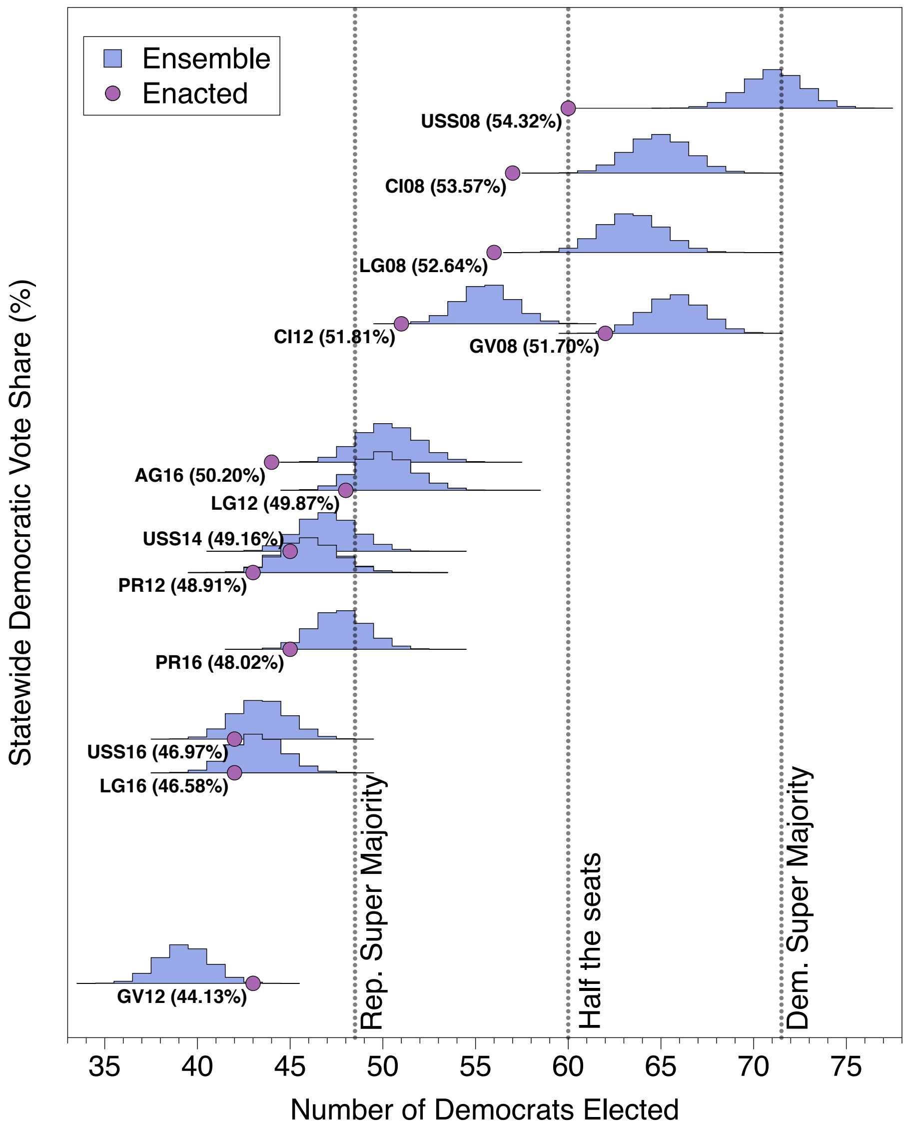


Figure 5: Each blue distribution represents the range of possible Democratic seats won in the ensemble of plans, given a set of historic votes; the height is the relative probability of observing the result. We only include a selection of the historic vote counts for clarity. Other vote counts either lead to qualitatively similar conclusions (e.g. USS10), or are well above the displayed range of statewide vote fractions (AG08). We label each distribution with an abbreviation for the votes used along with the Democratic statewide vote percentage. Abbreviations contain the year of the last two characters, and AG for Attorney General, USS for United States Senate, CI for Commissioner of Insurance, GV for Governor, LG for Lieutenant Governor, and PR for United States President.

(see the table of Figure 6). It is possible that some plans in the ensemble may consistently elect fewer Democrats than the median number of elected Democrats in the ensemble; alternatively, it is also possible that typical plans will alternate between electing more Democrats than the median in some elections, and fewer Democrats than the median in other elections. We now perform two analyses to shed light on which scenario is dominate.

We begin by testing if the enacted plan is atypical of the ensemble by determining the net number of elections for which a plan skews in favor of the Democratic or Republican Party. This tests if maps in the ensmble are typically biased (relative to the median) towards one party or the other or if they tend to switch back and forth. To calculate this number, we select a random plan and count the number of elections in which the plan elects more Democrats than the median number of elected Democrats; we then subtract the number of elections in which the plan elects fewer Democrats than the median number of elected Democrats. For example, in the enacted plan, 15 of the 17 elections lead to fewer elected Democrats than the median number of elected Democrats over the ensemble, and 2 of the 17 elections lead to more elected Democrats than the median number of elected Democrats: this would provide a value of $2 - 15 = -13$ (see the 'seat shift' column in the table of Figure 6). We repeat this procedure for each plan in the ensemble. By contextualizing the net median skew within the ensemble of plans, we measure partisan bias in a way that is adapted to the geographic structure in the votes across the state. The baseline for what is typical is set by the emergent ensemble of plans. To display this baseline, we plot the probability of seeing a certain net number of elections favoring each party and contextualize the enacted plan in this distribution (see the upper right figure in Figure 6).

The two plots in Figure 6 give interesting information about the structure of typical plans in the ensemble. Most plans have a net skewness close to zero with zero being the most frequent. This implies that most plans are biased towards one party about as often as towards the other party. Furthermore, the typical average deviation is well below one in magnitude. This implies that any biases which occur in the ensemble plans are not only largely equally distributed between the two parties but the sizes of the biases are comparable.

Comparing the enacted plan to the ensemble, we find that less than 1.4% of plans in the ensemble plans have a net median skew of -13 or less, meaning that it is very atypical to find a random plan which favors the Republican Party as consistently as does the enacted plan.

Although the enacted plan favors the Republicans in an atypically large number of elections, it may be the case that degree of skew within any particular election is typical of the ensemble. To test this we ask how often plans in the ensemble are as far, or farther, away from the median number of elected Democrats in the ensemble of plans. Under many historic vote counts, the number of Democrats elected in the enacted plan is abnormally low with respect to the ensemble. When considering the votes from elections PR08, AG16, GV16, CI12, LG08, CI08, USS08, and GV08, there are the same or fewer Democrats elected in the enacted plan than in less than 2% of the plans in the ensemble; when considering only the AG16, GV16, CI12, LG08, CI08, and USS08 elections, there are the same or fewer Democrats elected in the enacted plan than in less than 0.8% of the plans in the ensemble (see the '%' column in the table in Figure 6). In short, we find that the number of seats elected by the enacted plan is a significant outlier with respect to the ensemble in the majority of considered elections.

Besides just noting the extremity of the enacted plan in the context of the ensemble, we also quantify the *number* of seats by which the enacted plan typically differs from the results of the ensemble. To do so, we calculate the median number of seats elected in the ensemble of maps for each set of votes. For each election, we tabulate the difference between the number of Democrats elected by each map, and the median number of Democrats elected over the ensemble. We then average this difference in seats over all elections. We repeat this procedure for the enacted plan and, again, contextualize it in the ensemble of results.

We find that, on average, the enacted plan deviates from the median number of elected Democrats by just over three seats in favor of the Republicans (see the 'seat shift' column and last row in the table of Figure 6). We find that this difference is greater than all plans in the ensemble (see the bottom right figure of Figure 6). The ensemble shows a typical average deviation of plus or minus one seat, and can range as high as plus or minus two seats, but we never see any plan that favors the Republican Party to the same extent with which the enacted plan does according to this metric.

To understand the above results, we again examine the ranked-vote marginal distributions (see Section 3.1). In this case, we display the resulting ranked-vote marginal distributions for the middle 40 districts in two of the elections in Figure 7; we display the full ranked marginal distributions across all other elections in the appendix (see Section G).

Again, the ranges of each ranked district are represented by box-plots: 50% of all plans have a corresponding ranked district that lies within the box; the median is given by the line within the box; the ticks mark the 1% and 99% quartiles; the extent of the lines outside of the boxes represent the range of results observed in the ensemble. We compare the ranked-votes curve of the enacted plan with the ranked-votes marginal distributions. There are 50 seats; any dot (or box) that lies above the 50% line on the vertical axis will elect (or typically elect) a Democrat; any dot (or box) that lies below the 50% line will

Election	Median Dems. in Ensemble	Dems. elected in enacted plan	Seat shift	% of plans that are as far or farther from the median
USS10	36	39	D+3	5.36%
GV12	39	43	D+4	1.05%
LG16	43	42	R+1	29.4%
USS16	43	42	R+1	25.1%
PR12	46	43	R+3	4.19%
USS14	47	45	R+2	15.7%
PR16	48	45	R+3	5.47%
PR08	49	44	R+5	0.62%
LG12	50	48	R+2	14.1%
AG16	50	44	R+6	1.3e-2%
GV16	51	47	R+4	0.71%
CI12	56	51	R+5	0.20%
LG08	63	56	R+7	2.2e-3%
CI08	65	57	R+8	4.8e-4%
GV08	66	62	R+4	1.52%
USS08	71	60	R+11	6.1e-9%
AG08	94	92	R+2	22.9%
Average	-	-	-3.35	-

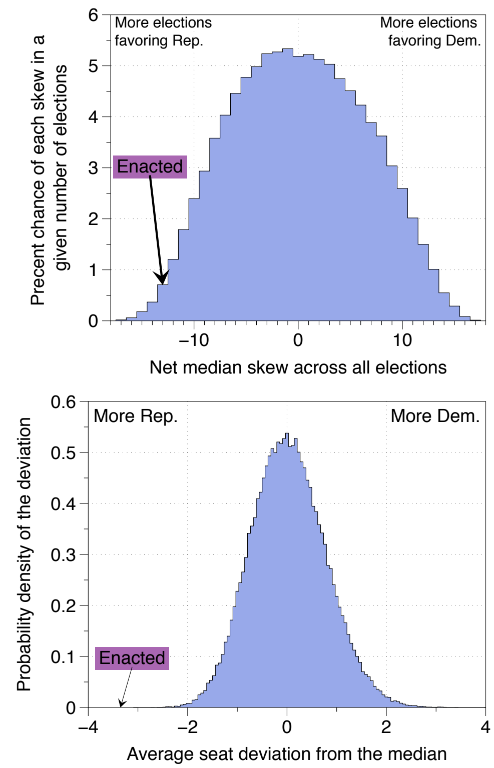


Figure 6: In the table (left), we list the median number of elected Democrats over the plans in the House ensemble for each considered election. We then list the number of Democrats that would have been elected by the enacted plan for each election. We calculate the difference between the median number of Democrats and the Democrats that would have been elected by the enacted plan, and then display the chance that a random plan from the ensemble would be as far or farther than the enacted plan is away from the median. We then plot the net median skew (out of 17 elections) that give a favorable result to the Democrats versus those that give a favorable result to the Republicans; a skewed result is one in which a party wins more than the median number of seats (top right; the number of skewed elections for the enacted plan is 15 for the Republicans and 2 for the Democrats over the 17 considered elections). Next, we show how the ensemble of plans deviates from the median number of votes averaged over each set of historic vote counts (bottom left; the enacted plan has an average difference of 3.35 seats in favor of the Republicans). In both cases, we find that the enacted plan is highly atypical of the plans in the ensemble.

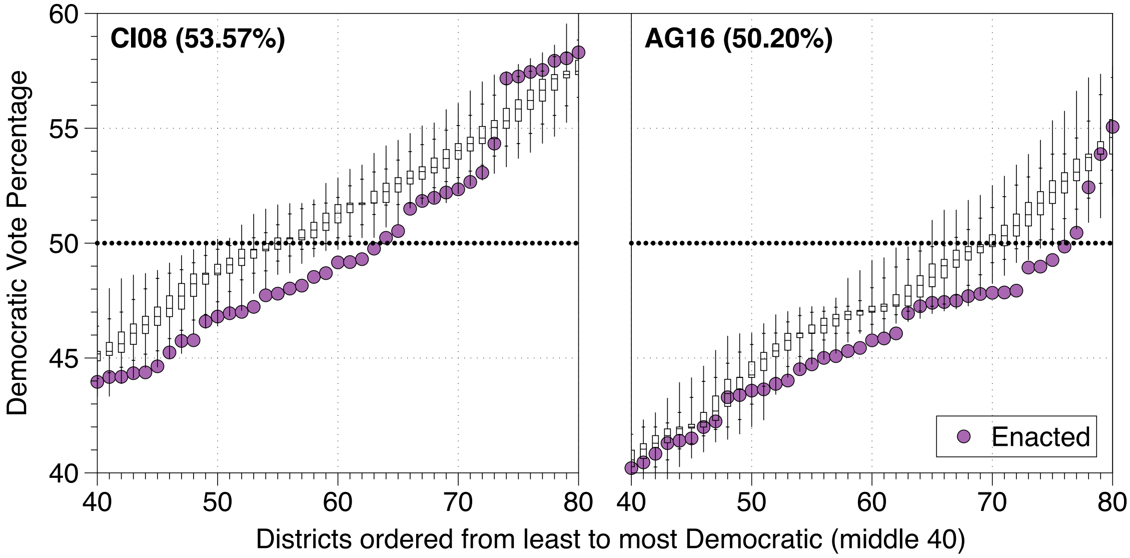


Figure 7: Over two elections, we plot the typical range of the 40th least Democratic district to the 80th least democratic district. The ranges are represented by box-plots: 50% of all plans have a corresponding ranked district that lies within the box; the median is given by the line within the box; the ticks mark the 1% and 99% quartiles; the extent of the lines outside of the boxes represent the range of results observed in the ensemble. We compare the ranked-votes curve of the enacted plan with the ranked-votes marginal distributions (purple dots). There are 120 seats; any dot (or box) that lies above the 50% line on the vertical axis will elect (or typically elect) a Democrat; any dot (or box) that lies below the 50% line will elect (or typically elect) a Republican.

elect (or typically elect) a Republican.

The enacted plan has significantly fewer votes in the middle districts than is typical of the ensemble. In the 2008 Commissioner of Insurance race, we see that the Democrats would typically win between 64 and 66 seats, as the (56th through 54th ranked marginal distributions are often above the 50% line), but that the enacted plan leads to corresponding districts with far fewer Democratic votes leading to strong Republican victories. In fact, the enacted plan does not cross the 50% line until the 64th seats, meaning that the Republican Party would have won a majority under these votes. When the statewide vote share shifts toward the Republicans, as in the 2016 Attorney General election, the Democrats win fewer seats, both in the ensemble and in the enacted plan, however the depression in the middle districts remains consistent.

To test the observation that the middle districts have an abnormally low Democratic vote fraction, we consider the 48th through 72th least Democratic districts and consider the average vote share of this range in the enacted plan compared with the ensemble. We choose this range because this is where power shifts to give a supermajority for either party. In 15 of the 17 elections, we find that there is less than a 0.0005% chance of finding a plan in the ensemble that has a smaller averaged Democratic vote share than the corresponding districts in the enacted plan. The only two exceptions to this are the 2016 Governor's race in which we find that less than 0.02% of the plans in the ensemble have a smaller Democratic vote share than the corresponding districts in the enacted plan, and the the 2008 Governor race in which we find that less than 0.3% of plans in the ensemble have a smaller Democratic vote share than the corresponding districts in the enacted plan. In terms of depletion of the Democratic vote in the middle (and more competitive districts) we find that the enacted plan is an extreme outlier, with significantly fewer Democratic voters than is expected within the context of the ensemble.

The above differences between the enacted plan and the ensemble can affect majority representation. In three of the elections we examine (USS08, CI08, LG08) the Democrats are expected to gain a majority of the seats in over 96% of the plans in the ensemble, yet in the enacted plan the Republicans would win a majority; In two of these elections (USS08, CI08) there is more than a 99.7% probability the Democrats would have achieved a majority of seats. In contrast, we find no elections under which the Republicans would have been expected to received a majority under the ensemble, but would not receive a majority in the enacted plan.

The differences in the ensemble and the enacted plan also can affect the supermajority. Under the LG12 election, less than 14.2% of plans in the ensemble lead to a Republican supermajority; under the AG16 election, less than 13% of plans in the ensemble lead to a Republican supermajority; and under the GV16 election, less than 4.4% of plans in the ensemble lead to a Republican supermajority. The enacted plan yields a supermajority for the Republicans in each of these cases.

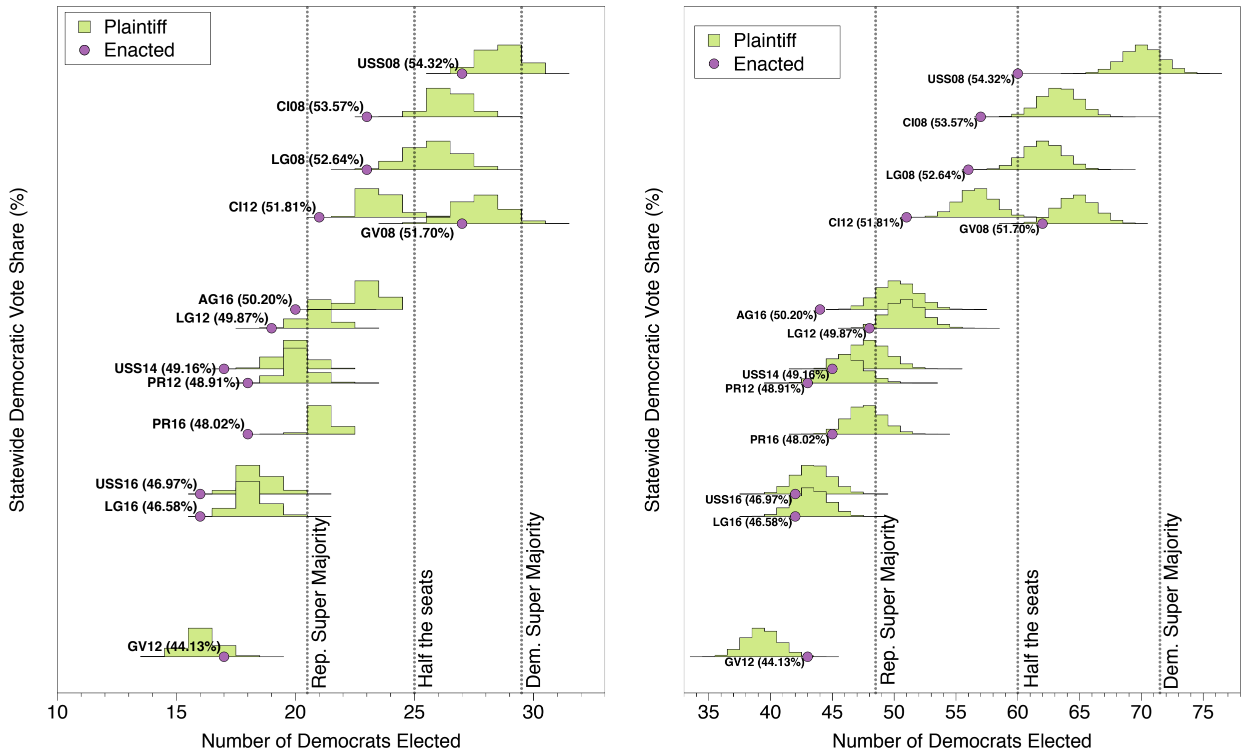


Figure 8: We plot the collection of histograms in the Senate (left) and House (right), when only considering sampling over the Plaintiff requested clusters. All other districts in other clusters are fixed to be the enacted districts.

In contrast, more than 40.1% of plans in the ensemble provide a supermajority for the Democrats under the USS08 votes, whereas the enacted plan does not even give the Democrats a majority of the seats.

4 Plaintiff Requested Cluster Analyses

The evidence for partisan gerrymandering found in Section 3 is the aggregated result of gerrymandering at the cluster level. It does not address *where* gerrymandering has occurred within the state. There are 16 county clusters in the House that the Plaintiffs have requested a cluster level analysis.[3] Similarly, there are seven clusters that the Plaintiffs have requested us to examine in the Senate.[4] We begin by asking if the anomalous effects seen at the statewide level are due largely to these clusters. To assess this, we generate new statewide ensembles that fix all other clusters and consider the ensembles of only these selected clusters. We plot this result in Figure 8. The similarity of these histograms with the results of the previous section (Figures 2 and 5) confirm that the majority of the deviation between the enacted plan and the ensemble of plans occurs in these clusters.

4.1 Summary of Cluster-by-Cluster Partisanship Analysis

The rank ordered vote histograms which were presented in Figures 2 and 5 will be repeated for each county-cluster. We are particularly interested in groups of districts which appear to have the percentage of a specific political party abnormally large or small when compared to the typical behavior revealed by the ensemble. Of course such statements depend on the particular structure of the election considered and hence should be understood under an election which might have been reasonably seen. Under a packing and cracking senario of gerrymandering, one often sees a group of districts that has an abnormally high vote margin for one party and a second group of districts that haas an abnormally low vote margin for the same party.

[3]These clusters in the House are Union-Anson, Nash-Franklin, Alamance, Yadkin-Forsyth, Columbus-Pender-Robeson, Mecklenburg, Duplin-Onslow, Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly, NewHanover-Brunswick, Guilford, Cumberland, Buncombe, Gaston-Cleveland, Wake, Person-Granville-Vance-Warren, and Pitt-Lenoir.

[4]These clusters in the Senate are Guilford-Alamance-Randolph, Bladen-Brunswick-Pender-NewHanover, Wake-Franklin, Mecklenburg, Lee-Sampson-Harnett-Duplin-Johnston-Nash, Davie-Forsyth, and Buncombe-Transylvania-Henderson

As the total number of votes for a particular party in a county cluster will not change once the election under consideration is fixed, creating a group of districts where one party is decreased often has the effect of creating a complimentary group of districts where the party is concentrated. Hence one often sees a large *jump* between the two groups. This effect is well illustrated in either the House or Senate for Mecklenburg County (see Figure 10 and Figure 40) or for the Wake-Franklin Cluster (see Figure 12 and Figure 52).

The large jump translates to a large range over which the results of the election would not change if one were to assume a uniform swing across all of the precincts in favor of one party or the other. This means that the partisan make up of the delegation from this county cluster will not change over a wider range of elections that would be seen in the ensemble of plans. County clusters which show this effect with a wide range of statewide partisan outcomes include Mecklenburg in the Senate (Figure 10) and House (Figure 40), Wake-Franklin in the Senate (Figure 12), Wake in the House (Figure 52), Yadkin-Forsyth in the House (Figure 54), Cumberland in the House (Figure 30), Davie-Forsyth in the Senate (Figure 14), Guilford in the Senate (Figure 20) and House (Figure 38), Columbus-Pender-Robeson in the House (Figure 28), New Hanover-Brunswick in the House (Figure 44), Pitt-Lenoir in the House (Figure 48), and Union-Anson in the House (Figure 50). To a lesser extent the phenomenon also occurs in Buncombe in the House (Figure 26), Buncombe-Henderson-Transylvania in the Senate (Figure 18), Bladen-Brunswick-Pender-New Hanover in the Senate (Figure 16), Duplin-Onslow in the House (Figure 34), and Nash-Franklin in the House (Figure 42). Lee-Sampson-Harnett-Duplin-Johnston-Nash in the Senate (Figure 22) shows a different, but related, effect where the enacted map tends to make the partisan make up of all of the districts the same while the maps in the ensemble of plans do not.

It is the combination of these atypical jumps across many county clusters which lock in the constantly biased results across a large number of election vote counts. Together these durable biases, even of just a seat or two in a given cluster, combine to produce the constantly and dramatically biased statewide results towards the Republican Party contained in Figures 3 and 6 and the accompanying tables.

4.2 Cluster-by-cluster Partisanship Analysis

For each of the clusters the plaintiffs requested analysed, we compare the ranked-votes marginal distributions of the ensemble, with the enacted plan. This analysis is very similar to the box-plots presented in Figures 2 and 5 above, however instead of using box plots to display our results, we use horizontal histograms. As an example, see Figure 10 which describes the Mecklenburg county cluster in the N.C. Senate: The third most Democratic district in the enacted plan is North Carolina Senate District 40. Over all six elections this district leads to a Democratic vote fraction that varies between around 75% and 80% in the Mecklenburg cluster. We can compare this result with the ensemble; in the USS14 race, the corresponding third most Democratic districts have a median of around 60% of their votes going to the Democrats, with a standard deviation of at most a few percentage points; we never see a plan in the ensemble that has three districts voting as overwhelmingly for the Democratic candidates in aggregate as in the enacted plan. The ordered histograms reveal this type of structure across all districts and all shown elections.

For clusters with districts drawn in 2011 and unchanged in 2017, we plot the enacted plan against the ensemble of plans when using the GV08, LG08, CI08, PR08, USS10, and AG16 elections. For clusters with districts that were redrawn in 2017, we use the CI12, USS14, GV16, AG16, PR16, and USS16 elections.

4.3 Mecklenburg Cluster (Senate)

We examine partisan gerrymandering in the Mecklenburg county cluster in the North Carolina state Senate. We contextualize the current district plan in Figure 9. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 10.

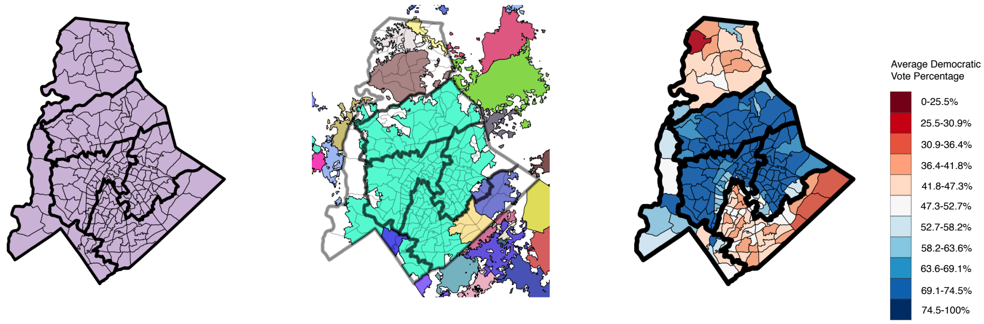


Figure 9: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

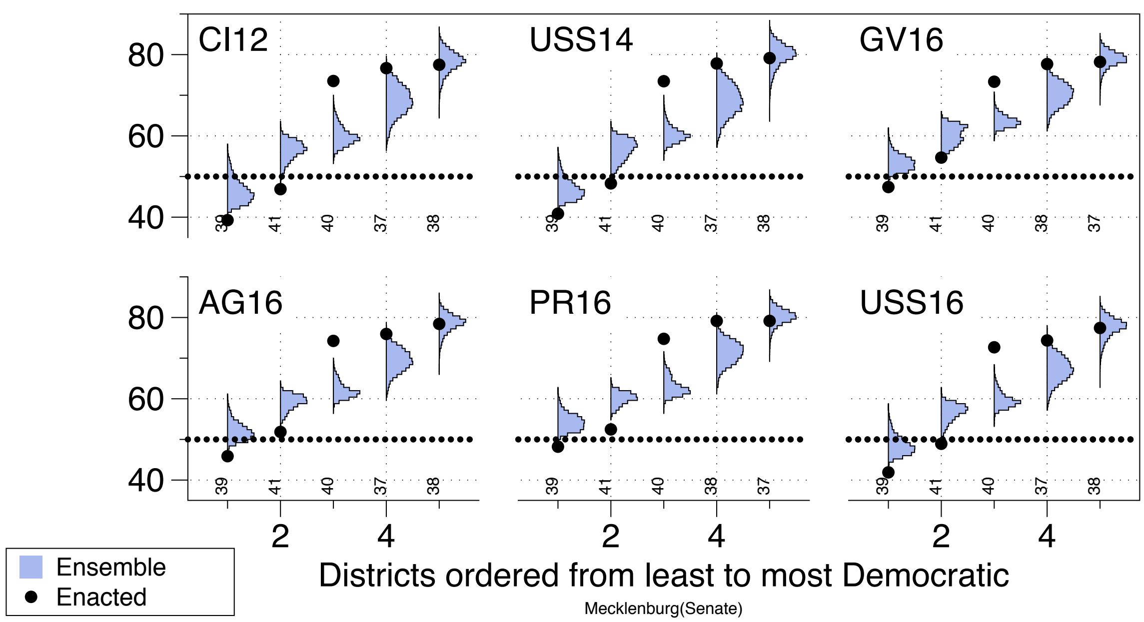


Figure 10: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 2 least Democratic districts are comprised of Districts 39 and 41 in all 17 elections, and the last 3 most Democratic districts are always comprised of Districts 37, 38, and 40. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the last 3 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 100% of the plans in the ensemble (all of the 6721 plans). Similarly, we find that the last 3 most Democratic districts have the same or more average fraction of Democrats than in 100% of the plans in the ensemble (all of the 6721 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.4 Wake-Franklin Cluster (Senate)

We examine partisan gerrymandering in the Wake-Franklin county cluster in the North Carolina state Senate. We contextualize the current district plan in Figure 11. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 12.

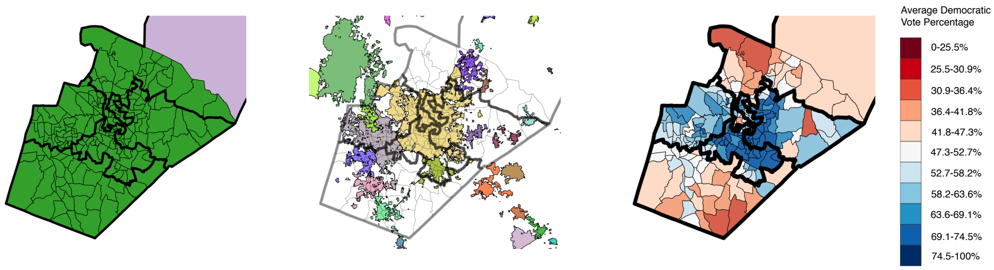


Figure 11: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

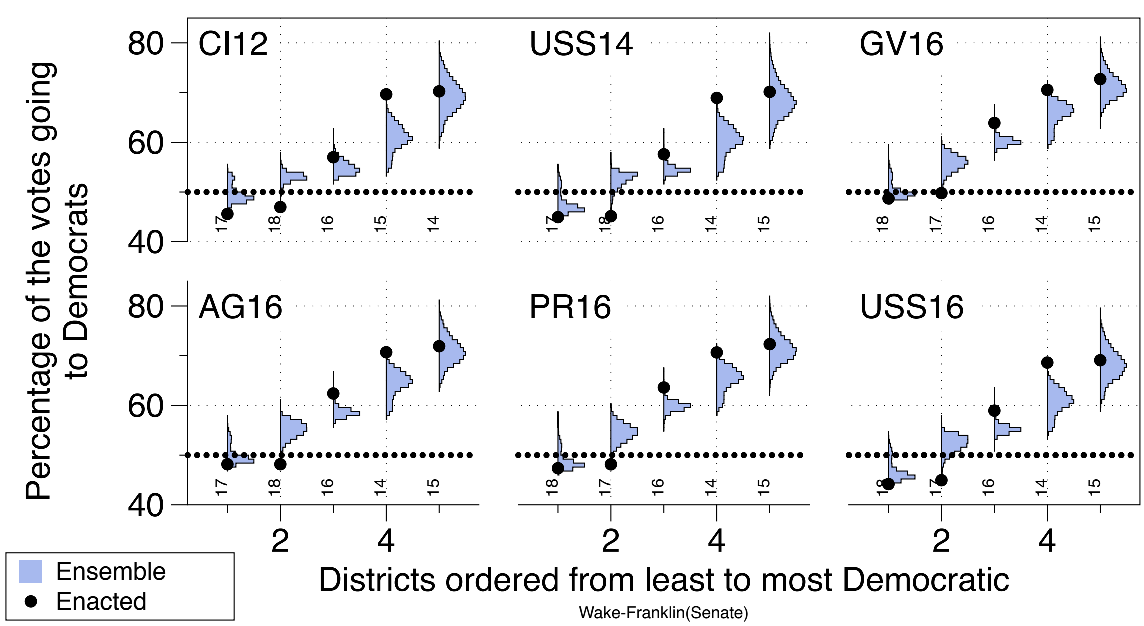


Figure 12: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 2 least Democratic districts are comprised of Districts 17 and 18 in all 17 elections, and the last 3 most Democratic districts are always comprised of Districts 14, 15, and 16. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the last 3 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 100% of the plans in the ensemble (all of the 14338 plans). Similarly, we find that the last 3 most Democratic districts have the same or more average fraction of Democrats than in 100% of the plans in the ensemble (all of the 14338 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.5 Davie-Forsyth Cluster (Senate)

We examine partisan gerrymandering in the Davie-Forsyth county cluster in the North Carolina state Senate. We contextualize the current district plan in Figure 13. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 14.

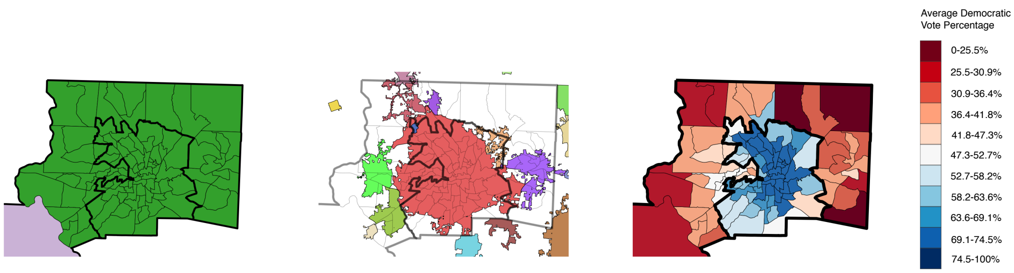


Figure 13: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

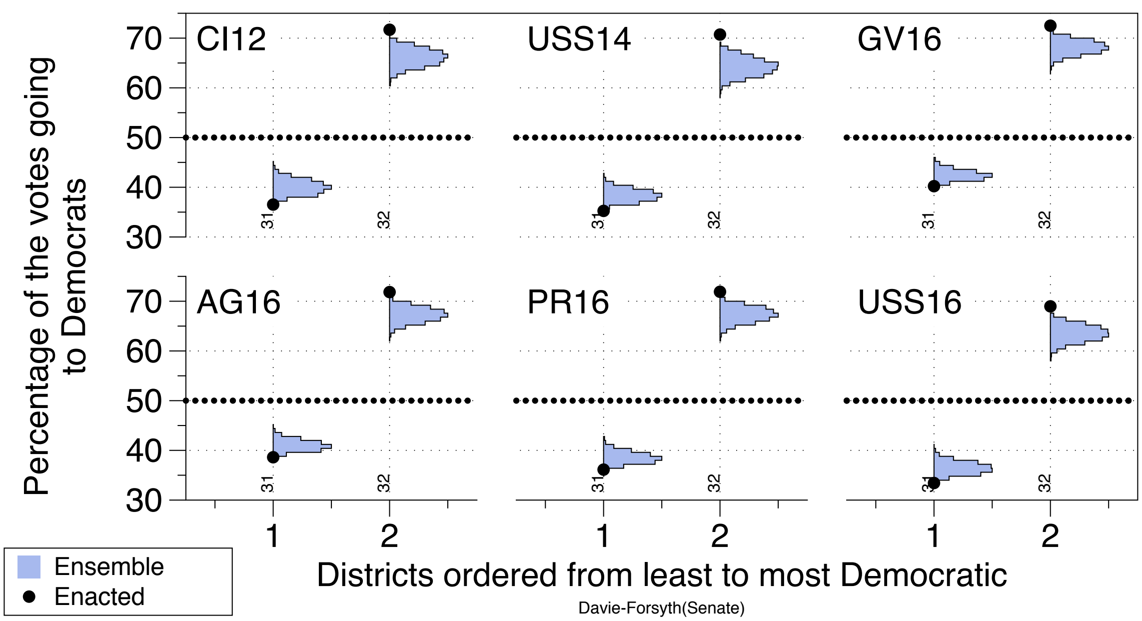


Figure 14: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the least Democratic district is District 31 in all 17 elections, and the most Democratic district is always District 32. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or fewer average fraction of Democrats than in 99.93% of the plans in the ensemble (approximately 2194 out of the 2196 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 100% of the plans in the ensemble (all of the 2196 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.6 Bladen-Brunswick-Pender-New Hanover Cluster (Senate)

We examine partisan gerrymandering in the Bladen-Brunswick-Pender-New Hanover county cluster in the North Carolina state Senate. We contextualize the current district plan in Figure 15. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 16.

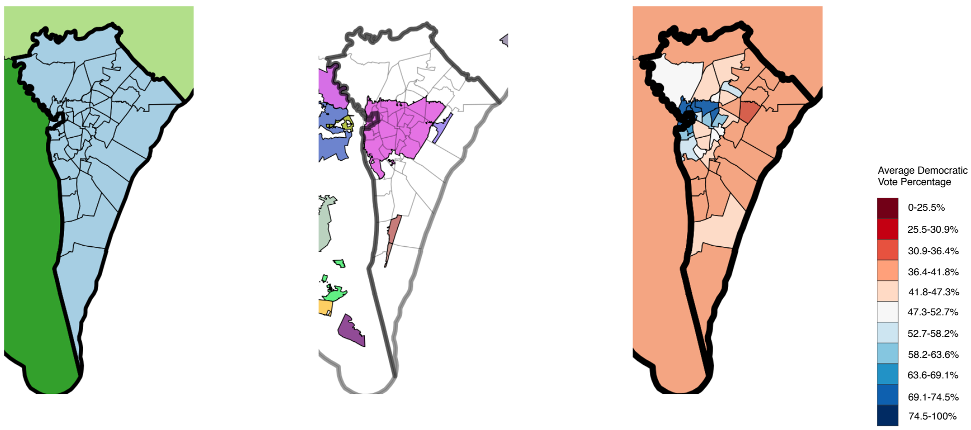


Figure 15: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

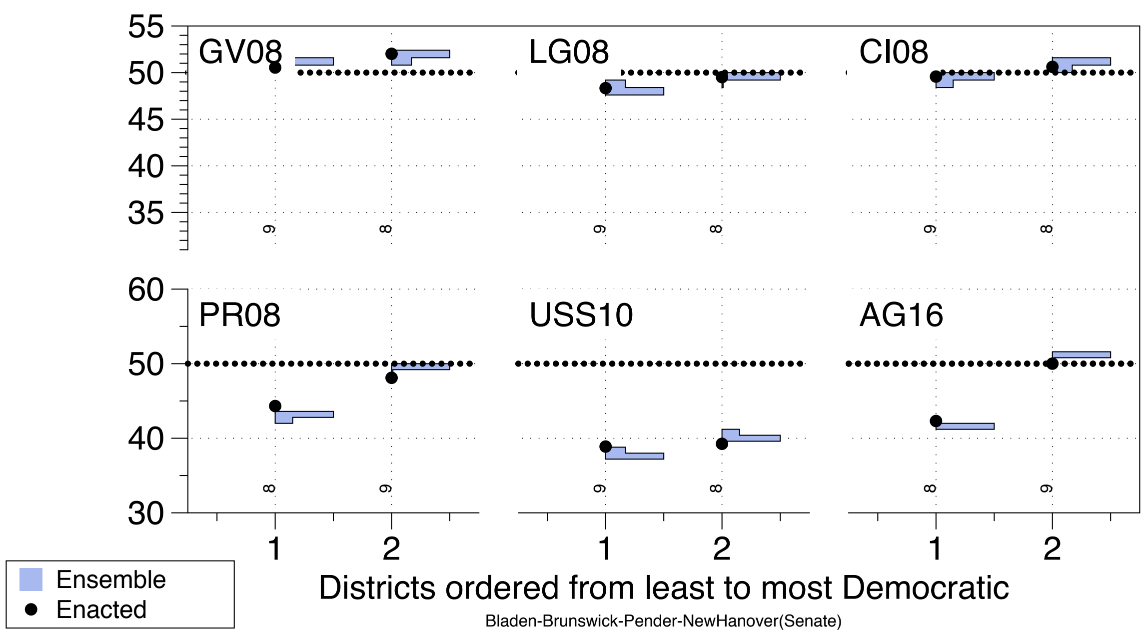


Figure 16: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, we group the least Democratic district, and the most Democratic district. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or more average fraction of Democrats than in 92.46% of the plans in the ensemble (approximately 47083 out of the 50921 plans). Similarly, we find that the most Democratic district has the same or fewer average fraction of Democrats than in 92.46% of the plans in the ensemble (approximately 47083 out of the 50921 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.7 Buncombe-Transylvania-Henderson Cluster (Senate)

We examine partisan gerrymandering in the Buncombe-Transylvania-Henderson county cluster in the North Carolina state Senate. We contextualize the current district plan in Figure 17. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 18.

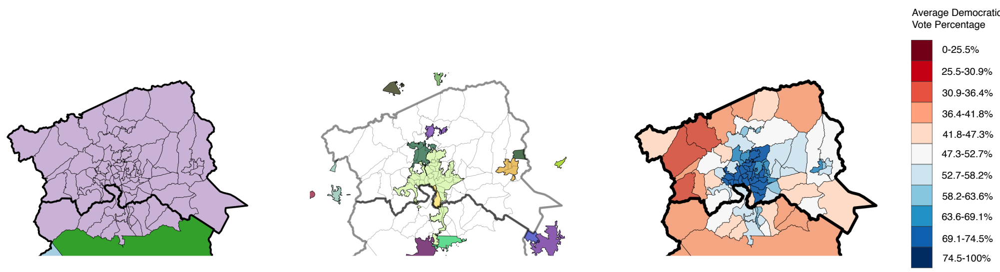


Figure 17: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

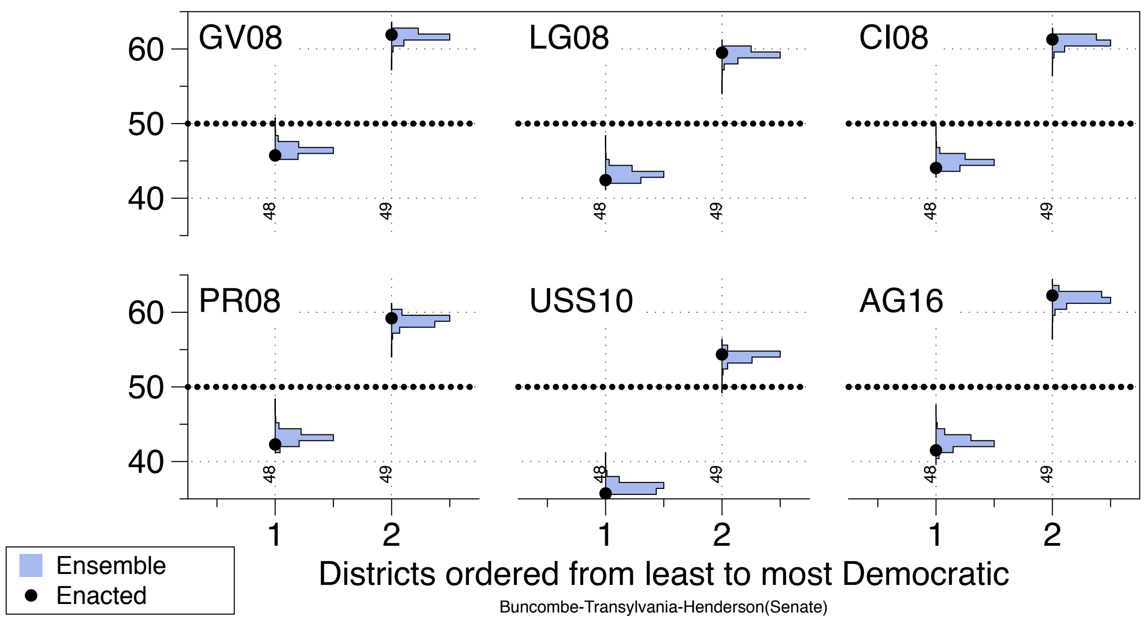


Figure 18: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the least Democratic district is District 48 in all 17 elections, and the most Democratic district is always District 49. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or fewer average fraction of Democrats than in 95.44% of the plans in the ensemble (approximately 37954 out of the 39764 plans). We also find that the least Democratic district has the same number of Democrats in 0.045% of plans in the ensemble (approximately 18 out of the 39764 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 72.34% of the plans in the ensemble (approximately 28767 out of the 39764 plans). We also find that the most Democratic district has the same number of Democrats in 0.045% of plans in the ensemble (approximately 18 out of the 39764 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.8 Guilford-Alamance-Randolph Cluster (Senate)

We examine partisan gerrymandering in the Guilford-Alamance-Randolph county cluster in the North Carolina state Senate. As metioned above, we freeze Districts 24 and 28 since these districts were drawn by the Special Master. We contextualize the current district plan in Figure 19. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 20.

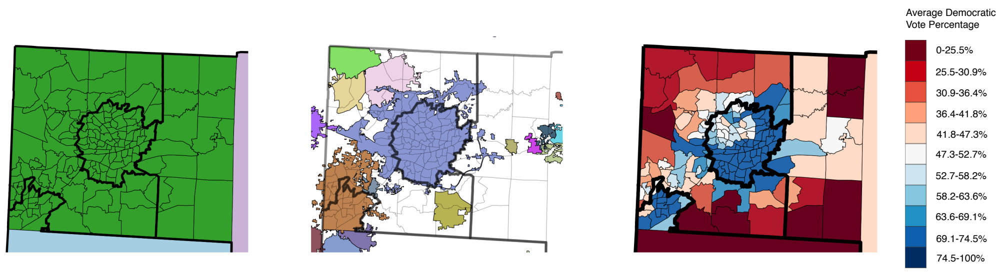


Figure 19: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

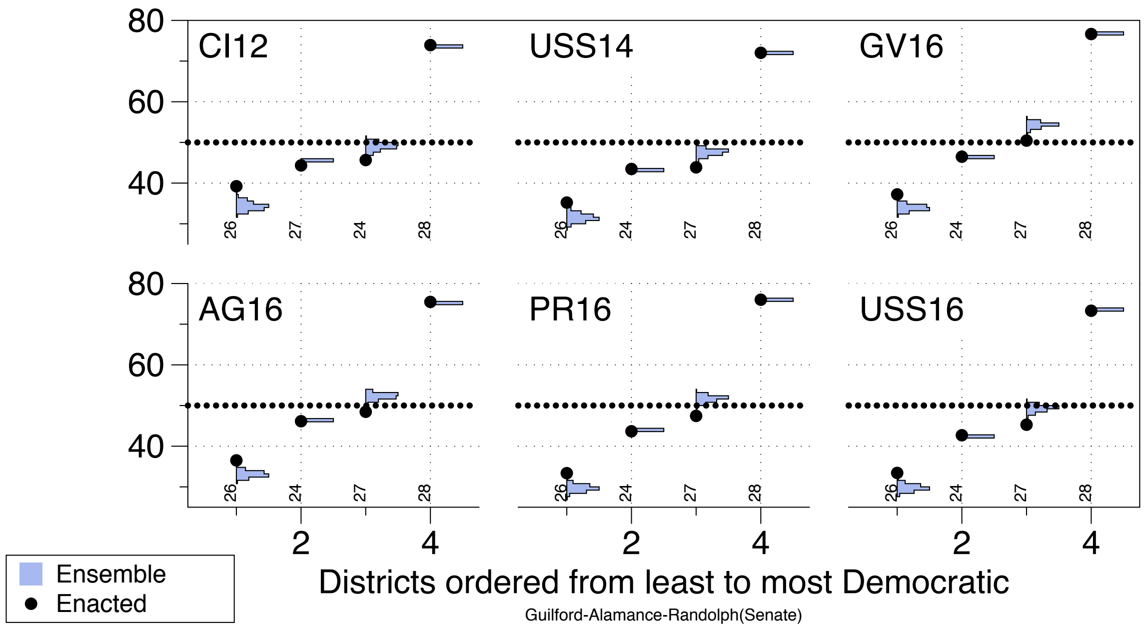


Figure 20: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, setting aside the frozen districts, the least Democratic district is District 26 in all 17 elections, and the most Democratic district is always District 27. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or more average fraction of Democrats than in 100% of the plans in the ensemble (all of the 711 plans). Similarly, we find that the most Democratic district has the same or fewer average fraction of Democrats than in 100% of the plans in the ensemble (all of the 711 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.9 Lee-Sampson-Harnett-Duplin-Johnston-Nash Cluster (Senate)

We examine partisan gerrymandering in the Lee-Sampson-Harnett-Duplin-Johnston-Nash county cluster in the North Carolina state Senate. We contextualize the current district plan in Figure 21. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 22.

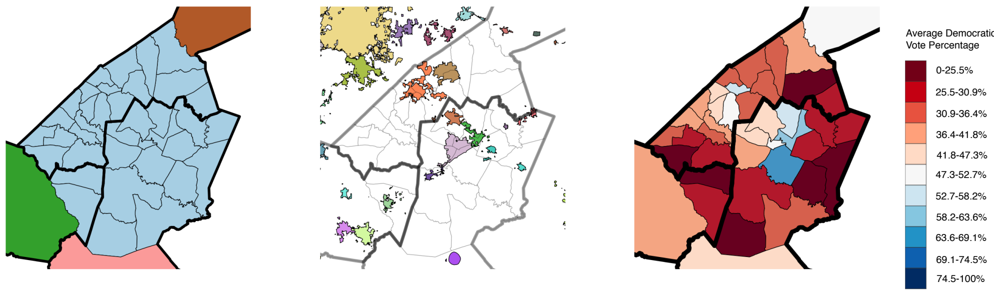


Figure 21: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

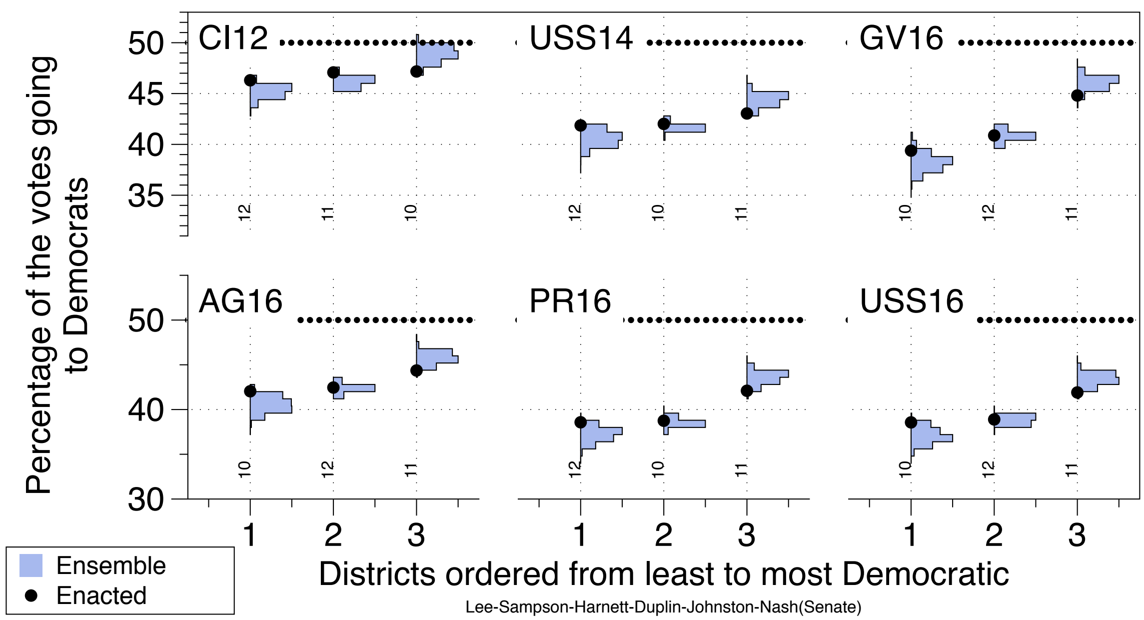


Figure 22: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, we group the first 2 least Democratic districts, and the most Democratic district. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or more average fraction of Democrats than in 77.21% of the plans in the ensemble (approximately 851 out of the 1103 plans). Similarly, we find that the most Democratic district has the same or fewer average fraction of Democrats than in 74.35% of the plans in the ensemble (approximately 820 out of the 1103 plans).

4.10 Alamance Cluster (House)

We examine partisan gerrymandering in the Alamance county cluster in the North Carolina state House. We contextualize the current district plan in Figure 23. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 24.

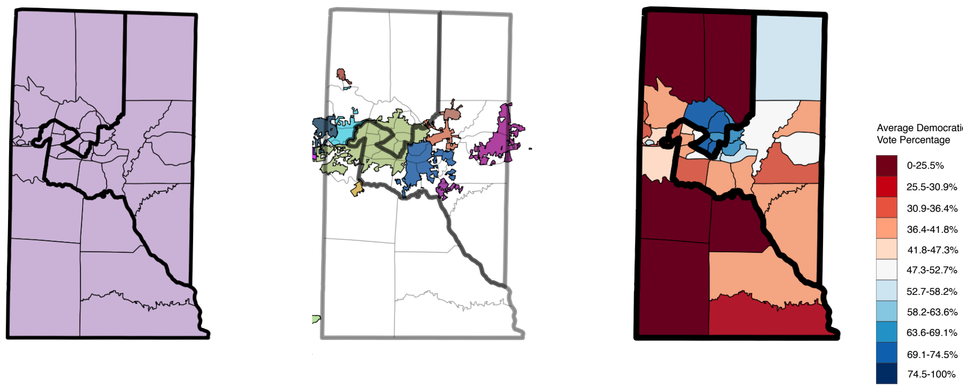


Figure 23: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

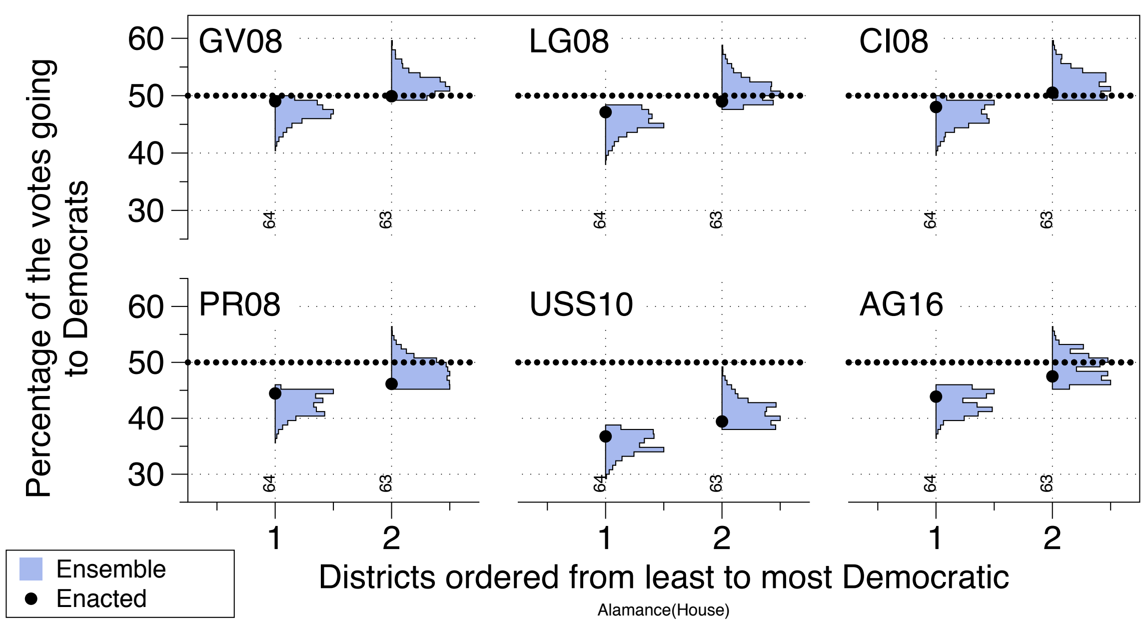


Figure 24: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the least Democratic district is District 64 in all 17 elections, and the most Democratic district is always District 63. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or more average fraction of Democrats than in 74.48% of the plans in the ensemble (approximately 1905 out of the 2558 plans). Similarly, we find that the most Democratic district has the same or fewer average fraction of Democrats than in 77.73% of the plans in the ensemble (approximately 1988 out of the 2558 plans).

4.11 Buncombe Cluster (House)

We examine partisan gerrymandering in the Buncombe county cluster in the North Carolina state House. We contextualize the current district plan in Figure 25. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 26.

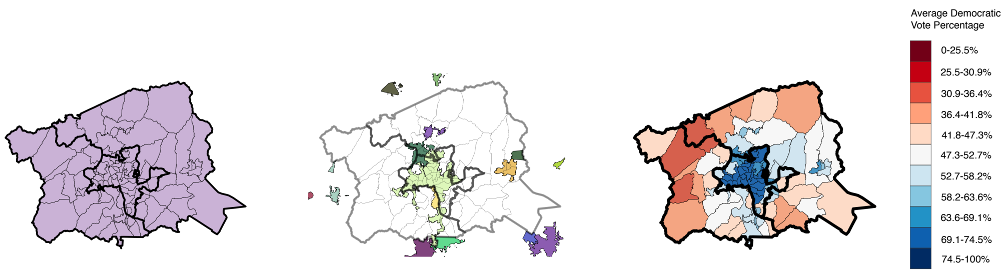


Figure 25: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

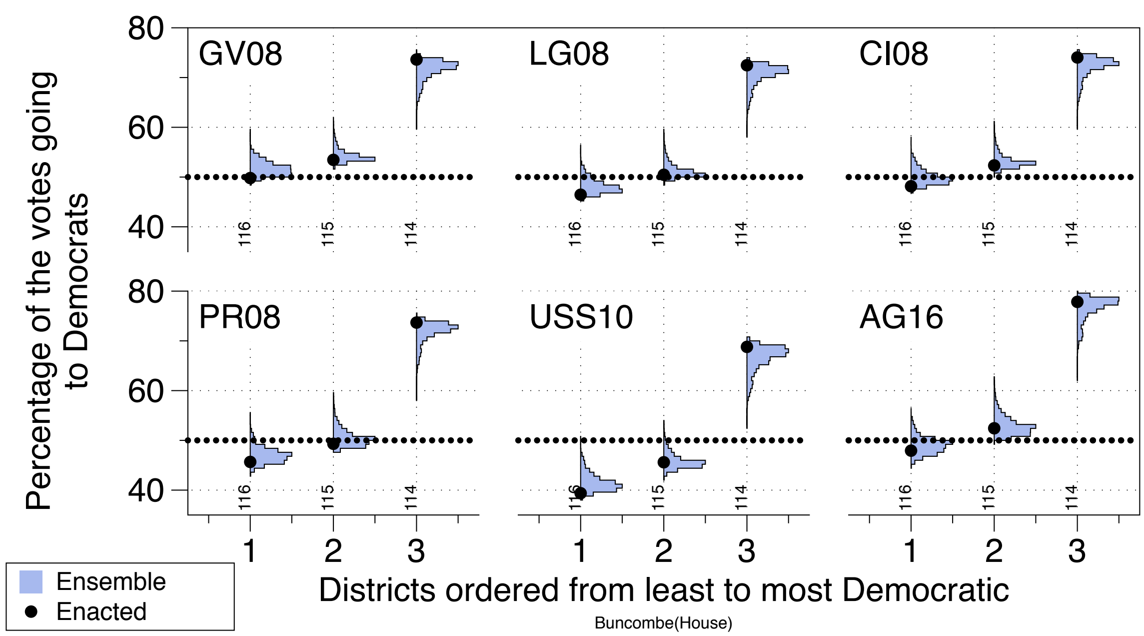


Figure 26: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 2 least Democratic districts are comprised of Districts 115 and 116 in all 17 elections, and the most Democratic district is always comprised of District 114. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 85.45% of the plans in the ensemble (approximately 3081 out of the 3606 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 78.69% of the plans in the ensemble (approximately 2837 out of the 3606 plans).

4.12 Columbus-Pender-Robeson Cluster (House)

We examine partisan gerrymandering in the Columbus-Pender-Robeson county cluster in the North Carolina state House. We contextualize the current district plan in Figure 27. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 28.

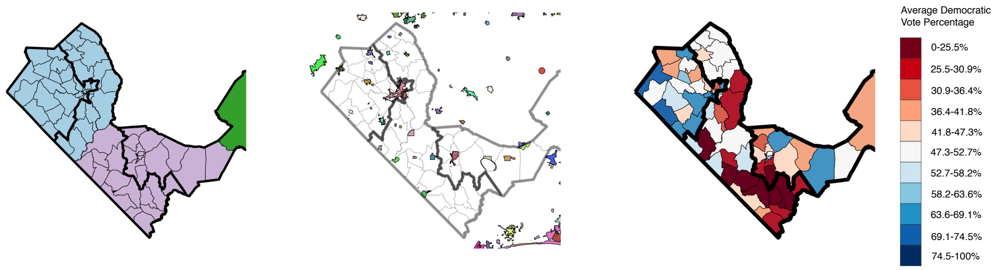


Figure 27: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

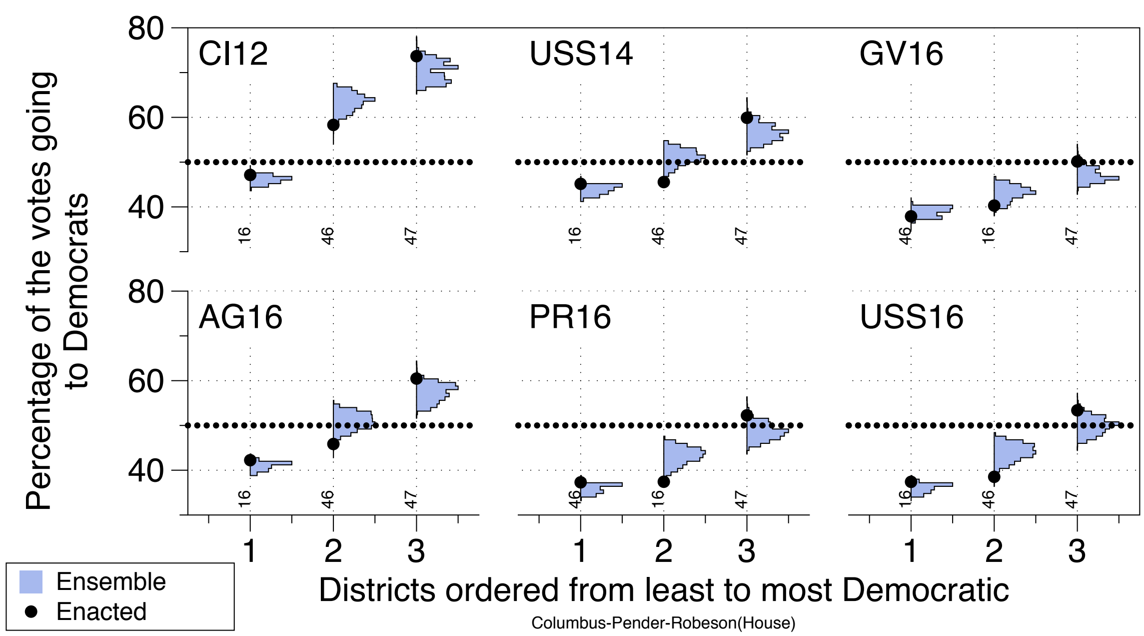


Figure 28: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 2 least Democratic districts are comprised of Districts 16 and 46 in all 17 elections, and the most Democratic district is always comprised of District 47. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 97.98% of the plans in the ensemble (approximately 15619 out of the 15941 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 93.77% of the plans in the ensemble (approximately 14948 out of the 15941 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.13 Cumberland Cluster (House)

We examine partisan gerrymandering in the Cumberland county cluster in the North Carolina state House. We contextualize the current district plan in Figure 29. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 30.

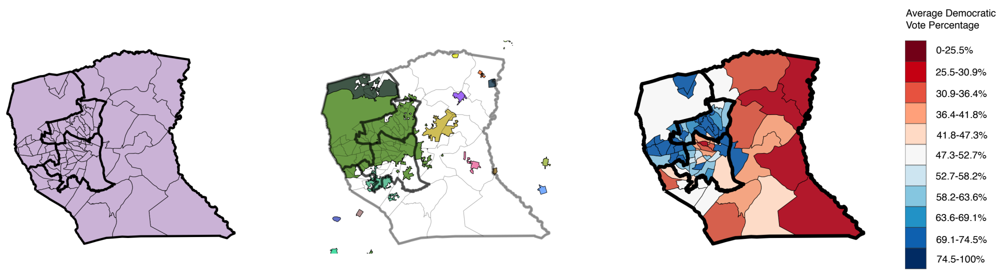


Figure 29: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

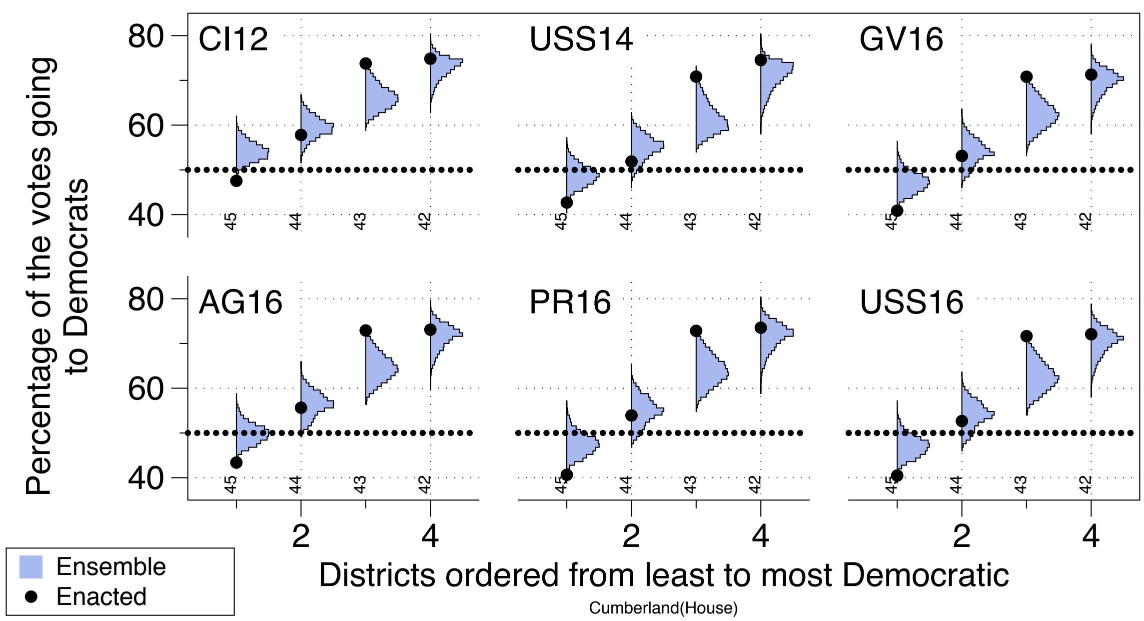


Figure 30: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 2 least Democratic districts are comprised of Districts 44 and 45 in all 17 elections, and the last 2 most Democratic districts are always comprised of Districts 42 and 43. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the last 2 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 99.73% of the plans in the ensemble (approximately 10340 out of the 10368 plans). Similarly, we find that the last 2 most Democratic districts have the same or more average fraction of Democrats than in 99.79% of the plans in the ensemble (approximately 10346 out of the 10368 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.14 Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly Cluster (House)

We examine partisan gerrymandering in the Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly county cluster in the North Carolina state House. We contextualize the current district plan in Figure 31. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 32.

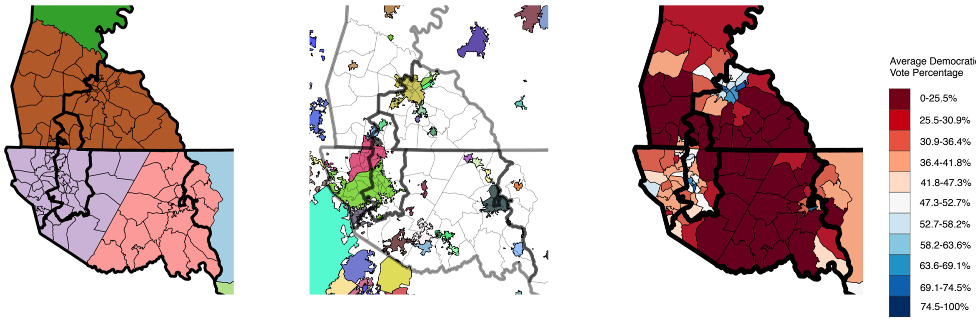


Figure 31: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

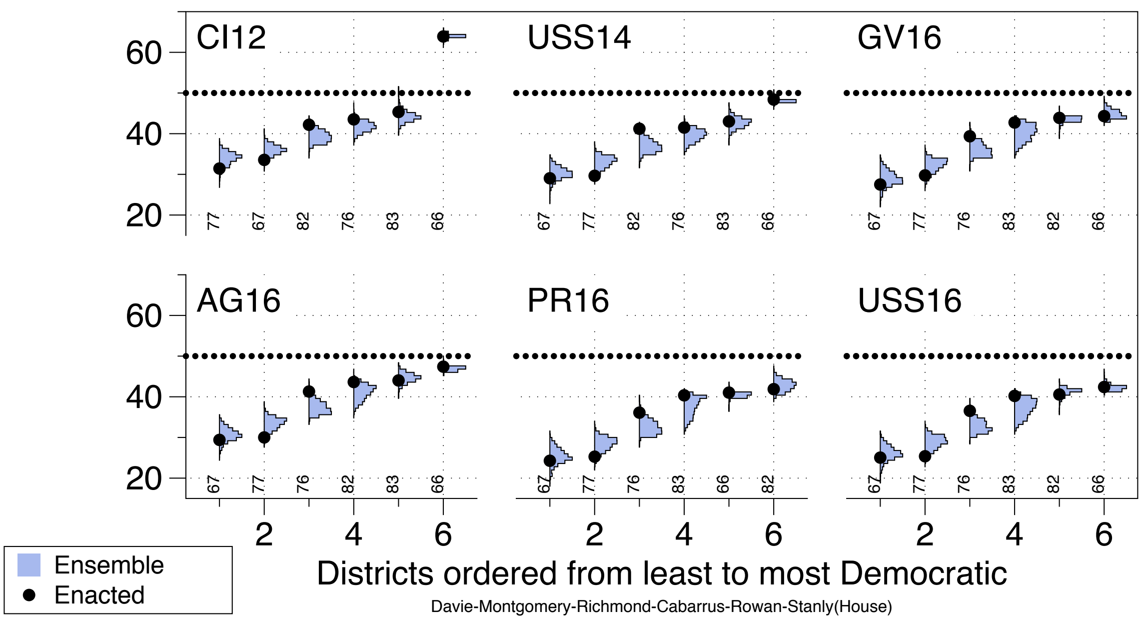


Figure 32: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, we group the first 2 least Democratic districts, and the last 4 most Democratic districts. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the last 4 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 94.96% of the plans in the ensemble (approximately 7336 out of the 7725 plans). Similarly, we find that the last 4 most Democratic districts have the same or more average fraction of Democrats than in 97.38% of the plans in the ensemble (approximately 7522 out of the 7725 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.15 Duplin-Onslow Cluster (House)

We examine partisan gerrymandering in the Duplin-Onslow county cluster in the North Carolina state House. We contextualize the current district plan in Figure 33. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 34.

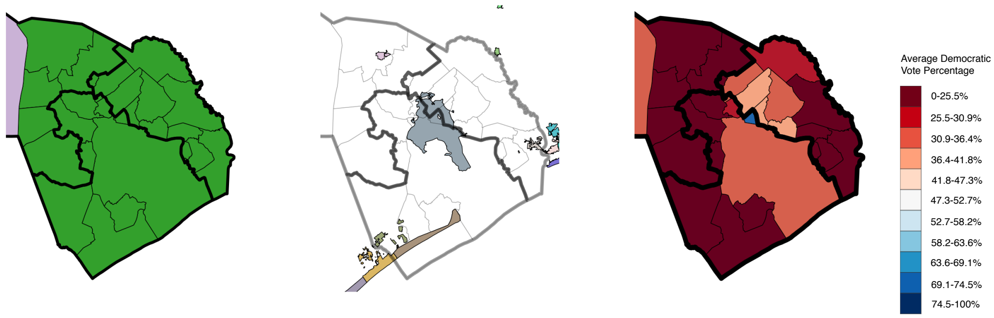


Figure 33: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

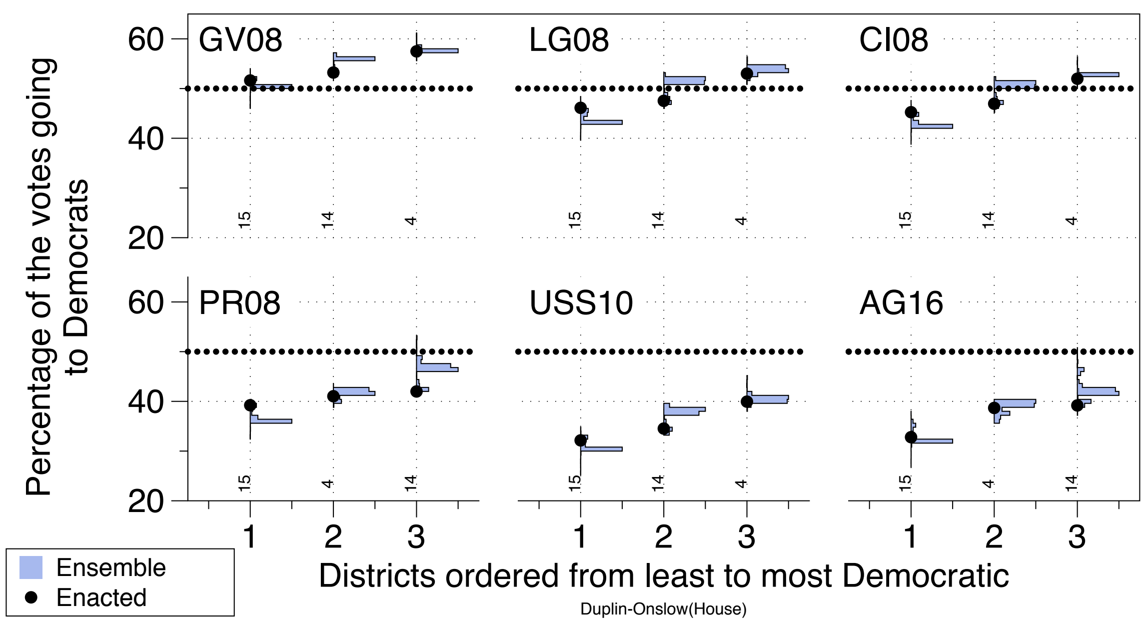


Figure 34: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the least Democratic district is District 15 in all 17 elections, and the last 2 most Democratic districts are always Districts 4 and 14. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the last 2 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or more average fraction of Democrats than in 87.79% of the plans in the ensemble (approximately 6749 out of the 7688 plans). Similarly, we find that the last 2 most Democratic districts have the same or fewer average fraction of Democrats than in 92.40% of the plans in the ensemble (approximately 7103 out of the 7688 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.16 Gaston-Cleveland Cluster (House)

We examine partisan gerrymandering in the Gaston-Cleveland county cluster in the North Carolina state House. We contextualize the current district plan in Figure 35. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 36.

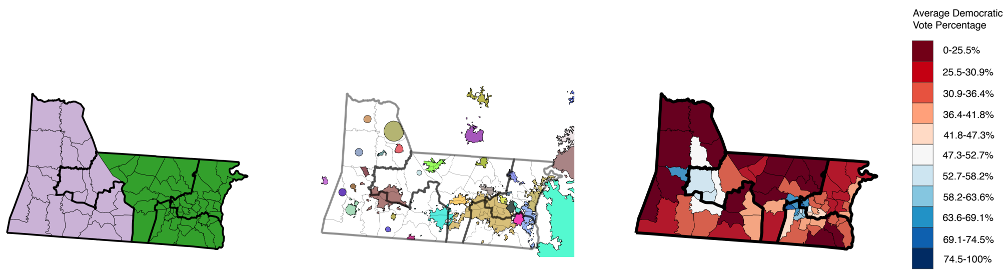


Figure 35: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

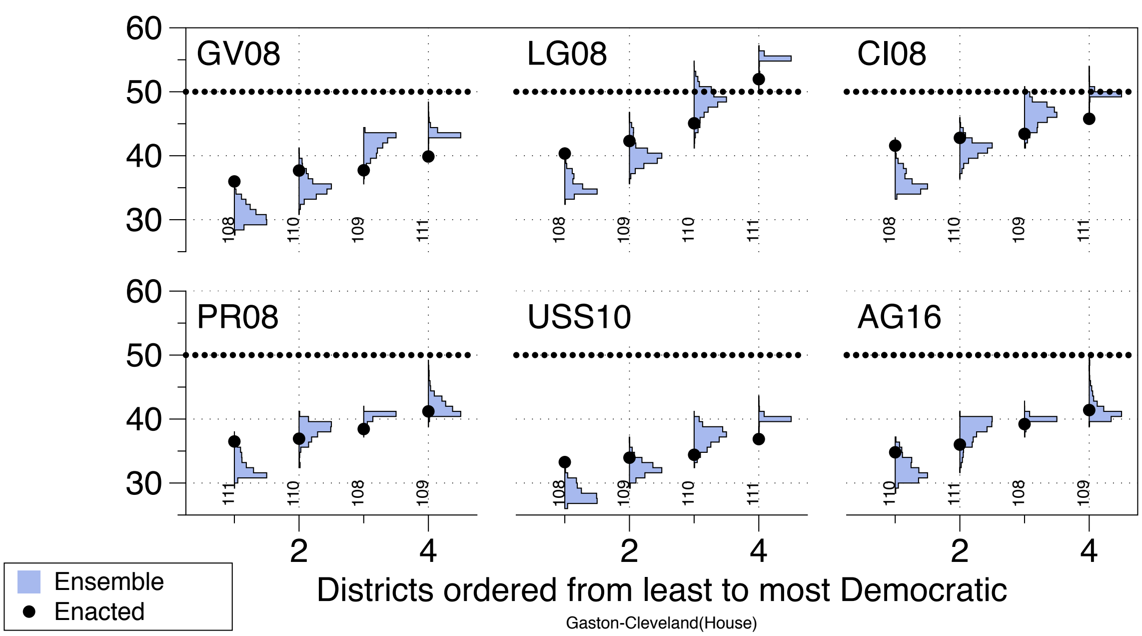


Figure 36: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, we group the first 2 least Democratic districts, and the last 2 most Democratic districts. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the last 2 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or more average fraction of Democrats than in 80.61% of the plans in the ensemble (approximately 7053 out of the 8750 plans). Similarly, we find that the last 2 most Democratic districts have the same or fewer average fraction of Democrats than in 82.86% of the plans in the ensemble (approximately 7250 out of the 8750 plans).

4.17 Guilford Cluster (House)

We examine partisan gerrymandering in the Guilford county cluster in the North Carolina state House. As mentioned above, we freeze Districts 57, 61, and 62 since these districts were drawn by the Special Master. We contextualize the current district plan in Figure 37. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 38.

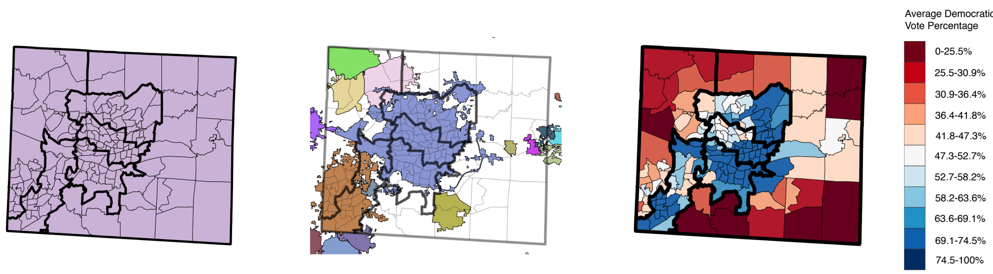


Figure 37: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

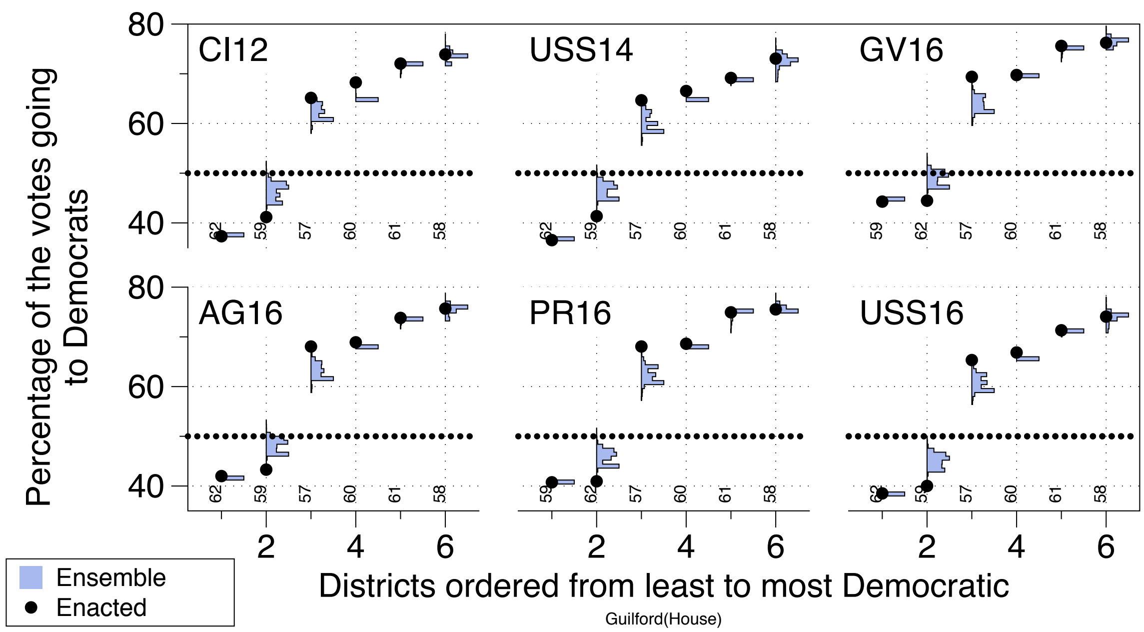


Figure 38: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, setting aside the frozen districts, the least Democratic district is District 59 in all 17 elections, and the last 2 most Democratic districts are always Districts 58 and 60. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the last 2 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or fewer average fraction of Democrats than in 99.89% of the plans in the ensemble (approximately 1440 out of the 1442 plans). Similarly, we find that the last 2 most Democratic districts have the same or more average fraction of Democrats than in 99.86% of the plans in the ensemble (approximately 1440 out of the 1442 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.18 Mecklenburg Cluster (House)

We examine partisan gerrymandering in the Mecklenburg county cluster in the North Carolina state House. We contextualize the current district plan in Figure 39. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 40.

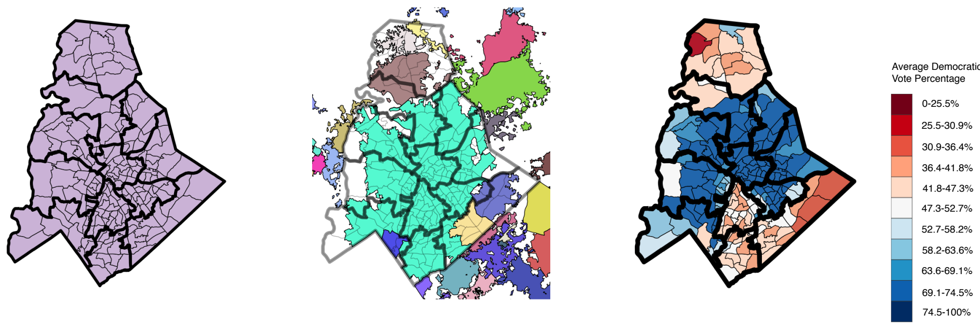


Figure 39: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

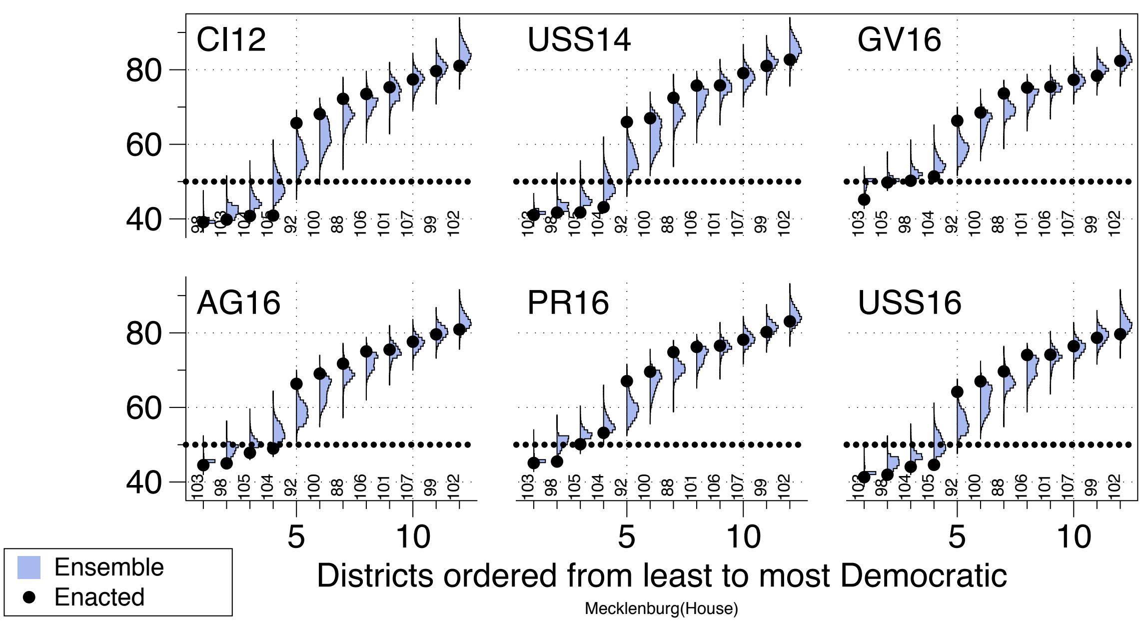


Figure 40: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 4 least Democratic districts are comprised of Districts 98, 103, 104, and 105 in all 17 elections, and the last 8 most Democratic districts are always comprised of Districts 88, 92, 99, 100, 101, 102, 106, and 107. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 4 least Democratic districts and the last 8 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 4 least Democratic districts have the same or fewer average fraction of Democrats than in 99.99% of the plans in the ensemble (approximately 40548 out of the 40549 plans). Similarly, we find that the last 8 most Democratic districts have the same or more average fraction of Democrats than in 99.95% of the plans in the ensemble (approximately 40528 out of the 40549 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.19 Nash-Franklin Cluster (House)

We examine partisan gerrymandering in the Nash-Franklin county cluster in the North Carolina state House. We contextualize the current district plan in Figure 41. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 42.

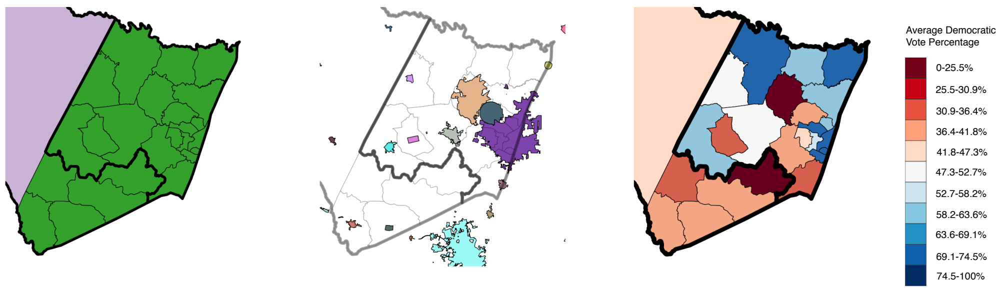


Figure 41: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

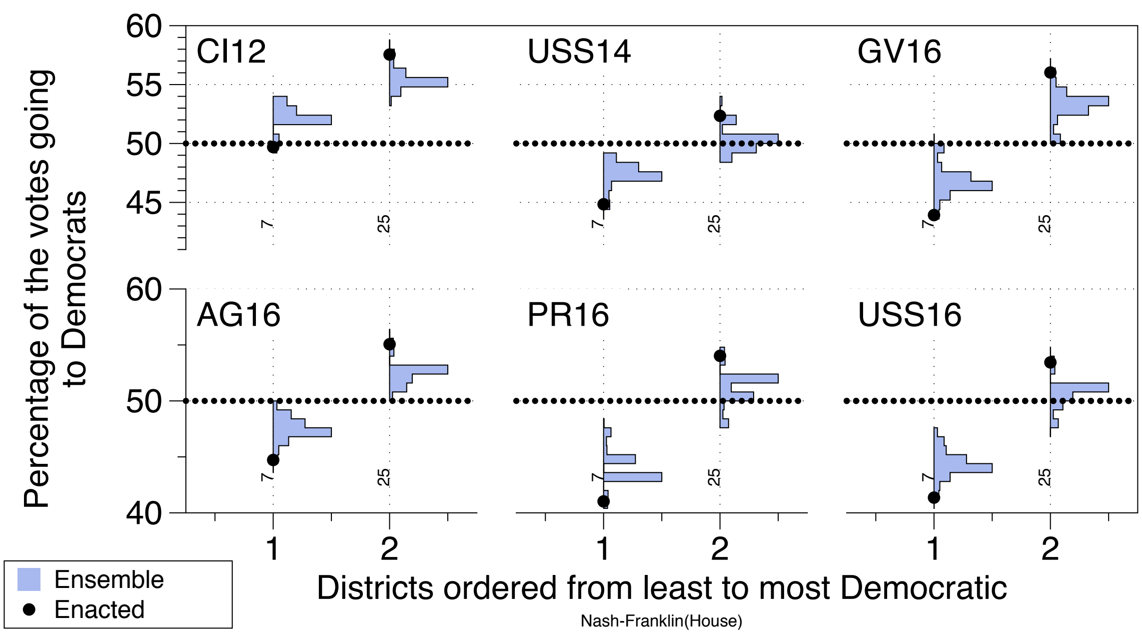


Figure 42: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the least Democratic district is District 7 in all 17 elections, and the most Democratic district is always District 25. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or fewer average fraction of Democrats than in 98.96% of the plans in the ensemble (approximately 4239 out of the 4284 plans). We also find that the least Democratic district has the same number of Democrats in 3.384% of plans in the ensemble (approximately 145 out of the 4284 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 97.89% of the plans in the ensemble (approximately 4194 out of the 4284 plans). We also find that the most Democratic district has the same number of Democrats in 3.384% of plans in the ensemble (approximately 145 out of the 4284 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.20 New Hanover-Brunswick Cluster (House)

We examine partisan gerrymandering in the New Hanover-Brunswick county cluster in the North Carolina state House. We contextualize the current district plan in Figure 43. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 44.

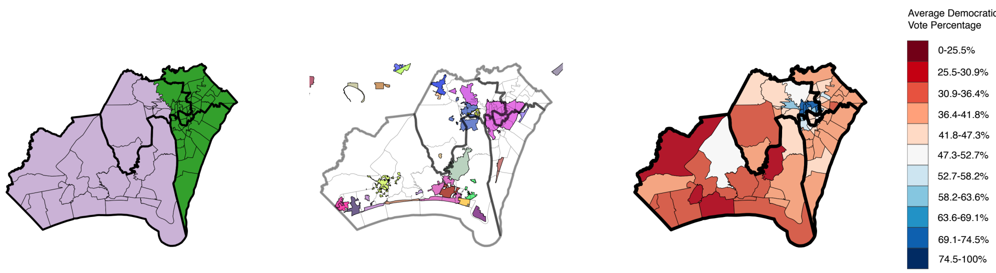


Figure 43: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

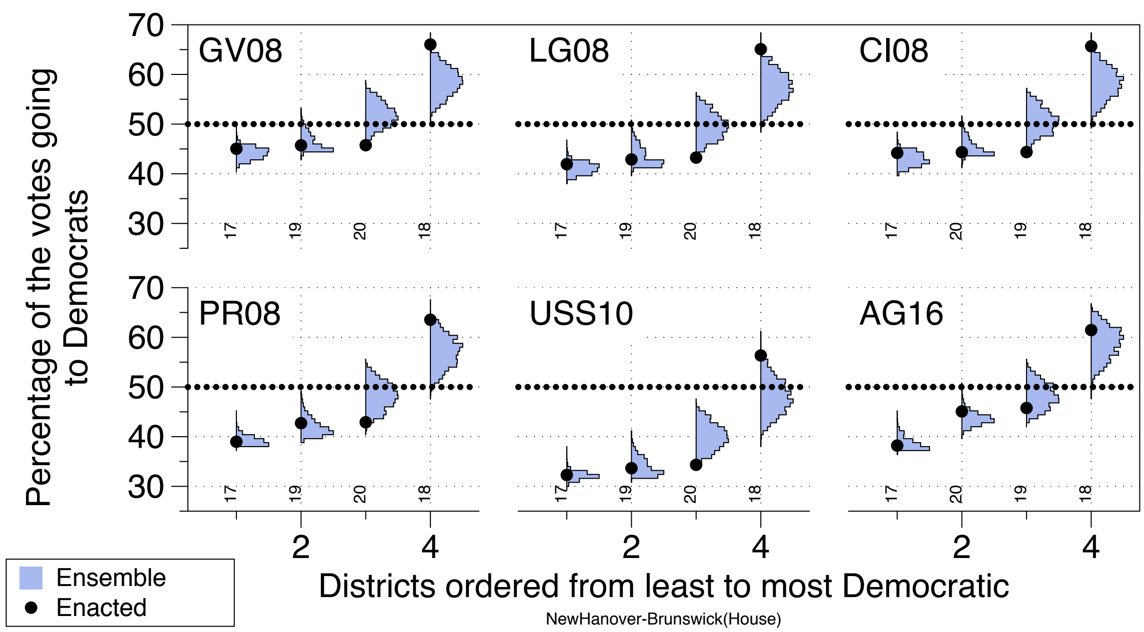


Figure 44: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 3 least Democratic districts are comprised of Districts 17, 19, and 20 in all 17 elections, and the most Democratic district is always comprised of District 18. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 3 least Democratic districts and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 3 least Democratic districts have the same or fewer average fraction of Democrats than in 88.44% of the plans in the ensemble (approximately 41254 out of the 46642 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 92.01% of the plans in the ensemble (approximately 42919 out of the 46642 plans). In short, we find that the enacted district plan is atypical of the ensemble of plans.

4.21 Person-Vance-Granville-Warren Cluster (House)

We examine partisan gerrymandering in the Person-Vance-Granville-Warren county cluster in the North Carolina state House. We contextualize the current district plan in Figure 45. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 46.

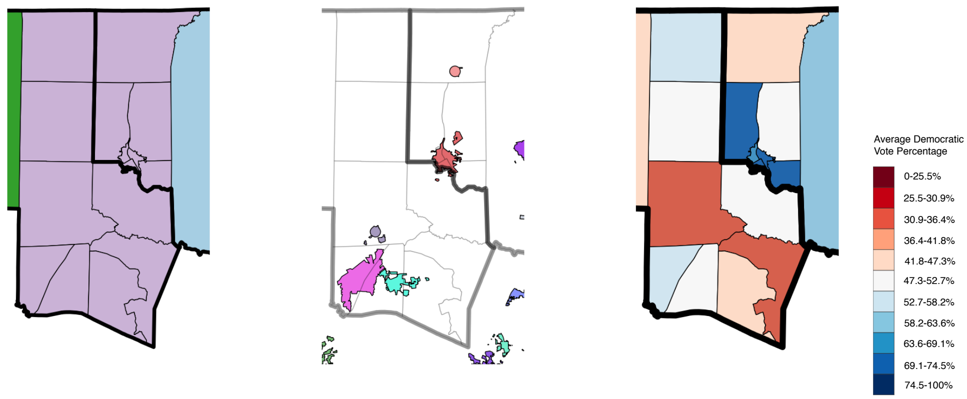


Figure 45: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

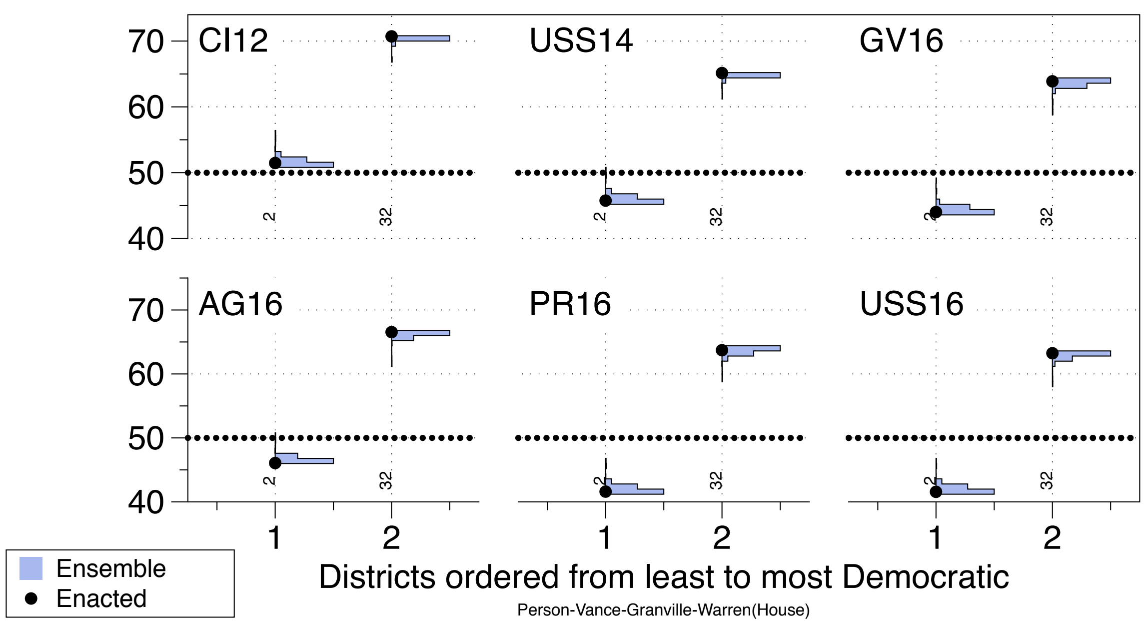


Figure 46: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the least Democratic district is District 2 in all 17 elections, and the most Democratic district is always District 32. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or fewer average fraction of Democrats than in 100% of the plans in the ensemble (all of the 4060 plans). We also find that the least Democratic district has the same number of Democrats in 59.87% of plans in the ensemble (approximately 2431 out of the 4060 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 100% of the plans in the ensemble (all of the 4060 plans). We also find that the most Democratic district has the same number of Democrats in 59.87% of plans in the ensemble (approximately 2431 out of the 4060 plans).

4.22 Pitt-Lenoir Cluster (House)

We examine partisan gerrymandering in the Pitt-Lenoir county cluster in the North Carolina state House. We contextualize the current district plan in Figure 47. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 48.

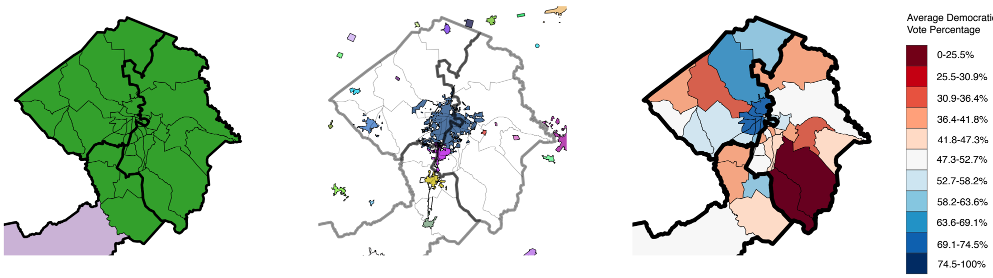


Figure 47: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

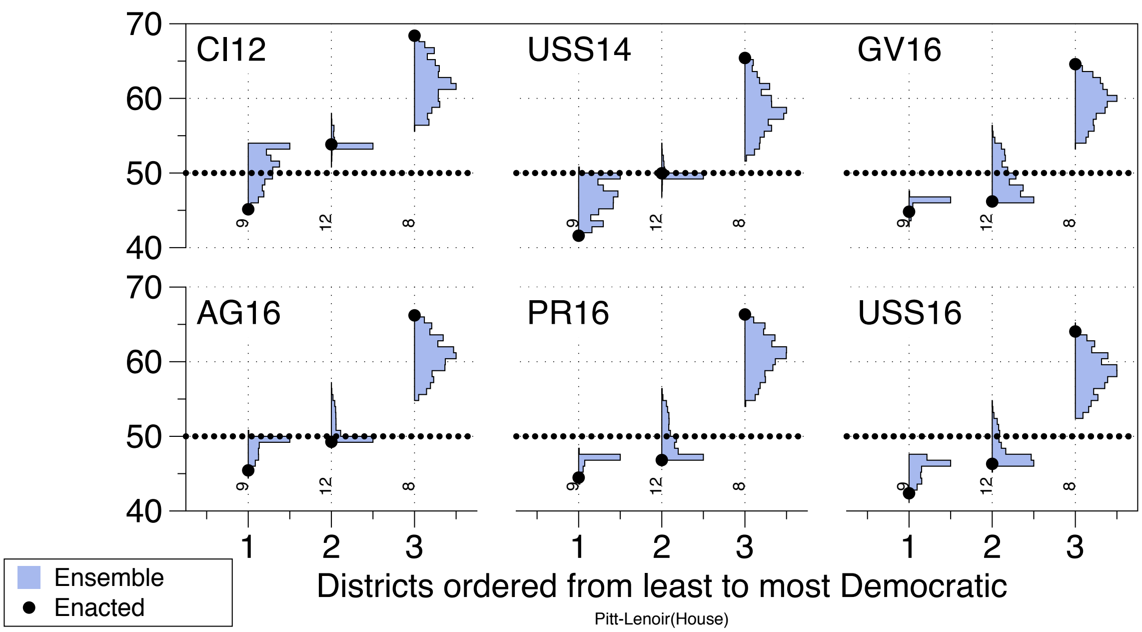


Figure 48: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 2 least Democratic districts are comprised of Districts 9 and 12 in all 17 elections, and the most Democratic district is always comprised of District 8. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 2 least Democratic districts and the most Democratic district and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 2 least Democratic districts have the same or fewer average fraction of Democrats than in 99.98% of the plans in the ensemble (approximately 3762 out of the 3763 plans). Similarly, we find that the most Democratic district has the same or more average fraction of Democrats than in 99.95% of the plans in the ensemble (approximately 3761 out of the 3763 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.23 Union-Anson Cluster (House)

We examine partisan gerrymandering in the Union-Anson county cluster in the North Carolina state House. We contextualize the current district plan in Figure 49. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 50.

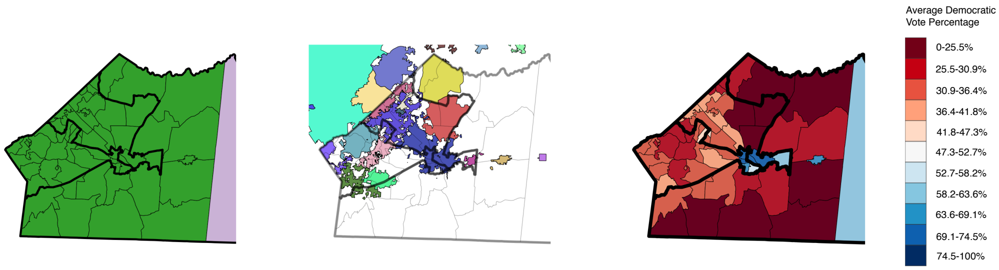


Figure 49: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

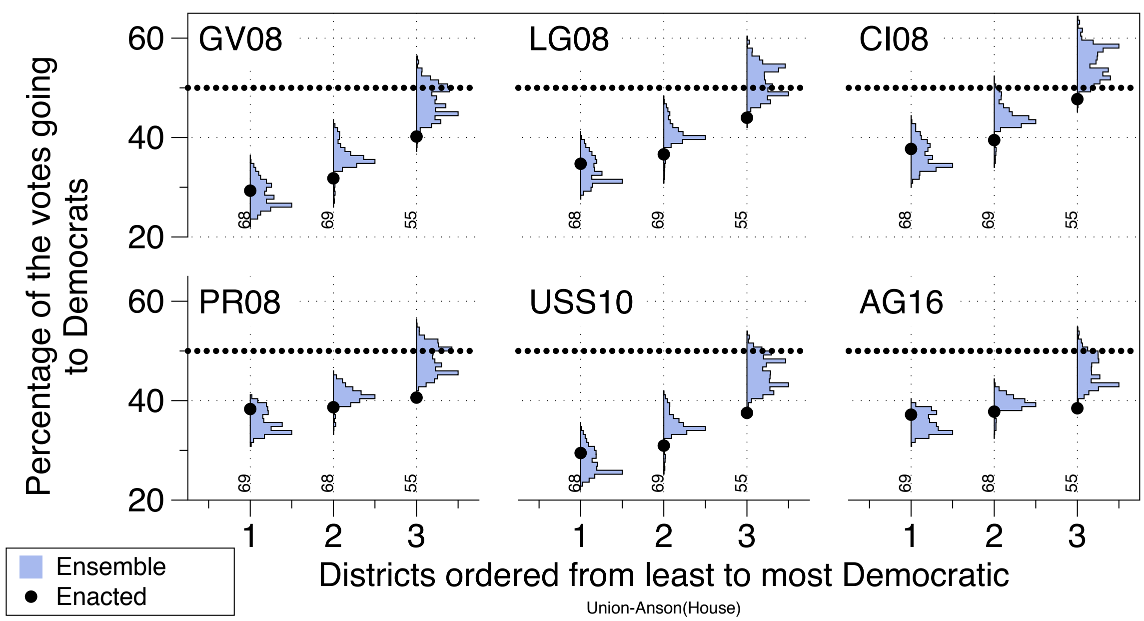


Figure 50: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, we group the least Democratic district, and the last 2 most Democratic districts. The enacted plan appears to remain relatively flat between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the least Democratic district and the last 2 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the least Democratic district has the same or more average fraction of Democrats than in 69.16% of the plans in the ensemble (approximately 4596 out of the 6645 plans). Similarly, we find that the last 2 most Democratic districts have the same or fewer average fraction of Democrats than in 100% of the plans in the ensemble (all of the 6645 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.24 Wake Cluster (House)

We examine partisan gerrymandering in the Wake county cluster in the North Carolina state House. We contextualize the current district plan in Figure 51. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 52.

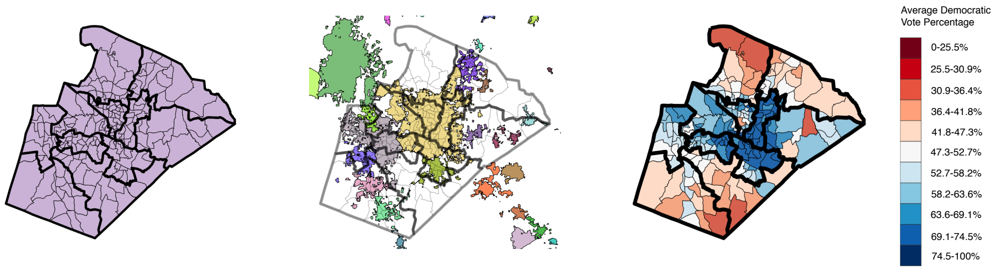


Figure 51: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

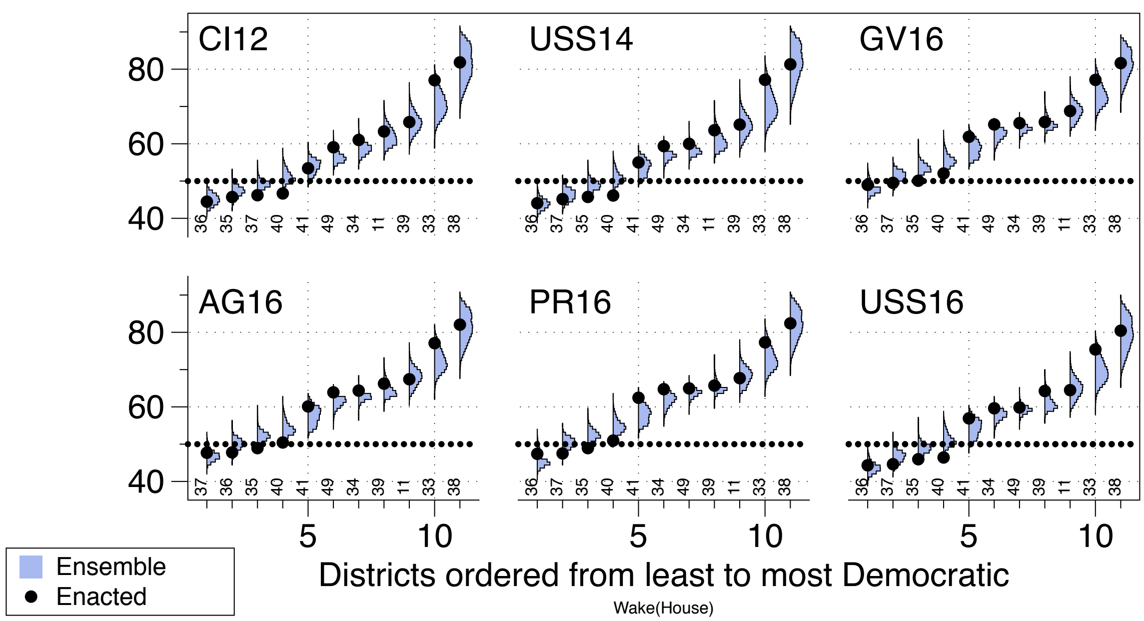


Figure 52: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 4 least Democratic districts are comprised of Districts 35, 36, 37, and 40 in all 17 elections, and the last 7 most Democratic districts are always comprised of Districts 11, 33, 34, 38, 39, 41, and 49. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 4 least Democratic districts and the last 7 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 4 least Democratic districts have the same or fewer average fraction of Democrats than in 99.98% of the plans in the ensemble (approximately 49994 out of the 50000 plans). Similarly, we find that the last 7 most Democratic districts have the same or more average fraction of Democrats than in 99.99% of the plans in the ensemble (approximately 49999 out of the 50000 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

4.25 Yadkin-Forsyth Cluster (House)

We examine partisan gerrymandering in the Yadkin-Forsyth county cluster in the North Carolina state House. We contextualize the current district plan in Figure 53. We examine the partisan performance in the ensemble of plans and compare with the enacted plan across six elections. For each of six elections, and each district in the ensemble, we tally the votes going to the Democratic and Republican candidates. For each ensemble we then consider the marginal distribution for each value of ranked-votes curve, ordering the districts from least to most Democratic. We compare the marginal distributions of the ensembles with the result found when using the enacted plan in Figure 54.

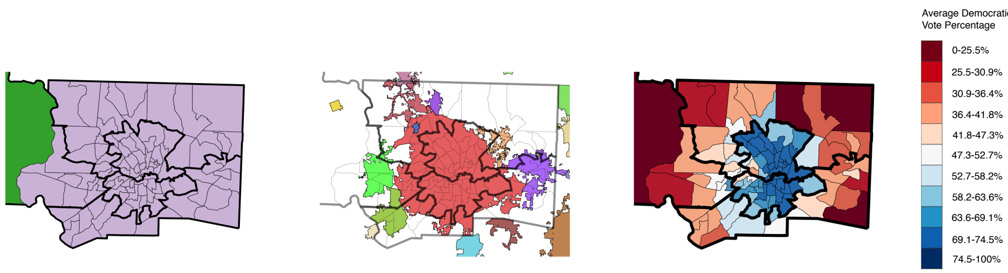


Figure 53: The enacted districting plan, colored by county (left), municipalities (center), and averaged Democratic vote fractions (right). Thin black lines show precincts or county lines; precincts are omitted in cases that counties are kept intact. Democratic vote fractions are found by taking an average of historic vote counts.

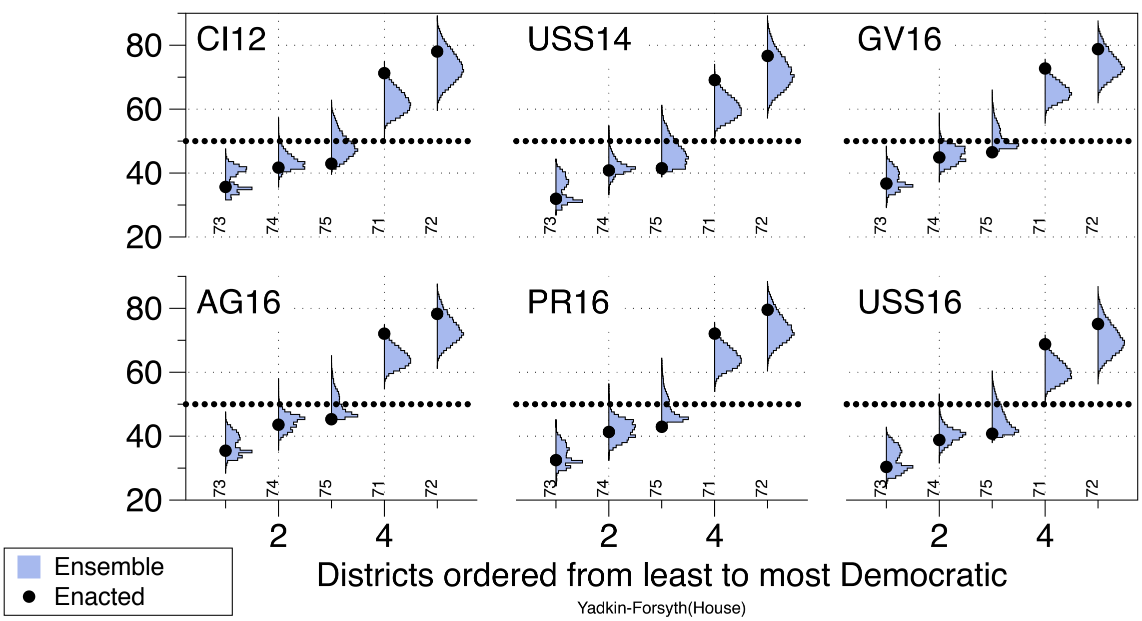


Figure 54: The districts in each plan of the ensemble are ordered from least to most Democratic for a given election. The results found in the ensemble of plans are contrasted with the enacted plan; the actual district numbers of the enacted plan are given.

In this cluster, the first 3 least Democratic districts are comprised of Districts 73, 74, and 75 in all 17 elections, and the last 2 most Democratic districts are always comprised of Districts 71 and 72. The enacted plan appears to jump between this first and second group in a way that may be atypical of the ensemble. To test this observation we examine the average number of Democrats in the first 3 least Democratic districts and the last 2 most Democratic districts and compare the enacted plan with the ensemble. Averaged over the 17 elections, we find that the first 3 least Democratic districts have the same or fewer average fraction of Democrats than in 99.46% of the plans in the ensemble (approximately 16970 out of the 17062 plans). Similarly, we find that the last 2 most Democratic districts have the same or more average fraction of Democrats than in 99.84% of the plans in the ensemble (approximately 17035 out of the 17062 plans). In short, we find that the enacted district plan is highly atypical of the ensemble of plans.

A Combining County Cluster Level Ensembles

Depending on quantity being calculated we use two methods to produce statewide statistics from our cluster level ensembles. In the first, one creates a collection of statewide maps by independently choosing a redistricting map in each of the county clusters and then combining the local county cluster level choices into statewide redistricting map. This ensemble built from the individual county cluster maps will be referred to as the *aggregated ensemble*. This method is used to calculate the statewide Ranked Votes Curve and Ranked Marginal Distributions. This amounts to re-sampling from the cluster level ensembles. Since this is very computationally inexpensive one can measure quantities relative to the aggregate ensemble with a high degree of accuracy.

In the second method, statistics are calculated at the cluster level and then combined at the statewide level. While this is not always feasible, in cases where it is, it produces an exact way of combining cluster level statistics.

The size of the aggregate ensemble which is the object being sampled in both cases is the product of all of the cluster by cluster sample sizes.

B Details on Statewide Analysis

We present detailed tables on statewide data for the quantiles of histograms along with the values of the histograms presented in Figures 2 and 5. Tables 1 and 2 show the 1%, 5% and 25% outliers for the histograms over the number of elected Democrats when using a variety of vote counts. Tables 3 and 4 report the numerical values of the histograms.

			Less than $P\%$ of plans have fewer than S seats			Less than $P\%$ of plans have more than S seats		
Elec.	Enacted	Ensemble Median	S ($P = 1\%$)	S ($P = 5\%$)	S ($P = 25\%$)	S ($P = 25\%$)	S ($P = 5\%$)	S ($P = 1\%$)
USS10	15	15	14	15	15	16	17	17
GV12	17	17	16	17	17	17	18	19
LG16	16	19	18	19	19	19	20	21
USS16	16	19	18	19	19	20	21	21
PR12	18	20	19	20	20	20	21	22
LG12	19	21	20	21	21	21	22	22
USS14	17	21	20	21	20	21	22	23
PR16	18	21	20	21	21	21	22	22
PR08	19	22	21	22	22	23	23	24
GV16	19	22	21	22	22	23	23	23
CI12	21	23	23	23	23	24	25	25
AG16	20	23	22	23	23	24	24	24
CI08	23	26	26	26	26	27	28	28
LG08	23	26	24	26	25	26	27	28
GV08	27	28	26	28	27	28	29	30
USS08	27	29	28	29	28	29	30	30
AG08	39	40	39	40	39	40	41	41

Table 1: We summarize the range of expected partisan results in the enacted plan and the ensemble for the North Carolina Senate. The results are from our primary enesemble that respects municipalities, population, compactness and whole county/traversal criteria.

			Less than $P\%$ of plans have fewer than S seats			Less than $P\%$ of plans have more than S seats		
Elec.	Enacted	Ensemble Median	S ($P = 1\%$)	S ($P = 5\%$)	S ($P = 25\%$)	S ($P = 25\%$)	S ($P = 5\%$)	S ($P = 1\%$)
USS10	39	36	34	36	35	37	39	40
GV12	43	39	38	39	38	40	42	43
LG16	42	43	41	43	42	44	46	47
USS16	42	43	42	43	42	44	46	47
PR12	43	46	44	46	45	47	49	50
USS14	45	47	45	47	46	48	50	51
PR16	45	48	46	48	47	49	50	51
PR08	44	49	47	49	48	50	52	53
LG12	48	50	48	50	49	51	53	54
AG16	44	50	48	50	49	51	53	54
GV16	47	51	49	51	50	52	54	55
CI12	51	56	54	56	55	57	58	59
LG08	56	63	61	63	62	65	66	67
CI08	57	65	63	65	64	66	68	69
GV08	62	66	64	66	65	67	68	69
USS08	60	71	69	71	70	72	74	75
AG08	92	94	92	94	93	95	96	97

Table 2: We summarize the range of expected partisan results in the enacted plan and the ensemble for the North Carolina House. The results are from our primary enesemble that respects municipalities, population, compactness and whole county/traversal criteria.

Elec.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
USS10	1.10	11.6	37.8	37.0	11.8	0.54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GV12	1e-5	0.09	2.69	21.5	53.3	20.1	1.96	0.05	1e-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LG16	0	0	0	0.03	2.26	20.9	54.6	19.5	2.44	0.10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
USS16	0	0	0	1e-3	1.31	15.6	48.7	28.1	5.75	0.39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PR12	0	0	0	1e-7	1e-3	0.22	13.3	65.4	19.3	1.59	0.02	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LG12	0	0	0	0	0	1e-2	1.28	20.7	64.0	13.4	0.50	1e-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PR16	0	0	0	0	1e-4	1e-2	0.44	11.2	70.0	18.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
USS14	0	0	0	0	1e-3	0.11	3.04	24.5	54.2	16.7	1.33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GV16	0	0	0	0	0	0	0	0.12	10.5	56.1	33.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PR08	0	0	0	0	0	0	0.01	1.20	19.7	49.5	25.8	3.71	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AG16	0	0	0	0	0	0	1e-6	1e-3	0.16	12.9	60.6	26.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CI12	0	0	0	0	0	0	0	0	0.01	1.89	50.2	38.7	8.48	0.62	1e-2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CI08	0	0	0	0	0	0	0	0	0	0	1e-2	0.43	8.67	45.3	36.0	8.94	0.54	0	0	0	0	0	0	0	0	0	0	0	0	0
LG08	0	0	0	0	0	0	0	0	0	0.03	1.45	10.3	28.2	35.5	20.2	4.07	0.04	0	0	0	0	0	0	0	0	0	0	0	0	0
GV08	0	0	0	0	0	0	0	0	0	0	1e-3	0.06	1.44	9.81	28.5	37.0	19.0	3.75	0.24	1e-3	1e-7	1e-12	0	0	0	0	0	0	0	0
USS08	0	0	0	0	0	0	0	0	0	0	0	0	0	0.15	9.17	34.1	40.8	15.7	1e-2	0	0	0	0	0	0	0	0	0	0	0
AG08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3.11	37.0	44.2	14.8	0.69

Table 3: We display the percent chance of electing a given number of Democrats for a given election in the North Carolina Senate. The column headers show the number of Democrats elected; the row headers show the election considered. We use our primary ensemble that respects municipalities, population, compactness and whole county/traversal criteria. The result observed in the enacted plan is highlighted in purple.

Elec.	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
USS10	0.25	2.04	8.57	20.1	27.7	23.3	12.4	4.26	0.94	0.13	0.01	1e-3	1e-4	1e-6	1e-8	1e-10	1e-12	1e-15	1e-18
GV12	1e-5	1e-3	0.01	0.18	1.57	7.36	18.9	28.0	24.8	13.4	4.49	0.92	0.11	1e-2	1e-3	1e-4	1e-6	1e-8	1e-10
LG16	0	0	0	0	1e-5	1e-3	0.01	0.23	1.83	7.95	19.3	27.5	23.9	13.1	4.68	1.09	0.17	0.01	1e-2
USS16	0	0	0	0	1e-5	1e-3	0.01	0.18	1.50	6.64	16.8	25.8	24.9	15.6	6.39	1.73	0.31	0.03	1e-2
PR12	0	0	0	1e-16	1e-12	1e-9	1e-6	1e-4	1e-2	0.04	0.59	3.55	11.3	21.5	25.7	20.4	11.0	4.20	1.12
USS14	0	0	0	0	0	0	1e-9	1e-6	1e-4	1e-2	0.05	0.65	3.69	11.3	21.0	25.2	20.4	11.4	4.50
PR16	0	0	0	0	0	0	0	1e-8	1e-5	1e-3	1e-2	0.07	0.81	4.58	14.2	25.2	26.9	17.9	7.62
PR08	0	0	0	0	0	0	1e-10	1e-7	1e-5	1e-3	1e-2	0.07	0.54	2.36	6.87	13.9	20.4	21.9	17.3
AG16	0	0	0	0	0	0	0	0	1e-14	1e-10	1e-7	1e-4	1e-2	0.04	0.51	2.94	9.42	18.7	24.4
LG12	0	0	0	0	0	0	0	0	1e-15	1e-11	1e-8	1e-5	1e-3	0.02	0.42	3.05	10.6	20.9	25.7
GV16	0	0	0	0	0	0	0	0	0	0	1e-12	1e-7	1e-4	1e-2	0.05	0.65	3.62	11.2	21.3
CI12	0	0	0	0	0	0	0	0	0	0	0	0	1e-13	1e-10	1e-8	1e-6	1e-4	1e-3	0.01
LG08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CI08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GV08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
USS08	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Elec.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76
USS10	1e-22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
GV12	1e-13	1e-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LG16	1e-4	1e-5	1e-7	1e-9	1e-11	1e-14	1e-17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
USS16	1e-3	1e-5	1e-6	1e-8	1e-11	1e-13	1e-16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PR12	0.21	0.02	1e-2	1e-3	1e-5	1e-6	1e-8	1e-10	1e-12	1e-15	1e-17	1e-21	0	0	0	0	0	0	0	0	0	0	0	0	0	0
USS14	1.26	0.25	0.03	1e-2	1e-3	1e-4	1e-6	1e-8	1e-9	1e-11	1e-13	1e-16	1e-18	1e-21	0	0	0	0	0	0	0	0	0	0	0	0
PR16	2.09	0.36	0.04	1e-2	1e-3	1e-5	1e-7	1e-9	1e-12	1e-15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PR08	10.1	4.40	1.40	0.32	0.05	1e-2	1e-3	1e-4	1e-6	1e-7	1e-9	1e-11	1e-14	1e-17	0	0	0	0	0	0	0	0	0	0	0	0
AG16	21.9	13.6	5.99	1.84	0.39	0.05	1e-2	1e-3	1e-4	1e-6	1e-8	1e-10	1e-12	1e-15	1e-19	0	0	0	0	0	0	0	0	0	0	0
LG12	20.9	11.7	4.71	1.38	0.30	0.04	1e-2	1e-3	1e-4	1e-5	1e-7	1e-8	1e-10	1e-13	1e-15	1e-18	0	0	0	0	0	0	0	0	0	0
GV16	25.9	20.8	11.2	4.00	0.93	0.13	0.01	1e-3	1e-4	1e-7	1e-9	1e-13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CI12	0.18	1.30	5.72	15.4	25.4	25.8	16.7	7.01	1.93	0.35	0.04	1e-2	1e-3	1e-5	1e-6	1e-8	1e-10	1e-13	1e-15	1e-19	1e-23	0	0	0	0	0
LG08	0	1e-13	1e-10	1e-7	1e-5	1e-3	1e-2	0.07	0.60	2.79	8.38	16.8	23.2	22.4	15.1	7.25	2.43	0.57	0.09	1e-2	1e-3	1e-4	1e-6	1e-8	1e-11	1e-14
CI08	0	0	1e-16	1e-12	1e-9	1e-6	1e-4	1e-2	0.02	0.23	1.40	5.21	12.7	21.1	24.0	19.0	10.5	4.12	1.11	0.20	0.02	1e-2	1e-4	1e-5	1e-7	1e-9
GV08	0	0	0	1e-14	1e-11	1e-8	1e-6	1e-4	1e-3	0.01	0.19	1.31	5.33	13.6	22.8	25.2	18.6	9.13	2.94	0.60	0.07	1e-2	1e-3	1e-7	0	0
USS08	0	0	0	0	1e-23	1e-19	1e-16	1e-13	1e-11	1e-9	1e-7	1e-5	1e-4	1e-2	0.01	0.16	0.94	3.79	10.4	19.5	24.9	21.4	12.4	4.78	1.22	0.20

Table 4: We display the percent chance of electing a given number of Democrats for a given election in the North Carolina House. The column headers show the number of Democrats elected; the row headers show the election considered. We use our primary ensemble that respects municipalities, population, compactness and whole county/traversal criteria. The result observed in the enacted plan is highlighted in purple. We omit the 2008 Attorney General election since it leads to an anomalously large number of elected democrats and including it would prevent the table from fitting on the page.

C Sampling methodology

C.1 The distribution on redistricting plans

We will encode non-partisan design criteria into a *score function* J which will measure the extent to which the criteria are satisfied. The lower the score the more compliant the district is. Using the score function, we define a measure P on the space of redistrictings ξ by

$$P(\xi) = \frac{1}{Z} e^{-\beta J(\xi)}\,.$$

Here β is the inverse temperature and Z is the normalization constant which makes the probabilities $P(\xi)$ sum to one. The inverse temperature controls the extent to which the measure is concentrated around the minimizer of J. The lower β the more it is concentrated.

C.2 The Score Function

To define the score function, we introduce several mathematical formalisms, the first of which represents the redistricting region (e.g. a given county cluster) as a graph G with edges E and vertices V. Each vertex represents the granularity from which we are sampling, meaning that it may represent an entire county, a precinct, a census block, or some combination of these elements; an edge between two vertices exists if the two vertices share boundaries with non-zero length. In general, vertices will be considered to be precincts in counties that are divided, or whole counties in counties that are not split. Occasionally we find bi-connected components in our graph and compress the vertex structure to eliminate these elements by allowing a single vertex to represent multiple precincts or whole counties.

Defining the graph this way allows us to formally define a redistricting plan: Assuming each VTD belongs to a single district, a redistricting plan is defined as a function from the vertices, V, to one of the possible districts, which are represented by sequential integers – for example, there are six House districts contained within Guilford County, so we would define a redistricting plan as a function $\xi : V \to \{1, 2, 3, 4, 5, 6\}$. The redistricting plan function ξ is interpreted as follows: If a geographical region in the county cluster is represented by a vertex $v \in V$, then $\xi(v) = i$ means that the VTD in question belongs to district i; similarly, supposing we have D districts, for any $i \in \{1, 2, \ldots, D\}$ and plan ξ, the i-th district, denoted $D_i(\xi)$, is given by the set $\{v \in V : \xi(v) = i\}$. We restrict the space of considered redistricting plans ξ such that each district $D_i(\xi)$ is a single connected component; this restriction, along with our edge criteria, ensures that districts are always contiguous.

A plan ξ is rated with a score function denoted J. J maps each redistricting $\xi \in \mathcal{R}$ to a nonnegative number. Lower scores signify redistricting plans that more closely adhere to stated redistricting goals. The score function J is the sum of several auxiliary functions that measure how well a given redistricting satisfies individual redistricting principles. We denote these auxiliary functions as J_{pop}, J_{PP}, and J_M: the *population score* $J_{pop}(\xi)$ measures how closely the redistricting ξ partitions the population into groups near the size of the ideal district population; the *Polsby-Popper score* $J_{PP}(\xi)$ measures how compact the districts are; the *municipality score* $J_M(\xi)$ measures how municipalities have been split. Once the individual auxiliary functions are specified, our score function J is defined as a weighted sum of J_{pop}, J_{PP}, and J_M; since all of the auxiliary score functions are not on the same scale, we use a weighted combination to balance the influence of each criteria. Specifically, we define

$$J(\xi) = w_{pop} J_{pop}(\xi) + w_{PP} J_{PP}(\xi) + w_M J_M(\xi), \tag{1}$$

where w_{pop}, w_{PP}, and w_M are a collection of positive weights.

To describe the individual auxiliary functions, data is associated to our graph G which allows the recovery of relevant features on each vertex. The functions $\mathrm{pop} : V \to \mathbb{N}$, $\mathrm{area} : V \to \mathbb{R}^+$, and $\mathrm{muni} : V \to \mathbb{N}^M$ are defined on a vertex $v \in V$, and represent (respectively) the total population, geographic area, county assignments, and the populations of each municipality contained within the vertex.

The municipal function maps each vertex to the number of people in the vertex who also live in the jth municipality, where j is an index from 1 to M, and M is the number of municipalities that have part of their population in the county cluster.

These functions, $\mathrm{pop}(v)$, $\mathrm{area}(v)$, and $\mathrm{muni}(v)$ are extended to a collection of vertices $B \subset V$ by

$$\mathrm{pop}(B) = \sum_{v \in B} \mathrm{pop}(v), \quad \mathrm{area}(B) = \sum_{v \in B} \mathrm{area}(v), \quad \mathrm{muni}(B) = \sum_{v \in B} \mathrm{muni}(v). \tag{2}$$

The boundary of a district $D_i(\xi)$, denoted $\partial D_i(\xi)$, is the subset of the edges E that connect vertices inside of $D_i(\xi)$ to vertices outside of $D_i(\xi)$. According to this definition, geographic regions that border another state, county cluster, or the ocean will not have an edge that signifies this fact. To incorporate state boundary information, we add the vertex o to V, which represents the "outside" and connect it with an edge to each vertex representing a vertex which shares perimeter with the boundary of the cluster. We assume that any redistricting ξ always satisfies $\xi(v) = 0$ if and only if $v = o$; since ξ always satisfies $\xi(o) = 0$, and $o \notin D_i(\xi)$ for $i \geq 1$, it does not matter that we have not defined $\mathrm{pop}(o)$, $\mathrm{area}(o)$, $\mathrm{cnty}(o)$, nor $\mathrm{muni}(o)$ as o is never included in the districts.

Given an edge $e \in E$ which connects two vertices $v, \tilde{v} \in V$, $\mathrm{boundary}(e)$ represents the length of common border of the vertices associated with the vertex v and $\tilde{v}$. As before, the definition is extended to the boundary of a set of edges $B \subset E$ by

$$\mathrm{boundary}(B) = \sum_{e \in B} \mathrm{boundary}(e)\,. \tag{3}$$

We use $\partial D_i(\xi)$ to denote the collection of edges $e = (v_k, v_l)$ such that $\xi(v_k) \neq \xi(v_l)$ and that either $\xi(v_k) \in D_i$, or $\xi(v_l) \in D_i$.

With these preliminaries out of the way, we define the score auxiliary functions used to assess the goodness of a redistricting.

C.2.1 The population score function

The population score, which measures how evenly populated the districts are, is defined by

$$J_{pop}(\xi) = \sqrt{\sum_{i=1}^{D} \Big(\frac{\mathrm{pop}(D_i(\xi))}{\mathrm{pop}_{\text{Ideal}}} - 1\Big)^2}, \quad \mathrm{pop}_{\text{Ideal}} = \frac{\mathrm{pop}_{\text{State}}}{D_{sw}},$$

where N_{pop} is the total population of North Carolina, $\mathrm{pop}(D_i(\xi))$ is the population of the district $D_i(\xi)$ as defined in equation (2), and $\mathrm{pop}_{\text{Ideal}}$ is the population that each district should have according to the 'one person one vote' standard, which is the statewide population ($\mathrm{pop}_{\text{State}}$) divided by the statewide number of districts (D_{sw}).

C.2.2 The Polsby-Popper score function

The Polsby-Popper score, which measures the overall compactness of a redistricting, is defined by

$$J_{PP}(\xi) = \sum_{i=1}^{D} \frac{\big[\mathrm{boundary}(\partial D_i(\xi))\big]^2}{\mathrm{area}(D_i(\xi))}\,,$$

where $\mathrm{area}(D_i(\xi))$ is as defined in Equation 2, $\mathrm{boundary}(\cdot)$ is as defined in Equation 3, and $\partial D_i(\xi)$ is as defined above. We omit the traditional 4π scaling factor of the Polsby-Popper score as it can naturally be incorporated into the weight w_{PP}. This score measures the sum of the ratios between the square perimeter to the total area of each district, which is proportional to the inverse of the Polsby-Popper score. The Polsby-Popper score is maximized for a circle, which is the most compact shape; taking it's inverse provides a score function that grows as districts become less compact.

We select the Polsby-Popper score because penalizes undulating boundaries, while also accounting for convexity measures and dispersive quantities. Dispersive or convexity measures do not consider perimeter, which can, in practice, lead to wildly undulating boundaries. There are over 20 measures to evaluate compactness (see, for example, [8]). We chose the *the Polsby-Popper score* both because of its historical significance and because it is consistent with the utilized compactness criteria.

C.2.3 The municipal score function

The municipal score function measures how many people in a given municipality have been separated from the district(s) that best represents their municipality (i.e. contain the largest population(s) of the municipality). For municipalities with a population that is less than the size of an ideal district, we first determine the district containing the greatest portion of this municipality and say anyone living in the municipality who is not in this district has been split out of their municipality. Formally, we define a municipal-district population matrix $MP_{ij} = \text{muni}(D_i) \cdot \vec{e}_j$, where $\vec{e}_j \in \mathbb{R}$, and the kth element of $\vec{e}_j$ is δ_{jk}, or the Kronecker delta, and then say the number of people who have been split from a smaller municipality as

$$M_j^{\text{split}} = \text{pop}_{M_j} - \max_i |MP_{ij}|$$

where pop_{M_j} is the population of the jth municipality intersected with the county cluster, and $\max_i |MP_{ij}|$ is the largest absolut value in the j-th column of the matrix MP. The quantity M_j^{split} represents how many people in the jth municipality reside in each district.

The story is more complicated for municipalities that must be split. We examine how these municipalities are split in the enacted plan and reveal the results in Table 5. We note that when municipalities can entirely contain multiple districts, they often either do, or do so to a high degree. For example, in the house Winston-Salem can contain two entire districts and nearly does; Greensboro in the House is similar, with three districts nearly entirely contained within it, as is Fayetteville in the House, and Durham, Winston-Salem, and Greensboro in the Senate.

A remaining question is how to consider people who are not in the few core districts. Should they be contained in a single district or should they be free to be split. Such remaining populations appear to be mostly maintained in High Point in the House, and Greensboro in the Senate; however, the converse is true in Winston-Salem and Greensboro in the House and Raleigh in the Senate, where remaining populations may be split evenly among remaining districts.

In this report, we make the decision not to account for populations outside of core districts, which is keeping in line with Winston-Salem and Greensboro in the House. We extend our previous score function by counting the people who are in this municipality but not part of one of the core districts.

To formally define the score of a districting plan on each large municipality, we first determine the number of core districts the municipality should have to be $N_{M_j}^{\text{core}} = \lfloor \text{pop}_{M_j} / \text{pop}_{\text{Ideal}} \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function; furthermore, we let $D_{M_j}^{\text{core}}$ be the set of the $N_{M_j}^{\text{core}}$ districts with the highest population of the jth municipality. For such large municipalities, we then define

$$M_j^{\text{split}} = \text{pop}_{M_j} - \sum_{i \in D_{M_j}^{\text{core}}}^{N_{M_j}^{\text{core}}} (MP_{;j})^T \cdot \vec{e}_i,$$

where the superscript T is the transpose operation. It is inevitable that this number not be zero since, since there will nearly always be some people excluded from the core district(s). From a sampling point of view, this is irrelevant as it appears as a constant in the energy term which does not effect the underlying Gibbs measure. However, when quantifying the amount of splitting that has occurred in the sections below, we will not count people that must be cut from core districts as having been excessively split.

For a cluster wide score, we sum all the people who have been split out of the municipality they reside in

$$J_M(\xi) = \sum_{i=1}^{M} M_j^{\text{split}}.$$

Municipality	Population	No. Dist. in Clust.	Dist. in Muni.	Body	Districts (ordered)	Percent (ordered)
Winston-Salem	229,622	5	2.889	House	(71, 72, 75, 74, 73)	(95.36, 92.45, 41.37, 37.85, 21.92)
Greenville	84,513	3	1.063	House	(8, 9, 12)	(61.36, 43.57, 1.415)
Charlotte	731,424	12	9.204	House	(102, 104, 105, 106, 88, 101, 99, 100, 107, 92, 103)	(97.39, 96.73, 95.60, 93.61, 90.66, 89.77, 84.31, 81.49, 79.55, 72.49, 38.82)
Durham	228,300	4	2.873	House	(29, 31, 30, 54)	(100.4, 90.17, 76.52, 20.18)
Wilmington	106,476	4	1.339	House	(18, 19, 20)	(52.30, 42.72, 38.96)
Cary	133,812	11	1.683	House	(41, 11, 49, 36, 37)	(58.18, 55.41, 27.85, 24.38, 2.547)
Raleigh	402,825	11	5.069	House	(38, 34, 33, 49, 40, 11, 39, 35)	(102.1, 97.31, 68.68, 64.28, 53.66, 46.83, 39.49, 34.49)
High Point	99,042	6	1.246	House	(60, 62, 59)	(75.07, 49.52, 0.041)
Greensboro	269,666	6	3.393	House	(61, 58, 57, 62, 60, 59)	(100.3, 95.85, 95.56, 20.78, 17.83, 8.955)
Fayetteville	200,548	4	2.523	House	(43, 42, 44, 45)	(93.61, 84.81, 64.60, 9.341)
Durham	228,300	2	1.197	Senate	(20, 22)	(87.36, 32.35)
Charlotte	731,424	5	3.835	Senate	(37, 38, 40, 39, 41)	(96.14, 91.17, 85.37, 72.65, 38.17)
Raleigh	402,825	5	2.112	Senate	(15, 14, 16, 18, 17)	(77.60, 72.13, 36.25, 25.22, 0.003)
Fayetteville	200,548	2	1.051	Senate	(21, 19)	(60.37, 44.78)
Greensboro	269,666	4	1.414	Senate	(28, 27, 24)	(103.7, 35.24, 2.364)
Winston-Salem	229,622	2	1.204	Senate	(32, 31)	(96.34, 24.06)

Table 5: Several municipalities must be split in any given plan. We examine how these municipalities are split in the currently enacted plan in order to inform our municipal score function. We show each municipalities population, the number of districts within the county cluster that contains the municipality, the fractional number of district populations within each municipality, the districts that intersect the municipality (ordered from most intersection to least intersection), and finally the percent of an ideal population that the municipality's population intersects each district (again ordered from the most intersection to the least intersection).

C.3 Constraints

In addition to the score function, we consider several constraints which (i) still allow the system to fully explore the space of redistricting plans, while (ii) ensure the plans more closely follow traditional and legal criteria. The first constraint has been discussed above: districts must always be contiguous. Any step that violates the contiguity of a district will be automatically rejected. We have added several other constraints to our systems.

- Counties which are kept whole in the enacted plan are kept whole in the ensemble of plans.
- Single district, double traversals are forbidden from occuring in the ensemble.
- Only one district may cross a county-to-county boundary.
- Whole counties are kept together where possible to avoid splitting a third county: this occurs in the Polk-Transylvania-Henderson Cluster in the House, and the Bladen-Brunswick-Pender-New Hanover Cluster, the Person-Durham-Granville Cluster, and the Lincoln-Gaston-Cleveland Cluster in the Senate.
- The following districts were kept fixed: Senate districts 19, 21, 24, and 28, and House districts 21, 22, 57, 61, and 62; this was done by the request of the plaintiffs.

In the case of keeping multiple whole counties together, Polk and Transylvania are not adjacent in the Polk-Transylvania-Henderson cluster. There are two options to connect these counties, either by taking the southern or northern part of Henderson to be contained in the same district as Polk and Transylvania. We tested both choices, and found that the northern path lead to highly non-compact districts so we omit this choice in our work; we instead examine variations of the enacted plan when examining this cluster.

C.4 Sampling the space; random walks on the space of redistricting plans

To sample the distribution defined by the score function, we take a random walk on the space of redistricting plans using a Markov Chain Monte Carlo algorithm. Our walk is defined as a collection of steps that swap graph vertices that are on district boundaries and are adjacent to other districts. Our chain is reversible. We propose moves uniformly on conflicted edges and then reweight the step so that we respect detailed balance. Our acceptance criteria is the standard Metropolis-Hastings algorithm.

We sample the space at constant temperature. We allow the walk to take 2000 steps before we seek to resample so that the plan has time to decorrelate between recording the state of the system. We ensure convergence of our random walks via the algorithm presented in Section F.1.

D Detailed Cluster-by-Cluster Analyses

In order to robustly sample the entire space of redistricting plans, we must ensure that we have robustly sampled each county cluster. In this section we demonstrate our sampling results for each county cluster. We examine the range of compactness scores and municipal splitting given by the ensemble in each cluster that accounts for population deviation, compactness, municipal splitting, and, where appropriate, other constraints such as forbidding single district double traversals and only allowing one district to cross a county-to-county border.

All counties that are kept whole in the enacted plan are kept whole in the ensemble of plans; when two whole counties can be in the same district, they are kept in the same district. This provision is in the Polk-Transylvania-Henderson cluster in the House, and the Bladen-Brunswick-Pender-New Hanover, Person-Durham-Granville, and Lincoln-Gaston-Cleveland clusters in the Senate. In the Polk-Transylvania-Henderson cluster, the Polk and Transylvania are joint but are not adjacent; there are two possible ways to connect them either with a northern or southern path through Henderson; we find that only the southern route (as enacted) leads to viably compact districts so we only examine this possibility.

To sample the space we first place a measure and score function on the space of plans; the measure placed on the space of redistricting plans is defined in Section C; the parameters of the score function for each cluster are given in Tables 6 and 7; the sampling procedure is described above in Section C. We ensure the convergence of our ensembles with the process described in Section F.1, and report convergence data in Tables 10 and 11 in Section F.1.3 below.

We find a significant number of compliant maps in nearly all clusters without splitting precincts. There are a few exceptions in the House, including Wake, Mecklenburg, the Bladen-Lee-Sampson-Harnett-Greene-Wayne-Johnston cluster, and the Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander cluster. This is due to the relative granularity between the precincts and the districts; the effect is that we cannot generate plans with all districts between $\pm 5\%$ population deviation. In these cases, we first sample the space of plans with whole precincts and accept districts with slightly elevated population deviations; we then show that if we split precincts until we would fall below below the required population deviation, that it would not change the partisan results we get from our ensemble. We find that splitting precincts does not alter our results (see Section E.24).

To validate that our ensembles are consistent with the redistricting criteria, we examine the extent to which our ensembles respect compactness, and preserving municipalities. We do this for *all* cluster ensembles. To examine the Polsby-Popper scores, we plot the ranked marginal distributions of Polsby-Popper scores for each ensemble (least to most compact). If the range of compactnesses encompass or are similar to the enacted plan, we conclude that the ensemble is representative of the criteria that went into the ensemble. We also determine how many municipalities each redistricting plan splits, along with the cluster-wide number of people who were removed from their municipality's core district(s) beyond what was necessary (see Section C.2.3 for how we define people being split from core districts).

Geographic precinct data was gathered from the North Carolina State Board of Elections (see [9]) from which the May 19th, 2017 precinct data was used, since this is what was available at the time the enacted precinct map was drawn. Other precinct and census block data, including population data, voting data, and municipal shapefiles, were taken from the NC legislature's website within the redistricting archives (see [4, 5]). The one exception to this is the 2016 voting data which was taken from [6, 7]. Census block shapefiles were taken from the Census Bureau [10].

Cluster	Weights $(w_{\text{pop}}, w_{\text{PP}}, w_{\text{M}})$	Thresholds $(t_{\text{pop}}, t_{\text{PP}})$
Bladen-Brunswick-Pender-New Hanover	(50.0, 0.1, 0.001)	(5.0, 0.179)
Buncombe-Transylvania-Henderson	(50.0, 0.5, 0.0002)	(5.0, 0.179)
Caswell-Wilkes-Rockingham-Alleghany-Watauga-Ashe-Stokes-Surry	(100.0, 0.3, 0.0005)	(5.0, 0.179)
Davie-Forsyth	(100.0, 0.1, 0.0005)	(5.0, 0.132)
Guilford-Alamance-Randolph	(60.0, 0.2, 0.0001)	(5.0, 0.179)
Lee-Sampson-Harnett-Duplin-Johnston-Nash	(100.0, 0.2, 0.0005)	(5.0, 0.179)
Lincoln-Gaston-Cleveland	(50.0, 0.1, 0.001)	(5.0, 0.179)
Mecklenburg	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Person-Durham-Granville	(50.0, 0.3, 0.0005)	(5.0, 0.179)
Union-Cabarrus	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Wake-Franklin	(50.0, 0.2, 0.0002)	(5.0, 0.179)

Table 6: Sampling parameters for the Senate clusters are presented for two ensembles we have generated - the primary ensemble that considers municipal splits, and a second that does not. Thresholds are presented as t_{pop} for the maximum allowable population deviation (in percent), and t_{PP} for the minimum allowable Polsby-Popper score. Plans that do not meet the thresholds are not saved if they are encountered during the output stage, or considered as part of the convergence criteria.

Cluster	$(w_{\text{pop}}, w_{\text{PP}}, w_{\text{M}})$	$(t_{\text{pop}}, t_{\text{PP}})$
Alamance	(80.0, 0.1, 0.0003)	(5.0, 0.179)
Bladen-Lee-Sampson-Harnett-Greene-Wayne-Johnston	(50.0, 0.2, 0.001)	(10.0, 0.179)
Buncombe	(50.0, 0.1, 0.001)	(5.0, 0.167)
Caswell-Orange	(25.0, 0.2, 0.001)	(5.0, 0.179)
Catawba	(50.0, 0.1, 0.001)	(5.0, 0.179)
Chatham-Durham	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Columbus-Pender-Robeson	(50.0, 0.1, 0.001)	(5.0, 0.147)
Craven-Beaufort	(100.0, 0.15, 0.0005)	(5.0, 0.179)
Cumberland	(50.0, 0.2, 0.0001)	(5.0, 0.179)
Davidson	(50.0, 0.1, 0.0005)	(5.0, 0.179)
Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly	(100.0, 0.2, 0.0001)	(5.0, 0.167)
Duplin-Onslow	(50.0, 0.2, 0.0005)	(5.0, 0.179)
Gaston-Cleveland	(50.0, 0.2, 0.0001)	(5.0, 0.179)
Guilford	(100.0, 0.15, 0.001)	(5.0, 0.167)
Iredell	(25.0, 0.1, 0.0001)	(5.0, 0.179)
Madison-Swain-Jackson-Haywood-Yancey	(100.0, 0.2, 0.001)	(5.0, 0.147)
Mecklenburg	(50.0, 0.2, 0.0001)	(10.0, 0.179)
Nash-Franklin	(50.0, 0.2, 0.0001)	(5.0, 0.179)
New Hanover-Brunswick	(100.0, 0.1, 0.0005)	(5.0, 0.179)
Person-Vance-Granville-Warren	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Pitt-Lenoir	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Polk-Transylvania-Henderson	(50.0, 0.4, 0.0001)	(5.0, 0.179)
Randolph-Moore	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Rutherford-Burke	(50.0, 0.2, 0.001)	(5.0, 0.179)
Union-Anson	(100.0, 0.2, 0.0001)	(5.0, 0.179)
Wake	(100.0, 0.2, 0.0001)	(12.0, 0.179)
Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander	(50.0, 0.1, 0.001)	(6.0, 0.147)
Yadkin-Forsyth	(50.0, 0.1, 0.0005)	(5.0, 0.157)

Table 7: Sampling parameters for the House clusters are presented for two ensembles we have generated - the primary ensemble that considers municipal splits, and a second that does not. Thresholds are presented as t_{pop} for the maximum allowable population deviation (in percent), and t_{PP} for the minimum allowable Polsby-Popper score. Plans that do not meet the thresholds are not saved if they are encountered during the output stage, or considered as part of the convergence criteria.

E Detailed Analysis on the Compliance of Sampled Plans

E.1 Mecklenburg Cluster (Senate)

We examine the quality of our ensemble in the Mecklenburg county cluster in the North Carolina Senate.[5] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 55 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 55, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 5884 of the 6721 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 4503 of the 6721 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 56 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 56. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

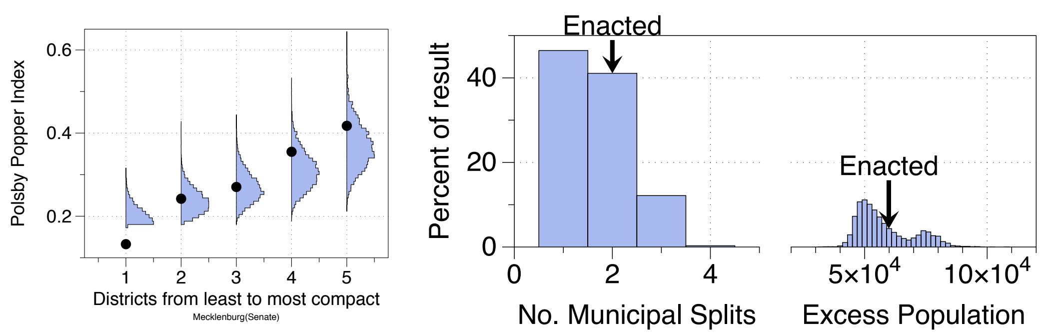
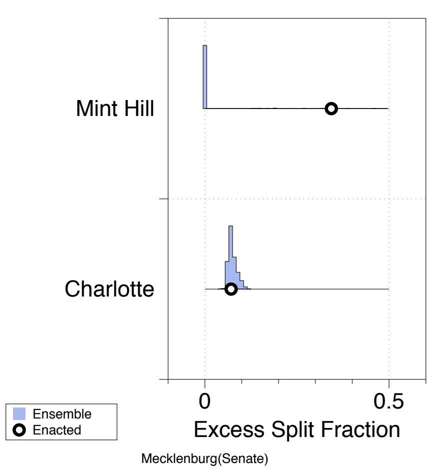


Figure 55: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Cornelius	0.0	0.0	0.0	-
Davidson	0.0	0.0	0.0	-
Huntersville	0.699	11.97	5.278	-
Mint Hill	3.957	26.16	11.72	Y
Matthews	29.72	27.94	11.43	-
Pineville	31.90	33.87	3.967	-
Charlotte	100.0	7.472	1.150	Y

Figure 56: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[5]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.2 Wake-Franklin Cluster (Senate)

We examine the quality of our ensemble in the Wake-Franklin county cluster in the North Carolina Senate.[6] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 57 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 57, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 7352 of the 14338 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 14280 of the 14338 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 58 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 58. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

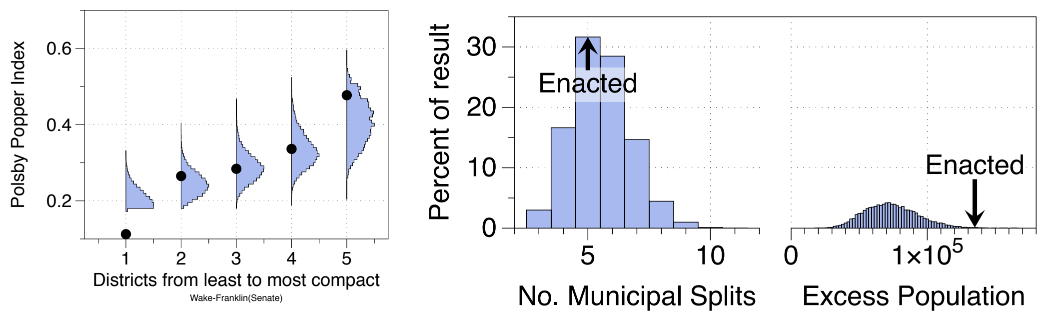
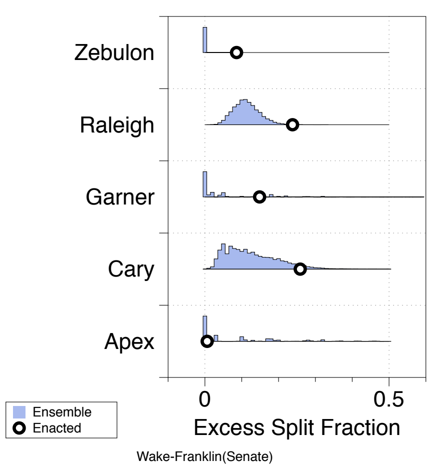


Figure 57: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Angier	1.457	4.854	0.336	-
Morrisville	2.754	22.18	7.905	-
Zebulon	4.303	8.594	0.346	Y
Rolesville	8.794	21.49	11.34	-
Fuquay-Varina	20.16	14.30	15.54	-
Wake Forest	27.67	13.96	11.62	-
Wendell	36.73	8.284	14.81	-
Garner	46.42	10.76	11.33	Y
Knightdale	53.57	12.01	9.087	-
Apex	65.39	12.50	12.08	Y
Holly Springs	85.91	19.92	13.80	-
Cary	100.0	12.67	7.112	Y
Raleigh	100.0	11.30	3.565	Y

Figure 58: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[6] For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.3 Davie-Forsyth Cluster (Senate)

We examine the quality of our ensemble in the Davie-Forsyth county cluster in the North Carolina Senate.[7] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 59 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 59, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 1449 of the 2196 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 1683 of the 2196 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 60 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 60. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

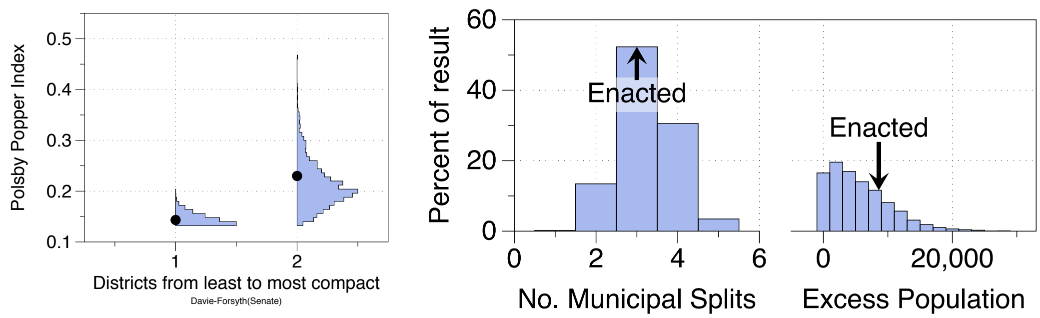
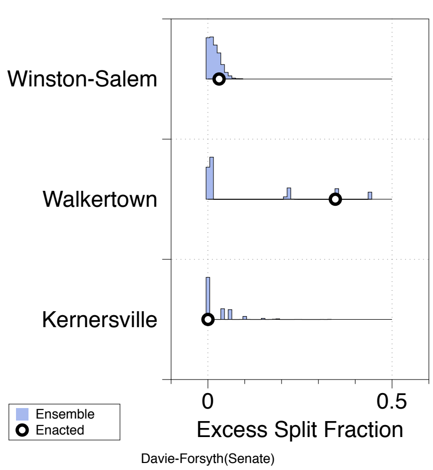


Figure 59: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
King	0.0	0.0	0.0	-
Rural Hall	0.0	0.0	0.0	-
Tobaccoville	0.0	0.0	0.0	-
Bethania	0.0	0.0	0.0	-
Clemmons	27.73	3.382	8.254	-
Lewisville	32.42	3.439	5.860	-
Walkertown	69.67	13.71	16.14	Y
Kernersville	93.76	2.702	4.138	Y
Winston-Salem	100.0	1.835	1.587	Y

Figure 60: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[7]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.4 Bladen-Brunswick-Pender-New Hanover Cluster (Senate)

We examine the quality of our ensemble in the Bladen-Brunswick-Pender-New Hanover county cluster in the North Carolina Senate.[8] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 61 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 61, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 50921 of the 50921 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 50921 of the 50921 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 62 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 62. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

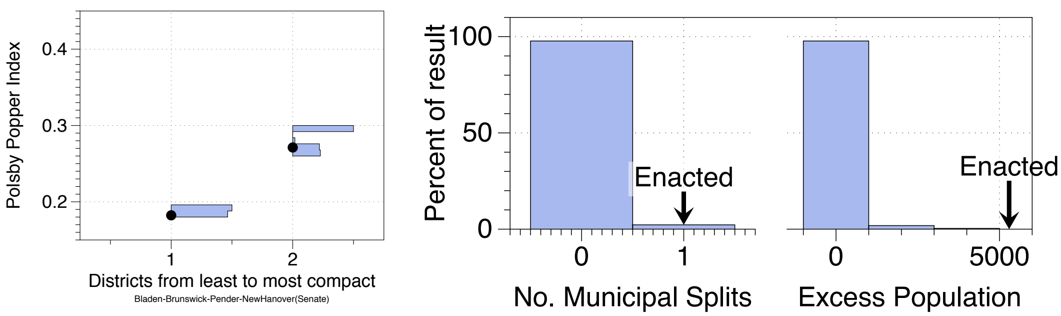
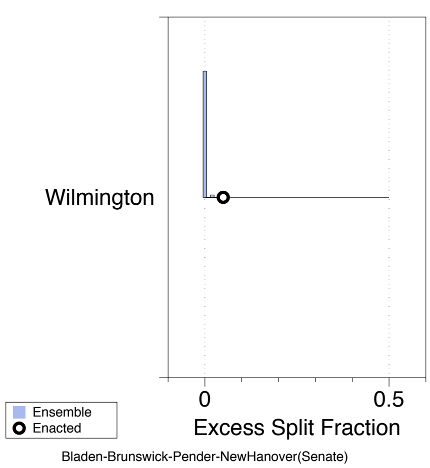


Figure 61: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Kure Beach	0.0	0.0	0.0	-
Wrightsville Beach	0.0	0.0	0.0	-
Wilmington	2.238	2.707	0.503	Y

Figure 62: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[8]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.5 Buncombe-Transylvania-Henderson Cluster (Senate)

We examine the quality of our ensemble in the Buncombe-Transylvania-Henderson county cluster in the North Carolina Senate.[9] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 63 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 63, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 39736 of the 39764 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 11317 of the 39764 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 64 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 64. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

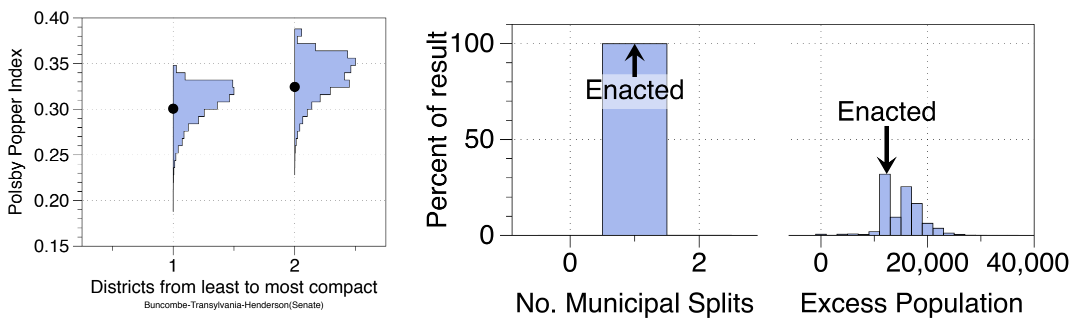
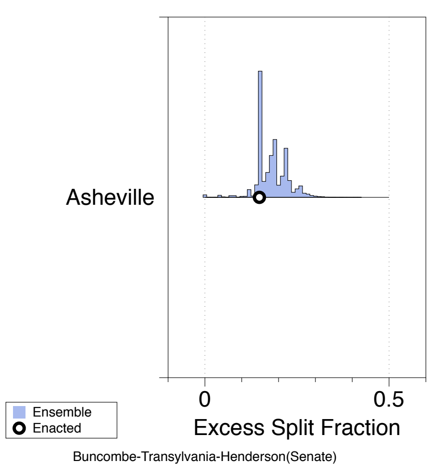


Figure 63: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Weaverville	0.0	0.0	0.0	-
Biltmore Forest	0.0	0.0	0.0	-
Black Mountain	0.012	33.50	19.78	-
Woodfin	0.057	3.206	0.706	-
Asheville	99.99	18.18	4.348	Y

Figure 64: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[9]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.6 Guilford-Alamance-Randolph Cluster (Senate)

We examine the quality of our ensemble in the Guilford-Alamance-Randolph county cluster in the North Carolina Senate.[10] As metioned above, we freeze Districts 24, and 28 since these districts were drawn by the Special Master. We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 65 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 65, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 604 of the 711 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 699 of the 711 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 66 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 66. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

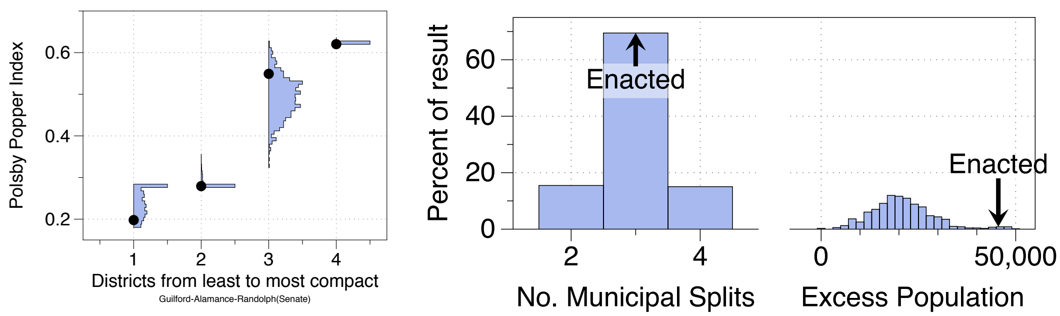
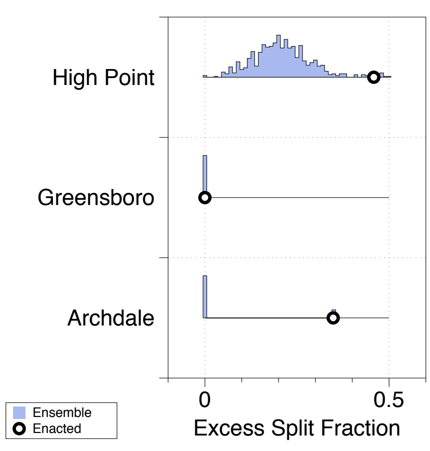


Figure 65: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Stokesdale	0.0	0.0	0.0	-
Burlington	0.0	0.0	0.0	-
Summerfield	0.0	0.0	0.0	-
Gibsonville	0.0	0.0	0.0	-
Kernersville	0.0	0.0	0.0	-
Oak Ridge	0.0	0.0	0.0	-
Whitsett	0.0	0.0	0.0	-
Sedalia	0.0	0.0	0.0	-
Pleasant Garden	0.421	49.98	35.34	-
Archdale	16.03	34.83	3.276	Y
Jamestown	83.12	0.235	0.127	-
High Point	100.0	21.05	8.103	Y
Greensboro	100.0	0.0	0.0	Y

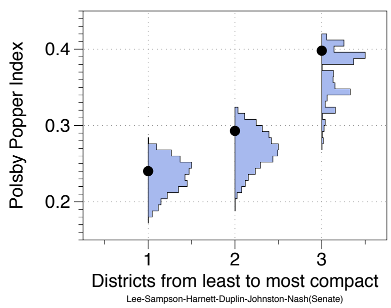
Figure 66: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[10]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.7 Lee-Sampson-Harnett-Duplin-Johnston-Nash Cluster (Senate)

We examine the quality of our ensemble in the Lee-Sampson-Harnett-Duplin-Johnston-Nash county cluster in the North Carolina Senate.[11] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 67 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 67, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 308 of the 1103 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 427 of the 1103 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 68 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 68. A 'Y' on the right most column denotes that the plan was split in the enacted plan.



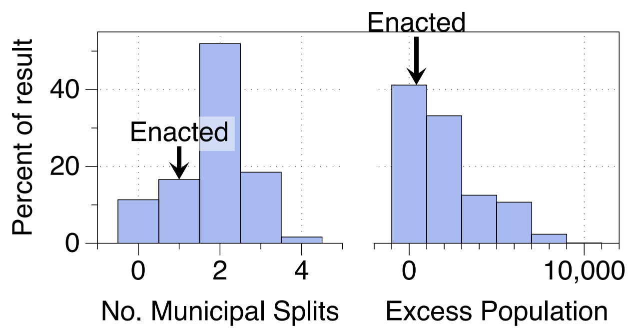
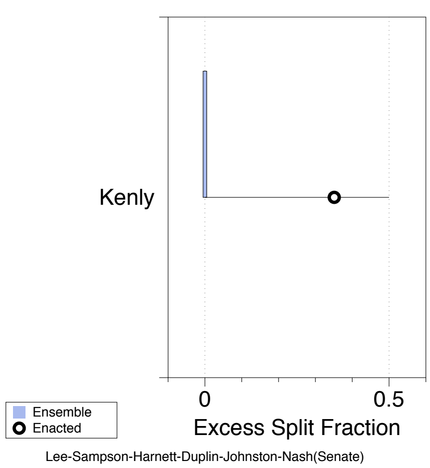


Figure 67: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Kenly	0.0	0.0	0.0	Y
Wilson's Mills	0.0	0.0	0.0	-
Archer Lodge	1.541	44.54	11.13	-
Clayton	20.58	13.95	10.45	-
Pine Level	43.51	0.352	0.016	-
Selma	46.14	10.15	14.38	-
Smithfield	70.71	17.16	15.23	-

Figure 68: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[11]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.8 Alamance Cluster (House)

We examine the quality of our ensemble in the Alamance county cluster in the North Carolina House.[12] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 69 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 69, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 632 of the 2558 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 2355 of the 2558 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 70 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 70. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

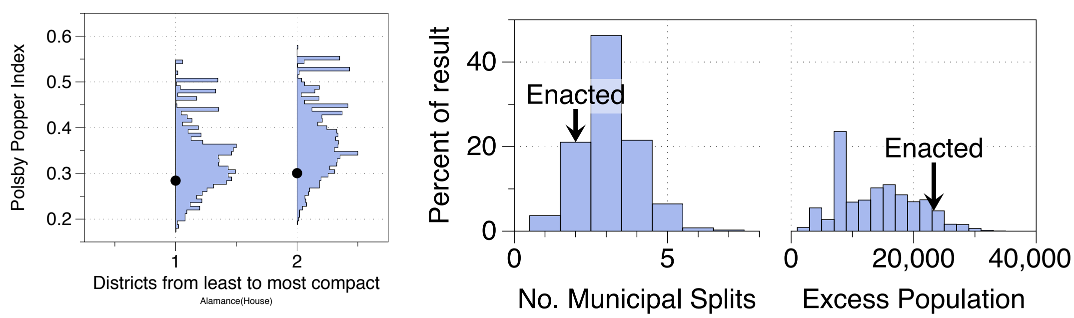
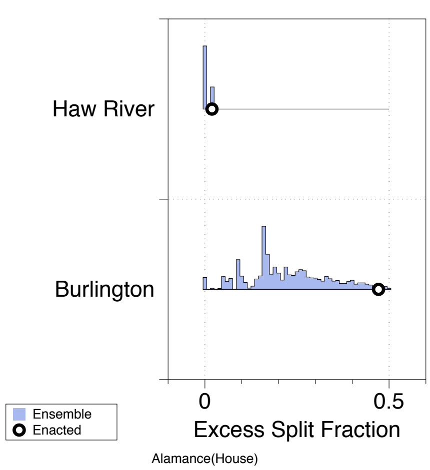


Figure 69: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Mebane	4.222	29.23	2.826	-
Ossipee	8.405	7.550	0.516	-
Gibsonville	19.42	0.254	0.011	-
Swepsonville	21.30	5.805	0.248	-
Green Level	22.20	8.428	0.353	-
Haw River	25.95	1.914	0.074	Y
Elon	43.08	15.06	15.54	-
Graham	66.96	22.41	15.52	-
Burlington	97.88	22.78	10.62	Y

Figure 70: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[12]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.9 Buncombe Cluster (House)

We examine the quality of our ensemble in the Buncombe county cluster in the North Carolina House.[13] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 71 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 71, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 2936 of the 3606 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 3151 of the 3606 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the numb er of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 72 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 72. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

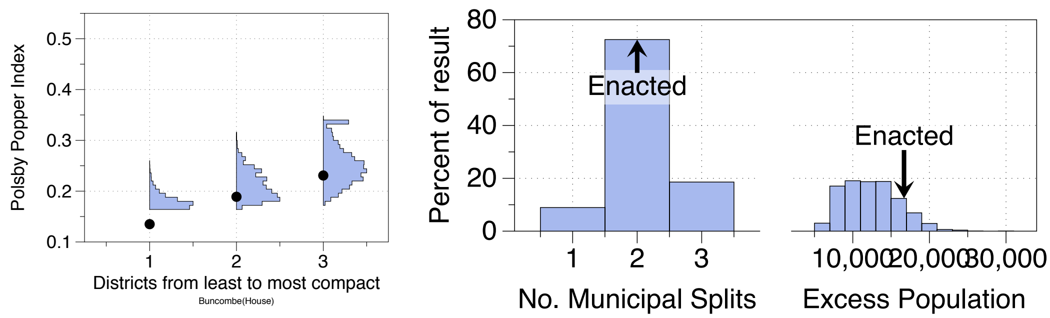
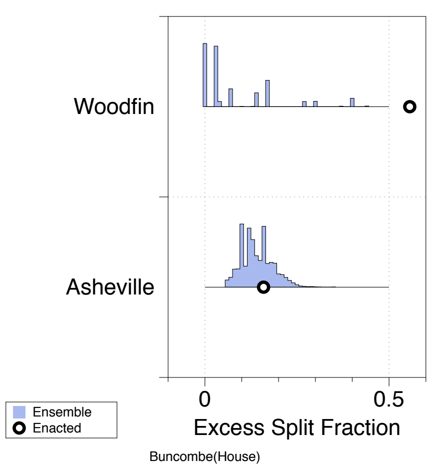


Figure 71: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Biltmore Forest	0.0	0.0	0.0	-
Black Mountain	0.027	30.41	0.0	-
Weaverville	40.04	5.900	2.424	-
Woodfin	69.57	11.49	10.75	Y
Asheville	100.0	14.14	4.226	Y

Figure 72: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[13]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.10 Columbus-Pender-Robeson Cluster (House)

We examine the quality of our ensemble in the Columbus-Pender-Robeson county cluster in the North Carolina House.[14] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 73 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 73, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 4975 of the 15941 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 15611 of the 15941 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 74 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 74. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

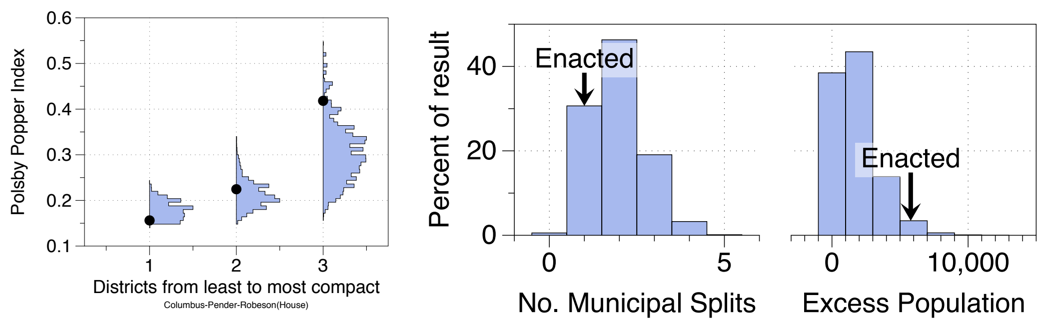
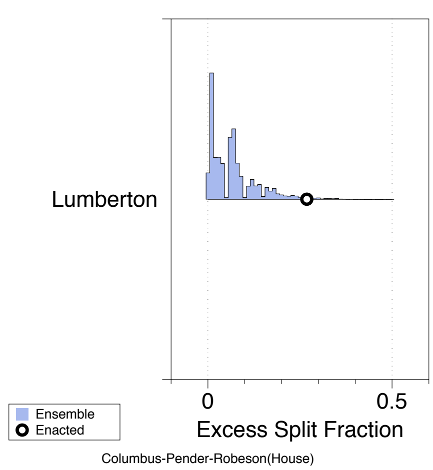


Figure 73: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Fairmont	0.0	0.0	0.0	-
Tabor City	0.0	0.0	0.0	-
Lake Waccamaw	3.230	3.175	0.140	-
Pembroke	10.32	38.58	0.951	-
Red Springs	18.29	0.029	0.000	-
Brunswick	26.20	38.24	0.591	-
Whiteville	39.10	3.741	9.764	-
Lumberton	97.08	6.977	6.290	Y

Figure 74: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[14]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.11 Cumberland Cluster (House)

We examine the quality of our ensemble in the Cumberland county cluster in the North Carolina House.[15] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 75 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 75, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 9275 of the 10368 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 6331 of the 10368 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 76 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 76. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

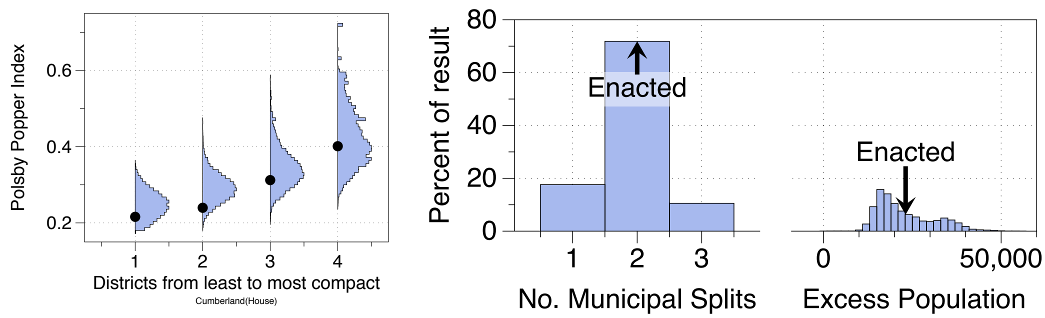
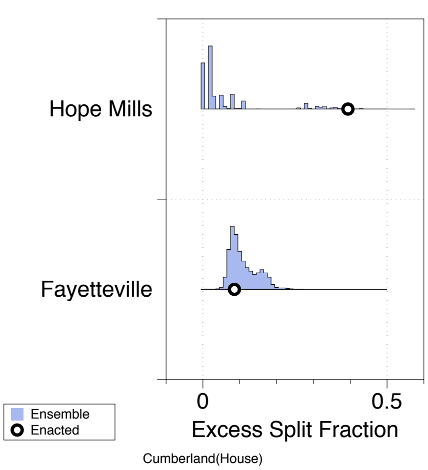


Figure 75: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Spring Lake	0.0	0.0	0.0	-
Eastover	11.30	24.31	13.42	-
Hope Mills	81.62	7.311	9.941	Y
Fayetteville	100.0	10.94	3.724	Y

Figure 76: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[15] For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.12 Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly Cluster (House)

We examine the quality of our ensemble in the Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly county cluster in the North Carolina House.[16] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 77 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 77, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 7353 of the 7725 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 7571 of the 7725 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 78 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 78. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

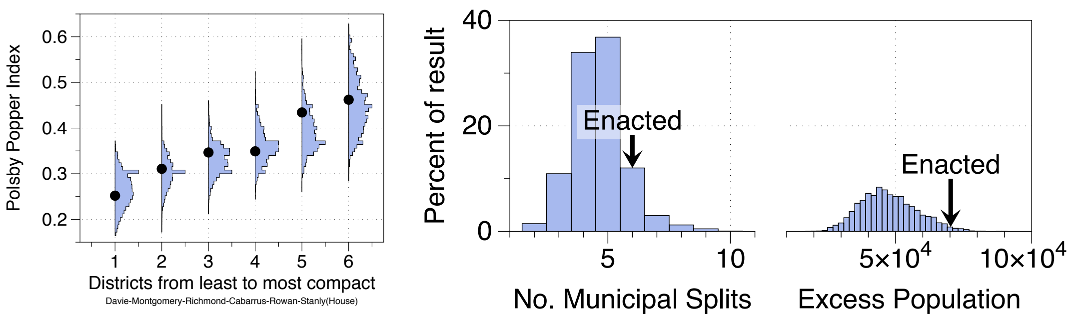
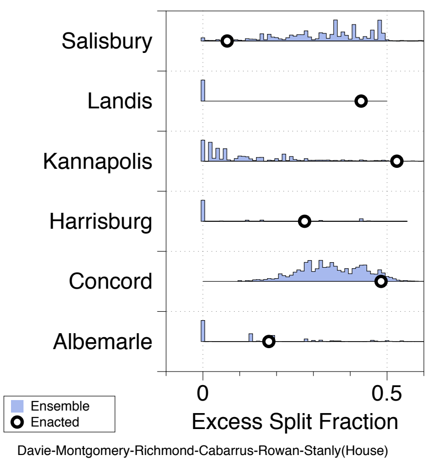


Figure 77: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).

[16]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
New London	0.0	0.0	0.0	-
Midland	0.0	0.0	0.0	-
Mount Pleasant	0.0	0.0	0.0	-
Landis	0.0	0.0	0.0	Y
Spencer	0.0	0.0	0.0	-
Badin	0.0	0.0	0.0	-
Oakboro	0.258	16.46	3.776	-
Red Cross	1.540	7.951	0.731	-
Stanfield	1.980	7.620	2.628	-
Faith	4.155	7.806	0.436	-
Granite Quarry	4.155	1.058	0.059	-
Norwood	5.592	31.77	1.530	-
East Spencer	20.37	0.238	0.089	-
Locust	37.42	3.173	3.232	-
Harrisburg	38.66	21.76	17.15	Y
Albemarle	58.45	23.88	12.05	Y
Kannapolis	91.75	13.85	13.35	Y
Salisbury	99.50	32.95	12.21	Y
Concord	100.0	35.13	8.857	Y

Figure 78: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

E.13 Duplin-Onslow Cluster (House)

We examine the quality of our ensemble in the Duplin-Onslow county cluster in the North Carolina House.[17] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 79 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 79, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 4933 of the 7688 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 7593 of the 7688 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 80 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 80. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

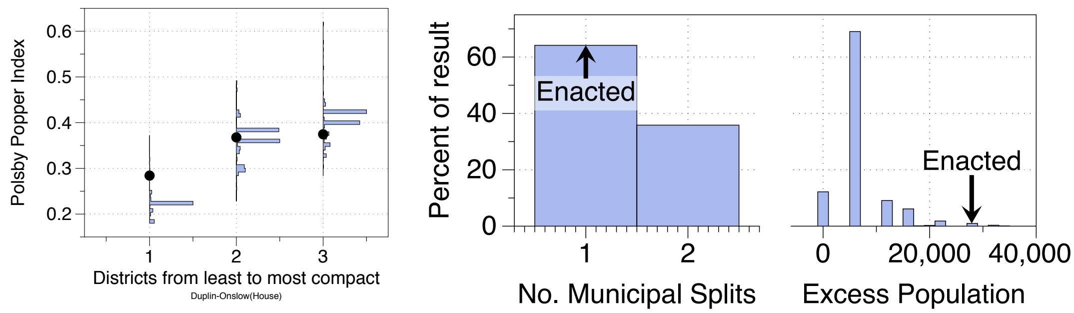
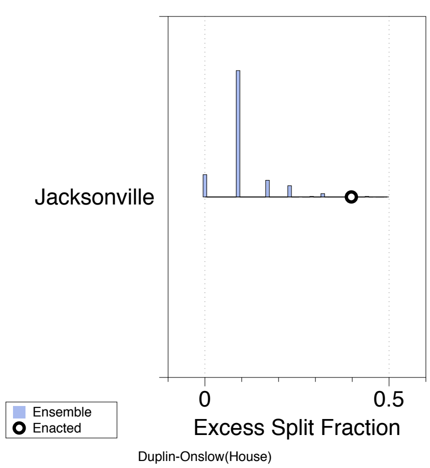


Figure 79: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Surf City	0.0	0.0	0.0	-
North Topsail Beach	0.0	0.0	0.0	-
Swansboro	0.0	0.0	0.0	-
Holly Ridge	35.83	0.236	0.004	-
Jacksonville	100.0	10.66	7.094	Y

Figure 80: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[17]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.14 Gaston-Cleveland Cluster (House)

We examine the quality of our ensemble in the Gaston-Cleveland county cluster in the North Carolina House.[18] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 81 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 81, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 8662 of the 8750 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 8204 of the 8750 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 82 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 82. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

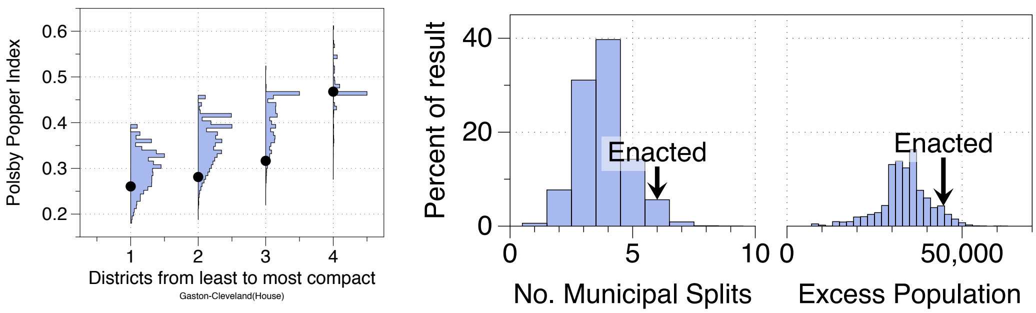
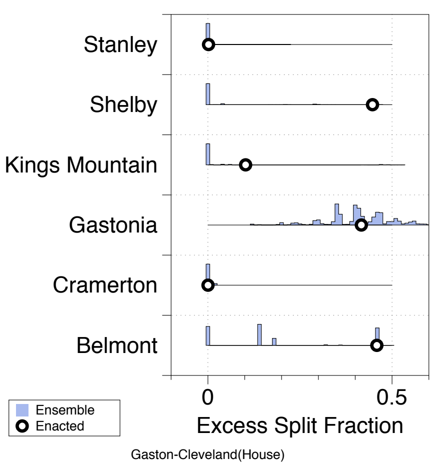


Figure 81: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Kingstown	0.0	0.0	0.0	-
Ranlo	0.0	0.0	0.0	-
High Shoals	0.0	0.0	0.0	-
Stanley	8.628	0.622	3.083	Y
Shelby	8.8	15.99	12.92	Y
Lowell	11.73	0.340	0.010	-
Bessemer City	13.53	38.52	1.119	-
Mount Holly	17.22	21.08	15.25	-
Cherryville	20.75	42.20	9.118	-
Kings Mountain	21.09	11.06	16.36	Y
Dallas	33.14	35.11	0.652	-
Belmont	71.08	27.05	14.93	Y
Cramerton	74.24	0.221	0.599	Y
Gastonia	100.0	40.90	8.599	Y

Figure 82: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[18]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.15 Guilford Cluster (House)

We examine the quality of our ensemble in the Guilford county cluster in the North Carolina House.[19] As metioned above, we freeze Districts 57, 61, and 62 since these districts were drawn by the Special Master. We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 83 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 83, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 1442 of the 1442 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 1406 of the 1442 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 84 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 84. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

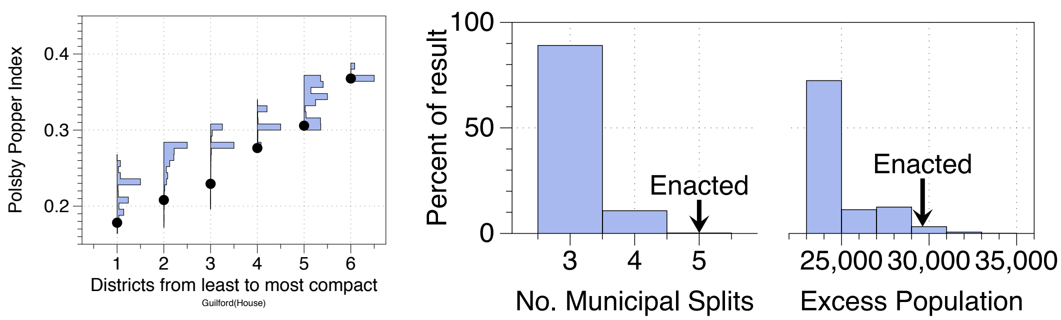
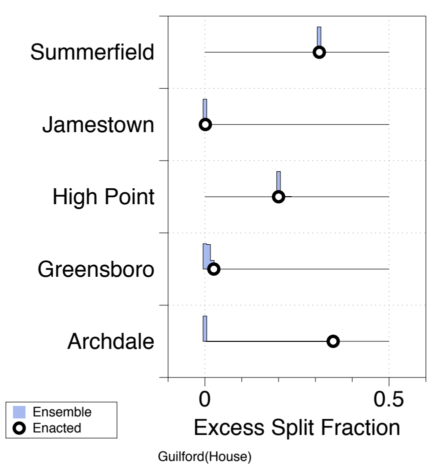


Figure 83: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Stokesdale	0.0	0.0	0.0	-
Burlington	0.0	0.0	0.0	-
Whitsett	0.0	0.0	0.0	-
Gibsonville	0.0	0.0	0.0	-
Kernersville	0.0	0.0	0.0	-
Oak Ridge	0.0	0.0	0.0	-
Sedalia	0.0	0.0	0.0	-
Pleasant Garden	0.485	49.98	20.40	-
Archdale	0.554	34.83	13.16	Y
Jamestown	10.12	0.199	0.040	Y
Summerfield	100.0	31.06	0.818	Y
High Point	100.0	20.00	0.618	Y
Greensboro	100.0	0.779	0.602	Y

Figure 84: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[19]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.16 Mecklenburg Cluster (House)

We examine the quality of our ensemble in the Mecklenburg county cluster in the North Carolina House.[20] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 85 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 85, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 28543 of the 40549 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 27037 of the 40549 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 86 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 86. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

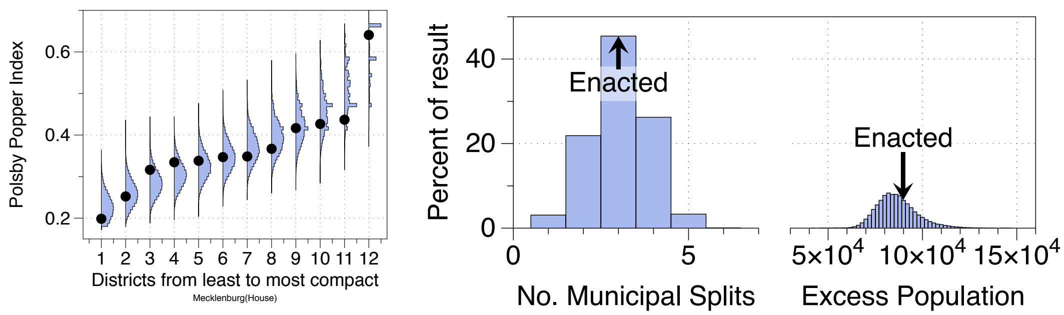
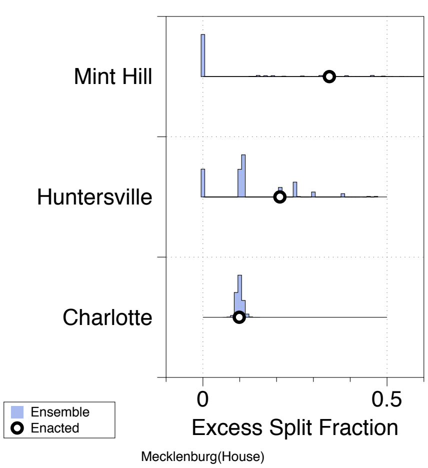


Figure 85: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Cornelius	1.750	27.80	14.97	-
Mint Hill	18.69	29.39	11.96	Y
Pineville	42.22	35.25	8.502	-
Matthews	63.40	30.23	11.48	-
Huntersville	78.90	16.10	8.331	Y
Charlotte	100.0	9.951	0.813	Y
Davidson	0.004	19.17	33.09	-

Figure 86: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[20]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.17 Nash-Franklin Cluster (House)

We examine the quality of our ensemble in the Nash-Franklin county cluster in the North Carolina House.[21] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 87 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 87, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 2634 of the 4284 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 2634 of the 4284 plans in the ensemble.

To continue to examine municipal splitting, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 88. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

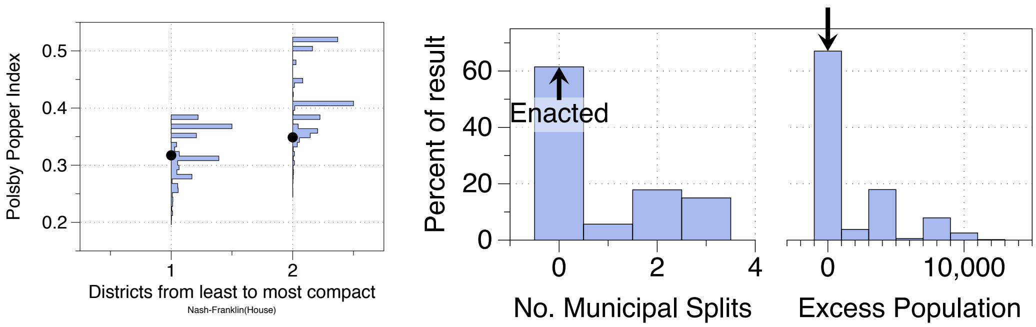


Figure 87: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).

Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Nashville	1.237	0.056	0.007	-
Red Oak	15.10	0.651	0.195	-
Dortches	32.84	0.838	1.166	-
Rocky Mount	37.16	10.88	7.509	-

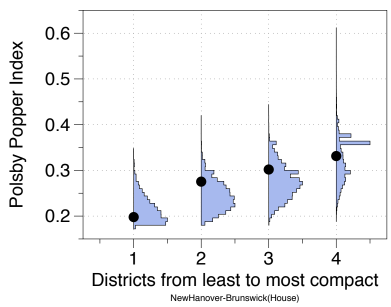
Figure 88: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[21]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.18 New Hanover-Brunswick Cluster (House)

We examine the quality of our ensemble in the New Hanover-Brunswick county cluster in the North Carolina House.[22] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 89 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 89, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 21738 of the 46642 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 46642 of the 46642 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 90 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 90. A 'Y' on the right most column denotes that the plan was split in the enacted plan.



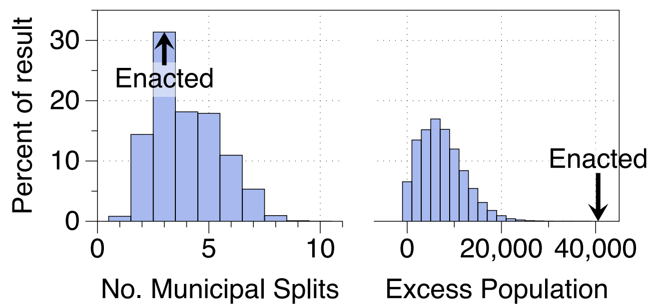
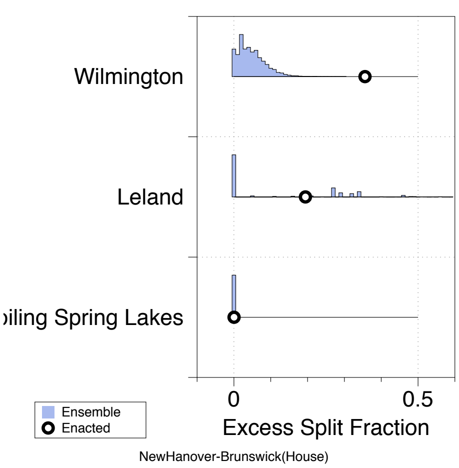


Figure 89: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Kure Beach	0.0	0.0	0.0	-
Carolina Shores	0.0	0.0	0.0	-
Holden Beach	0.0	0.0	0.0	-
Sunset Beach	0.917	1.035	0.050	-
Calabash	1.029	39.58	1.808	-
Navassa	3.949	38.93	0.907	-
Southport	7.152	36.99	0.640	-
Shallotte	12.53	20.42	21.56	-
Belville	26.06	8.249	2.914	-
St. James	26.39	9.984	0.089	-
Leland	40.56	30.03	9.078	Y
Oak Island	53.02	7.147	12.74	-
Boiling Spring Lakes	60.04	0.045	0.015	Y
Wrightsville Beach	65.76	0.161	0.000	-
Wilmington	100.0	4.776	3.661	Y

Figure 90: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[22]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.19 Person-Vance-Granville-Warren Cluster (House)

We examine the quality of our ensemble in the Person-Vance-Granville-Warren county cluster in the North Carolina House.[23] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 91 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 91, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 4060 of the 4060 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 0 of the 4060 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 92 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 92. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

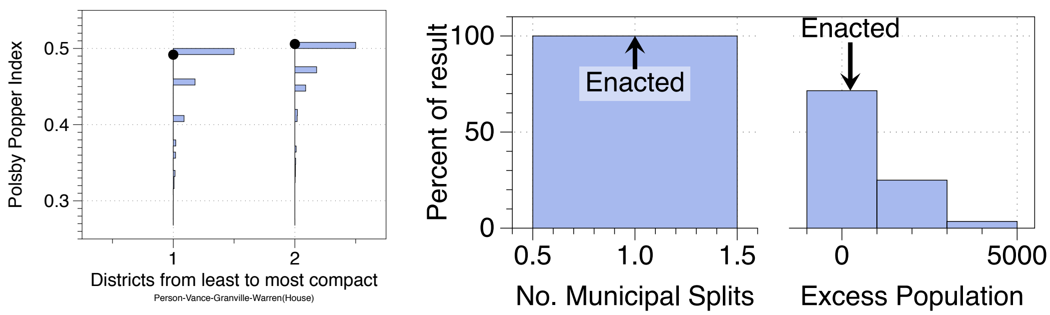
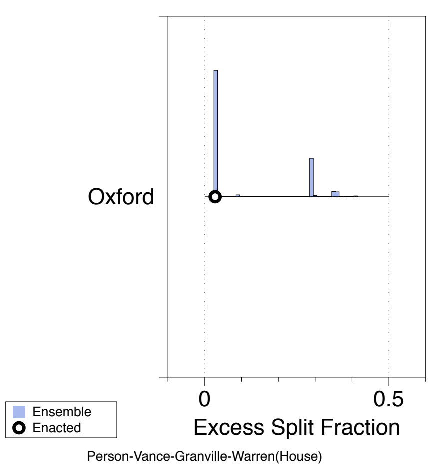


Figure 91: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Creedmoor	0.0	0.0	0.0	-
Butner	0.0	0.0	0.0	-
Oxford	100.0	10.90	12.69	Y

Figure 92: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[23] For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.20 Pitt-Lenoir Cluster (House)

We examine the quality of our ensemble in the Pitt-Lenoir county cluster in the North Carolina House.[24] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 93 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 93, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 3363 of the 3763 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 2046 of the 3763 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 94 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 94. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

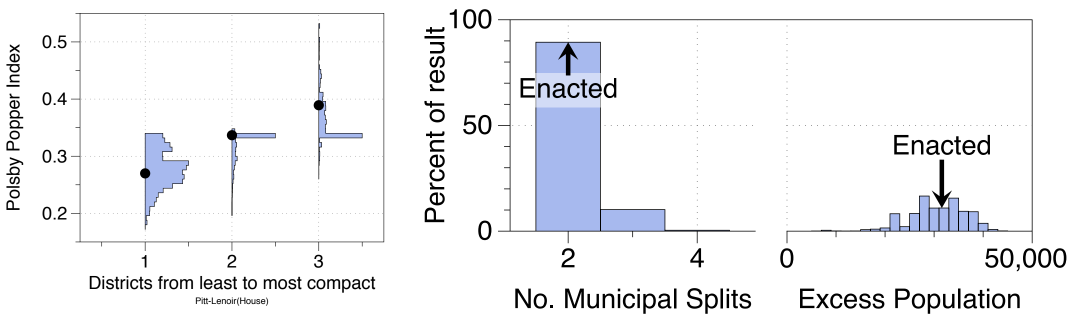
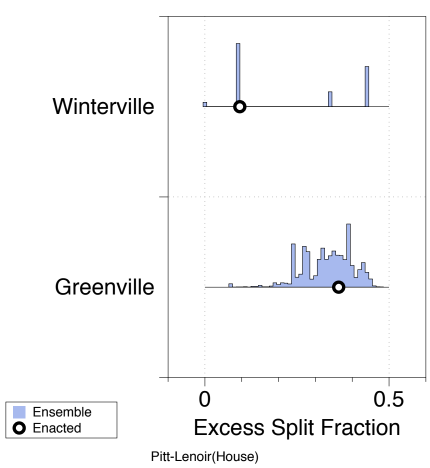


Figure 93: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Grimesland	1.567	0.907	0.119	-
Farmville	2.391	47.01	4.983	-
Ayden	10.68	35.87	19.33	-
Winterville	96.41	24.23	16.08	Y
Greenville	100.0	33.32	6.708	Y

Figure 94: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[24]For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.21 Union-Anson Cluster (House)

We examine the quality of our ensemble in the Union-Anson county cluster in the North Carolina House.[25] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 95 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 95, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 6569 of the 6645 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 4807 of the 6645 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 96 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 96. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

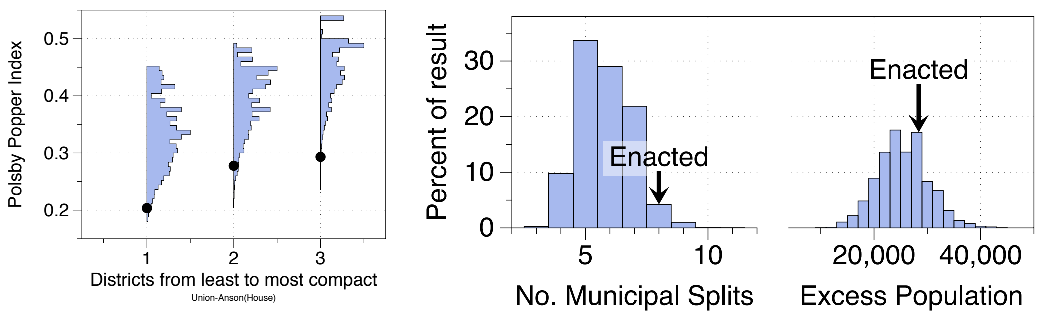
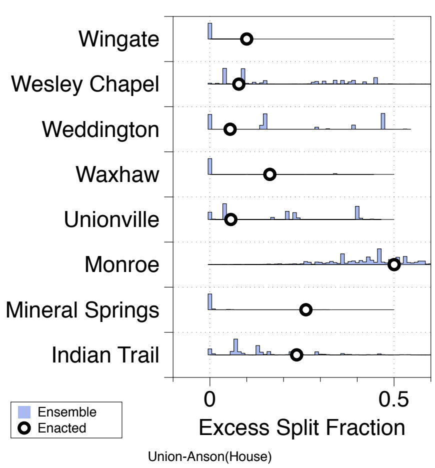


Figure 95: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Fairview	4.725	41.00	2.317	-
Marshville	6.139	0.359	0.105	-
Mineral Springs	7.389	4.215	6.490	Y
Waxhaw	7.720	24.73	12.73	Y
Wingate	9.285	9.997	0.402	Y
Marvin	10.35	20.08	17.57	-
Hemby Bridge	25.04	1.587	3.083	-
Stallings	48.26	23.36	13.39	-
Weddington	78.01	28.79	16.20	Y
Unionville	87.75	20.78	14.33	Y
Indian Trail	95.69	15.02	11.35	Y
Wesley Chapel	99.54	20.53	15.20	Y
Monroe	99.95	42.66	10.84	Y

Figure 96: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[25] For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.22 Wake Cluster (House)

We examine the quality of our ensemble in the Wake county cluster in the North Carolina House.[26] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 97 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 97, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 18269 of the 50000 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 49815 of the 50000 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 98 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 98. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

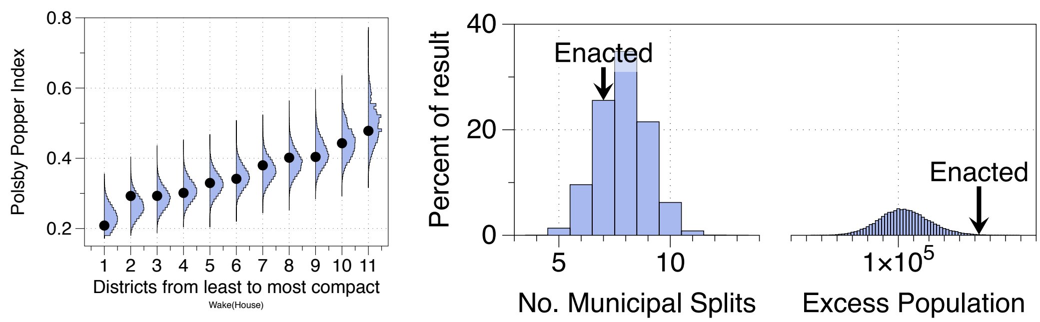
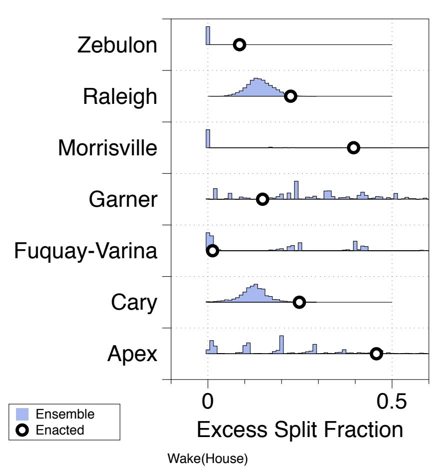


Figure 97: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Zebulon	4.258	8.594	0.186	Y
Morrisville	15.79	19.72	16.91	Y
Angier	18.76	4.854	0.050	-
Wendell	20.45	9.127	15.28	-
Rolesville	31.10	30.64	10.80	-
Knightdale	44.02	16.80	8.967	-
Wake Forest	72.80	21.27	13.47	-
Fuquay-Varina	85.29	21.30	16.89	Y
Holly Springs	95.66	25.81	14.48	-
Garner	99.66	28.92	15.23	Y
Apex	99.9	19.99	15.31	Y
Cary	100.0	12.38	3.772	Y
Raleigh	100.0	14.09	3.473	Y

Figure 98: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[26] For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

E.23 Yadkin-Forsyth Cluster (House)

We examine the quality of our ensemble in the Yadkin-Forsyth county cluster in the North Carolina House.[27] We examine the marginal distributions on compactness (Polsby-Popper), and municipal splitting. Compactness comparisons are shown in Figure 99 (left). We display how many people, cluster wide, are cut out of their municipality's primary district(s), and investigate how many different municipalities were split (see Figure 99, two right most). The ensemble splits the same or fewer municipalities than the enacted plan in 16315 of the 17062 plans in the ensemble. The ensemble splits fewer or the same number of people from their core district(s) than the enacted plan in 16888 of the 17062 plans in the ensemble.

To continue to examine municipal splitting, we examine all municipalities that were split within the enacted plan. For each of these municipalities, and in each district plan of the ensemble, we quantify the number of people who were cut out of each municipality as described in Section C.2.3. We then contextualize how the enacted plan cut people out of these split municipalities within the ensemble in Figure 100 (left). Next, we classify how often each municipality in the ensemble was cut by the ensemble in the table in Figure 100. A 'Y' on the right most column denotes that the plan was split in the enacted plan.

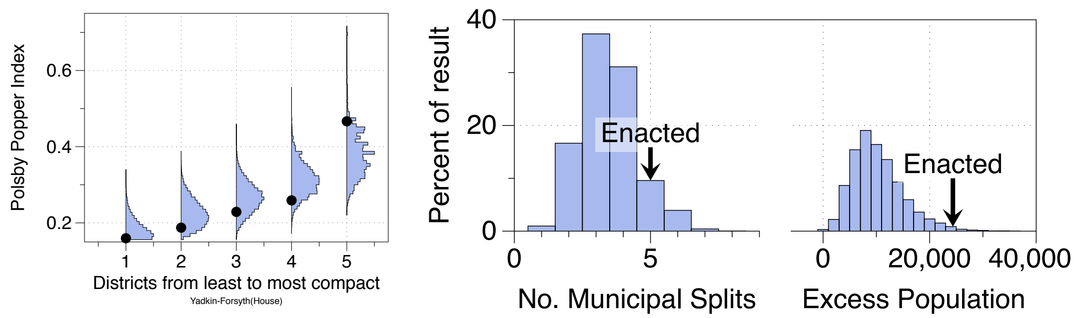
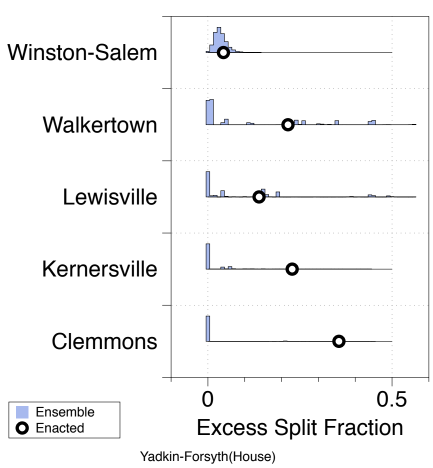


Figure 99: The districts in each plan of the ensemble are ordered from least to most compact; marginal distributions are then shown for the least and most compact districts within the ensemble of plans; the districts from the enacted plan are also sorted and then situated in the marginal distributions as dots (left). We compare the total number of people cut out of the municipality's primary district across the ensemble and enacted plan (center). We also compare the total number of municipal divisions (right).



Municipality	Split %	Excess Avg % split	St. dev.	Split in enacted
Bethania	0.0	0.0	0.0	-
King	6.511	29.07	0.872	-
Tobaccoville	7.701	15.73	7.980	-
Clemmons	11.36	5.685	10.94	Y
Rural Hall	13.21	13.44	0.283	-
Lewisville	57.50	17.48	13.92	Y
Walkertown	73.64	15.93	15.84	Y
Kernersville	75.34	1.943	3.973	Y
Winston-Salem	100.0	3.430	1.665	Y

Figure 100: We compare municipal divisions of the enacted plan with the ensemble. Split % reports the fraction of plans in the ensemble in which the given municipality was split. Avg. Split reports the fraction of the population that was removed from the core district(s) and St. dev. is the standard deviation of this split in the ensemble.

[27] For a description of the sampling procedure, see Section C; for the parameters used, see Section E; for information on validating the ensembles, see Section F.1

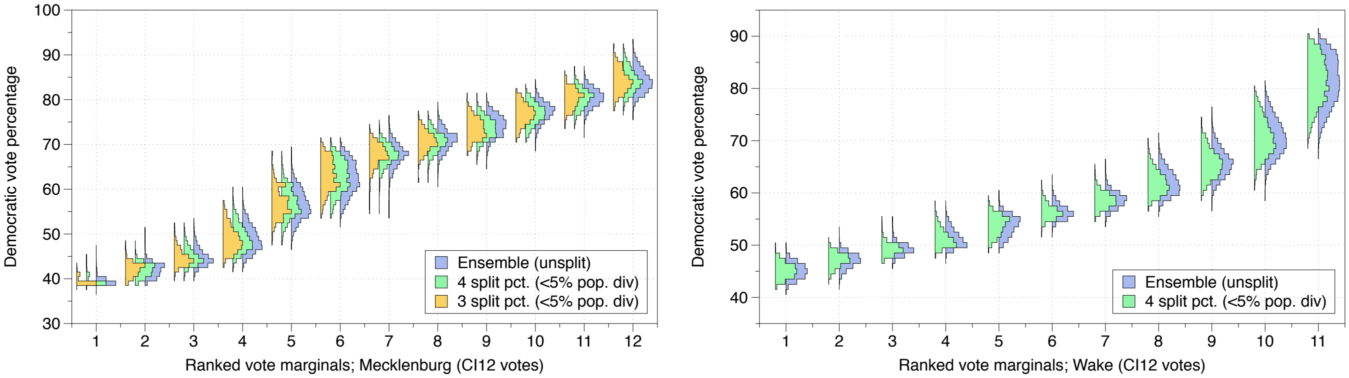


Figure 101: We plot ranked marginal distributions in the ensemble of House districts for Mecklenburg (left) and Wake (right). We compare our original distribution that does not split precincts, with the plans from the ensemble that have been altered to achieve compliant population deviation and split fewer than four precincts. In Mecklenburg, we also display the collection of split-precinct plans that split 3 or fewer precincts.

E.24 Splitting Precincts to Achieve Compliant Population Deviation

In four of the county clusters we found few or no ensemble plans that kept population deviation below 5%. These clusters are Wake, Mecklenburg, the Bladen-Lee-Sampson-Harnett-Greene-Wayne-Johnston cluster, and the Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander cluster in the House. For the latter two clusters we have already seen that there is no statewide effect when considering the enacted plan or the sampled plans (see Section 4, and Figure 8).

In the House clusters of Wake and Mecklenburg, we examine whether splitting precincts in order to achieve population deviation below 5% makes any difference in our results. To examine this we determine how much population adjacent precincts would have to exchange in order to achieve an acceptable deviation. We then randomly clump census blocks within each precinct, keeping them compact, so that there are roughly 100 people in a clump. We treat these clumps as our new, sub-precinct, graph. We then adapt the second part of the algorithm described in Section F.1.1 which seeks to reduce population deviations: First we choose a precinct to exchange; instead of exchanging the whole precinct, we exchange the closest compliant sub-precinct clump from this precinct until we have either transferred enough population, or the entire precinct has been transfered; in the latter case we repeat.

In the House districts in Wake there are 4 split precincts; in Mecklenburg there are 3. We subsample all plans which split four or fewer precincts in both Wake and Mecklenburg and plot the election results for the 2012 Commissioner of Insurance election in Figure 101. We see that splitting precincts has either no effect or, at most, only a small effect on the overall structure of the election outcomes.

E.25 Evaluating the effects of incumbency, only examining unique plans, and considering the space of 'more' compliant plans.

Our primary ensemble does not preserve incumbents. Furthermore it provides a distribution on the districting space that may lead to different results than other distributions. Finally, some of our sampled plans are less compact, split more municipalities, and split more people out of municipalities than the enacted plan. In this section we examine whether our results change when considering incumbency, reweighting the sampled space, and introducing more stringent redistricting criteria. Because we have previously shown that the majority of partisan effects on the statewide plan occur in a subset of the clusters (see Section 4), we only focus on this subset of clusters.

To consider the effect of incumbency, we first note that for clusters with districts drawn in 2011 and not changed in 2017, we use 2011 incumbents; for clusters whose districts were all redrawn in 2017, we use incumbents from 2017. For each cluster, we first see how many plans preserve incumbents to the same extent, or better, than they are preserved in each cluster (see Tables 13 and 14)[28]. In all but six of the subset of clusters, we find a significant number of plans that preserve incumbents, and thus we subsample these plans that preserve incumbents.

[28]Only Buncombe and the Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly county cluster had a single double bunking in the House; only the Davie-Forsyth, Guilford-Alamance-Randolph and Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander clusters had a single double bunking. All other clusters had no double bunking

Chamber	Cluster	$(w_{\text{pop}}, w_{\text{PP}}, w_{\text{M}})$	$(t_{\text{pop}}, t_{\text{PP}})$
Senate	Wake-Franklin	(50.0, 0.3, 0.0002)	(5.0, 0.179)
Senate	Mecklenburg	(100.0, 0.2, 0.0001)	(5.0, 0.179)
House	Mecklenburg	(50.0, 0.2, 0.0001)	(10.0, 0.179)
House	Wake	(100.0, 0.2, 0.0001)	(12.0, 0.179)
House	Guilford	(100.0, 0.15, 0.001)	(5.0, 0.167)
House	Yadkin-Forsyth	(50.0, 0.1, 0.0005)	(5.0, 0.157)

Table 8: Parameters used when resampling the above clusters to preserve incumbents. The third column is a list of the weights used in the score functions. The second column is a list of thresholds; the population threshold t_{pop} is an upper bound, and the compactness threshold t_{PP} is a lower bound.

For the Mecklenburg and Wake-Franklin clusters in the Senate, and the Mecklenburg, Guilford, Yadkin-Forsyth, and Wake clusters in the House, we generate new ensembles that preserve incumbents. When generating these ensembles, we use the parameters found in Table 8. In these ensembles, we allow no double bunking of incumbents. The only exceptions to this are in the Wake-Franklin cluster in the Senate, and the Yadkin-Forsyth cluster in the House; in these two exceptions we constrain the space so that no incumbents are double bunked except for Donny Lambeth(R) in Forsyth county and Tamara Barringer(R) in Wake county; these exceptions were made to avoid 'jamming' when sampling the space. After the new ensembles are collected, we then subsample the new ensembles and only include those that that do not double bunk *any* incumbents including the two mentioned above. We report the number of plans sampled in Table 15; we report convergence data in Table 12.

To consider the effects of the weighted distribution, we examine the collection of unique plans, so that all plans are treated as equally important. The number of unique plans versus total plans (with redundancy) is shown in Tables 13 and 14.

To examine the effect of more stringent redistricting criteria, we consider only districting plans that are more compact, split fewer municipalities, and split fewer people out of their municipalities core district(s). For compactness, we accept any district with a Polsby-Popper score of more than 0.3 as we have empirically seen that anything above this score tends to look quite compact. If a district within a plan does not have a Polsby-Popper score above 0.3, we examine the ranked Polsby-Popper scores in the enacted plan and compare them with the ranked scores of the districts in a plan from the ensemble. If the districts in the corresponding ranks of the plan from the ensemble are not more compact than the corresponding districts in the enacted plan, we do not include this plan. We also do not consider the plan if it splits more municipalities than the enacted plan, nor if it cuts more people out of core districts than the enacted plan. In short, we consider ensemble plans that are as, or more, compact than the enacted plan, that split fewer people from their municipality's core district(s) and split fewer districts.

Finally, we consider the joint effect of both ensuring incumbents are preserved and requiring more stringent redistricting criteria described in the preceding paragraph. We give the number of plans found in each of these processes in Tables 16 and 17.

We examine the ranked marginal distributions on all four ensembles using the CI12 election data in Figures 102, 103, and 104. The vast majority of these comparisons lead to remarkably similar marginal distributions. There are a few exceptions: For example, the House districts in Alamance appears to provide a greater advantage to the Democrats; this effect, however, vanishes once we examine the joint effect of incumbency and more stringent redistricting criteria. In the Person-Granville-Vance-Warren cluster of the House, we don't find any districts other than the enacted plan under the more stringent redistricting criteria. Examining only unique plans also can occasionally change the structure, as it does in the Duplin-Onslow cluster.

We test if these small differences affect our overall conclusions that the enacted plan is an extreme outlier in favor of the Republican Party. In the 17 considered elections, we count the number of elections in which the enacted plan leads to a number of Democratic seats that is an extreme outlier in favor of either the Republicans or the Democrats. We display the results in Table 9. In all cases we find a significant number of elections in which the enacted plan leads to more Republican seats than in either 99% or 99.9% of the ensemble; conversely, we find no elections in which the enacted plan leads to more Democrats than 99% of plans in the ensemble. Furthermore, the number of outlying elections we see for each of the four new ensembles remains consistent with the number of outlying elections present in the original ensemble.

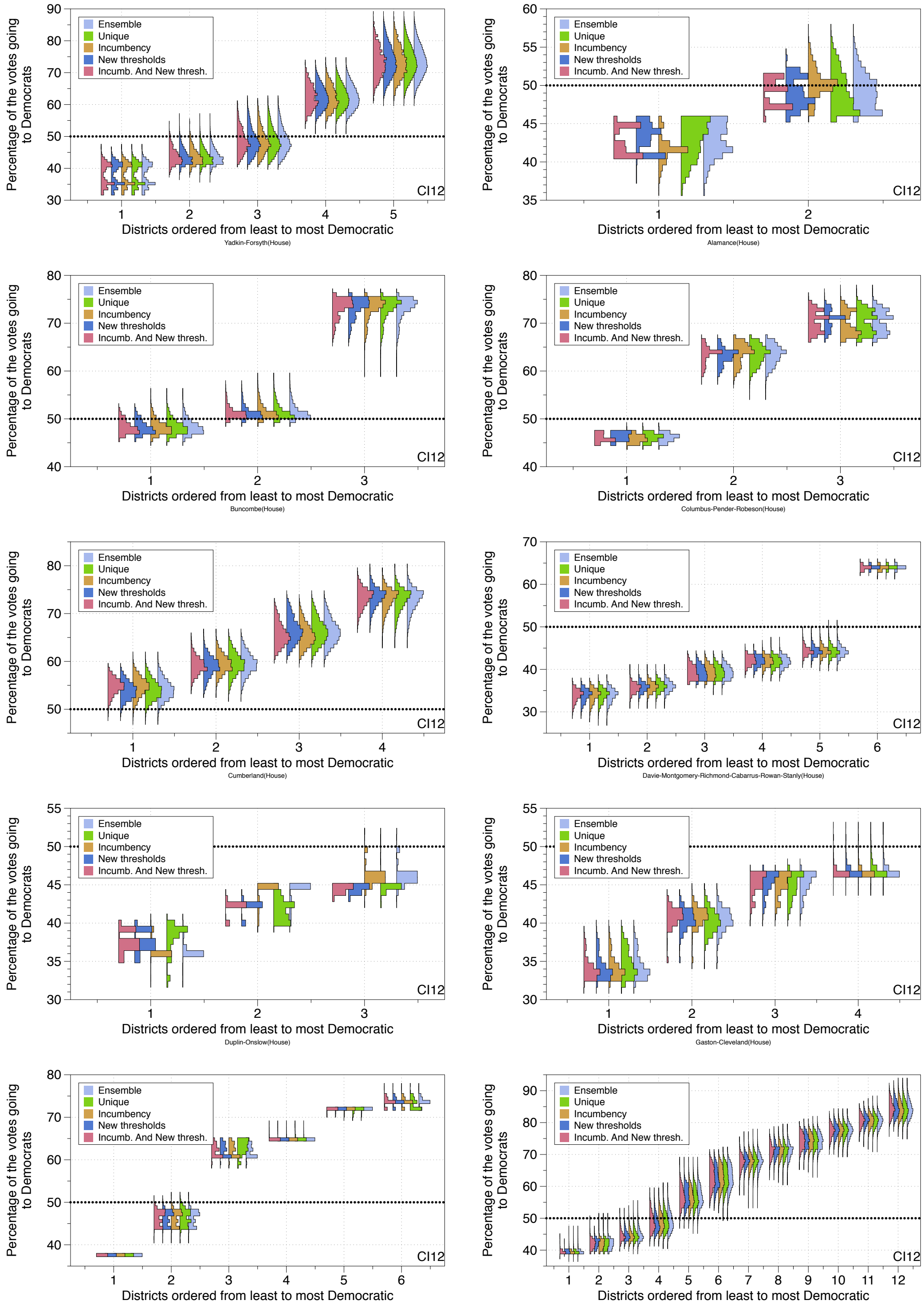


Figure 102: We examine the difference in the marginal distributions when considering incumbency, reweighting the space, and when making more stringent redistricting criteria. We examine a collection of House clusters presented in Section 4

Chamber	Ensemble	No. elections atypically favoring Rep. Prob$<1\%$	Prob$<0.1\%$	No. elections atypically favoring Dem. Prob$<1\%$	Prob$<0.1\%$
Senate	Original	10	9	0	0
Senate	Unique	10	8	0	0
Senate	Incumb	11	8	0	0
Senate	Thresh	10	8	0	0
Senate	Incmb. & Thresh.	10	8	0	0
House	Original	7	4	0	0
House	Unique	8	5	0	0
House	Incumb.	7	6	0	0
House	Thresh.	7	5	0	0
House	Incmb. & Thresh.	10	6	0	0

Table 9: Of the 17 considered elections, we count the number of elections in which the enacted plan is a 1% outlier and a 0.1% outlier favoring either the Republicans or the Democrats. We find no election in which the Democrats elect more seats than expected to the extreme extent. We find a significant number of elections in which the Republicans elected more seats than expected to an extreme extent.

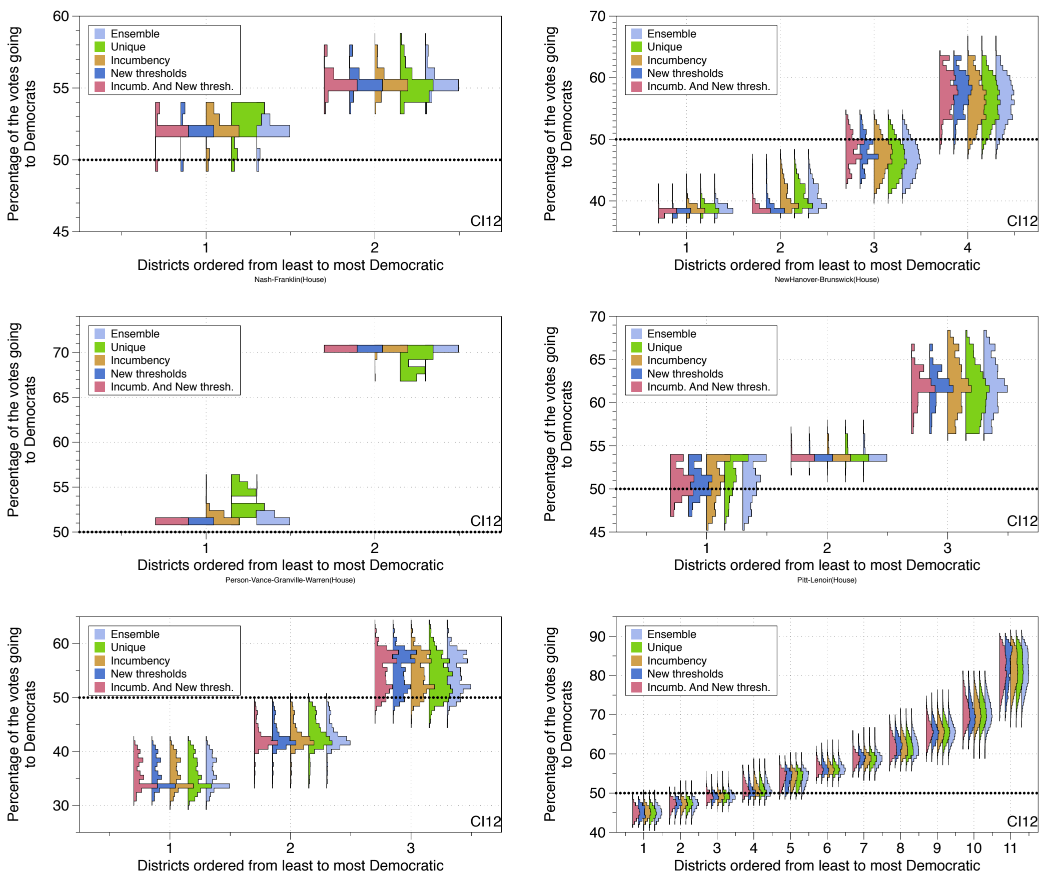


Figure 103: We examine the difference in the marginal distributions when considering incumbency, reweighting the space, and when making more stringent redistricting criteria. We examine the remaining House clusters presented in Section 4

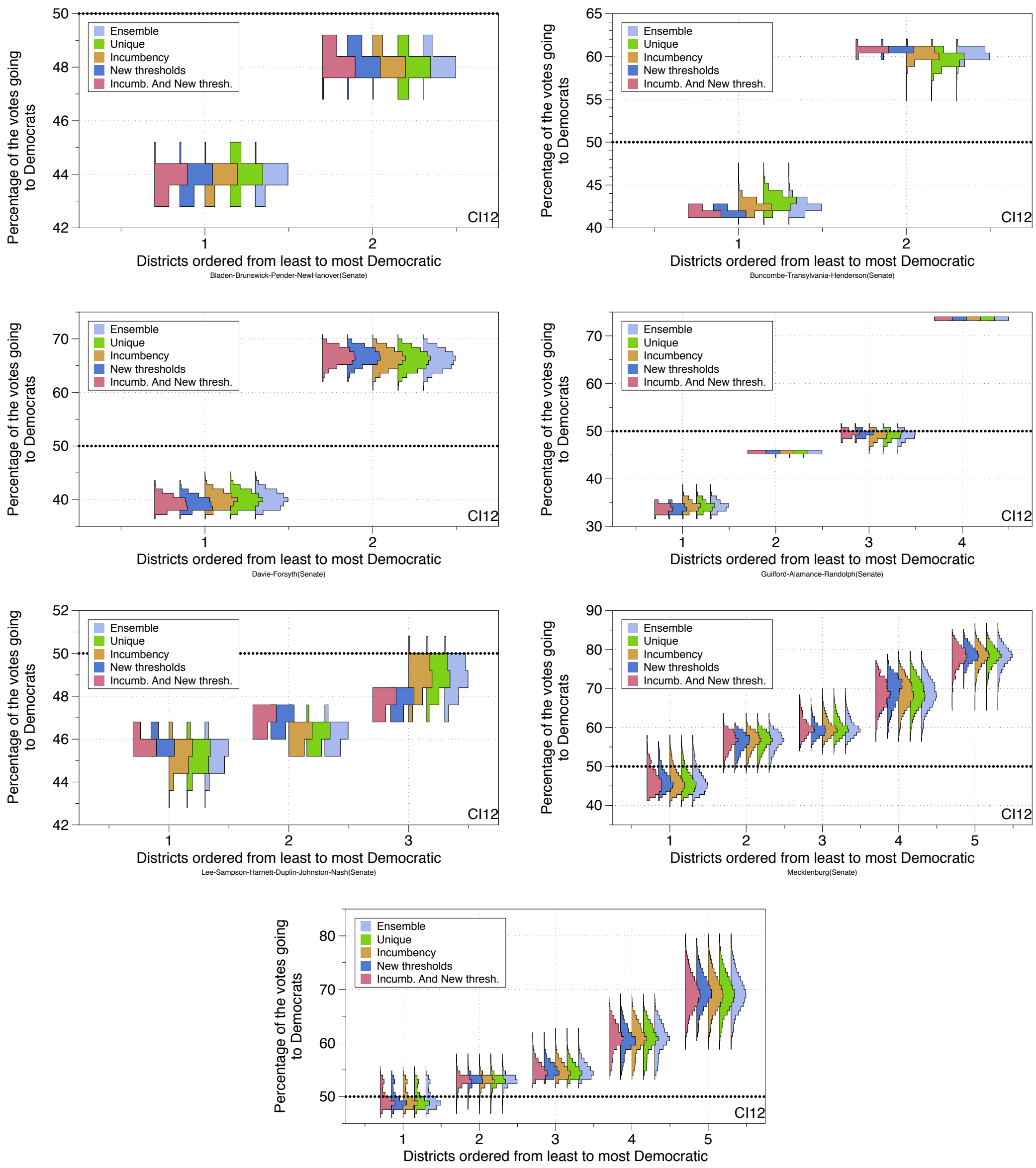


Figure 104: We examine the difference in the marginal distributions when considering incumbency, reweighting the space, and having more stringent redistricting criteria. We examine the Senate clusters presented in Section 4

F Validations

F.1 Convergence of samplers

To test for convergence, we draw from [11]. The idea is to begin a random walk from a variety of initial conditions, each with the same invarient measure. We then choose some observable quantities we care about, and consider their distribution in each of the random walks. In general, there will be differences in these distributions since each walk begins by exploring different areas of the space. As the walk explores the space of redistrictings, we expect for each of the distributions on the observables to converge to the same distribution. When, in the worst case, all of these distributions are similar, we report to have reliably sampled the space of redistricting plans. These methods have already been successfully used in testing for chain convergence for random walks on the space of redistricting plans [12].

We launch five chains for each county cluster, each with its own initial condition. One of these intial conditions is the enacted plan. The remaining four initial conditions are generated using an adaptation of the algorithm developed in [13] (for details see Section F.1.1 below).

We then consider the marginal distributions on the ranked votes when using the 2012 and 2016 Presidential elections. To estimate the difference between these ranked marginal distributions, approximate each distribution to be normal, and then use Jeffery's divergence to estimate the difference between the marginal distributions. Jeffery's divergence is defined to be the symmetrized KullbackLeibler divergence, or

$$\langle p, q\rangle_J = \langle p, q\rangle_{KL} + \langle q, p\rangle_{KL},$$

where p and q are two probability distributions, and the KullbackLeibler divergence is defined as

$$\langle p, q\rangle_{KL} = \int p(x)\log\left(\frac{p(x)}{q(x)}\right)dx$$

With the normal approximation, Jeffery's divergence may be explicitly calculated as

$$\langle \mathcal{N}(\mu_1, \sigma_1^2), \mathcal{N}(\mu_2, \sigma_2^2)\rangle_J = \frac{(\mu_1-\mu_2)^2\left(\sigma_1^2+\sigma_2^2\right)+\left(\sigma_1^2-\sigma_2^2\right)^2}{2\sigma_1^2\sigma_2^2},$$

where $\mathcal{N}(\mu, \sigma^2)$ is a normal distribution with mean μ and standard deviation σ.

We declare convergence when every considered marginal distribution across all combinations of walks is within some threshold, ϵ; i.e.

$$\frac{(\mu_1-\mu_2)^2\left(\sigma_1^2+\sigma_2^2\right)+\left(\sigma_1^2-\sigma_2^2\right)^2}{2\sigma_1^2\sigma_2^2} \leq \epsilon,$$

for every marginal distribution, where the subscripts 1 and 2 refer to an arbitrary choice of two of the five random walks.

We note that many of the marginal distributions we have presented may be well approximated with a normal distribution. The above convergence criteria, however, does not require a close approximation to a normal. It may be viewed as the a bound on the error of the variance and the mean. We develop exact and approximate bounds on these errors, as a function of ϵ below in Section F.1.2.

We demonstrate how these ideas work when considering the Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly county cluster for the North Carolina House and two initial conditions when using the 2012 presidential votes. The run to generate this data used the parameters reported in Section D. We show how the marginal distributions begin to look identical as more samples are gathered in Figure 105.

F.1.1 An algorithm to generate random initial conditions

To generate randomized initial conditions, we adapt the random district generating algorithm from [13]. To detail our implementation, we first

1. assign a unique district to each vertex of the graph.
2. Any vertices that are required to be in the same district (such as multiple whole counties) are then merged into the same district (merging these vertices must satisfy the other constraints).

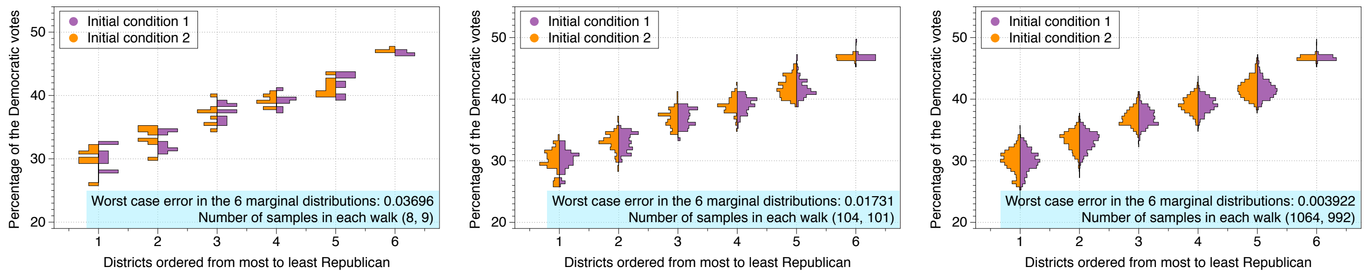


Figure 105: Using the 2012 presidential data along with two alternative initial conditions, we display the ranked marginal distributions of each district in the ensemble from the most to least Republican district. We show differences in the marginal distributions after roughly 10 plans (left), 100 plans (center) and 1000 plans (right) are sampled in the random walk. The corresponding worst case error is shown in the bottom of each figure.

3. Select a random district.
4. Find the nearest neighboring district that we have not already examined (centroid to centroid). If merging the randomly selected district with the neighboring district satisfies all constraints, continue. Otherwise repeat this step. If there are no more nearby districts to examine return to the previous step; if we have returned to the previous step having examined all district pairs without merging, restart from the beginning.
5. Repeat the previous step until there are the desired number of districts.
6. At this point we will have the correct number of districts, but have done nothing to control for their population. To do this select the district pair with the largest population disparity that we have not yet examined.
7. Taking the centroid of the smaller district, find the nearest vertex in the larger district that we have not yet examined. If moving this vertex to the smaller district complies with any contraints, move the precinct to the smaller district. If it does not comply with the constraints, find the next nearest vertex. If there are no more districts to examine, return to the previous step.
8. Repeat the previous two steps until either the population devation is below some desired threshold (we use the population threshold), or we have attempted to generate a districting plan a certain number of times (which ever comes first).

F.1.2 Error bounds implied by the convergence criteria

The convergence criteria in Equation 4 is a sum of two possitive numbers; one of these numbers is proportional to the squared error of the means and the second is related to the squared error of the standard deviations. If we find that the convergence criteria is satisfied, then we may also conclude that each of the two numbers in the sum is also less than ϵ. For the error in the standard deviation, we have

$$\frac{\left(\sigma_1^2 - \sigma_2^2\right)^2}{2\sigma_1^2\sigma_2^2} \leq \epsilon, \quad \text{or} \tag{4}$$

$$\frac{1}{2}\left|\frac{\sigma_1}{\sigma_2} - \frac{\sigma_2}{\sigma_1}\right| \leq \frac{\sqrt{2\epsilon}}{2}. \tag{5}$$

The left hand side of the re-expressed inequality may be rewritten as the average relative error between the standard deviations

$$\frac{1}{2}\left|\frac{\sigma_1}{\sigma_2} - \frac{\sigma_2}{\sigma_1}\right| = \frac{1}{2}\left(\left|\frac{\sigma_1 - \sigma_2}{\sigma_2}\right| + \left|\frac{\sigma_2 - \sigma_1}{\sigma_1}\right|\right),$$

meaning that this part of the inequality gives us a bound on the relative error between the standard deviations of two distributions.

The error for the mean may also be bounded by the convergence criteria since it implies that

$$\frac{(\mu_1-\mu_2)^2\left(\sigma_1^2+\sigma_2^2\right)}{2\sigma_1^2\sigma_2^2} \leq \epsilon, \quad \text{or} \tag{6}$$

$$|\mu_1-\mu_2| \leq \frac{\sqrt{2\epsilon}\sigma_1\sigma_2}{\sqrt{\sigma_1^2+\sigma_2^2}}. \tag{7}$$

When ϵ is small we have $\sigma_1 \approx \sigma_2$, so we will replace both with the common approximation of σ. Plugging in σ for σ_1 and σ_2 gives an approximate bound on the error between the means of the distributions as

$$|\mu_1-\mu_2| \lessapprox \sqrt{\epsilon}\sigma. \tag{8}$$

We can quantify the error in the above approximation noting that Equation 5 implies

$$\left|\frac{\sigma_1-\sigma_2}{\sigma_1}\right| < \sqrt{2\epsilon}, \quad \left|\frac{\sigma_1-\sigma_2}{\sigma_2}\right| < \sqrt{2\epsilon}, \quad \text{which implies}$$
$$(1-\sqrt{2\epsilon})\sigma_2 < \sigma_1 < (1+\sqrt{2\epsilon})\sigma_2.$$

Utilizing these inequalities in Equation 7, in turn implies

$$|\mu_1-\mu_2| \leq \frac{(\sqrt{\epsilon}+\sqrt{2}\epsilon)}{\sqrt{1-\sqrt{2\epsilon}+\epsilon}}\sigma_2 = \sqrt{\epsilon}\sigma_2 + O(\epsilon) < \sqrt{\epsilon}\sigma_1 + O(\epsilon). \tag{9}$$

In short, Equation 8 provides an increasingly accurate bound on the error between the means as $\epsilon \to 0$, which can be made exact by the left most inequality in Equation 9.

F.1.3 Convergence data

Cluster	PR12; Max Err(%)) means	PR12; Max Err(%)) st. dev.	PR16; Max Err(%) means	PR16; Max Err(%) st. dev.	Avg. No Plans	Used Plans
Bladen-Brunswick-Pender-New Hanover	0.628	0.264	0.057	0.030	50783.0	50921
Buncombe-Transylvania-Henderson	0.039	0.022	0.041	0.024	39775.4	39764
Caswell-Wilkes-Rockingham-Alleghany-Watauga-Ashe-Stokes-Surry	0.598	0.267	0.844	0.417	55833.4	59915
Davie-Forsyth	0.047	0.027	0.030	0.017	2188.4	2196
Guilford-Alamance-Randolph	0.051	0.029	0.114	0.065	717.4	711
Lee-Sampson-Harnett-Duplin-Johnston-Nash	0.105	0.058	0.083	0.047	1119.6	1103
Lincoln-Gaston-Cleveland	0.164	0.082	0.161	0.082	245.0	247
Mecklenburg	0.268	0.152	0.187	0.107	6679.6	6721
Person-Durham-Granville	0.125	0.071	0.082	0.046	974.6	963
Union-Cabarrus	0.073	0.040	0.107	0.057	248.8	248
Wake-Franklin	0.170	0.097	0.124	0.072	14499.0	14338

Table 10: Convergence Statistics for cluster-level ensemble of maps for N.C. Senate. Sampling runs starting from 5 different initial conditions were made. The first two columns give the maximum deviations in the mean and standard deviation between each of the pairs of runs. The second column gives the total number of maps generated in the 5 runs and the last column gives the number of maps which from the one run which was included in our ensemble.

Cluster	PR12; Max Err(%)) means	st. dev.	PR16; Max Err(%) means	st. dev.	Avg. No Plans	Used Plans
Alamance	0.118	0.068	0.060	0.035	2602.2	2558
Bladen-Lee-Sampson-Harnett-Greene-Wayne-Johnston	0.023	0.013	0.039	0.023	3677.0	3639
Buncombe	0.197	0.112	0.251	0.142	3564.2	3606
Caswell-Orange	0.093	0.054	0.097	0.057	3886.2	3881
Catawba	0.025	0.014	0.024	0.014	9954.4	9930
Chatham-Durham	0.221	0.124	0.232	0.129	2471.2	2437
Columbus-Pender-Robeson	0.088	0.051	0.043	0.025	15831.8	15941
Craven-Beaufort	0.314	0.160	0.098	0.054	190.2	187
Cumberland	0.142	0.081	0.171	0.098	10407.0	10368
Davidson	0.120	0.068	0.112	0.063	17538.4	17614
Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly	0.110	0.063	0.121	0.069	7609.2	7725
Duplin-Onslow	0.103	0.059	0.058	0.033	7866.4	7688
Gaston-Cleveland	0.100	0.056	0.192	0.109	8712.2	8750
Guilford	0.065	0.037	0.083	0.048	1486.8	1442
Iredell	0.050	0.028	0.067	0.038	50118.0	50115
Madison-Swain-Jackson-Haywood-Yancey	0.055	0.031	0.034	0.020	3253.2	3166
Mecklenburg	0.110	0.063	0.091	0.053	40692.2	40549
Nash-Franklin	0.040	0.023	0.144	0.082	4294.2	4284
New Hanover-Brunswick	0.085	0.049	0.117	0.067	46420.4	46642
Person-Vance-Granville-Warren	0.148	0.081	0.156	0.085	4027.6	4060
Pitt-Lenoir	0.237	0.135	0.195	0.111	3832.0	3763
Polk-Transylvania-Henderson	0.038	0.022	0.004	0.002	41058.0	41049
Randolph-Moore	0.023	0.013	0.016	0.009	8099.2	8078
Rutherford-Burke	0.051	0.029	0.043	0.025	5042.8	5026
Union-Anson	0.087	0.050	0.108	0.062	6534.2	6645
Wake	0.401	0.229	0.355	0.203	50074.8	50000
Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander	0.084	0.046	0.085	0.047	4463.6	4649
Yadkin-Forsyth	0.324	0.185	0.329	0.188	17050.4	17062

Table 11: Convergence statistics for cluster-level ensemble of maps for N.C. House. Sampling runs starting from 5 different initial conditions were made. The first two columns give the maximum deviations in the mean and standard deviation between each of the pairs of runs. The second column gives the total number of maps generated in the 5 runs and the last column gives the number of maps which from the one run which was included in our ensemble.

Chamber	Cluster	PR12; Max Err(%)) means	PR12; Max Err(%)) st. dev.	PR16; Max Err(%) means	PR16; Max Err(%) st. dev.
Senate	Mecklenburg	0.123	0.071	0.143	0.083
Senate	Wake-Franklin	0.427	0.242	0.596	0.337
House	Guilford	0.201	0.114	0.237	0.134
House	Mecklenburg	0.156	0.089	0.371	0.211
House	Wake	0.495	0.277	0.605	0.341
House	Yadkin-Forsyth	0.237	0.136	0.218	0.125

Table 12: Convergence Statistics for cluster-level ensemble of maps that preserve incumbents. Sampling runs starting from 5 different initial conditions were made. The first two columns give the maximum deviations in the mean and standard deviation between each of the pairs of runs.

Cluster	Unique	Ensemble	Unique-Incumbent	Incumbent	Plaintiff selected
Bladen-Brunswick-Pender-New Hanover	5	50921	5	50921	Y
Buncombe-Transylvania-Henderson	5748	39764	5748	39764	Y
Caswell-Wilkes-Rockingham-Alleghany-Watauga-Ashe-Stokes-Surry	1202	59915	1202	59915	N
Davie-Forsyth	2196	2196	1868	1868	Y
Guilford-Alamance-Randolph	710	711	710	711	Y
Lee-Sampson-Harnett-Duplin-Johnston-Nash	855	1103	855	1103	Y
Lincoln-Gaston-Cleveland	37	247	7	8	N
Mecklenburg	6721	6721	122	122	Y
Person-Durham-Granville	785	963	105	108	N
Union-Cabarrus	41	248	41	248	N
Wake-Franklin	14338	14338	942	942	Y

Table 13: The total number of maps in the ensemble for the North Carolina Senate, along with the total number of unique maps. We also present the number of total and unique maps in the ensemble that preserve incumbents to the same extent or better than the enacted plan. These numbers are for the primary ensembles; for the ensembles that were built to expressly preserve incumbents, see Table 15.

Cluster	Unique	Ensemble	Unique-Incumbent	Incumbent	Plaintiff selected
Alamance	1267	2558	333	798	Y
Bladen-Lee-Sampson-Harnett-Greene-Wayne-Johnston	654	3639	444	2244	N
Buncombe	3301	3606	2857	3146	Y
Caswell-Orange	381	3881	364	3852	N
Catawba	513	9930	335	6243	N
Chatham-Durham	2355	2437	15	15	N
Columbus-Pender-Robeson	14659	15941	10548	11378	Y
Craven-Beaufort	9	187	9	187	N
Cumberland	10368	10368	5048	5048	Y
Davidson	2752	17614	1728	8673	N
Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly	7725	7725	3822	3822	Y
Duplin-Onslow	238	7688	214	7590	Y
Gaston-Cleveland	4090	8750	1834	4666	Y
Guilford	332	1442	8	9	Y
Iredell	370	50115	370	50115	N
Madison-Swain-Jackson-Haywood-Yancey	295	3166	295	3166	N
Mecklenburg	40549	40549	103	103	Y
Nash-Franklin	67	4284	67	4284	Y
New Hanover-Brunswick	42386	46642	25684	28507	Y
Person-Vance-Granville-Warren	14	4060	14	4060	N
Pitt-Lenoir	1590	3763	1590	3763	Y
Polk-Transylvania-Henderson	289	41049	248	39286	N
Randolph-Moore	224	8078	130	5373	N
Rutherford-Burke	288	5026	288	5026	N
Union-Anson	2886	6645	2773	6495	Y
Wake	50000	50000	27	27	Y
Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander	4366	4649	2265	2461	N
Yadkin-Forsyth	17062	17062	1136	1136	Y

Table 14: The total number of maps in the ensemble for the North Carolina House, along with the total number of unique maps. We also present the number of total and unique maps in the ensemble that preserve incumbents to the same extent or better than the enacted plan. These numbers are for the primary ensembles; for the ensembles that were built to expressly preserve incumbents, see Table 15.

Chamber	Cluster	Unique	Ensemble	Unique-Incumbent	Incumbent
House	Yadkin-Forsyth	10134	10134	2111	2111
House	Mecklenburg	6459	6459	6459	6459
House	Wake	50399	50399	50399	50399
House	Guilford	580	2766	580	2766
Senate	Mecklenburg	13252	13252	13252	13252
Senate	Wake-Franklin	50324	50324	49286	49286

Table 15: We display the number of samples gathered for the few additional ensembles we have gathed in order to test for the effects on the ensemble when preserving incumbency. We also present the unqiue number of plans, the unique number of plans that preserve incumbents, and the number of plans that preserve incumbents. As discussed in Section E.25, many of these ensembles strictly preserve incumbents, which is why the numbers often match.

Cluster	New threshold	Unique	New thresh. w/ Incumb.	Unique
Bladen-Brunswick-Pender-NewHanover	38919	3	38919	3
Buncombe-Transylvania-Henderson	8689	55	8689	55
Caswell-Wilkes-Rockingham-Alleghany-Watauga-Ashe-Stokes-Surry	22209	52	22209	52
Davie-Forsyth	462	462	446	446
Guilford-Alamance-Randolph	134	134	134	134
Lee-Sampson-Harnett-Duplin-Johnston-Nash	62	9	62	9
Lincoln-Gaston-Cleveland	226	19	3	3
Mecklenburg	1580	1580	1580	1580
Person-Durham-Granville	38	36	8	8
Union-Cabarrus	156	14	156	14
Wake-Franklin	26458	26458	26040	26040

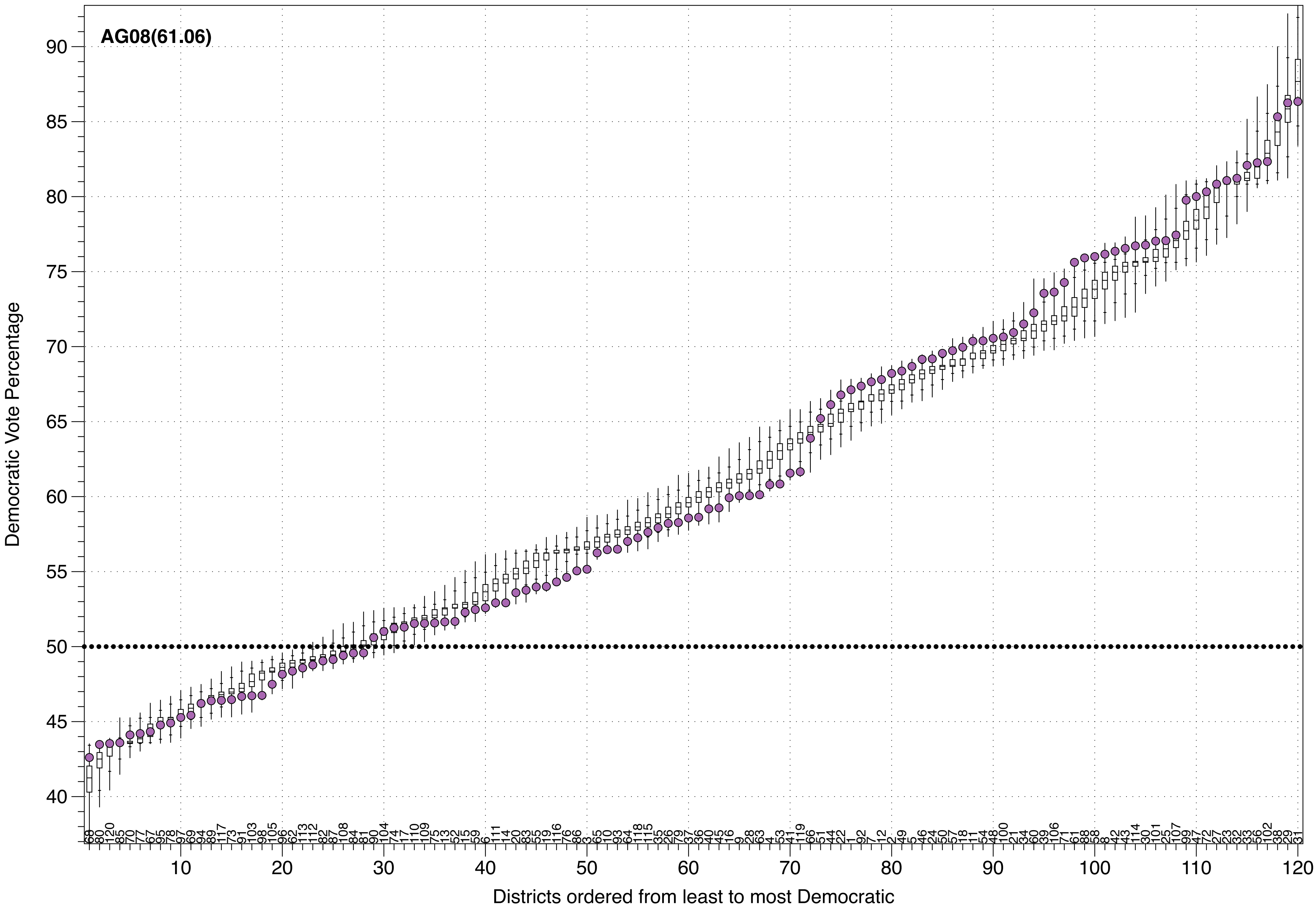
Table 16: We display the number of sub-samples in the North Carolina Senate when testing for the effects of more stringent redistricting thresholds and when joining these new thresholds with incumbency. We report the number of plans with and without resampling; the later is the number of unique plans.

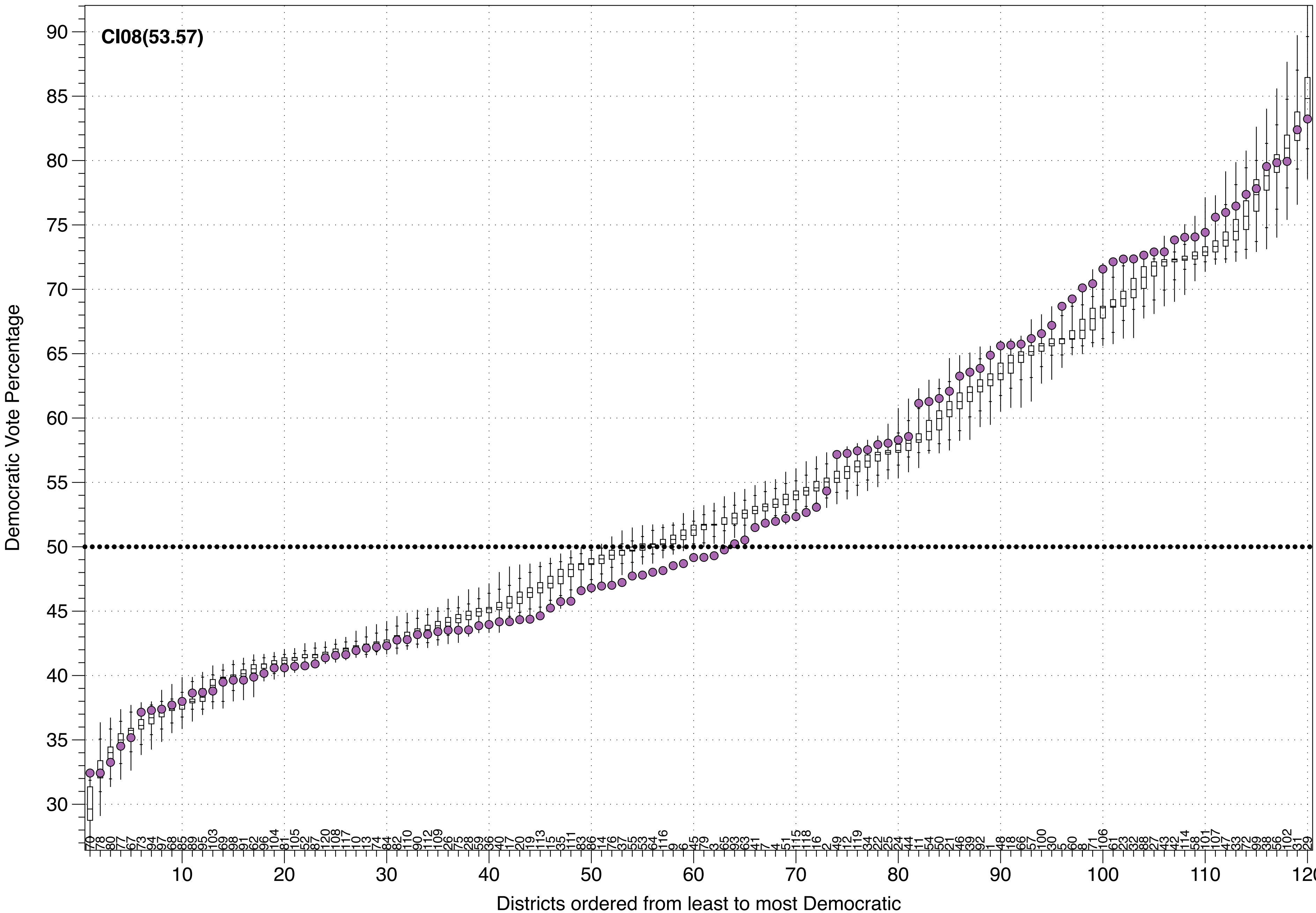
Cluster	New threshold	Unique	New thresh. w/ Incumb.	Unique
Buncombe	1403	1195	1257	1061
Columbus-Pender-Robeson	1284	952	828	631
Cumberland	4420	4420	2339	2339
Davie-Montgomery-Richmond-Cabarrus-Rowan-Stanly	3413	3413	1778	1778
Duplin-Onslow	81	17	81	17
Gaston-Cleveland	6527	2135	3734	1013
Guilford	1368	264	1209	95
Mecklenburg	5960	5960	1036	1036
Nash-Franklin	2538	9	2538	9
NewHanover-Brunswick	4046	2047	3211	1687
Person-Vance-Granville-Warren	2431	1	2431	1
Pitt-Lenoir	937	121	937	121
Union-Anson	4660	1634	4583	1577
Wake	3146	3146	4167	4167
Wilkes-Stokes-Surry-Rockingham-Alleghany-Alexander	1200	1007	987	819
Yadkin-Forsyth	10744	10744	1272	1272

Table 17: We display the number of sub-samples in the North Carolina House when testing for the effects of more stringent redistricting thresholds and when joining these new thresholds with incumbency. We report the number of plans with and without resampling; the later is the number of unique plans.

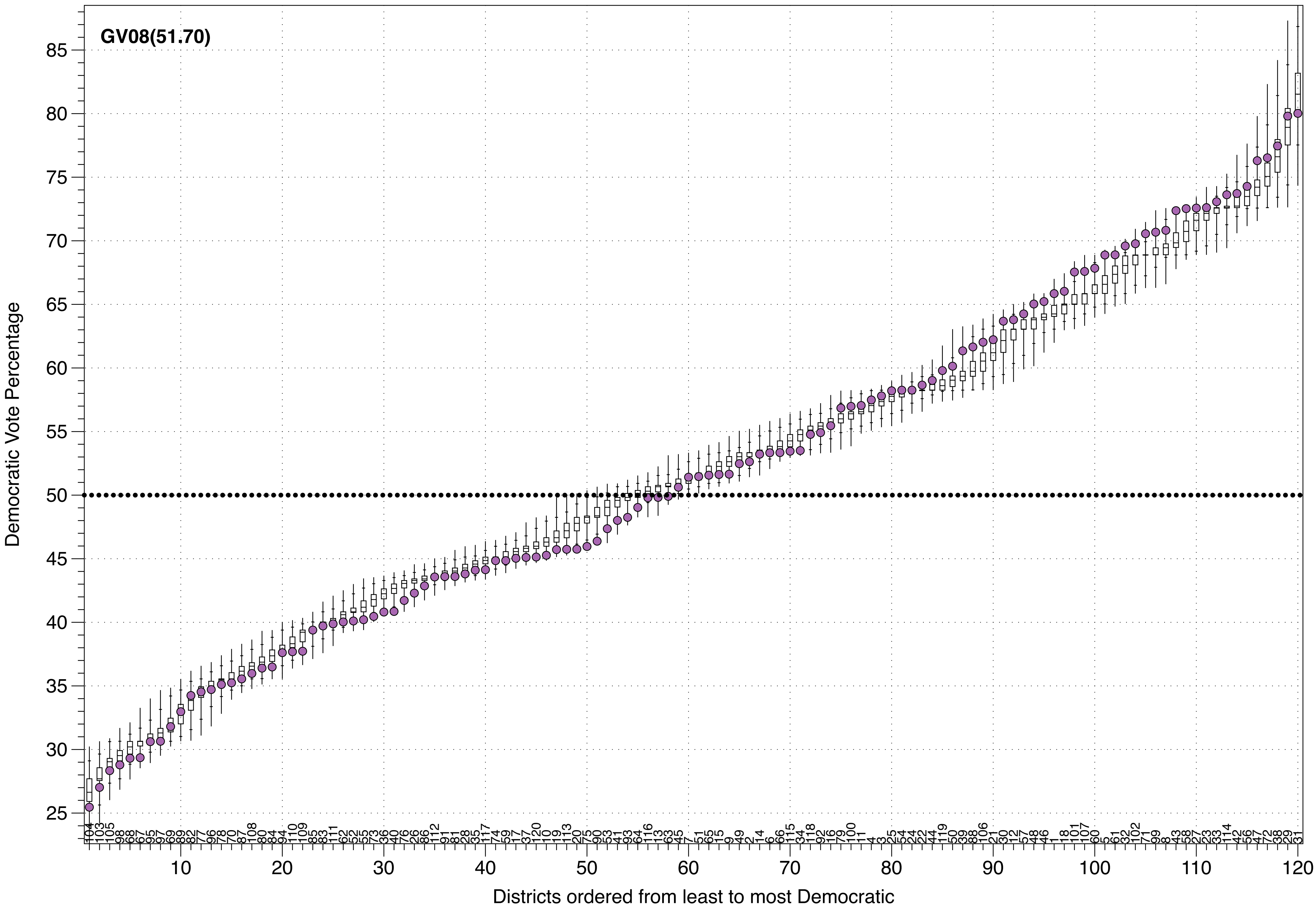
G Full Marginal Distributions

Although tremendously rich in structural detail, ranked marginal distributions on the state may be slightly overwhelming. We conclude by including them here, for each chamber and each considered election using the primary ensemble. We label the enacted districts along the x-axis.

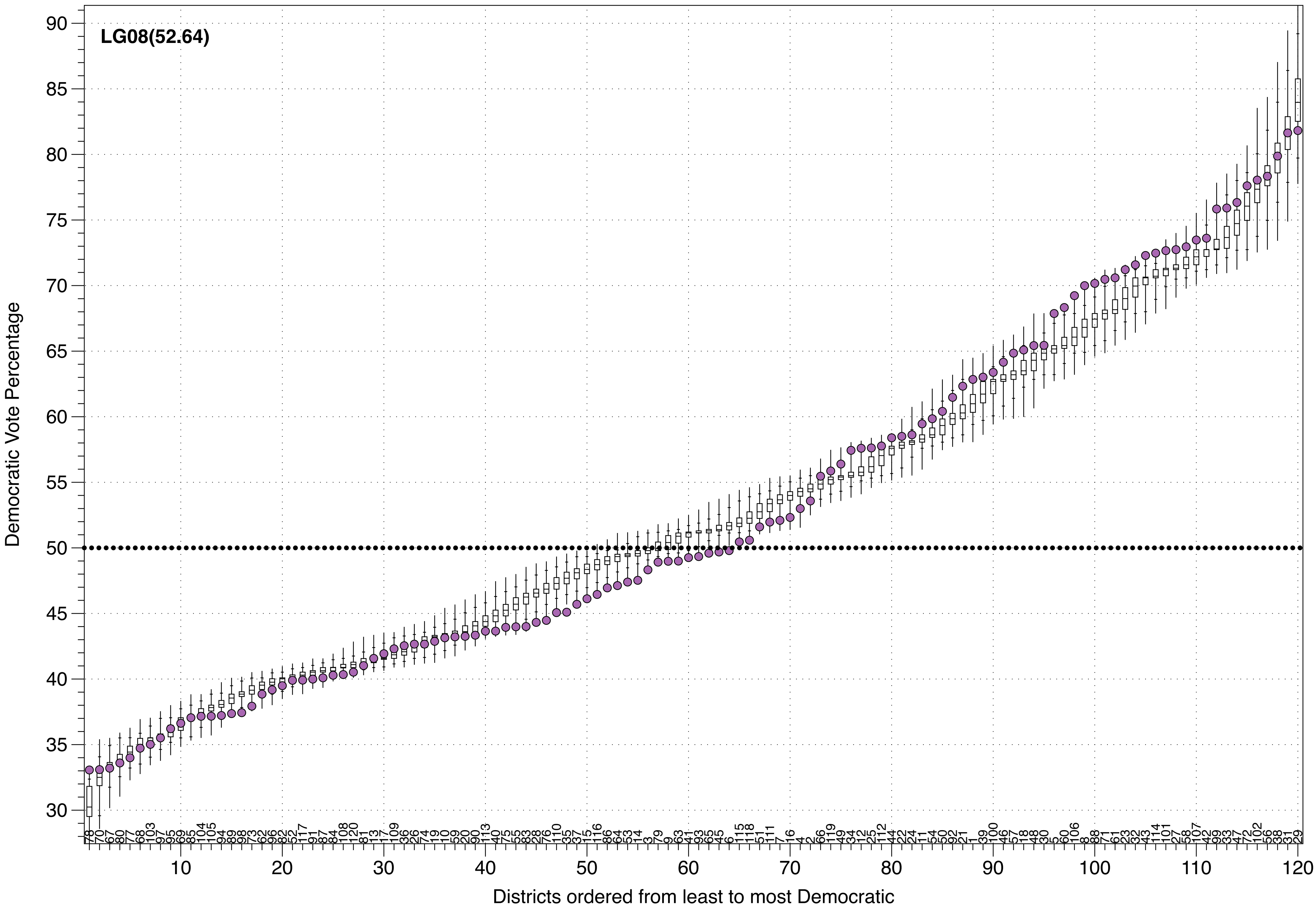

AG08(61.06)
Democratic Vote Percentage
Districts ordered from least to most Democratic

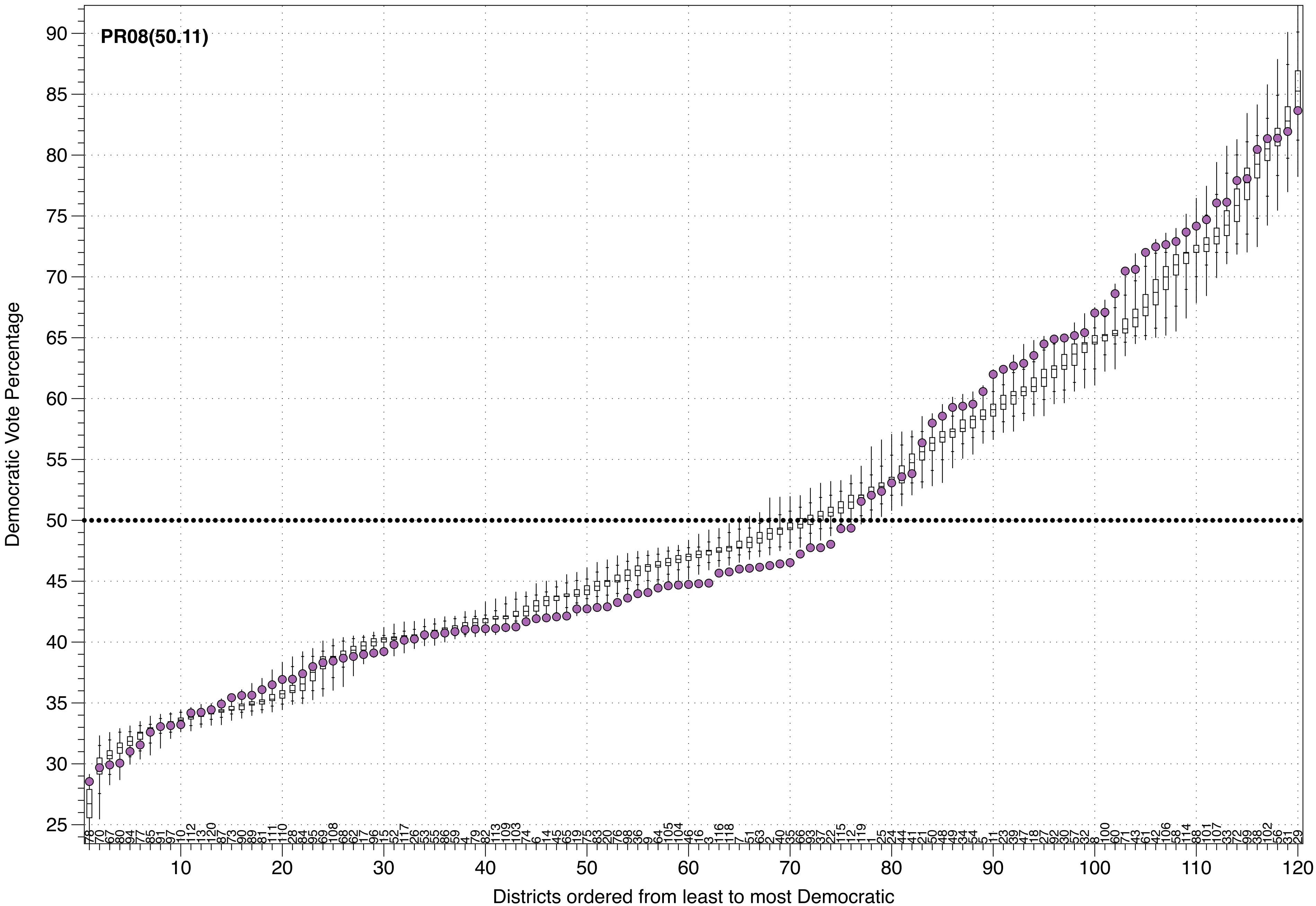


CI08(53.57)
Democratic Vote Percentage
Districts ordered from least to most Democratic



GV08(51.70)
Democratic Vote Percentage
Districts ordered from least to most Democratic


LG08(52.64)
Democratic Vote Percentage
Districts ordered from least to most Democratic
30
35
40
45
50
55
60
65
70
75
80
85
90
10
20
30
40
50
60
70
80
90
100
110
120


PR08(50.11)
Democratic Vote Percentage
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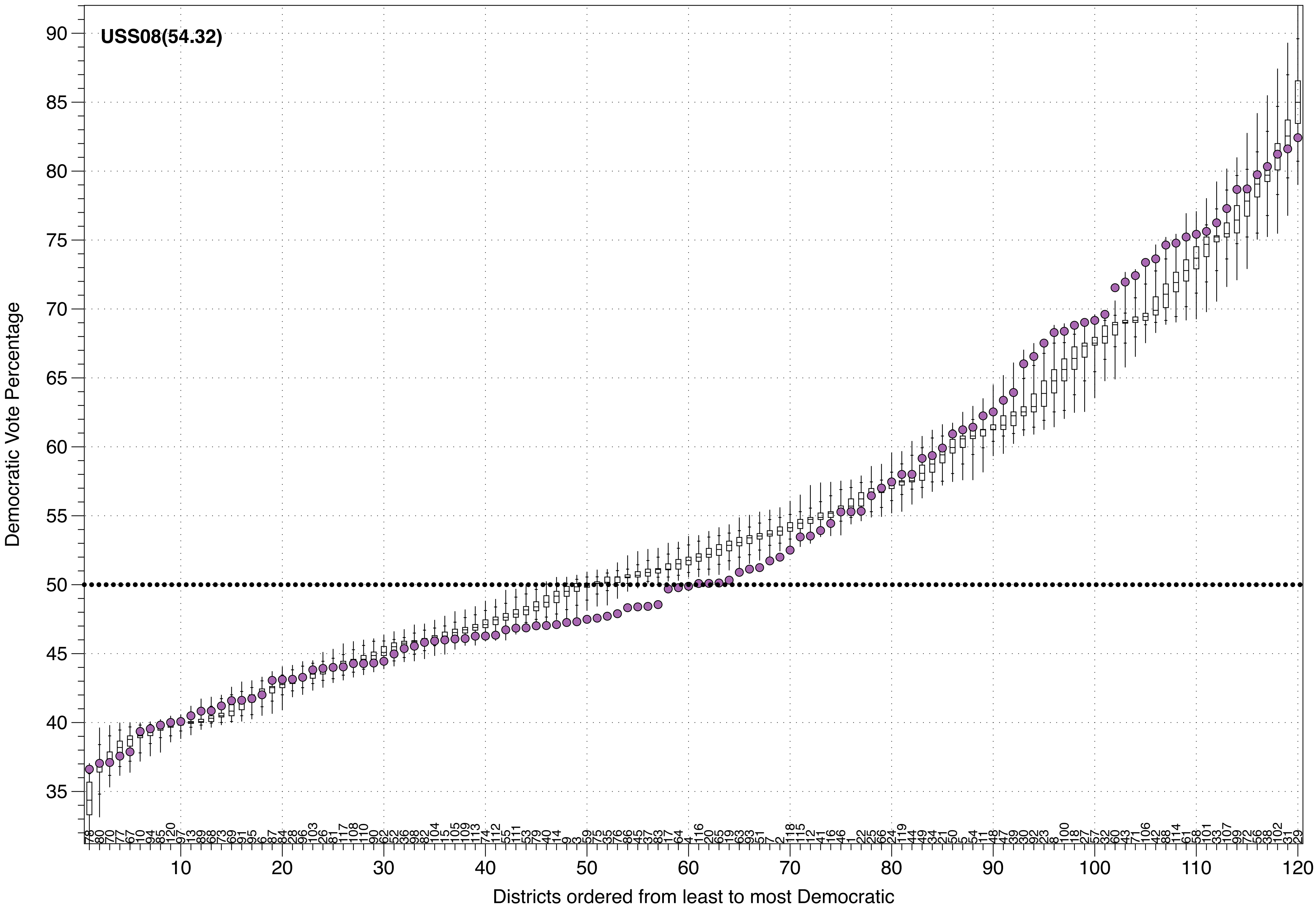
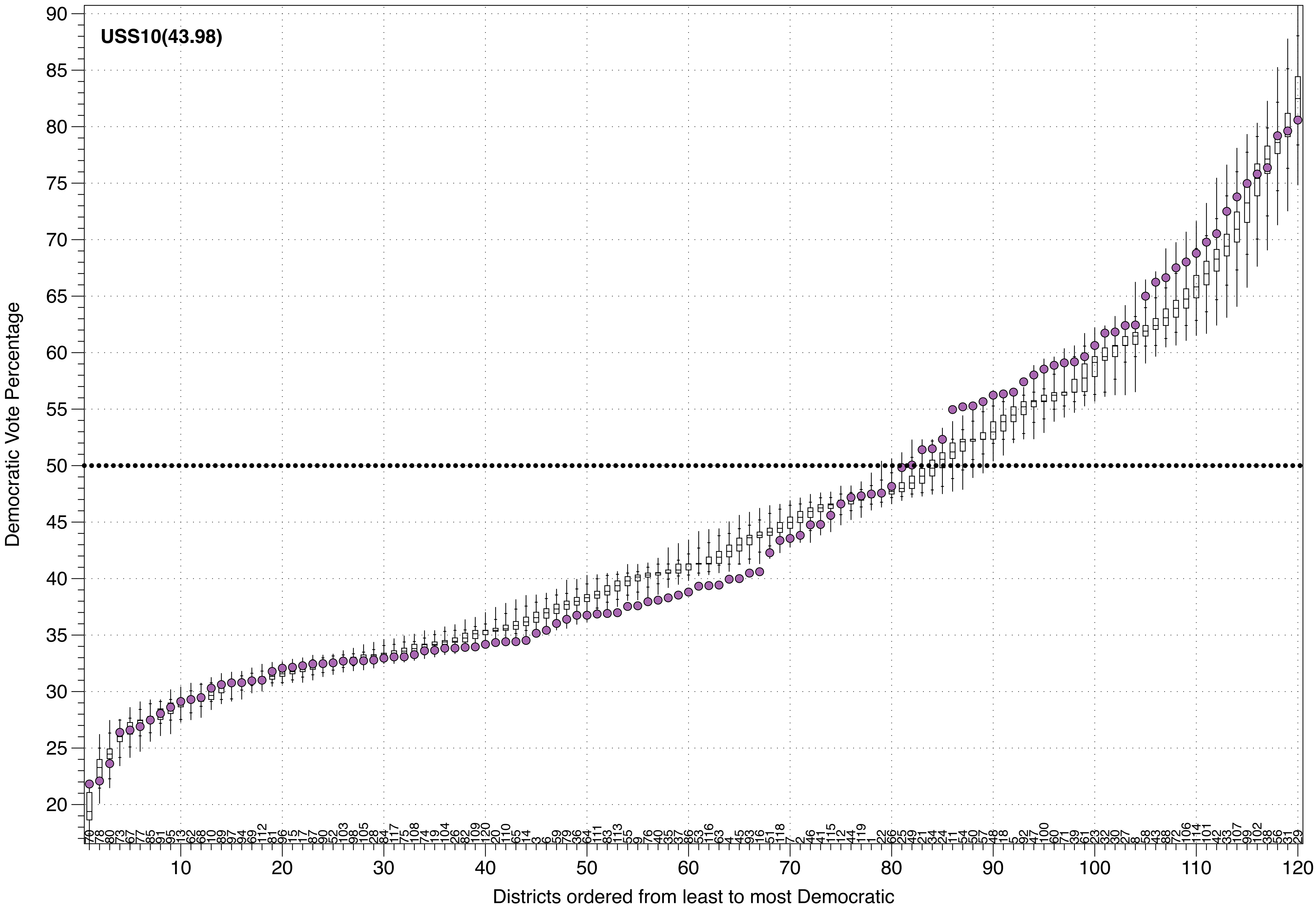
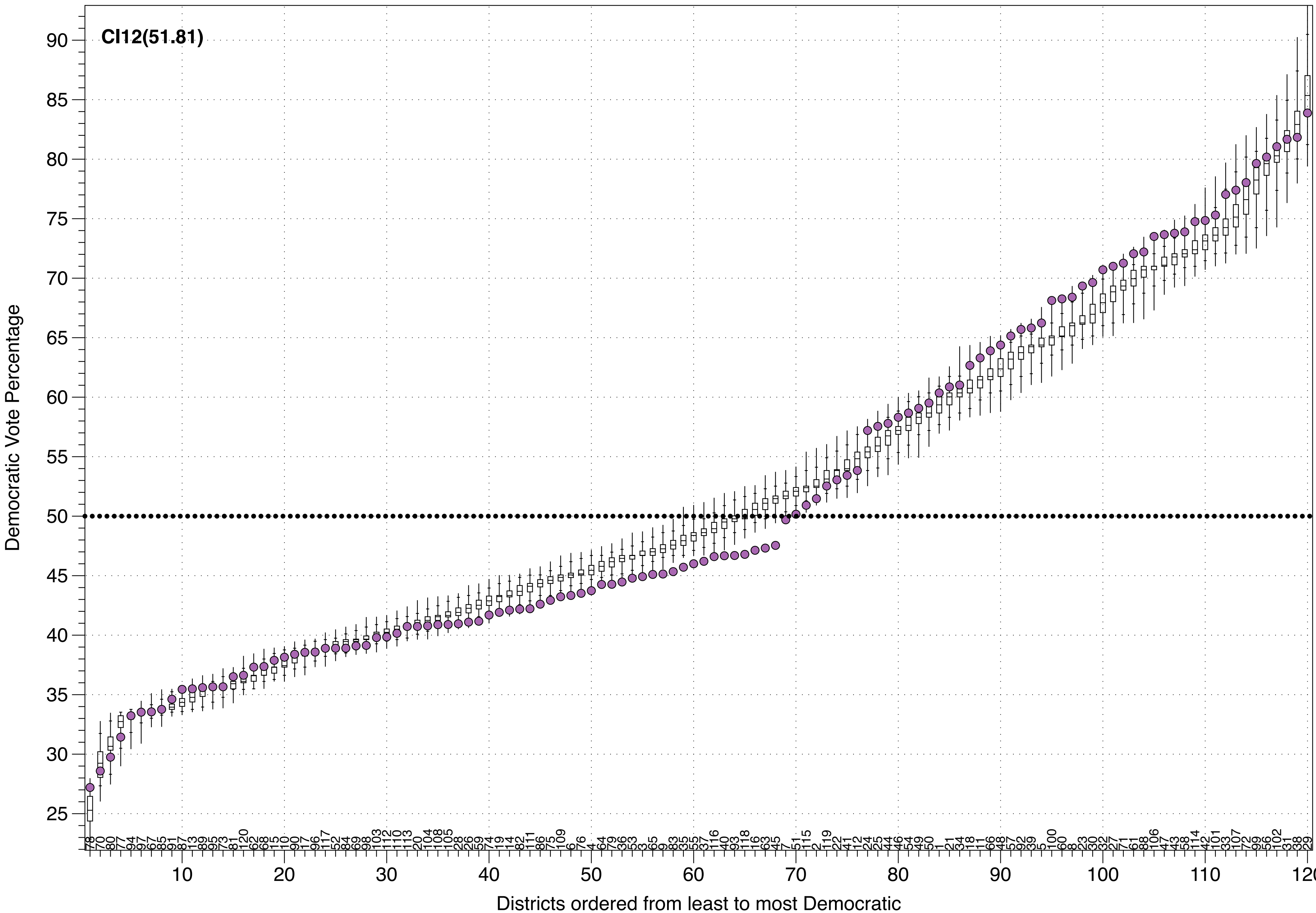
30
35
40
45
50
55
60
65
70
75
80
85
90
10
20
30
40
50
60
70
80
90
100
110
120
Districts ordered from least to most Democratic


USS08(54.32)
Democratic Vote Percentage
Districts ordered from least to most Democratic


USS10(43.98)
Democratic Vote Percentage
Districts ordered from least to most Democratic



CI12(51.81)
Democratic Vote Percentage
Districts ordered from least to most Democratic
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30
35
40
45
50
55
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75
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85
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10
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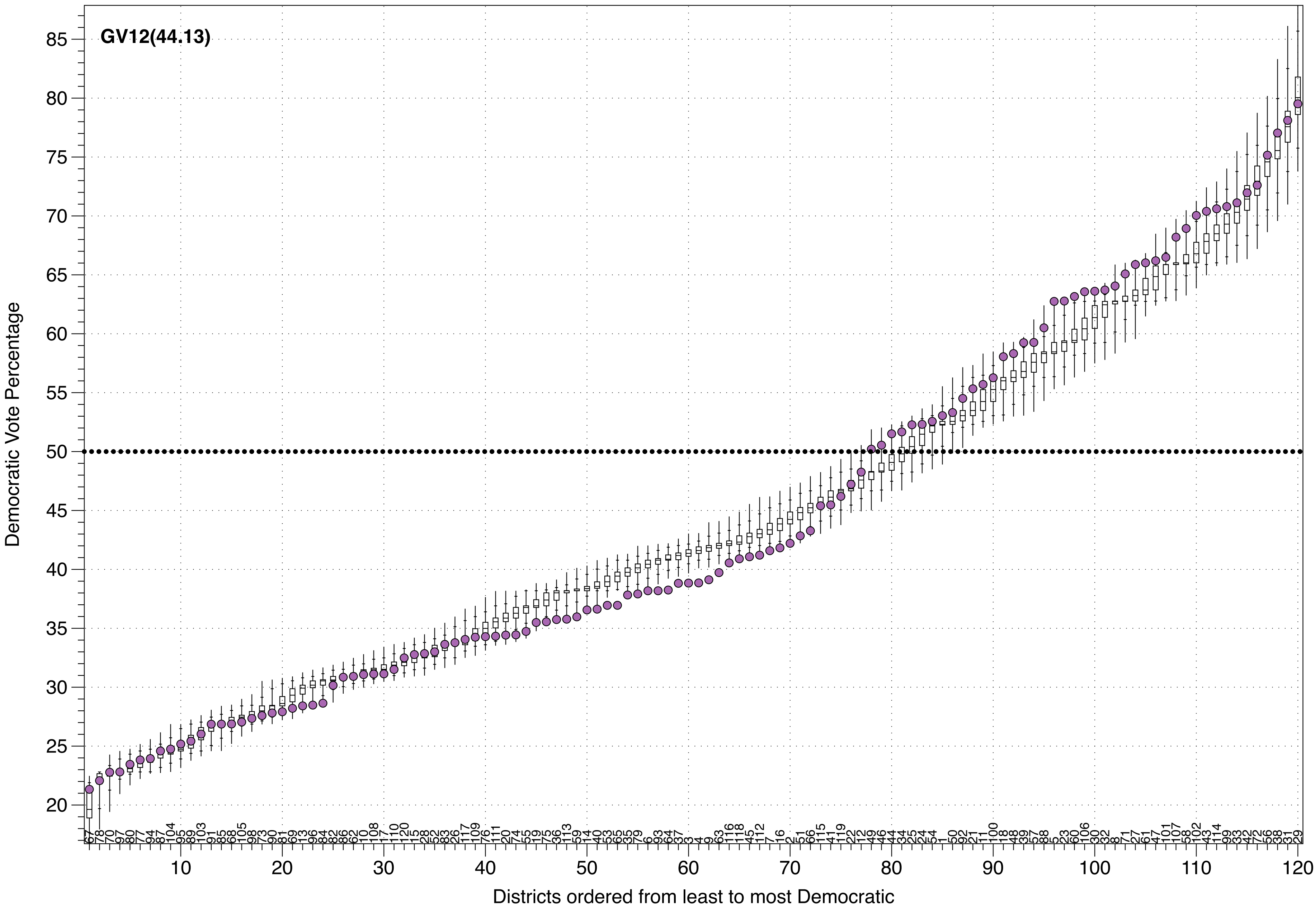
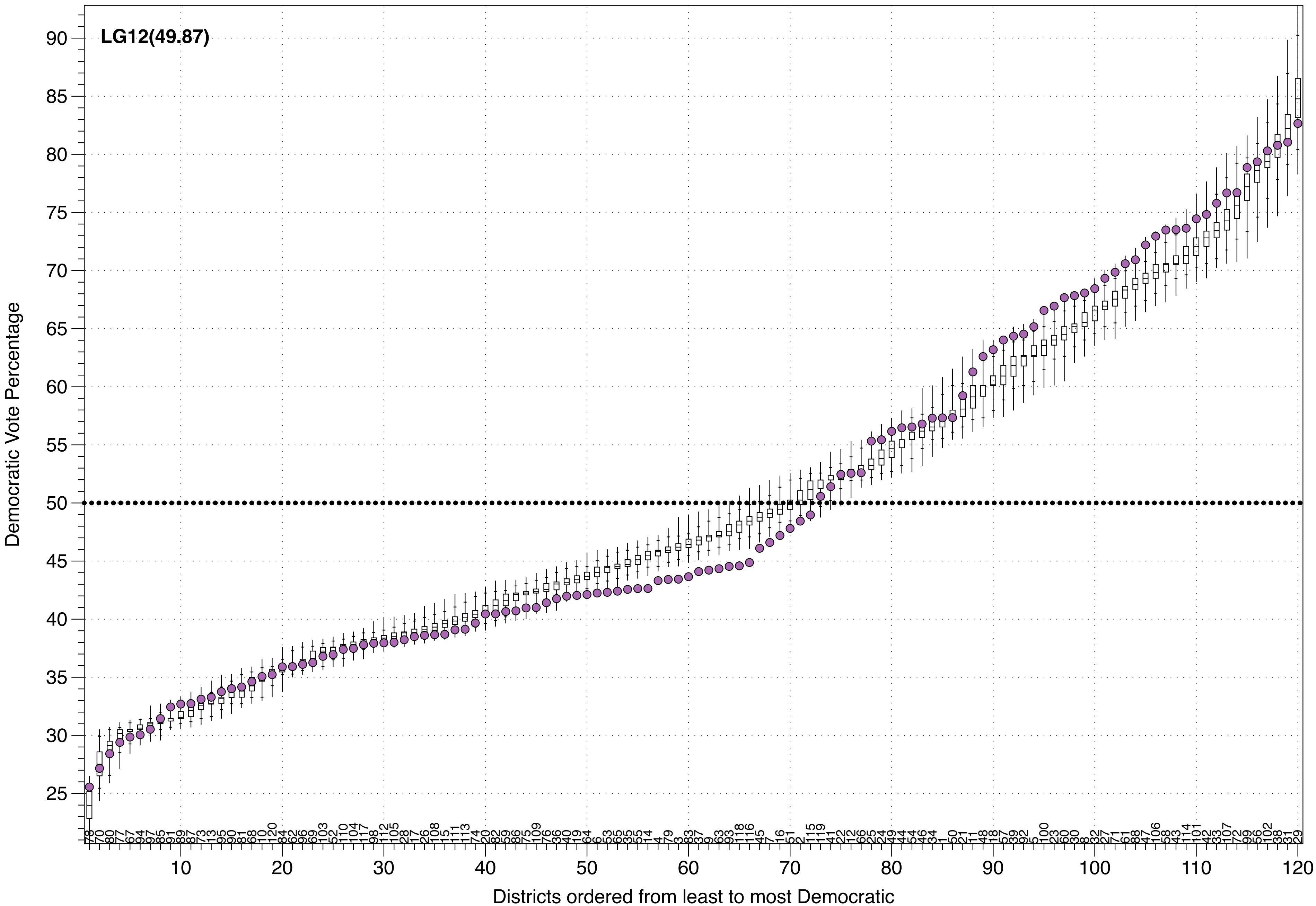
80
90
100
110
120


GV12(44.13)
Democratic Vote Percentage
Districts ordered from least to most Democratic


LG12(49.87)
Democratic Vote Percentage
Districts ordered from least to most Democratic
90
85
80
75
70
65
60
55
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40
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30
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30
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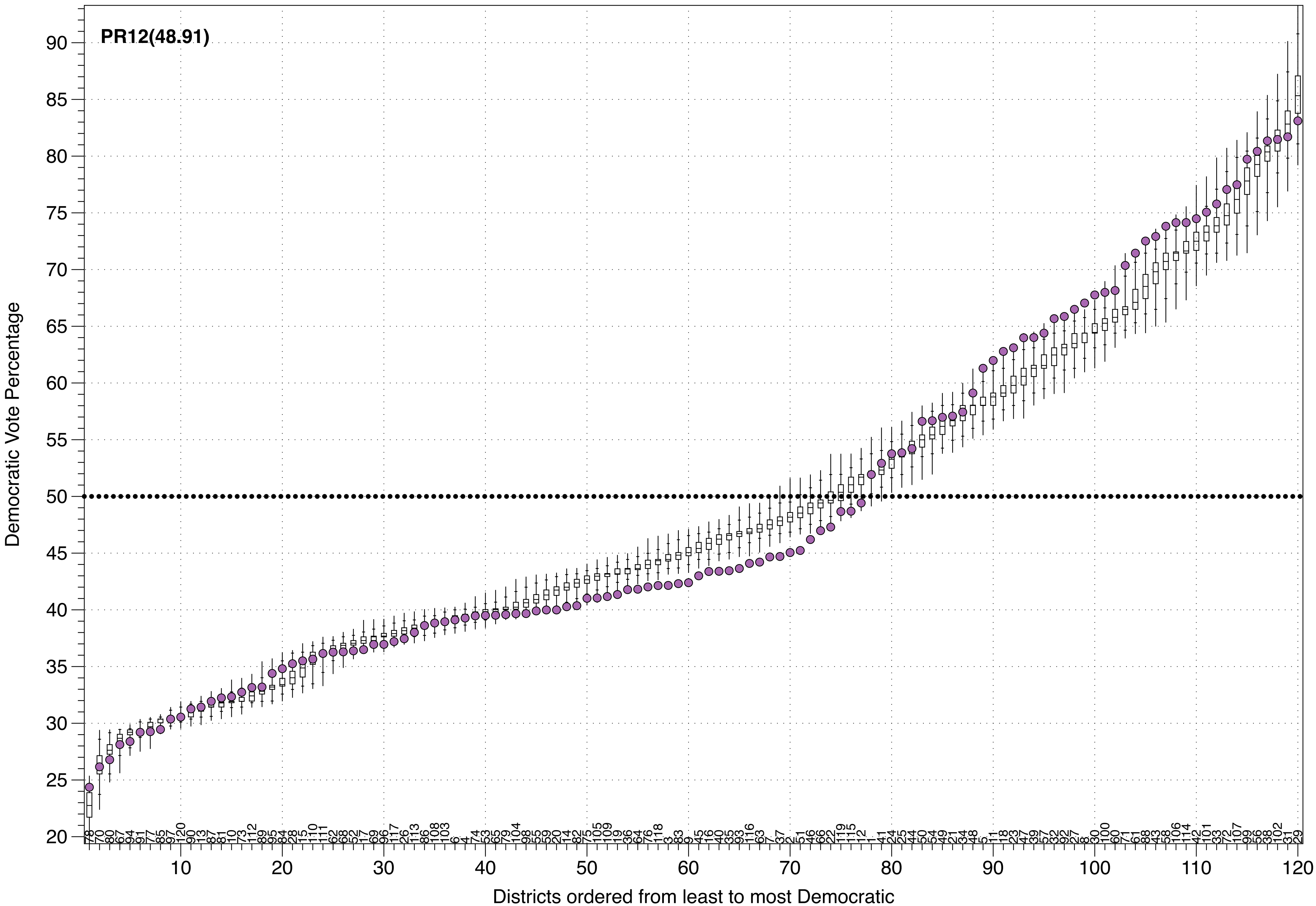
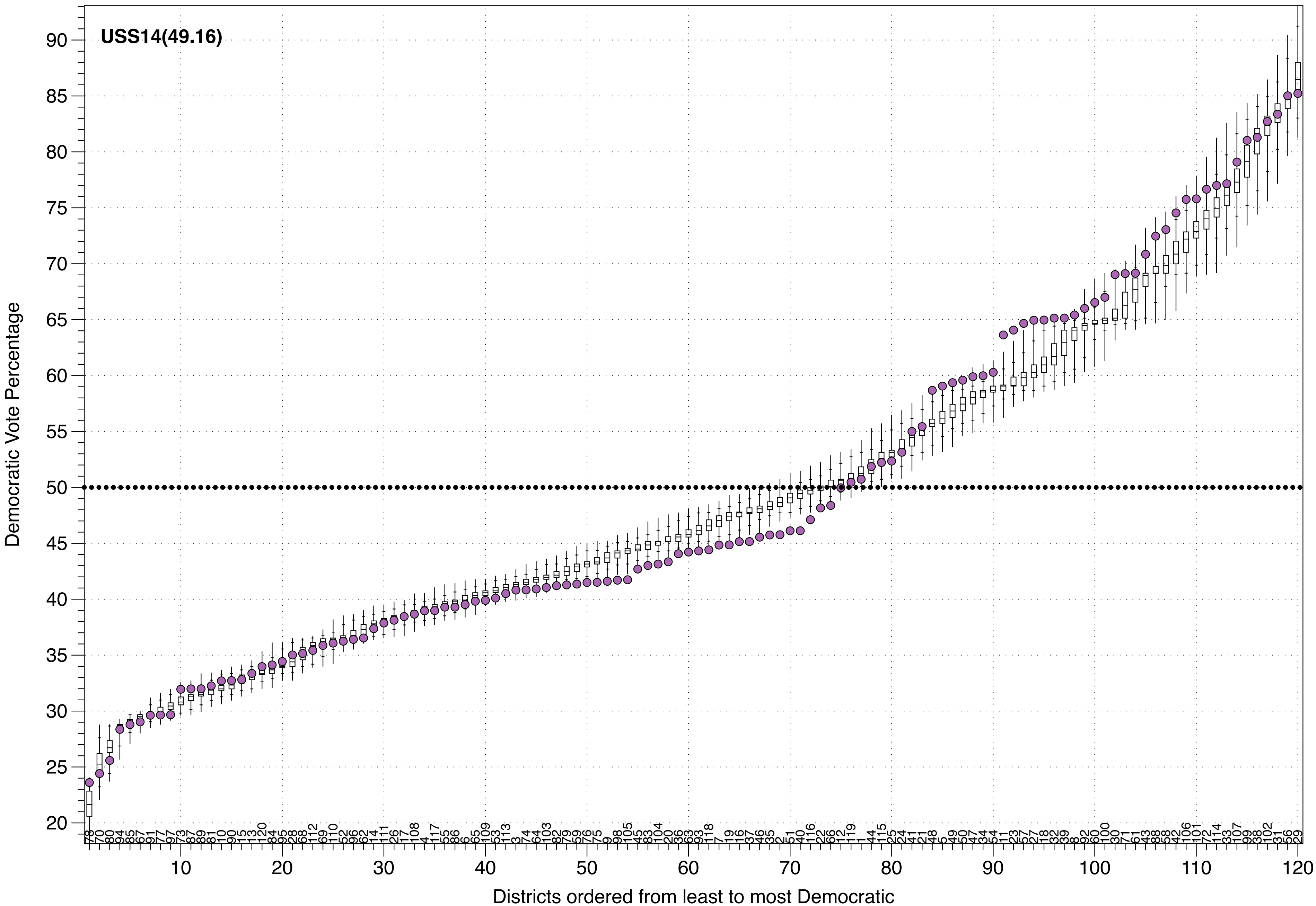
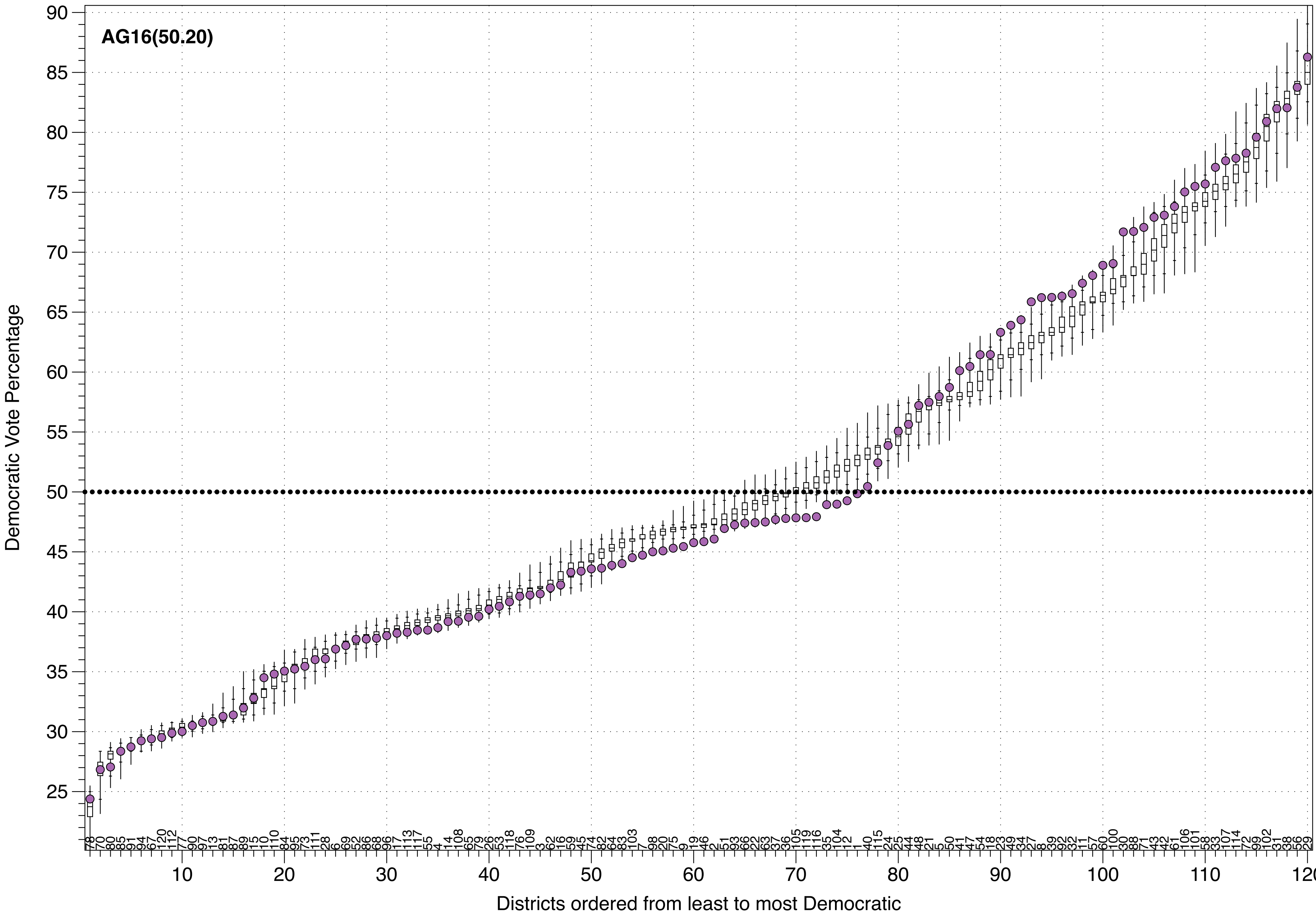
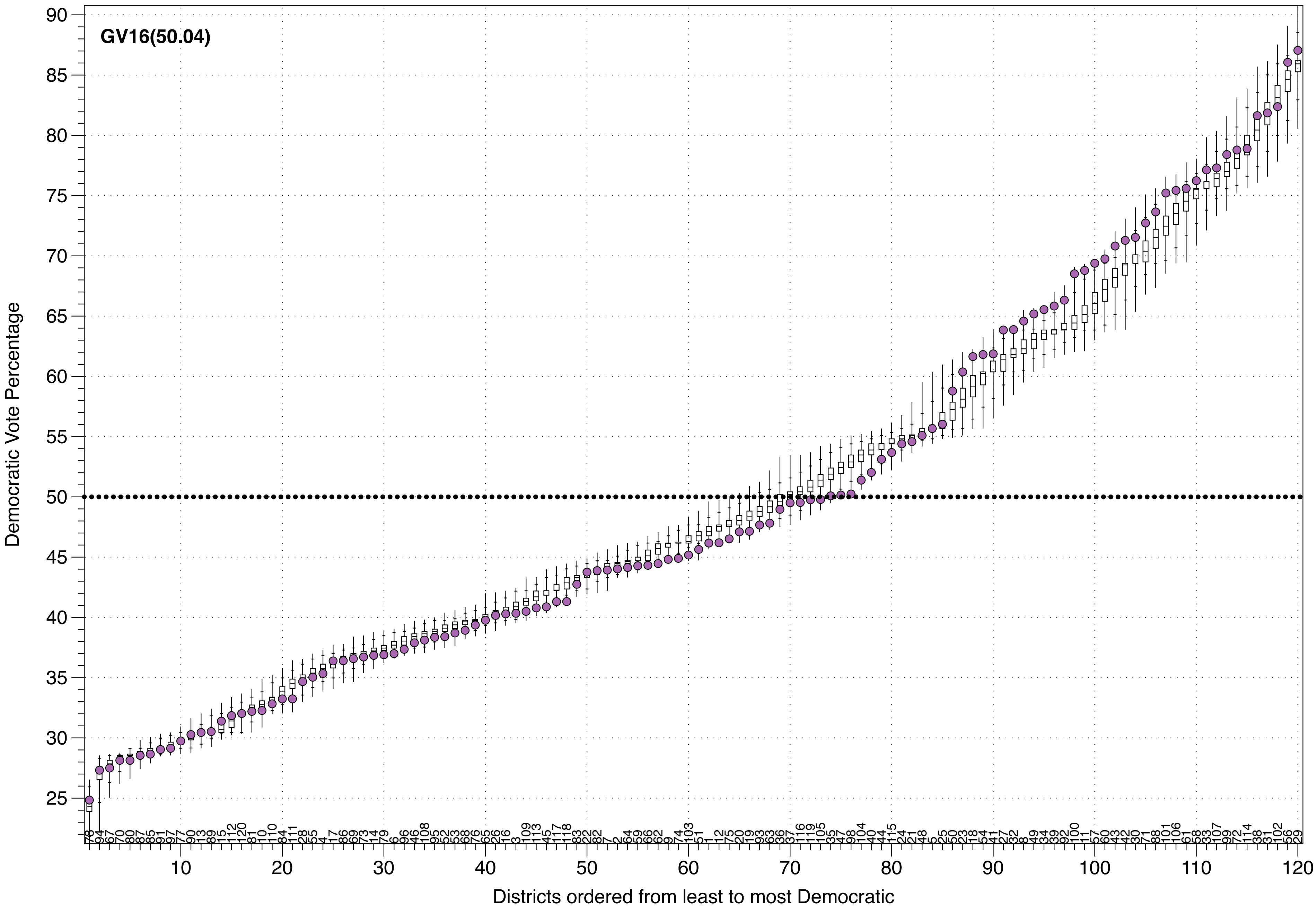
80
90
100
110
120


PR12(48.91)
Democratic Vote Percentage
Districts ordered from least to most Democratic


USS14(49.16)
Democratic Vote Percentage
Districts ordered from least to most Democratic


AG16(50.20)
Democratic Vote Percentage
Districts ordered from least to most Democratic


GV16(50.04)
Democratic Vote Percentage
Districts ordered from least to most Democratic
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40
45
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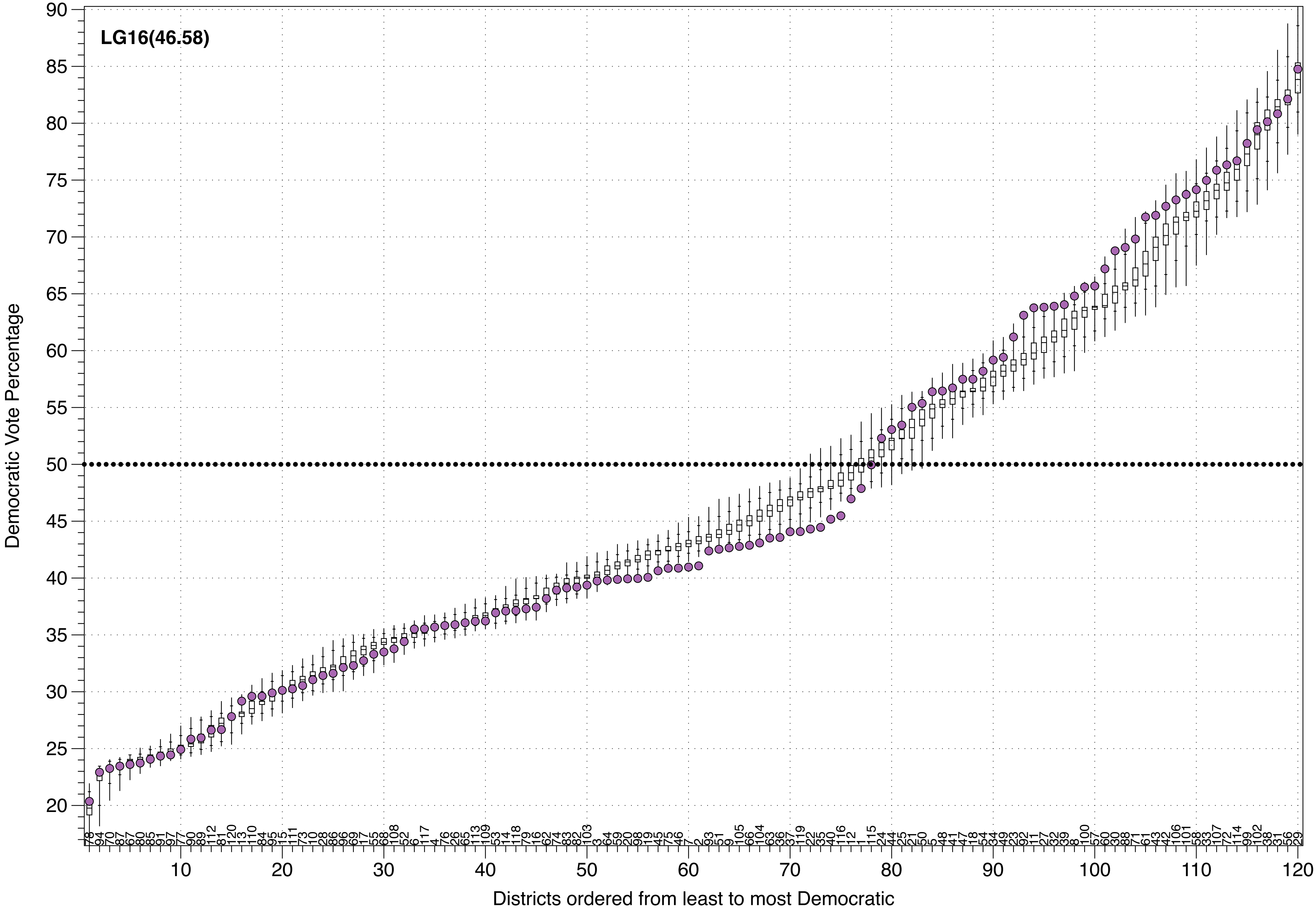
80
90
100
110
120


LG16(46.58)
Democratic Vote Percentage
Districts ordered from least to most Democratic
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25
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35
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45
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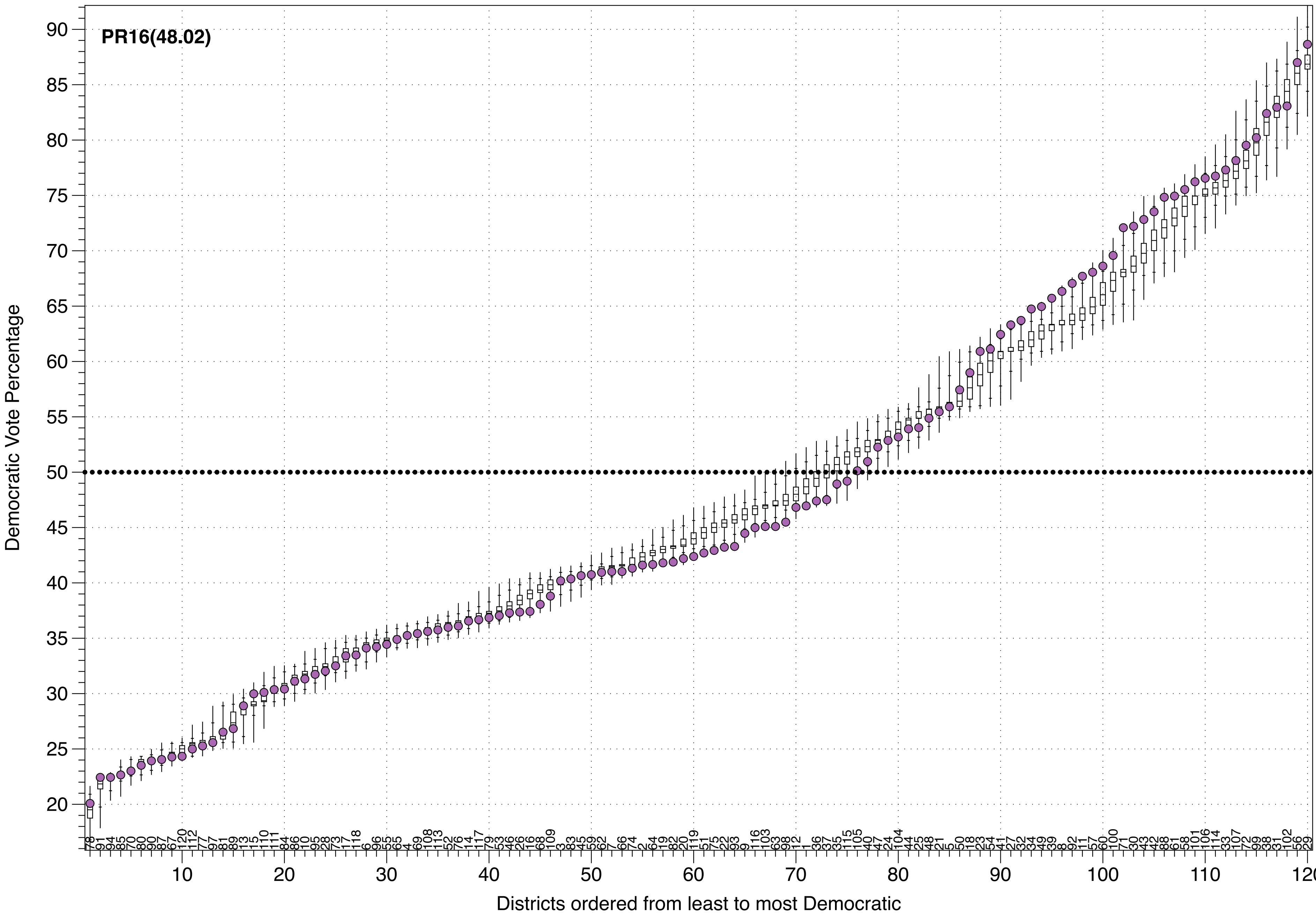
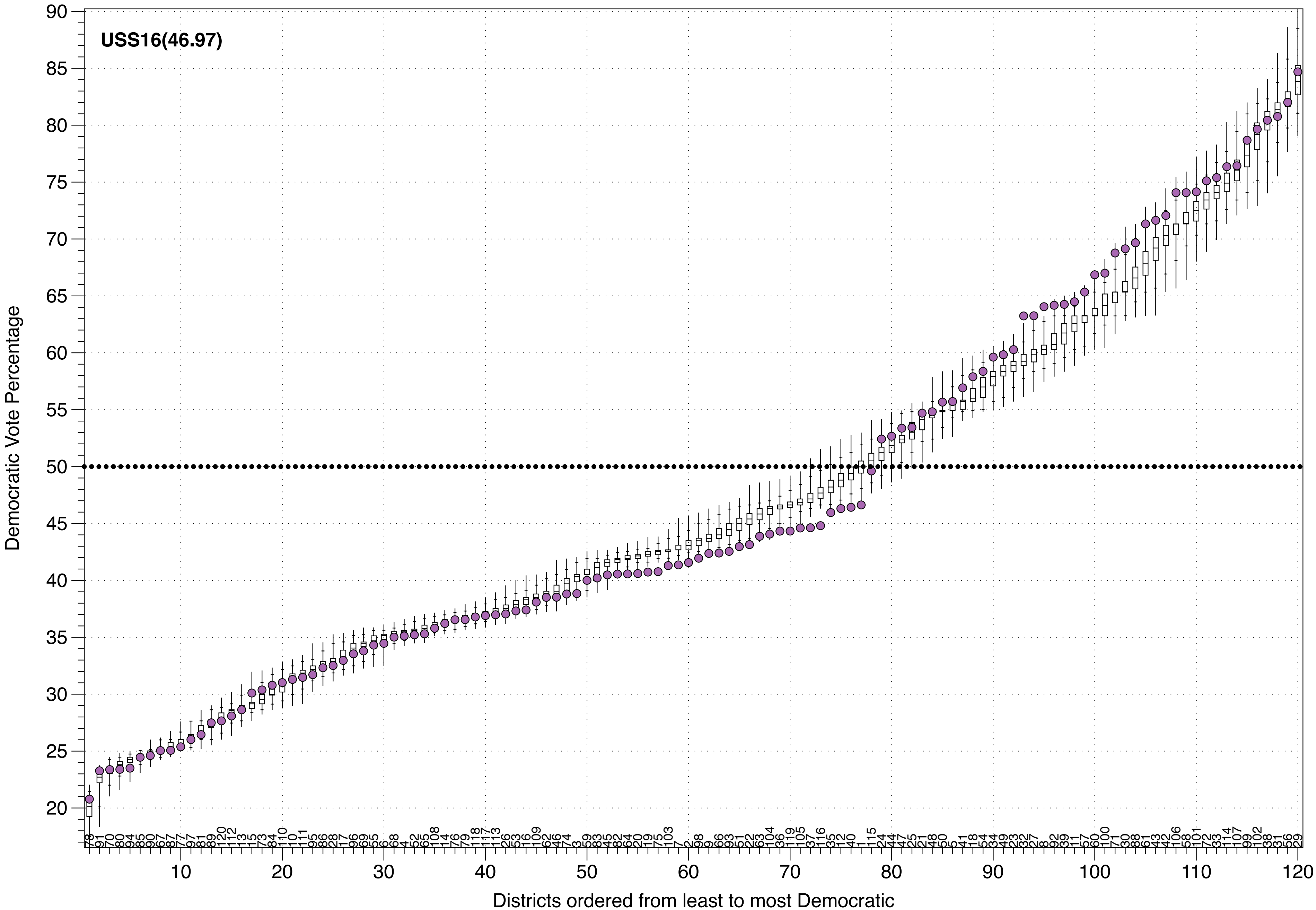
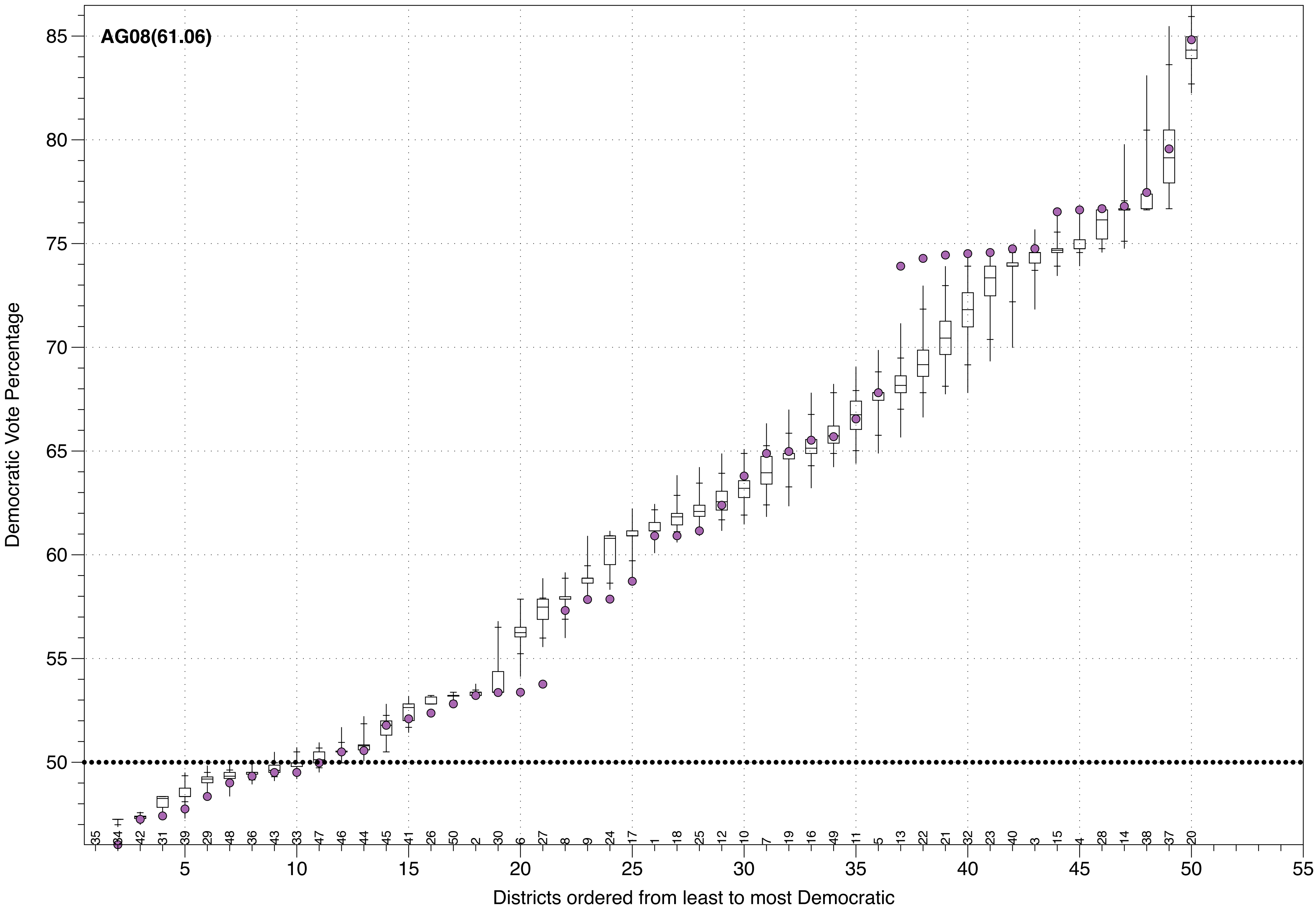
80
90
100
110
120


PR16(48.02)
Democratic Vote Percentage
Districts ordered from least to most Democratic


USS16(46.97)
Democratic Vote Percentage
Districts ordered from least to most Democratic


AG08(61.06)
Democratic Vote Percentage
Districts ordered from least to most Democratic
85
80
75
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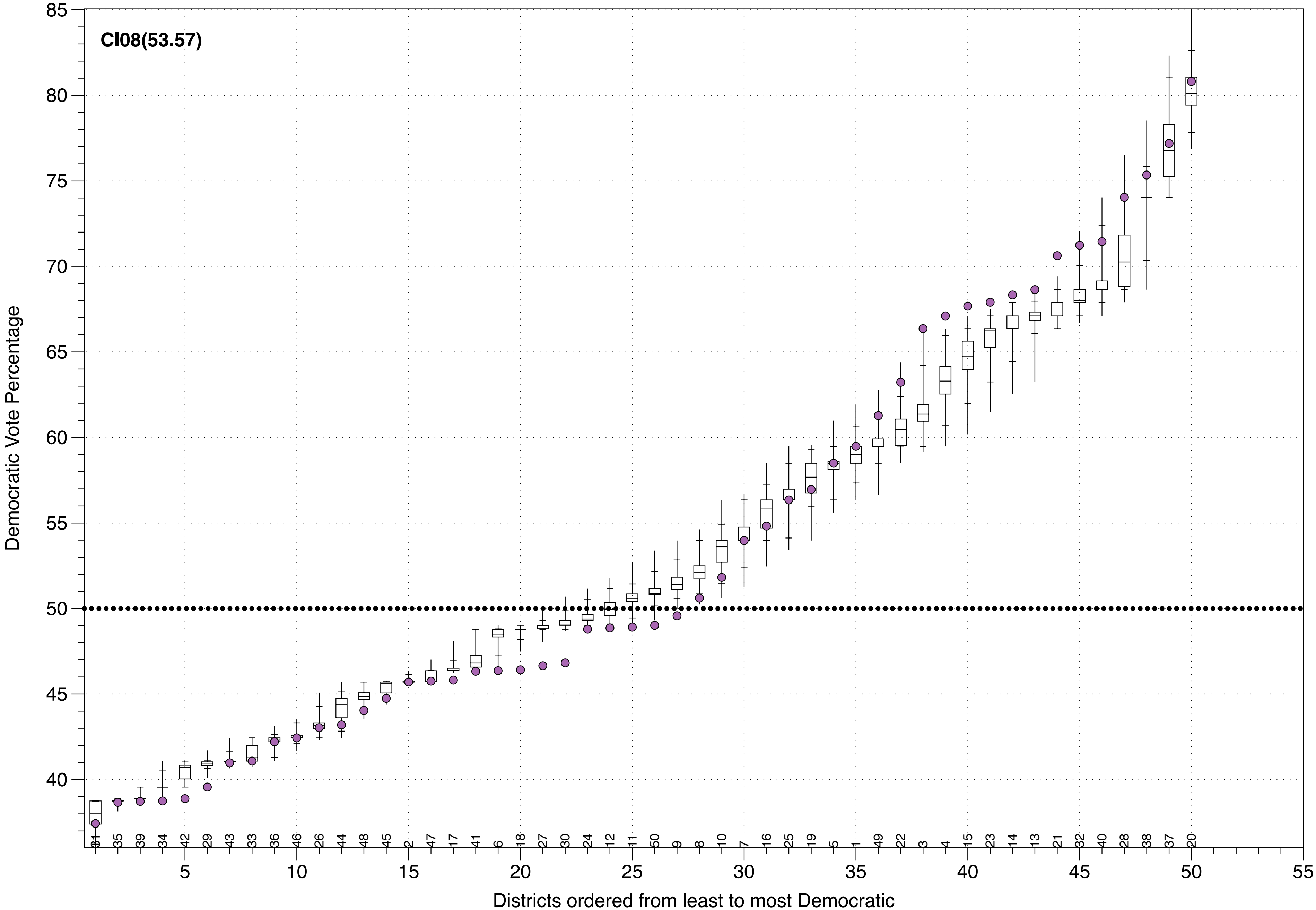
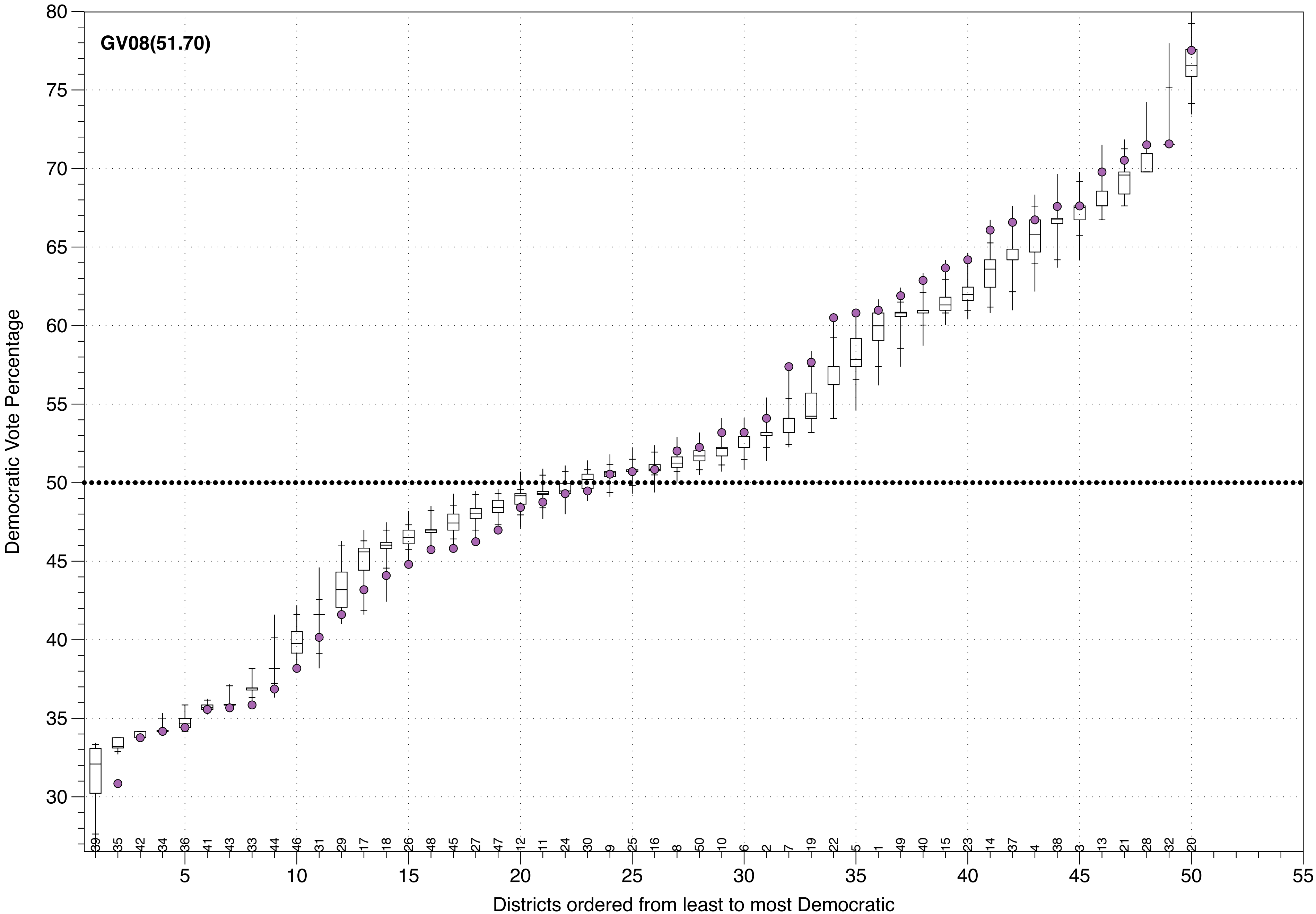

CI08(53.57)
Democratic Vote Percentage
Districts ordered from least to most Democratic


GV08(51.70)
Democratic Vote Percentage
Districts ordered from least to most Democratic
30
35
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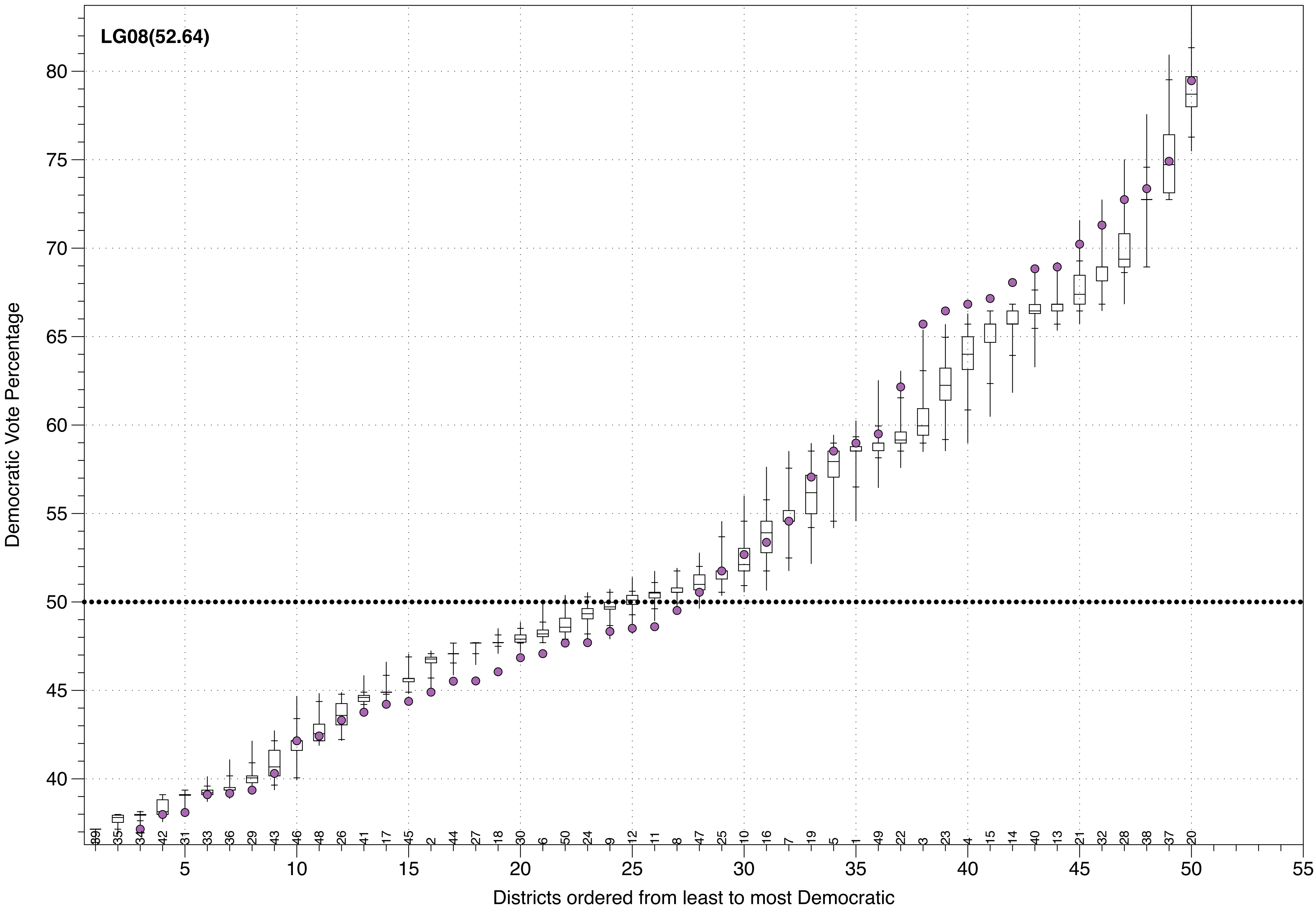
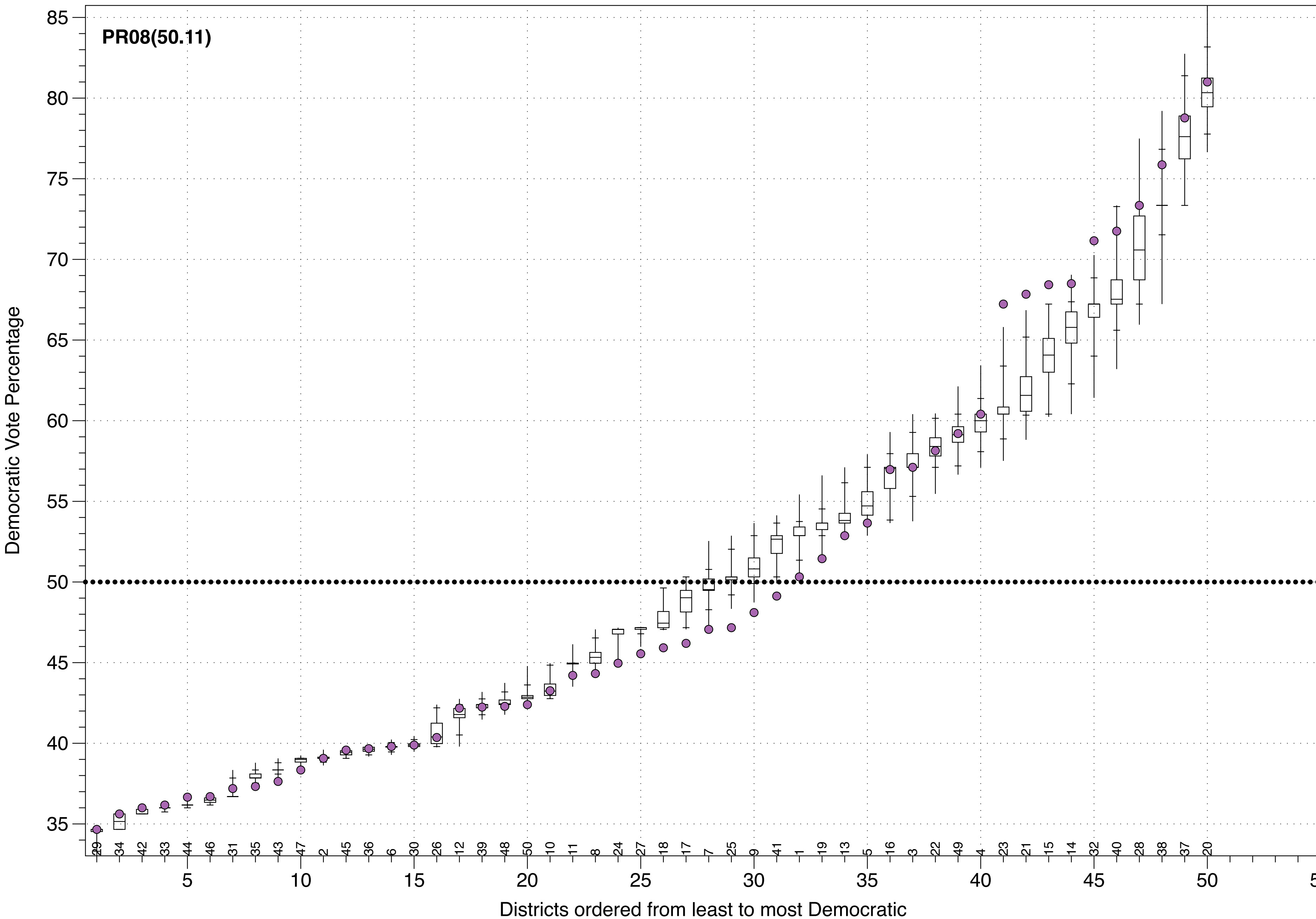
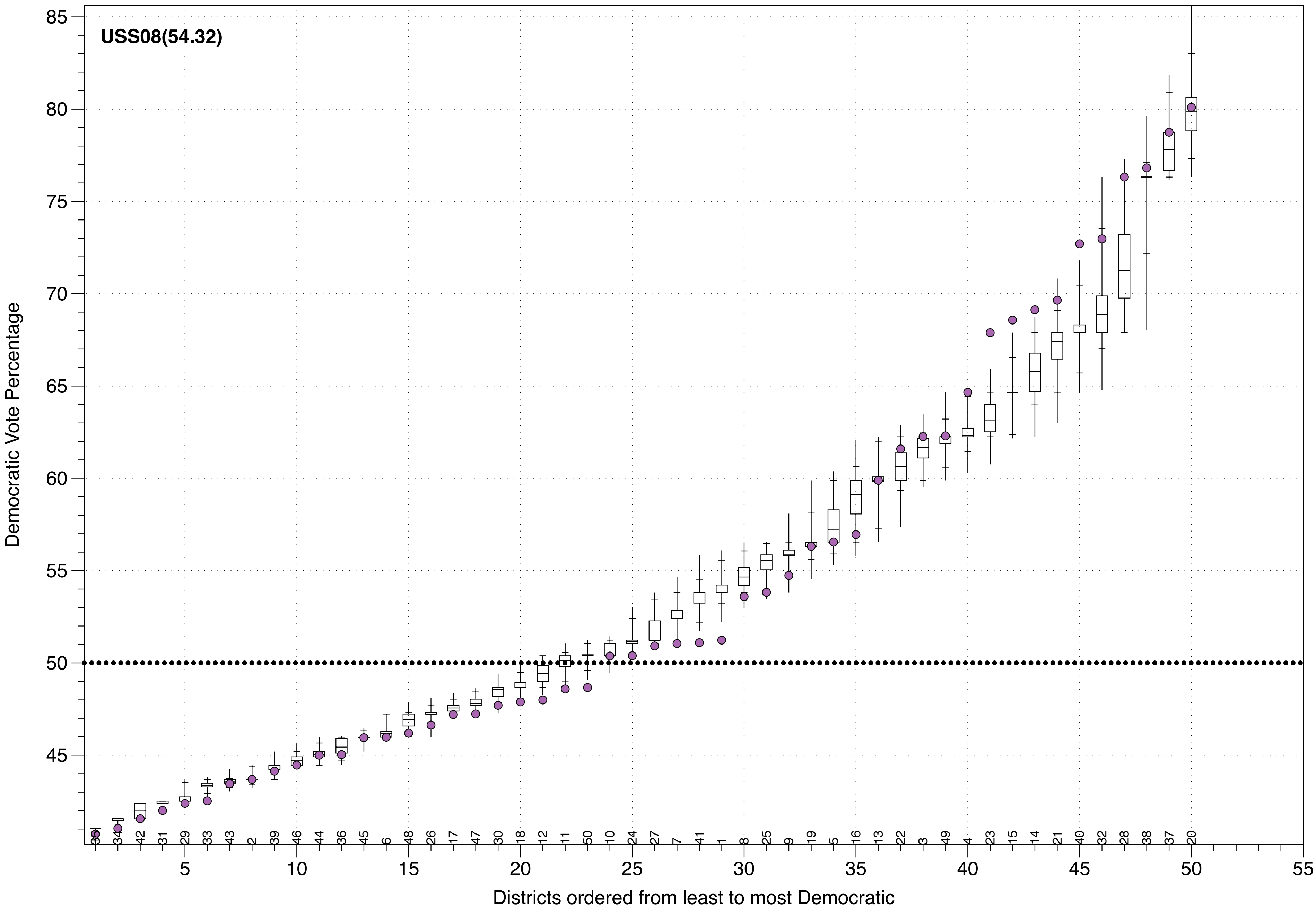
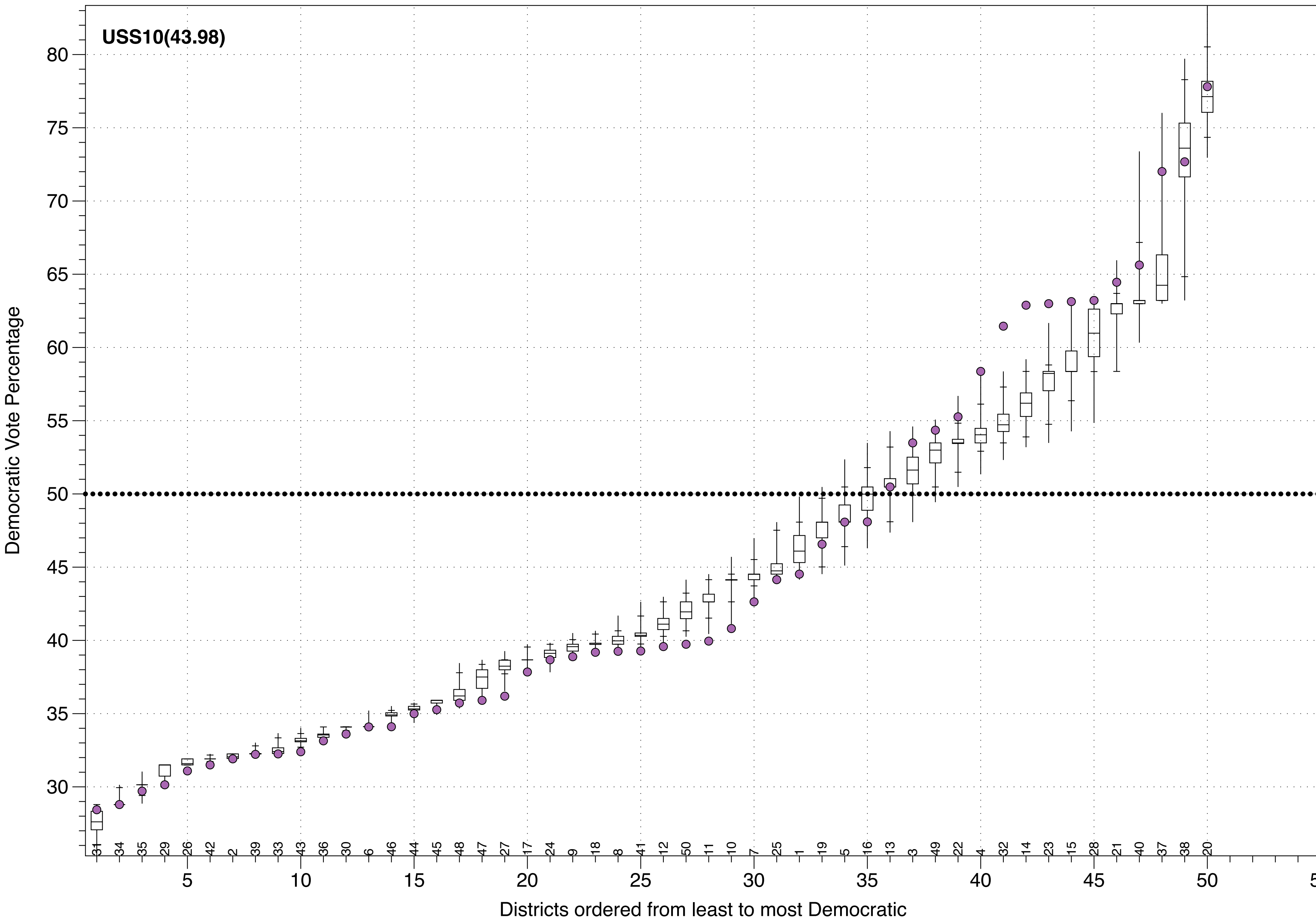

LG08(52.64)
Democratic Vote Percentage
40
45
50
55
60
65
70
75
80
39 35 34 42 31 33 36 29 43 46 48 26 41 17 45 2 44 27 18 30 6 50 24 9 12 11 8 47 25 10 16 7 19 5 1 49 22 3 23 4 15 14 40 13 21 32 28 38 37 20
5
10
15
20
25
30
35
40
45
50
55
Districts ordered from least to most Democratic



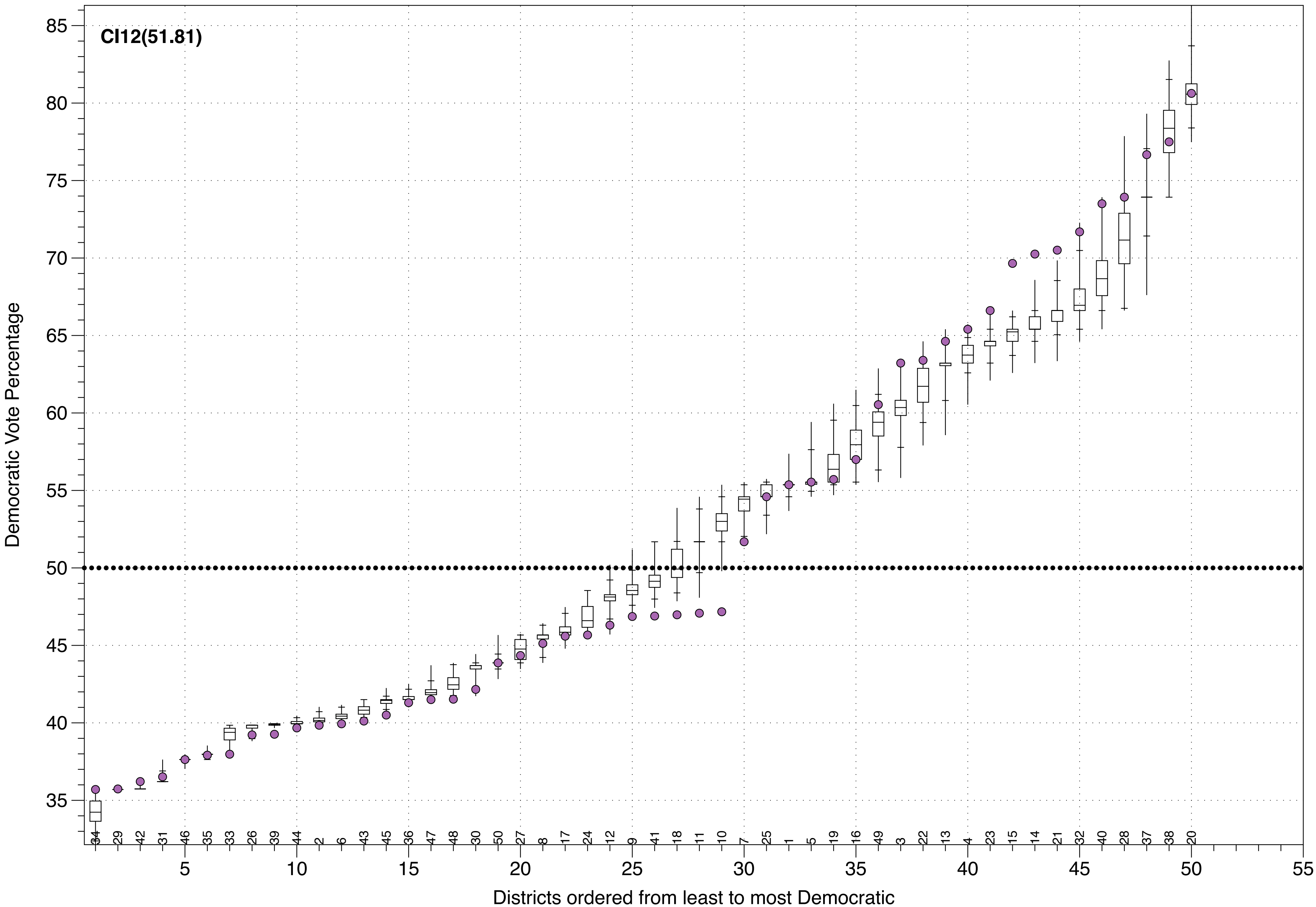
PR08(50.11)
Democratic Vote Percentage
35
40
45
50
55
60
65
70
75
80
85
29 34 42 33 44 46 31 35 43 47 2 45 36 6 30 26 12 39 48 50 10 11 8 24 27 18 17 7 25 9 41 1 19 13 5 16 3 22 49 4 23 21 15 14 32 40 28 38 37 20
5
10
15
20
25
30
35
40
45
50
55
Districts ordered from least to most Democratic

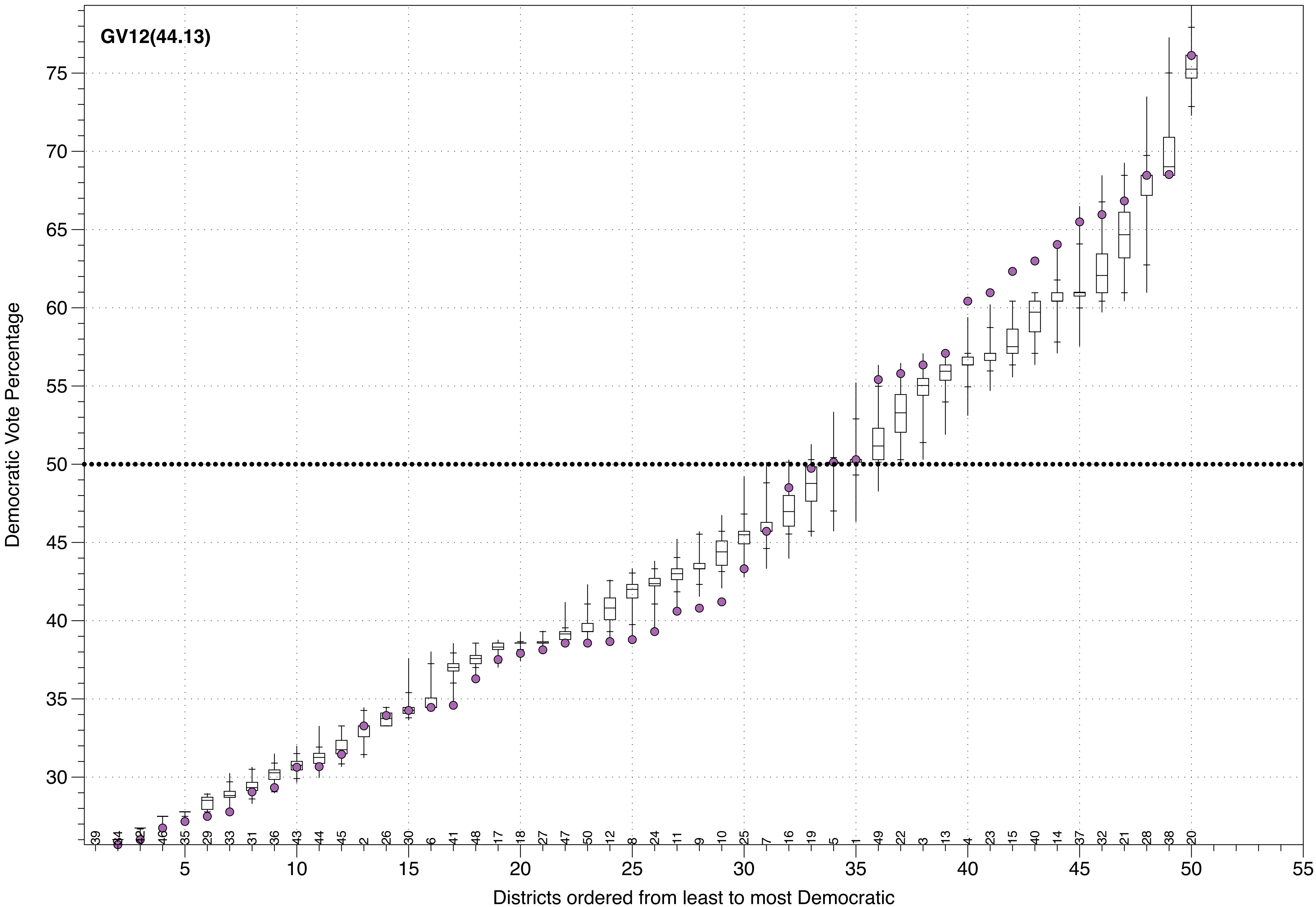


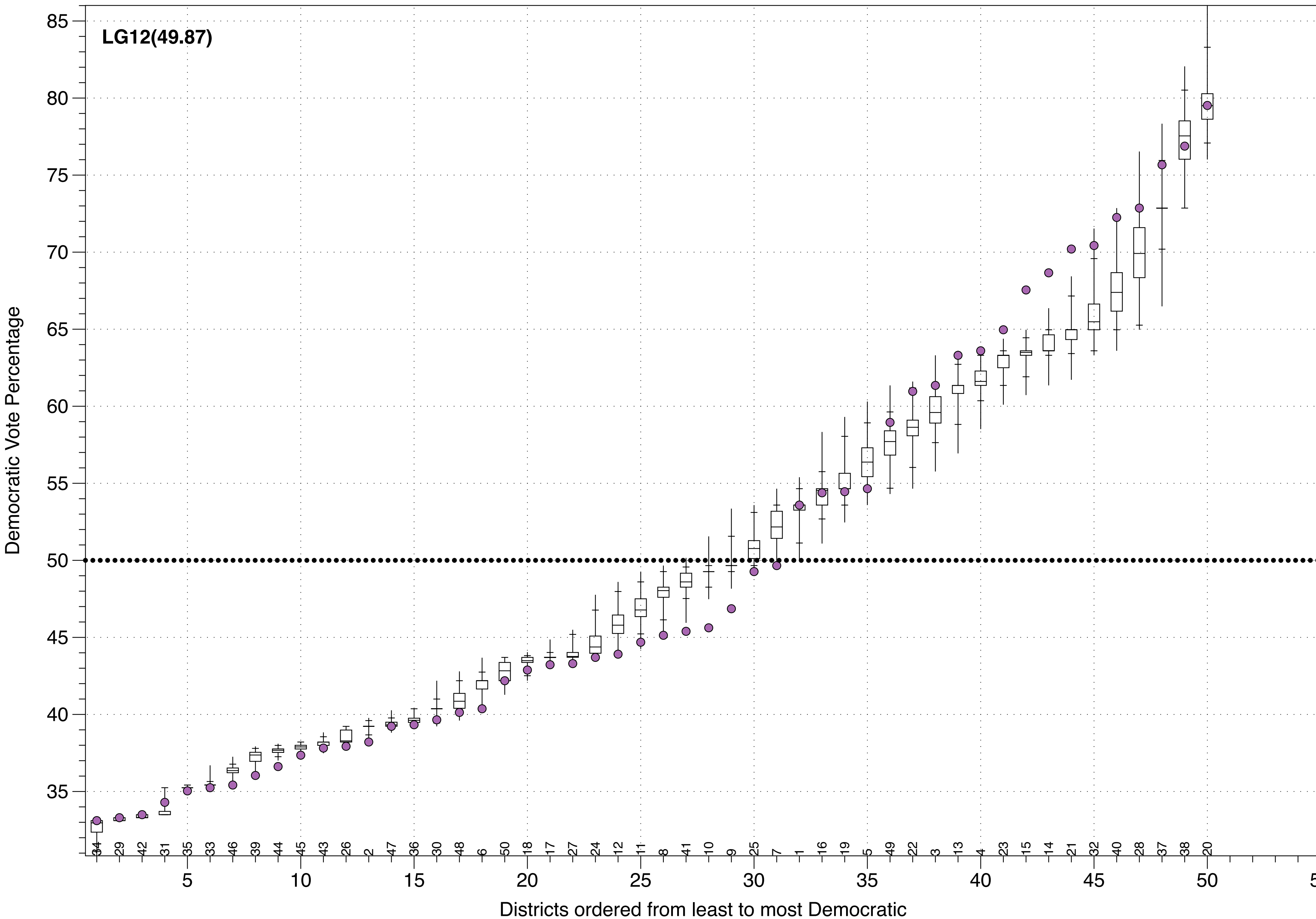
USS08(54.32)
Democratic Vote Percentage
Districts ordered from least to most Democratic

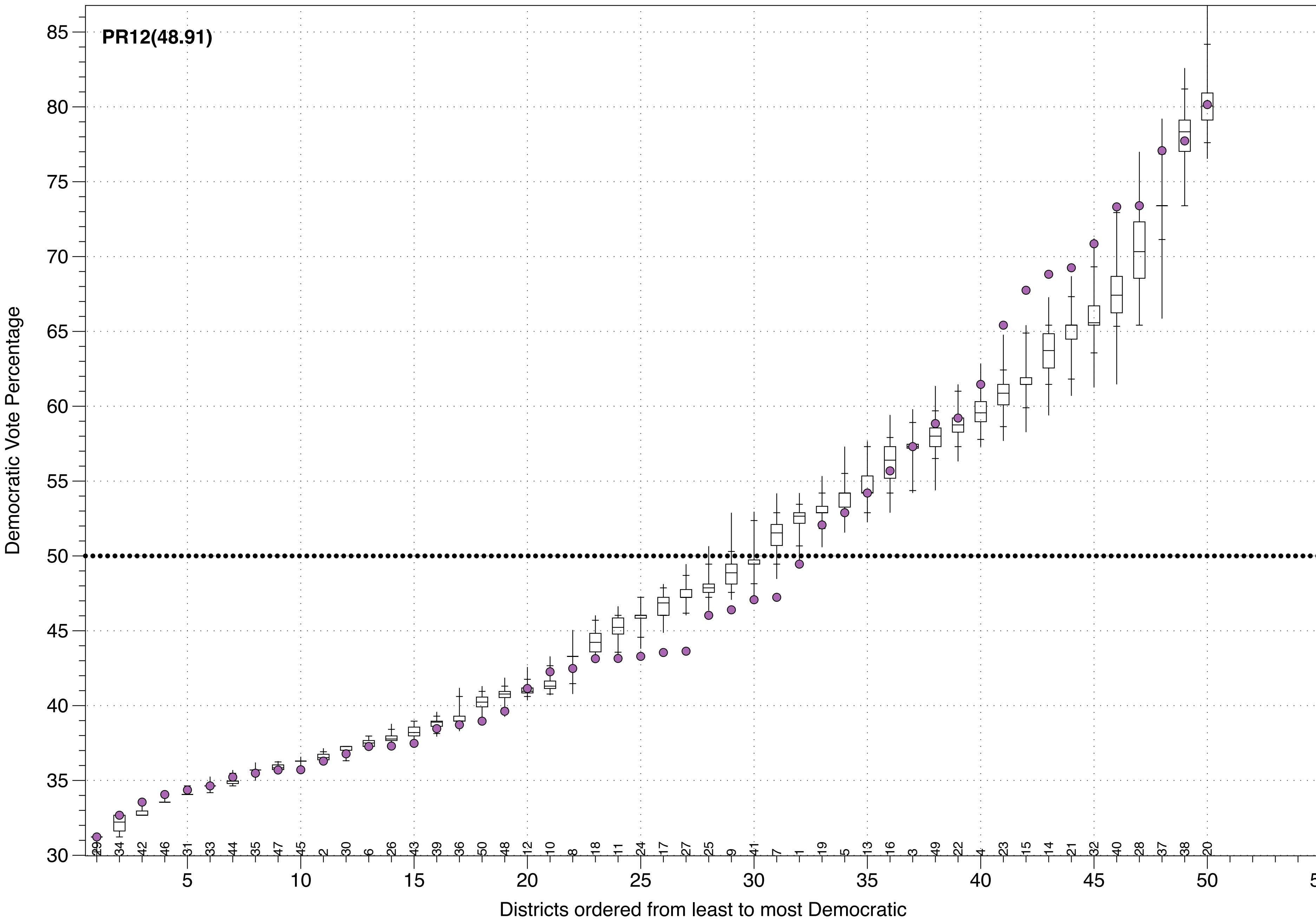


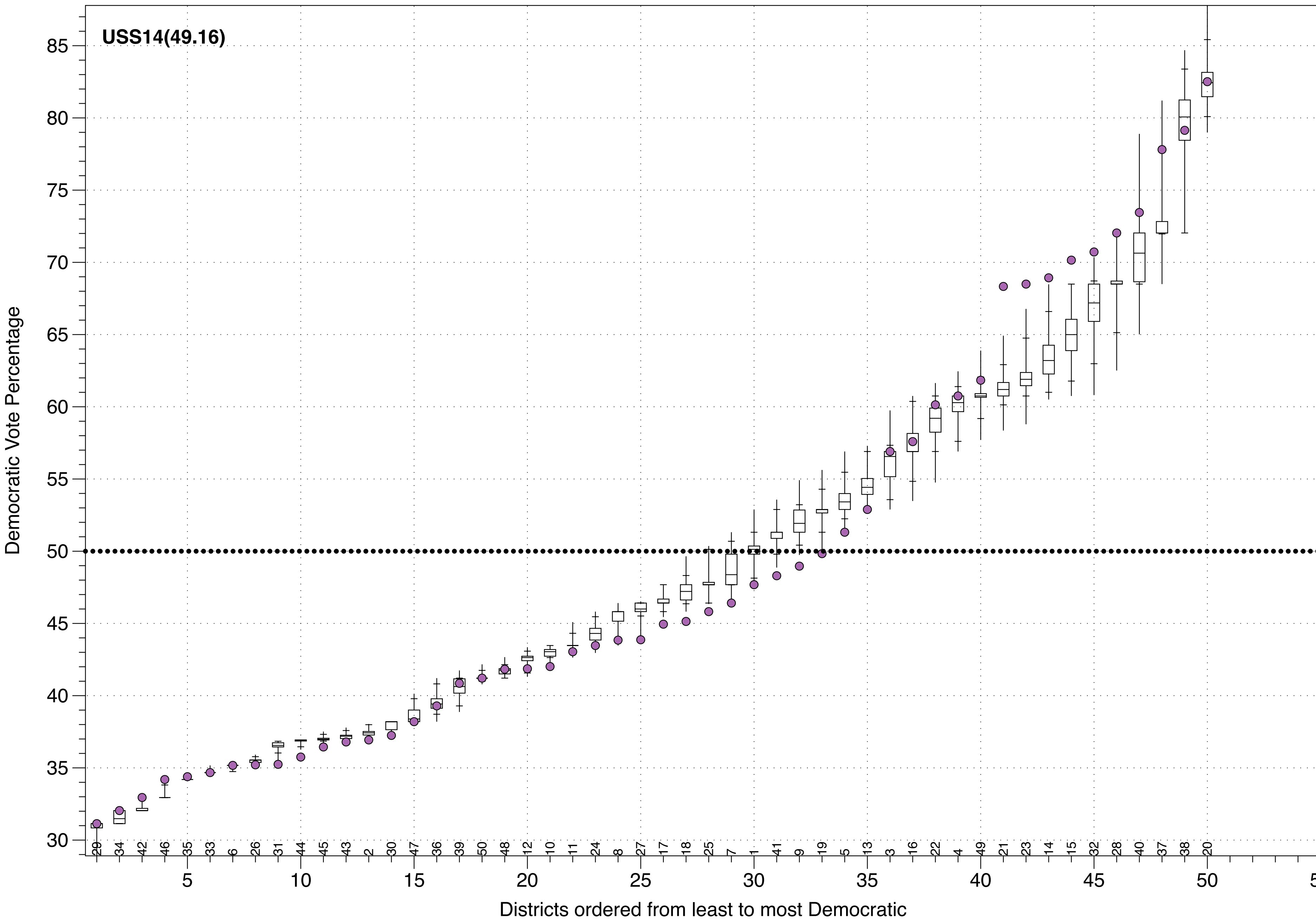
USS10(43.98)
Democratic Vote Percentage
Districts ordered from least to most Democratic
30
35
40
45
50
55
60
65
70
75
80
5
10
15
20
25
30
35
40
45
50
55
31 34 35 29 26 42 2 39 33 43 36 30 6 46 44 45 48 47 27 17 24 9 18 8 41 12 50 11 10 7 25 1 19 5 16 13 3 49 22 4 32 14 23 15 28 21 40 37 38 20


CI12(51.81)
Democratic Vote Percentage
Districts ordered from least to most Democratic


GV12(44.13)
Democratic Vote Percentage
30
35
40
45
50
55
60
65
70
75
39 34 42 46 35 29 33 31 36 43 44 45 2 26 30 6 41 48 17 18 27 47 50 12 8 24 11 9 10 25 7 16 19 5 1 49 22 3 13 4 23 15 40 14 37 32 21 28 38 20
5 10 15 20 25 30 35 40 45 50 55
Districts ordered from least to most Democratic


LG12(49.87)
Democratic Vote Percentage
Districts ordered from least to most Democratic
85
80
75
70
65
60
55
50
45
40
35
5
10
15
20
25
30
35
40
45
50
55
34 29 42 31 35 33 46 39 44 45 43 26 2 47 36 30 48 6 50 18 17 27 24 12 11 8 41 10 9 25 7 1 16 19 5 49 22 3 13 4 23 15 14 21 32 40 28 37 38 20


PR12(48.91)
Democratic Vote Percentage
30
35
40
45
50
55
60
65
70
75
80
85
29 34 42 46 31 33 44 35 47 45 2 30 6 26 43 39 36 50 48 12 10 8 18 11 24 17 27 25 9 41 7 1 19 5 13 16 3 49 22 4 23 15 14 21 32 40 28 37 38 20
5
10
15
20
25
30
35
40
45
50
55
Districts ordered from least to most Democratic


USS14(49.16)
Democratic Vote Percentage
Districts ordered from least to most Democratic
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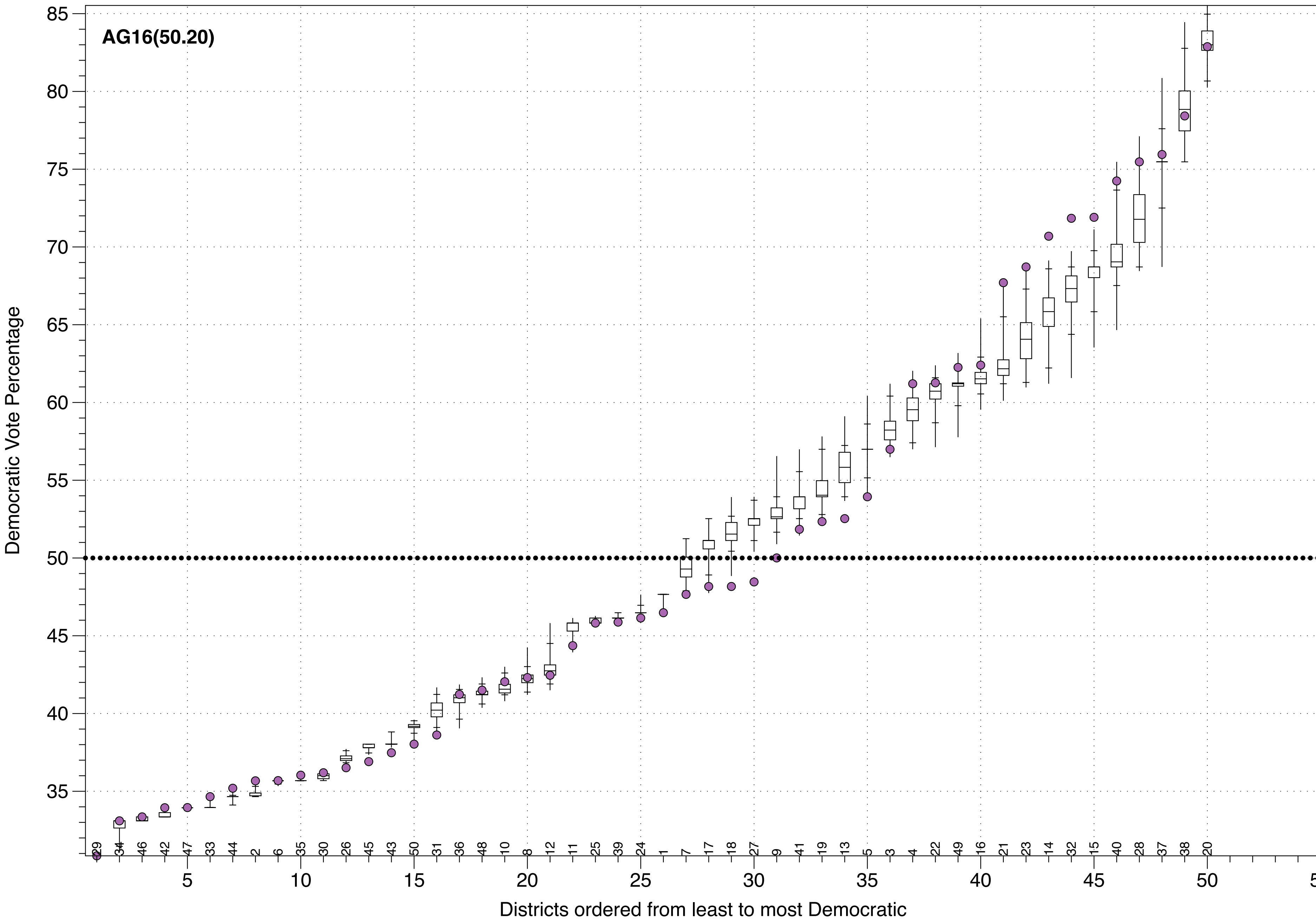
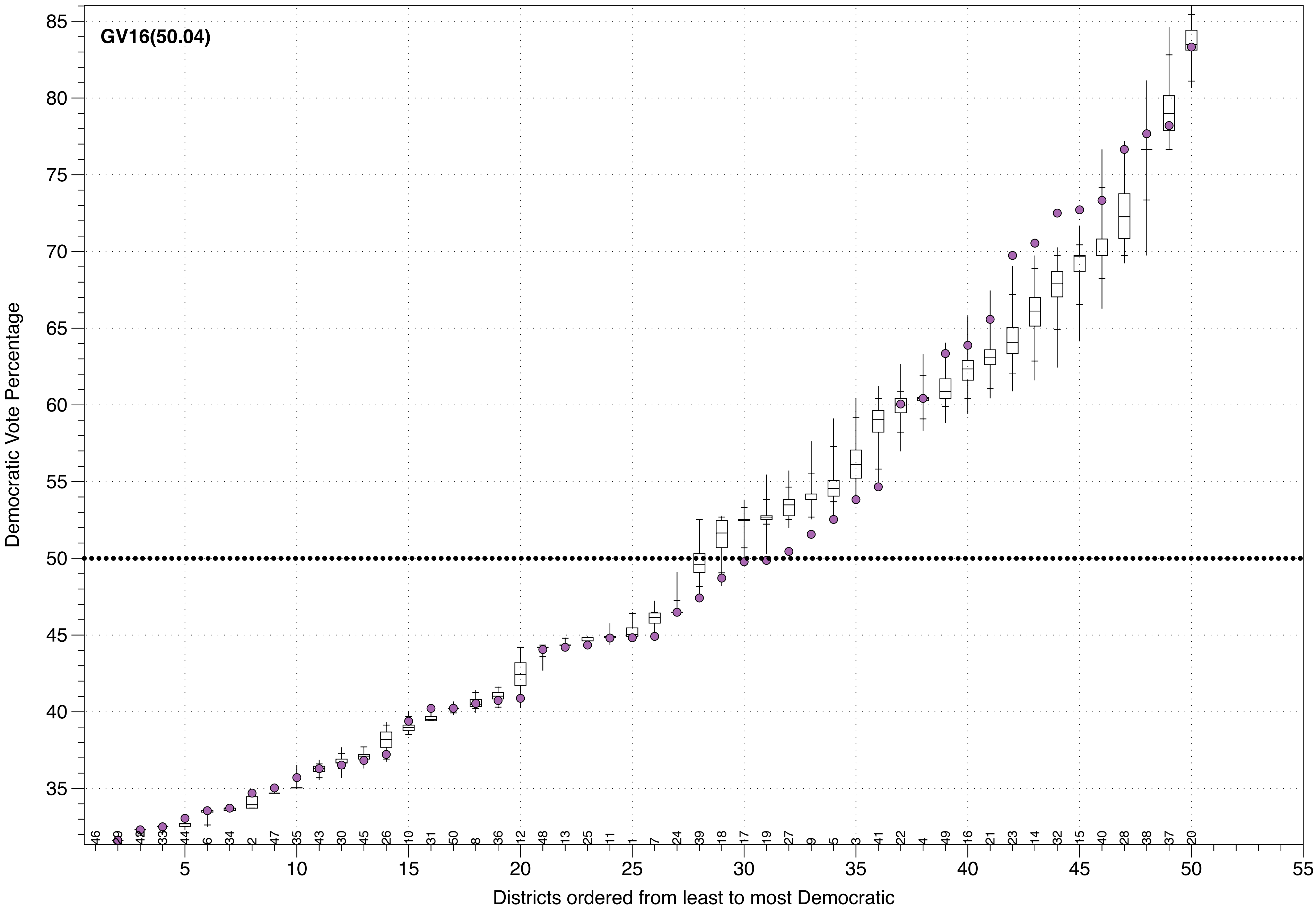
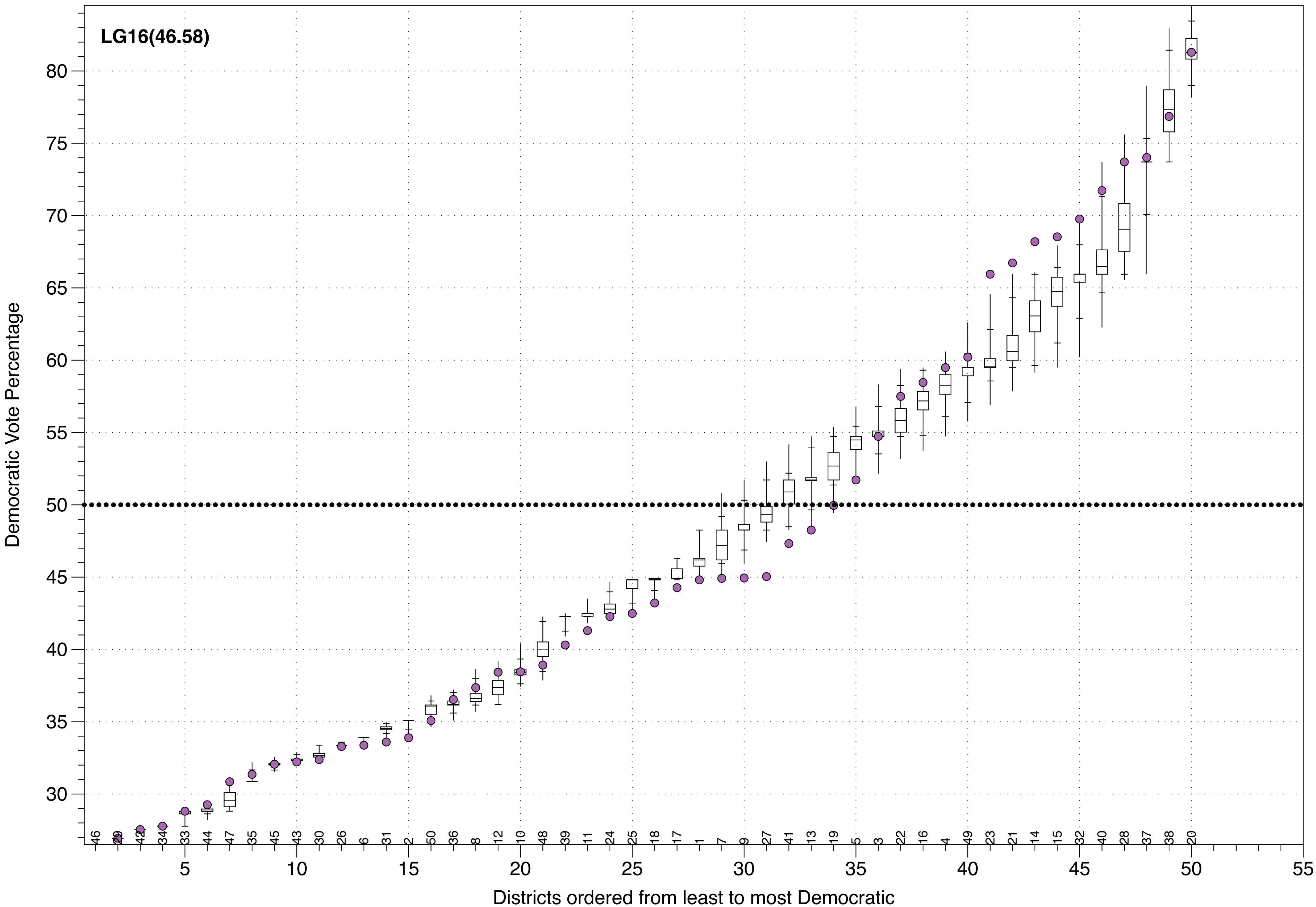
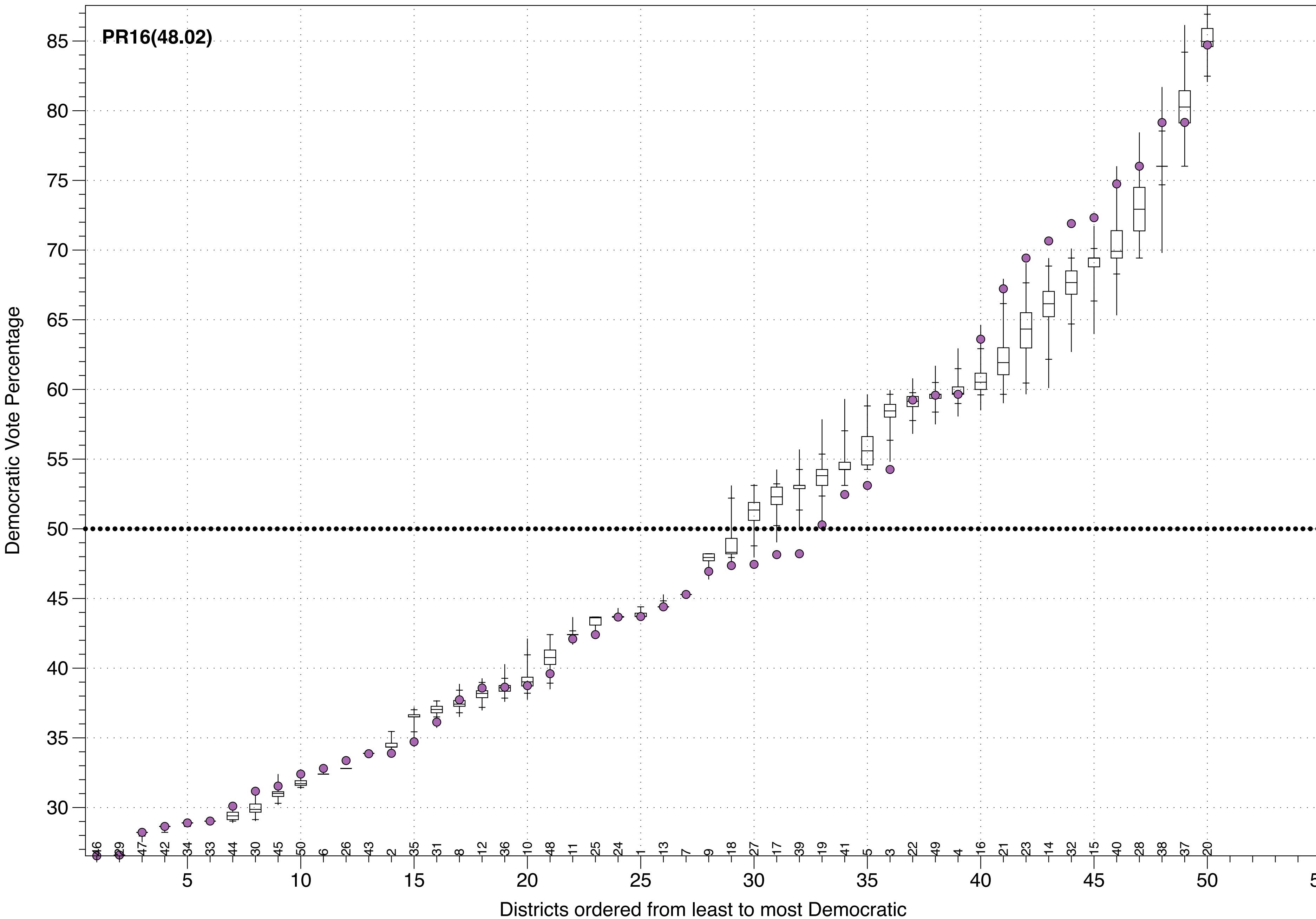

AG16(50.20)
Democratic Vote Percentage
85
80
75
70
65
60
55
50
45
40
35
29 34 46 42 47 33 44 2 6 35 30 26 45 43 50 31 36 48 10 8 12 11 25 39 24 1 7 17 18 27 9 41 19 13 5 3 4 22 49 16 21 23 14 32 15 40 28 37 38 20
5 10 15 20 25 30 35 40 45 50 55
Districts ordered from least to most Democratic


GV16(50.04)
Democratic Vote Percentage
35
40
45
50
55
60
65
70
75
80
85
46 29 42 33 44 6 34 2 47 35 43 30 45 26 10 31 50 8 36 12 48 13 25 11 1 7 24 39 18 17 19 27 9 5 3 41 22 4 49 16 21 23 14 32 15 40 28 38 37 20
5
10
15
20
25
30
35
40
45
50
55
Districts ordered from least to most Democratic


LG16(46.58)
Democratic Vote Percentage
Districts ordered from least to most Democratic
30
35
40
45
50
55
60
65
70
75
80
5
10
15
20
25
30
35
40
45
50
55
46
42
34
33
44
47
35
45
43
30
26
6
31
2
50
36
8
12
10
48
39
11
24
25
18
17
1
7
9
27
41
13
19
5
3
22
16
4
49
23
21
14
15
32
40
28
37
38
20


PR16(48.02)
Democratic Vote Percentage
Districts ordered from least to most Democratic
30
35
40
45
50
55
60
65
70
75
80
85
5
10
15
20
25
30
35
40
45
50
55
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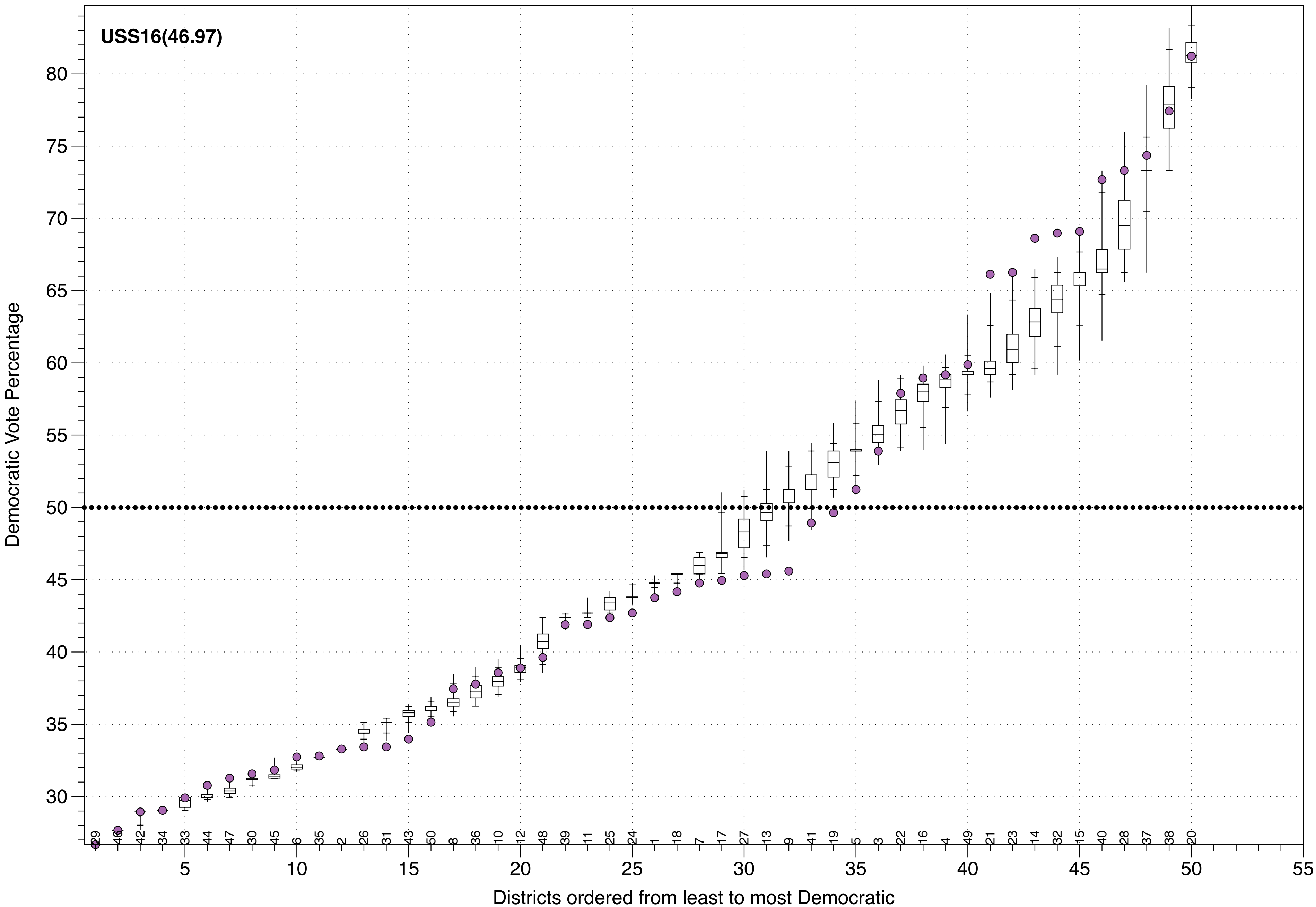
USS16(46.97)
Democratic Vote Percentage
Districts ordered from least to most Democratic
30
35
40
45
50
55
60
65
70
75
80
5
10
15
20
25
30
35
40
45
50
55
29 46 42 34 33 44 47 30 45 6 35 2 26 31 43 50 8 36 10 12 48 39 11 25 24 1 18 7 17 27 13 9 41 19 5 3 22 16 4 49 21 23 14 32 15 40 28 37 38 20

H Signature

I declare that the foregoing is true and correct to the best of my knowledge
Jonathan C. Mattingly,

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