Risk Sharing and Strategic Choice*

Brendan Daley† Philipp Sadowski‡

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Abstract

We undertake a decision-theoretic analysis of a model of bilateral risk sharing, conceptualized in two stages: first, agents choose risky endowments (Savage acts), and second, they form a risk sharing arrangement. Only the first-stage choices are observable to the analyst. We formulate axioms that put joint restrictions on best responses in the first stage and provide a representation result according to which agents behave as if they are risk averse expected utility maximizers who anticipate the subsequent sharing arrangement. All the parameters of the model, including the sharing arrangement, can be identified from this first-stage choice data. Applications are discussed.

Keywords: Risk sharing, Informal insurance, Choice under risk, Cooperative bargaining theory

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†Johns Hopkins University. e-mail: brendan.daley@jhu.edu

‡Duke University. e-mail: p.sadowski@duke.edu

1 Introduction

This project proposes a novel, decision-theoretic approach to the study of informal risk sharing arrangements between individuals. We assume that individual choices of risky prospects (i.e., of Savage acts) precede a second, risk sharing stage. The decision-theoretic nature of the study is twofold. One, the analyst only observes first-stage choices and tries to infer properties of the unobserved second stage. Two, we describe first-stage choices via behavioral axioms.

We aim to capture environments in which the motive for risk sharing is mutual insurance stemming from a diversification benefit to pooling risks (as opposed to risk shifting, which could lead one individual to insure another even without any diversification benefit from pooling risks). We argue that this focus on hedging idiosyncratic risks, and our resultant findings, are in line with much of the literature on risk sharing in developing economies.[1]

Our theory consists of *i*) behavioral axioms describing observable choices of first-stage Savage acts; and *ii*) a parameterized model that describes the agents' utility functions and the precise rule that governs how risk is allocated depending on the agents' pre-sharing choices of risky acts. We then establish that the axioms tightly characterize the model: a data set satisfies the axioms if and only if there exist parameters under which the model generates that data set.[2]

The axioms that found our benchmark model can be motivated on normative grounds: they capture behavior that one would want to see if individuals sought to hedge idiosyncratic risk and indeed managed to achieve this via informal risk sharing. The axioms can also be useful on positive grounds. Here, we do not argue that they are unassailable, but rather view them as plausible and empirically testable properties. Our model can then be tested entirely in terms of choice data, as opposed to describing outcomes as in co-operative game theory and relying on data about transfers. Moreover, the parameters of the model, including the sharing arrangement, can be uniquely identified (given sufficient choice data), meaning our model can in principle be estimated based only on first-stage choices.

For concreteness, consider the following instantiations of the elements in this envi-

[1]The parsimony of our axiomatization and representation benefits from the focus on diversification, but our approach could be adapted to environments where risk shifting is relevant as well.

[2]This work follows Daley and Sadowski (2017) in applying axioms to behavioral data directly in the multi-agent context in order to characterize aspects of the multi-agent outcome (e.g., the sharing rule), rather than applying axioms exclusively to the single-agent context and importing the single-agent representation into an assumed model of how multiple agents interact.

ronment, using the example of two neighboring farmers for illustration. The objects of observable choice are acts that lead to a consumption outcome for every payoff relevant state of the world, ω. For instance, one act may be planting a crop that produces high yield only when it rains a lot, and another act may be planting a crop that does better when the weather is dry. Suppose that each agent i evaluates consuming the output of any act f via a risk averse expected utility function $u_i(f)$.

The key is that agents are not consigned to consume the output of their chosen act. Instead, the acts chosen by Agents 1 and 2, respectively, serve as risky endowments in the subsequent sharing stage, where risk pooling may reduce exposure to idiosyncratic risk. For example, if Agent 1 plants the crop that is better suited to wet conditions, and Agent 2 plants the crop that does well in dry conditions, they might then agree on a transfer from the first to the second when the weather is wet, and vice versa when it is dry, providing some insurance to both. More generally, given respective endowments f and g, a *sharing rule* specifies a feasible state-contingent allocation $a(\omega)$ to Agent 1 and $b(\omega)$ to Agent 2 such that $a(\omega) + b(\omega) \leq f(\omega) + g(\omega)$ for all ω.

There are many possible sharing rules. For example, given (f, g), the three prominent cooperative bargaining solutions of Nash (1950); Kalai and Smorodinsky (1975); Kalai (1977) each prescribe an allocation of the pooled risk. In the case of the Nash bargaining solution (NBS), that allocation is

$$(a^*, b^*) = \arg \max_{(a,b)\in A(f,g)} \left(u_1\left(a\right) - u_1\left(f\right)\right)\left(u_2\left(b\right) - u_2\left(g\right)\right), \tag{1}$$

where $A(f, g)$ is the set of all feasible allocations given choices f and g. Under the NBS, holding fixed Agent 2's choice of act g, it is clear that Agent 1's choice of act f will affect the allocation each agent receives.

On the one hand, the endowments f and g determine the *size* of the possible surplus from mutual insurance. If $f = g$, there is no idiosyncratic risk to hedge through pooling. In the example this could occur in the hypothetical of two farmers with the same technologies and comparable plots of land choosing to grow the same crop. Of course, choices $f \neq g$ that better hedge one another increase the potential surplus from risk pooling.

On the other hand, f affects Agent 1's disagreement value, $u_1(f)$, and under the NBS the *share* of the surplus Agent 1 receives is increasing in this disagreement value. When choosing a risky act in the first stage, forward-looking agents will trade off the size of the potential surplus from risk pooling and their individual share.

For many sharing rules, including those based on other prominent bargaining solutions, shares also depend on disagreement values, but the manner in which they do so varies. As a consequence, the first-stage choice of act will depend on the particular sharing rule that governs the second stage, as well as on risk preferences. Our representation and identification theorems together imply that choice data that satisfies our axioms will reveal via our model (with uniquely identified sharing rule and risk preferences) the specific tradeoff the agents face.[3]

Mutual insurance through informal risk sharing has been a topic of ongoing interest in the study of developing economies for several decades. Quoting the seminal work of Townsend (1994):

"[T]hroughout much of the underdeveloped world, [people] live in poor, high-risk environments. Per capita income and per capita consumption are low, and the risk to agriculture from erratic monsoon rains is high. Crop and human diseases are also prevalent. Various policy issues turn on this level of risk and on the presence or absence of risk reduction mechanisms . . . In an optimal arrangement, both [agents] would coinsure the fluctuations of each, . . . "

The preeminent question of interest in this field is whether pooling these risks among agents provides diversification that efficiently shields them from idiosyncratic shocks. Notable additional examples that focus on this motive for risk sharing include Ligon et al. (2002); Belhaj and Deroïan (2012); Munshi and Rosenzweig (2016). Since risk sharing arrangements—especially informal ones that are prevalent in many developing economies—are difficult to observe directly, efficiency is often empirically tested based on consumption and income data obtained primarily from surveys. Pareto efficient sharing is rejected if individual consumption varies not only with common but also with idiosyncratic income shocks.[4] If this test rejects Pareto efficiency, then pooling must be incomplete, indicating that diversification benefits are left on the table. The test is insensitive to how pooled risk is distributed across agents, in line with a focus on the diversification benefits of risk sharing.

[3]In the motivations of our axioms, agents anticipate how they would agree to divide the surplus from risk sharing given endowments f and g, for instance through subsequent bargaining as in the example above. In an alternative timing, bargaining could take place earlier and include the choices of f and g. The timing in which agents take endowments as given when deciding on how to share risk is well suited to situations where the productive choices of f and g are difficult to enforce, for instance due to noncontractable effort, or because the length of time between the choices of f and g and the realization of output is too large to expect an agreement to last that long.

[4]Papers in the substantial literature that employs versions of this test include Cochrane (1991); Mace (1991); Altonji et al. (1992); Townsend (1994); Hayashi et al. (1996); Ravallion and Chaudhuri (1997); Laczó (2015); Bold and Broer (2021); Meghir et al. (2020).

Accepting the normative appeal of risk sharing that provides mutual insurance via diversification, we too focus our analysis on such situations, and motivate our axioms accordingly. Our approach has a number of potential benefits. First, it can identify properties of the sharing arrangement (e.g., does the sharing arrangement adhere to a known cooperative bargaining solution such as the NBS?) and specifies the distributional consequences of different sharing arrangements beyond whether they are Pareto efficient. Second, understanding how agents' pre-sharing choices of risky prospects (e.g., choices of crops to plant, seasonal migration, profession, education level, reproduction, etc.) depend on the sharing arrangement is crucial when evaluating efficiency of the arrangement not just in terms of transfers at the sharing stage, but also in terms of prior productive choices.[5] In particular, how much do agents prioritize their disagreement values at the cost of social surplus? Third, our axioms provide a basis for testing Pareto efficiency at the sharing stage jointly with auxiliary assumptions about individual risk preferences. Finally, in contexts where consumption and transfer data are hard to observe directly, the pre-sharing choice data on which our approach is based may provide a useful alternative or supplemental data source.

The paper is structured as follows. Section 2 formally introduces the environment. Section 3 explains and motivates our axioms, provides the representation and identification theorems for our risk sharing model, and discusses features of the model in connection to evidence from, and modeling assumptions in, the applied literature. In particular, it discusses how the behavioral data we rely on could be observed in practice in order to test the axioms. According to our identification result, each sharing arrangement has unique behavioral implications for first-stage choices. As examples, Section 4 discusses sharing according to the prominent cooperative bargaining solutions of Nash (1950); Kalai and Smorodinsky (1975); Kalai (1977), and Supplement S.1 provides additional axioms that characterizes each in turn.[6]

We often refer to neighboring farmers for illustration, but, as we mention again in Sections 3 and 7, hedging benefits also realize when risky income is pooled within the household. Further, it should be clear that our model can be applied when the risk from other labor income is pooled across households. Section 5 illustrates how our model of risk

[5]The most basic implication of this trade-off—that a subsequent opportunity for risk sharing should increase individual risk taking—is in line with empirical and experimental evidence (Angelucci et al., 2018; Attanasio et al., 2012).

[6]Hence, in the context of our model, our approach allows us to provide behavioral foundations for these bargaining solutions, which are usually motivated only via the normative appeal of the outcomes they produce.

sharing may help understand the observable decisions of rural workers whether or not to engage in seasonal migration, which alters their productive opportunities. In particular, we show how the endogenous concern for disagreement values in our model can generate the perceived cost of migration that has been assumed in order to explain the observed lack of seasonal migration (see, for instance, Meghir et al., 2020).

While our main model captures efficient risk sharing, which is the benchmark case in the literature, evidence suggests that risk sharing is often less than efficient.[7] Different types of sharing frictions have been proposed, and each of them could affect the choice of risky prospects in a different way. Analyzing the range of possible frictions within our model would be interesting but is beyond the scope of this paper. Instead, in Section 6, we consider a particularly parsimonious type of friction as one example, namely the probability of a "breakdown" of sharing. We show that in principle this friction can also be identified from our data. We provide comparative statics results that relate the magnitude of the friction to the strength of agents' preferences for sharing, and to their incentives to prioritize their disagreement values over the generation of social surplus.[8]

We believe our results demonstrate that in the risk sharing context, multi-agent best-response data strikes an appropriate balance between observability, tractability, and power for testing and estimation. In particular, if agents choose risky acts in sequence or opportunities for each agent to change acts arise stochastically, and if agents heavily discount the future, then static best responses to the risky acts currently held by others may reasonably model their behavior.[9] Throughout, we focus on the bilateral risk sharing case for ease of exposition. An extension to larger groups (*aka* "syndicates" in Wilson, 1968) is conceptually straightforward. Section 7 concludes by briefly suggesting a potential alternative extension where multiple agents share risk bilaterally along the ties of a network.

[7]The null-hypothesis of efficient sharing is rejected in several studies in the Economic Development literature; see Ligon, Thomas, and Worrall (2002) for references, which begin with Townsend (1994).

[8]Supplement S.2 provides the relaxed set of axioms that characterize this model.

[9]For simultaneous-move games virtually all solution concepts build on the analysis of best response correspondences, and hence the decision-theoretic study of our model can serve as a foundation for further analysis. Supplement S.3 formally models the situation where the two farmers mentioned above have to decide simultaneously which crops to plant. We investigate the inefficiencies induced by different sharing protocols in equilibrium. In particular, the three prominent bargaining solutions are ranked in terms of the efficiency loss they induce, perhaps providing a novel criterion to compare those solutions.

2 The Risk Sharing Environment

Overview. There are two agents, $i \in \{1,2\}$, and three time stages. In the first stage, each agent strategically chooses a risky endowment. In the second stage, the agents have the opportunity to share risk by agreeing to a feasible reallocation given their first-stage choices. In the third stage, uncertainty realizes, transfers are made in accordance with the second-stage agreement, and consumption occurs.

We are explicit about the first-stage choices of risky endowments, corresponding to an assumption that this is the data available to the analyst and therefore the domain of our axioms. In contrast, the procedure by which the agents bargain over risk sharing in the second stage is unspecified as in the cooperative game theory literature (most prominently Nash, 1950). Notice that if the agents fail to agree on a reallocation, then each agent is left to consume her first-stage choice, meaning it serves as the agent's *disagreement allocation*. In the first stage, agents are aware of the manner by which agreements are reached in the second stage (though it is unobservable to the analyst).

First-stage Acts. Let $(\Omega, \mathcal{A}, \mu)$ be an infinitely divisible probability space where $\mathcal{A}$ is the Borel sigma algebra and $\mu \in \Delta(\Omega)$ is a probability measure on Ω. An *act* $f : \Omega \to \mathbb{R}_{++}$ is a mapping that is measurable in $\mathcal{A}$ and assigns a consumption outcome to each $\omega \in \Omega$.[10] We use $f = g$ to mean the two acts are equal almost surely. Let $\chi(f)$ be the range of f, and say that f is simple if $\chi(f)$ is finite. Denote by $\mathcal{F}$ the set of all simple acts.

Theories of individual choice often follow Savage (1954) and employ acts on a state space to study subjective probabilities. In contrast, the probability measure μ in our model is objective, but states and acts are used in order to track the joint distribution generated by the two acts chosen by the two agents—given μ, every pair of acts endows a lottery over *pairs* of outcomes. Infinite divisibility of the probability space means that any simple lottery over pairs of outcomes can be generated this way.

Second-stage Agreements. Let f and g be the acts held by Agents 1 and 2, respectively, at the outset of the second stage. A *sharing rule* $\Gamma : \mathcal{F}^2 \to \mathcal{F}^2$ is a mapping from (f, g) to a consumption allocation (a, b) in the set of feasible reallocations $A(f, g) := \{(a, b) \in \mathcal{F}^2 \,|\, a + b \leq f + g\}$. Here, "feasibility" signifies only that there are sufficient resources to construct (a, b) from the pooled (f, g) in the abstract, not that such a reallocation among the agents is necessarily attainable. For example, physical or informational frictions may impede the required transfers, but whether such frictions exist is

[10]Restricting to strictly positive consumption averts well known issues for common utility functions, such as $u(c) = \log(c)$, in which the zero consumption yields $u(0) = -\infty$.

unobservable to the analyst. Looking ahead, our representation in Section 3 captures the frictionless environment, and one example of frictions is incorporated in Section 6.

3 Axioms, Model, and Representation

3.1 The Choice Domain

The observable-choice domain for our axioms is best-response data. The analyst observes each agent i's preferences over $\mathcal{F} \times \{0,1\}$, holding fixed the act held by $j \neq i$ to be $g \in \mathcal{F}$.[11] We denote these preferences by $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$. For alternative $\langle f, o\rangle \in \mathcal{F} \times \{0,1\}$ the interpretation of the second component $o \in \{0,1\}$ is that

- $\langle f, 1\rangle$ indicates the agents will have the opportunity to pool and reallocate their respectively chosen risky endowments, f and g (as described in the preceding section).
- $\langle f, 0\rangle$ indicates the agents *cannot* pool their risks, and must consume their own risky endowments. We refer to this situation as *autarky*.

For concreteness, $\langle f, 0\rangle \succsim_g^i \langle f', 0\rangle$ means that i prefers f over f' in autarky when agent j holds act g (below we will assume that autarky preferences are independent of g, ruling out social comparisons such as envy). Similarly, $\langle f, 1\rangle \succsim_g^i \langle f', 1\rangle$ means that i prefers f over f' when agent j holds act g, but now with the understanding that the two agents will have the opportunity to reallocate risk in the second stage. In this case, we should expect i's ranking over acts to depend on the act j holds. Note that the primitive encodes whether agents have the *opportunity* to pool risk; for instance, farmers may be able to pool risk only if they live in the same village, or a pair of marriageable individuals only if they have the potential to form a household with each other. To what extend they actually do so is an empirical question which our axioms can help address. Finally, our primitive also captures i's freedom to opt out of the sharing arrangement entirely: $\langle f, 1\rangle \succsim_g^i \langle f', 0\rangle$ means that i prefers to hold f when j holds g and they have the opportunity to reallocate in the second stage, rather than opting out of the sharing arrangement (e.g., a farmer may choose to permanently relocate to another village, a potential partner may choose to decline a marriage proposal, etc.) and consuming f' in autarky. Note that while i likely also has preferences over the act held by j, we do not observe those preferences because i cannot choose for j. Our *model* (Definition 3.1), however, does induce a complete preference over $\mathcal{F}^2 \times \{0,1\}$ for each agent.

[11] Summarizing best responses with conditional preferences is analogous to the standard approach in the decision theory literature, where choice data is summarized by a preference.

3.2 General Axioms

We now present axioms on the domain just described, which will tightly characterize our behavioral model. Notably, some axioms impose a joint structure on the agents' best responses. We refer to axioms in this section as "general" because they do not prescribe a particular sharing rule (e.g., one that adheres to the NBS).[12] Our first two axioms specify that all preference relations are well behaved, and that in autarky each agent has standard preferences that are concerned only with her own consumption.

Axiom 1 (Preference)
The binary relations $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ *for* $i \in \{1,2\}$ *are preference relations on* $\mathcal{F} \times \{0,1\}$.

Axiom 2 (Expected Utility Preferences in Autarky)
On $\mathcal{F} \times \{0\}$*, the binary relation* $\succsim_g^i$ *is independent of* g *and satisfies the von Neumann Morgenstern (vNM) axioms for a continuous, monotonic expected utility representation.*[13]

The next axiom captures voluntary participation: each agent likes being in the sharing arrangement at least as well as her disagreement allocation. Moreover, there is at least one instance in which an agent is made better off by the sharing arrangement compared to autarky.

Axiom 3 (Voluntary Participation) $\langle f,1\rangle \succsim_g^i \langle f,0\rangle$ *for all* $(f,g)\in\mathcal{F}^2$ *and* $i\in\{1,2\}$*, with strict preference for some* (f,g) *and* i.

As discussed at the outset, we consider situations where agents may choose to pool their resources, f and g, if doing so has the potential benefit of mutual insurance via diversification. Note that for scalar $\beta > 0$, the act βf is a "scaled replica" of the act f. Hence the pair $(f, \beta f)$ constitutes a situation without any idiosyncratic shocks to hedge via pooling.[14] The next axiom captures that the agents have no strict preference for

[12]Supplement S.1 provides additional axioms for each of the sharing rules discussed in Section 4.1.

[13]Throughout, any reference to expected utility maximization on the domain of simple acts, $\mathcal{F}$, assumes that only the lotteries induced via the measure μ matter for the ranking of the acts. Formally, for $c \in \mathbb{R}_{++}$ and $f \in \mathcal{F}$, let $p_f(c) = \mu(\{\omega\in\Omega | f(\omega)=c\})$, and assume that the decision maker is indifferent between acts $f,g\in\mathcal{F}$ that induce the same lottery: $p_f = p_g$ implies $f \sim_i g$. The induced ranking of simple lotteries is then required to satisfy the von Neumann Morgenstern (1944) axioms of Transitivity, Completeness, Continuity in probabilities and payoffs, and Independence, and to rank larger risk-free amounts over smaller ones.

[14]Consider the following criterion for pooling risks to have even a potential benefit from diversification: *Is the pooled risk less risky than at least one of the original individual risks*? To make this precise, isolate the riskiness of an act apart from its scale via the random variable $R_f := \frac{f}{n(f)}$, where $n : \mathcal{F} \to \mathbb{R}_{++}$ is any normalization factor satisfying $n(f+g) = n(f)+n(g)$ for all f, g. For example, $n(f)$ could be $\mathbb{E}[f]$, or if prices were introduced then R_f could be the gross percentage return of f. Now ask: is $variance(R_{f+g}) < \max\{variance(R_f), variance(R_g)\}$? When f and g are scaled replicas, pooling risk fails even this extremely weak criterion for a potential benefit of diversification.

sharing over autarky in this case.

Axiom 4 (Hedging) *$\langle f,0\rangle \succsim^i_{\beta f} \langle f,1\rangle$ for all $f\in\mathcal{F}$, $\beta>0$ and $i\in\{1,2\}$.*

For the next axiom, consider an act-pair (f,g) and remember that Agent 1's preference for f given g is based not on the consumption value of f, but on the second-stage reallocation that will be implemented starting from (f,g). If there are no waste or frictions in the reallocation process, then it cannot be that both agents prefer a feasible final reallocation $(a,b)\in A(f,g)$ to bargaining starting from (f,g). The axiom captures this requirement via its contrapositive.[15]

Axiom 5 (No Waste)
If $\langle a',0\rangle \succsim^1_g \langle f,1\rangle$ and $\langle b',0\rangle \succsim^2_f \langle g,1\rangle$, at least one of them strict, then $(a',b')\notin A(f,g)$.

The final axiom captures that resource constraints are binding, meaning that reallocating resources from the chosen acts is the only "joint production technology" available to the two agents, and that agents do not derive a non-instrumental (for instance, psychological) benefit from participating in risk sharing. Within a utility representation, such an assumption would be clear enough. But we need to capture this idea in terms of our observable domain: $\left\{\succsim^i_g\right\}_{g\in\mathcal{F}}$ for $i\in\{1,2\}$. To do so, we define the set of Pareto efficient reallocations of (f,g) as

$$PS\,(f,g) := \{(a,b)\in A(f,g) | \langle a,0\rangle \succsim^1_g \langle a',0\rangle \ or\ \langle b,0\rangle \succsim^2_f \langle b',0\rangle\ \forall\ (a',b')\in A\,(f,g)\}.$$

Axiom 6 (Resource Constraint)
If $(a,b)\in PS\,(f,g)$, then $\langle a,0\rangle \succsim^1_g \langle f,1\rangle$ or $\langle b,0\rangle \succsim^2_f \langle g,1\rangle$.

3.3 Model Preliminaries

Our behavioral model provides structure on two dimensions: a) the nature of risk sharing arrangements, and b) how first-stage preferences over acts are determined taking (a) into account. Taking the two components in turn:

a) A particularly parsimonious set of sharing rules are those that deliver each agent a constant share, $\alpha^i\in(0,1)$, of the total output in each state: for first-stage acts (f,g),

$$\Gamma(f,g) = \left(\alpha^1_{f,g}(f+g), \alpha^2_{f,g}(f+g)\right)\in A(f,g), \quad where\ \ \alpha^1_{f,g}+\alpha^2_{f,g}=1.$$

[15] The more general model in Section 6 accommodates possible frictions in the reallocation process, and Supplement S.2 relaxes Axiom 5 accordingly.

In finance parlance, in such an arrangement the stochastic "cash flows" of f and g are securitized exclusively using equity claims, and each agent holds some fraction of this "market portfolio." We refer to a sharing rule that has this structure (up to changes on μ-measure-zero events) as *proportional*. Notice that the shares are constant across states ω given (f, g), but critically, can vary with the acts (f, g). The choices of first-stage acts can influence the bargaining outcome in the second stage—but the bargaining outcome is some proportional sharing agreement for any chosen pair of acts.

b) As foreshadowed in Section 1 (and by Axiom 2), in our model agents are standard expected utility maximizers that care only about their own consumption. Let u_i be i's utility for consumption c, and slightly abusing notation, denote the linear extension of u_i to $\mathcal{F}$ also by $u_i(f) := \sum_{c \in \chi(f)} u_i(c)\mu(f^{-1}(c))$.

As discussed in the next subsection, the literature on informal risk sharing often assumes homogeneous risk tolerances, and (among expected utility preferences) constant relative risk aversion (CRRA) has support in empirical evidence. Explicitly, there is $\eta \geq 0$ such that for each agent i, up to a positive affine transformation,

$$u_i\left(c\right) = \begin{cases} \frac{c^{1-\eta}-1}{1-\eta} & \eta \neq 1, \eta \geq 0 \\ \ln(c) & \eta = 1. \end{cases} \tag{2}$$

Recall that $\eta = 0$ implies risk neutrality, and greater η implies more aversion to risk. Finally, as is standard, agents are forward looking and have common beliefs about how second-stage bargaining will reallocate from any first-stage acts (f, g).

The following proposition shows that Pareto efficient bargaining generates a tight connection between (a) and (b).

Proposition 3.1 *For two expected utility maximizers with continuous and monotonic consumption utility functions u_1, u_2, the following are equivalent.*

i) u_1 and u_2 are CRRA utilities with the same η-value.

ii) For all $f, g \in \mathcal{F}$ and $\alpha \in (0, 1)$, the proportional reallocation $(\alpha(f+g), (1-\alpha)(f+g))$ is Pareto efficient in $A(f, g)$.

Moreover, if $\eta > 0$, then (i) implies that: iii) for all $f, g \in \mathcal{F}$, if (a, b) is Pareto efficient in $A(f, g)$, then (a, b) is a proportional reallocation of (f, g).[16]

[16] At least the equivalence between proportional sharing rules and Pareto efficiency when agents have common CRRA utilities and $\eta > 0$ is familiar (Back, 2016, ch. 4).

For a specific example, suppose that for both agents, $u_i(c) = \ln(c)$. For any act-pair (f, g), the Egalitarian bargaining solution (EBS) of Kalai (1977) selects the Pareto efficient reallocation (a, b) such that $u_1(a) - u_1(f) = u_2(b) - u_2(g)$ to capture the idea that each agent "gains equally" from their agreement.[17] For this case an explicit solution exists: each agent's share of the total output corresponds to the relative value of consuming her chosen act (i.e., her disagreement allocation). This simple form makes clear each agent's incentive to increase her disagreement value, even at the possible expense of decreasing available surplus.

Lemma 3.1 *If $u_i = \ln$ for both agents and sharing is in accordance with the EBS, then, letting $ce(f)$ denote the certainty equivalent of act f, we have*

$$\alpha^1_{f,g} = \frac{ce(f)}{ce(f) + ce(g)} \quad \textit{and} \quad \alpha^2_{f,g} = \frac{ce(g)}{ce(f) + ce(g)}.$$

3.4 Representation Theorem

We now turn to the connection between the model and the axioms. Since η fully describes risk preferences, it is convenient to normalize agent utilities as in (2) and to describe familiar bargaining solutions in terms of those normalized utilities.

Definition 3.1 *A* ***two-stage model of proportional risk sharing*** *is summarized by a pair (η, α) where the parameter $\eta > 0$ specifies a common CRRA utility function u from (2), and α is a proportional sharing rule that respects disagreement values: $u\big(\alpha^1_{f,g}(f+g)\big) \geq u(f)$ and $u\big(\alpha^2_{f,g}(f+g)\big) \geq u(g)$.*

In the model, Agent 1's expected utility of sharing f with g is $u(\alpha^1_{f,g}(f+g))$. Hence, in this situation she prefers f to f' if and only if $u(\alpha^1_{f,g}(f+g)) \geq u(\alpha^1_{f',g}(f'+g))$, and analogously for Agent 2. Notice there are two components that could affect this ranking: the total output $(f+g)$ versus $(f'+g)$, and her individual share $\alpha^1_{f,g}$ versus $\alpha^1_{f',g}$. Recall that our domain includes the possibility of engaging in the sharing arrangement, $o = 1$, or opting out, $o = 0$, so that we have the following.

Definition 3.2 *A model (η, α)* ***explains*** *preferences $\left\{\succsim^i_g\right\}_{g \in \mathcal{F}}$ for $i \in \{1, 2\}$, if*

$$\langle f, o\rangle \succsim^i_g \langle f', o'\rangle \iff ou\left(\alpha^i_{f,g}(f+g)\right) + (1-o)u(f) \geq o'u\left(\alpha^i_{f',g}(f'+g)\right) + (1-o')u(f').$$

[17] Unlike other prominent bargaining solutions (Section 4.1), the interpretation of the EBS depends on cardinal utility values.

Theorem 3.1 (Representation) *Preferences* $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ *for* $i \in \{1,2\}$ *satisfy Axioms 1-6 if and only if they can be explained by a two-stage model of proportional risk sharing* (η, α).

The theorem establishes our axioms as a complete description of the testable implications of our model for first-stage choice of acts. To put the model into an applied context, empirical work on risk sharing in developing economies often follows Townsend (1994) and assumes that agents who share risk have homogeneous risk preferences. It may be the case that individuals in these populations have rather similar risk preferences. In addition, to the extent that there is heterogeneity along this dimension, Attanasio et al. (2012) document that there is assortative matching on risk tolerance in the formation of risk sharing relationships. In terms of observed risk attitudes, among the prominent classes of expected utility preferences, CRRA appears to be descriptively most accurate (for example, Pope and Just, 1991; Brunnermeier and Nagel, 2014; Chiappori and Paiella, 2011)[18] and most widely assumed (Eeckhoudt et al., 2005).[19]

In alignment with our representation, quite recent and top general-interest publications in the field of Economic Development that assume homogenous CRRA utility functions include Lagakos et al. (2023); Meghir et al. (2020); Brooks and Donovan (2020); Morten (2019); Munshi and Rosenzweig (2016). Indeed, in this literature, the homogenous-CRRA convention appears sufficiently strong that it is often deployed without further justification (similar to footnote 19).

In the Introduction, we discussed that the literature on informal risk sharing in developing countries is primarily concerned with mutual insurance: participating in these arrangements should allow agents to hedge risk. This is at the core of the normative justificaion for our axioms. Theorem 3.1 thus tightly connects mutual insurance as a motive for risk sharing to homogeneous CRRA utilities. Our result thereby offers a supporting rationale for the specific combination of sharing motive and other modeling assumptions commonly found in the literature, even if such a rationale is not discussed there. We now briefly discuss the type of behavioral data that could be used to test our axioms.

[18] Chiappori and Paiella (2011) demonstrate the importance of panel data and find strong support for CRRA in portfolio choice. In the agricultural context Pope and Just (1991) find evidence for CRRA preferences among potato farmers in Idaho.

[19] From their textbook (page 21): "Finally, one set of [expected utility] preferences that has been by far the most used in the literature is the set of power utility functions [CRRA]. Researchers in finance and in macroeconomics are so accustomed to this restriction that many of them do not even mention it anymore when they present their results."

Discussion of Data Requirements

As discussed in the Introduction, the literature typically relies on survey data regarding consumption and income to test whether sharing is consistent with Pareto efficiency: Individual consumption should vary with common, but not with idiosyncratic, income shocks.[20] In our two-stage environment, Theorem 3.1 provides an alternative approach to testing efficient risk sharing. It establishes the choice of risky act (e.g., the crops farmers plant) and the choice to participate in informal risk sharing as appropriate behavioral data to test Pareto efficiency at the sharing stage *jointly* with the assumption of homogeneous CRRA preferences. In situations where data on productive choices is available, our results may thus help researchers (i) make progress when there is no reliable consumption or income data, (ii) test the presupposition of homogeneous preferences, and (iii) verify constant relative risk aversion directly in the risk sharing context. Of course, many applied contexts appear to feature only partial mutual insurance, violating Pareto efficiency. Theorem S.2 in the Supplement axiomatically characterizes the generalization of our model from Section 6 that allows for inefficiencies due to sharing frictions.[21]

As usual, testing our model based on the behavioral axioms does not require observing entire preference relations for both agents. For instance, based on Axiom 4 (Hedging) our model can be refuted with just one observed binary choice: If one agent strictly prefers being able to share with another agent who holds the same act up to scaling, then our model cannot explain the data. That said, our identification result (Theorem 4.1 below) suggests that our model can also be estimated based on the choice data in our primitive. The usual concern that it is hard to observe the same individual choose in a number of different situations is relevant. This concern is aggravated for our model because choice situations depend on the act chosen by the other agent in the sharing arrangement. In practice, the lack of individual choice data often leads analysts to consider cross-sectional data instead, where different agents happen to face different choice situations. The typical caveat is that preferences are *assumed* to be homogenous (up to additive noise) across agents.[22] Here, our multi-agent setup may be beneficial as homogeneity of preferences, at least across the sharing parties, is part of our model. That is, heterogeneity within a pair should lead to a violation, for instance of the hedging axiom as discussed in the previous

[20]See footnote 4 for prominent works that employed this testing strategy.

[21]Mazzocco and Saini (2012) argue for heterogeneity in risk preferences as a possible explanation for the apparent inefficiency. Recall that our theory does not presuppose homogeneity, but rather suggests how to jointly test homogeneity and efficiency.

[22]For example, in consumer choice, following Varian (1982), various versions of the Axiom of Revealed Preferences have been tested based on aggregate data. In terms of estimation, structural models that are used in applied contexts often feature one parameter of risk aversion for a poulation of individuals.

paragraph.

As an illustrative example, we now suggest how one might observe different choice situations in cross-sectional data from rural economies in the developing world. Often times, taking up a productive activity unrelated to the farming of the local staple crop may require temporary or permanent migration, either to a village with different resources or production technology, or to an urban area with altogether different productive opportunities. In that sense, observing the migration choices of individuals from different villages can amount to observing the choice of act in different situations. Recall that our primitive requires that we observe not just the choice of an act, but of an act together with whether or not it is possible to share risk. Here, the distinction between temporary seasonal migration and permanent migration can help: Munshi and Rosenzweig (2016) argue that permanent migration might be a way to improve the set of available acts (e.g., better land or climate, better markets, other job opportunities, etc.) but at the cost of losing the local (caste based) risk sharing arrangement. They find this cost can partly explain low mobility in India. At the same time, a motivation for temporary migration may be to improve the distribution over joint outcomes available for sharing (by increasing diversification and/or having high payoffs in some states), which is only relevant in light of the existing and continued risk sharing arrangement (see Morten (2019) and also the application and literature discussed in Section 5). Based on these notions, permanent migration may amount to choosing some act paired with $o = 0$, while temporary migration amounts to choosing an act paired with $o = 1$.[23]

Consider, then, one example of how a researcher might employ our axioms to test the model: By Voluntary Participation (Axiom 3), for any act-pair, both agents weakly prefer participation in the sharing arrangement to opting out entirely. Hence, any preference for opting out requires that doing so alters the set of available acts. And from No Waste (Axiom 5), if two agents do prefer to opt out of the sharing arrangement, then this must be justified by increased total output at least in some set of states. To test our model one could, therefore, check whether permanent migration (opting out) can be explained by improvements in the set of available actions (an improvement in terms of autarky preferences).[24] Similarly, temporary migration—choosing a different act while

[23]See Fafchamps (2011) for a survey of the literature on risk sharing between households, including several papers that document households opting out of sharing arrangements.

[24]A potential alternative explanation of opting out would entail some violation of Voluntary Participation (Axiom 3): the risk sharing agreement is individually disadvantageous. For instance, wealthy individuals may opt out if the sharing arrangement is based on social norms that require wealth redistribution.

opting in—should not occur unless the chosen productive activity either improves the disagreement value (an improvement in terms of autarky preferences) or the total surplus (an improvement of the value of joint production in terms of autarky preferences).

Discussion of the Proof of the Representation Theorem

As is common in representation results, that the model generates preferences that satisfy the axioms is relatively straightforward. To gain some intuition for how the axioms imply the model, recall that Proposition 3.1 links the Pareto efficiency of proportional sharing rules to homogeneous CRRA utilities. What remains to be shown is that under the axioms, sharing must indeed be both Pareto efficient *and* proportional.

By Resource Constraint (Axiom 6), sharing (f, g) does not allow Pareto improvements over what can be achieved via distributions of the total endowment $f + g$ (i.e., final allocations in $A(f, g)$). By No Waste (Axiom 5), no final allocation in $A(f, g)$ Pareto dominates sharing (f, g). Hence, both agents are indifferent between sharing (f, g) and at least some Pareto efficient final reallocation in $A(f, g)$. Next, for arbitrary act h and $\alpha \in (0, 1)$, Hedging (Axiom 4) implies that $\langle \alpha h, 0\rangle \sim^i_{(1-\alpha)h} \langle \alpha h, 1\rangle$. Setting $h = f + g$ then implies that there is no scope for Pareto improvements when first-stage acts (f, g) are proportional. Consequently, final allocations that are shares of the total endowment $f + g$ must be Pareto efficient. Next, notice that the best allocation for agent i would be to give her the entire total output, and the worst would be to give her none of it, which would correspond to $\alpha^i = 1, 0$, respectively. By Continuity and Monotonicity of autarky preferences (Axiom 2), proportional allocations are then order dense in the space of all Pareto efficient allocations in $A(f, g)$. Putting everything together, we have that sharing any (f, g) corresponds to some proportional (and also Pareto efficient) reallocation of (f, g).

It is worth noting that only expected utility assumptions were imposed on autarky preferences (Axiom 2). Homogenous CRRA preferences in the model are implied by the remaining axioms, which capture plausible qualitative features of behavior in the presence of risk sharing. For instance, the commonality of risk preferences in autarky does not derive from a symmetry assumption, but rather from the Pareto efficiency of risk pooling. In fact, preferences over acts when sharing is possible may not be symmetric across the two agents because the sharing rule may be asymmetric. Similarly, none of our axioms directly impose "differentiability" on preferences, and deriving differentiability of utility functions is a crucial step in establishing that autarky preferences are CRRA.

4 Identification and Specific Sharing Arrangements

While the representation result (Theorem 3.1) provides a foundation for testing our model, the following identification result establishes that it is in principle possible to estimate its parameters, and in particular the sharing arrangement, from observable behavior.

Theorem 4.1 (Identification) *If models (η, α) and (η', α') both explain preferences $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ for $i \in \{1, 2\}$, then $(\eta, \alpha) = (\eta', \alpha')$.*

Informal sharing arrangements can be difficult to observe directly. In our approach, Theorem 4.1 derives the proportional sharing arrangement as well as the coefficient of risk aversion from the choices of risky actions (e.g., crop choices). By part (*iii*) of Proposition 3.1, the focus on proportional sharing arrangements is without loss, as non-proportional sharing rules cannot be Pareto efficient when individual preferences are homogeneous CRRA.

The class of proportional sharing rules featured in our model is very general: No constraints connect $\alpha_{f,g}$ to any other $\alpha_{f',g'}$, not even symmetry or continuity. Neither of these features would be difficult to capture axiomatically, but since the focus in the empirical literature has been on testing the Pareto efficiency of sharing, and because the set of proportional sharing rules coincides with the set of Pareto efficient sharing rules in the context of common CRRA utilities, the generality is desirable. That said, our identification theorem allows us to probe deeper into the particulars of the sharing arrangement. Next we discuss sharing based on the most prominent bargaining solutions.

4.1 Specific Bargaining Solutions

The second stage of our model fits within the class "bargaining problems" studied in cooperative game theory under the header *cooperative bargaining theory* (see Kibris, 2010, for a survey). In this approach, agents' utilities for each feasible bargaining outcome, as well as their disagreement values, are taken as inputs. A *solution* is found by imposing some set of normatively appealing properties regarding the utility surpluses (i.e., utilities in excess of disagreement values) generated for the agents. Chief among these are 1) Pareto Efficiency and 2) Symmetry: in the solution, the surplus to each agent is equal when the Pareto frontier of achievable surpluses is itself symmetric.

Beyond these two properties, three others have been deemed desirable: 3) Scale Invariance says that rescaling one agent's utility should not affect the bargaining solution as interpersonal comparisons of cardinal utilities are meaningless in the context of revealed preferences. As in individual choice, 4) Independence of Irrelevant Alternatives (IIA) requires, roughly, that removing unchosen outcomes from the feasible set should not change the bargaining solution. Finally, 5) Resource Monotonicity requires that if the set of feasible surplus profiles expands (in the sense of set inclusion), every agent should be weakly better off. It is, famously, impossible to satisfy all these requirements simultaneously.

The three most prominent bargaining solutions each satisfy Pareto Efficiency and Symmetry and are tightly characterized by adding two of the three other desiderata, as summarized in the following Table.

	Pareto	Symmetry	Scale inv.	IIA	Monotonicity
Nash, 1950 (NBS)	✓	✓	✓	✓	X
Kalai and Smorodinsky, 1975 (KSS)	✓	✓	✓	X	✓
Kalai, 1977 (EBS)	✓	✓	X	✓	✓

Table 1: Properties of prominent bargaining solutions

The manifestations of both the NBS and EBS in the context of risk sharing have been described above (see (1) and the discussion following Proposition 3.1, respectively). The KSS relies on each agent's "aspiration payoff": the maximum payoff an agent can get in an agreement that respects disagreement values. For first-stage acts (f, g), define

$$\bar{a}_{f,g} = \arg\max_{a \in \mathcal{F}} \; u_1(a), \quad s.t. \;\; u_2(f + g - a) \geq u_2(g),$$

Define $\bar{b}_{f,g}$ analogously for Agent 2. Then the KSS is the allocation $(a, b) \in PS(f, g)$ such that

$$\frac{u_1(a) - u_1(f)}{u_1(\bar{a}_{f,g}) - u_1(f)} = \frac{u_2(b) - u_2(g)}{u_2(\bar{b}_{f,g}) - u_2(g)}. \tag{3}$$

Given our identification result (Theorem 4.1), we can investigate the first-stage behavioral manifestations of each of these bargaining solutions applied in the second stage. In Supplement S.1, for each bargaining solution, we provide testable behavioral axioms (rather than the normative desiderata described above) that, together with Axioms 1-6, tightly characterize the special case of our general model in which α corresponds to that solution.

In general, the family of proportional sharing rules can accommodate arbitrary dependence of shares on disagreement allocations (up to voluntary participation). We next note a sensible property of sharing rules generated by any of the three prominent bargaining solutions (in the context of our model): an agent who simultaneously increases the size of the pie and her own disagreement value must also achieve a better outcome from the sharing arrangement. For ease of exposition, we formalize this mild restriction on the sharing rule in terms of our representation rather than preferences.

Definition 4.1 *In the context of our representation, proportional sharing rule α is **sensible** if, for any act g held by Agent $j \neq i$,*

$$u(f) > u(f') \text{ and } u(f+g) > u(f'+g) \implies u(\alpha^i_{f,g}(f+g)) > u(\alpha^i_{f',g}(f'+g)).$$

Notice that an agent's *share* may decrease even if she simultaneously increases the size of the pie and her own disagreement value, but the *utility* of her (possibly smaller) share of the larger pie must not decline under a sensible rule.

Proposition 4.1 *In our model, the NBS, KSS, and EBS all generate sensible sharing rules.*

In the application explored in Section 5, it will be useful to confine attention to sensible sharing rules.

5 Application and Toy Example: Migration

Several studies in Economic Development have documented that individuals migrate too little compared to what would appear economically efficient. Recall from Section 3.4 the important distinction between permanent migration and temporary migration (see, for instance, Kennan and Walker, 2011). In the context of rural India, permanent migration typically involves losing established risk sharing relationships, which can explain why observed levels are exceedingly low (see Banerjee and Newman, 1998; Munshi and Rosenzweig, 2016). In contrast, temporary seasonal migration often complements existing risk sharing arrangements by improving diversification and is more widespread (see Banerjee and Duflo, 2007; Morten, 2019). Nonetheless, empirical evidence from rural Bangladesh (see Bryan et al., 2008; Akram et al., 2017) shows the following.

Empirical Observations:

i) A small migration subsidy can be effective in raising seasonal migration.

ii) A small migration subsidy can lead to substantial welfare gains.

It follows that even seasonal migration is inefficiently low. Meghir et al. (2020) further investigate this data and find in addition that

iii) A small migration subsidy can lead to increased risk sharing over all.[25]

iv) A small migration subsidy can lead to more risk sharing even between agents who do not migrate and remain in the village.

Meghir et al. (2020) proceed to estimate a structural model that includes risk sharing as well as migration and that can match Empirical Observations (i)-(iv) relying on an unobservable utility cost of migration. The aim of this section is to show that our two-stage conceptualization of risk sharing can provide a qualitative microfoundation for the utility cost of migration and a justification for Empirical Observations (i)-(iv) above.[26] The intuition is that even if migration is socially efficient (due to having a high expected production and/or diversification benefit), it may reduce the disagreement value of the migrating agent (due to being risky). Consequently, under many sharing rules (including those that adhere to the prominent bargaining solutions in Section 4.1), migrating would reduce the share of total output the migrating agent receives and manifest as a utility cost.

The nature of the utility cost from migrating is central for policy evaluation: A utility loss that stems directly from the act of migrating reduces welfare. In contrast, a worsened bargaining position for the migrating party amounts to a state-dependent transfer of resources from one party to the other that harms (benefits) the migrator (nonmigrator) in aggregate. In this case, "correcting" incentives using a migration-contingent transfer policy results in zero deadweight loss. In order to focus on the change in bargaining position, we abstract away from any direct utility cost to migration, but of course both forces could be present simultaneously.

5.1 Inefficiently Low Seasonal Migration

The possibility of inefficiently low levels of migration is natural in our model. For illustration, consider a risk sharing pair of agents, 1 and 2, from the same village, in which Agent 2 has only a single act, g, available to her, interpreted as remaining in the village and carrying on her usual productive activity (e.g., farming a staple crop). Agent 1 has two available acts: v (remain in the village and carry on her usual productive activity) and

[25]The paper uses several measures, including the relationship between household income and household consumption as in the literature referenced in our Footnote 4.

[26]The model in Meghir et al. (2020) is dynamic and allows the value of migration to depend on past migration outcomes, which is important in matching observed migration and risk sharing data over time. Our model of risk sharing is static, and so cannot address such dynamic phenomena. Furthermore, estimating the parameters of our model is beyond the scope of this decision-theoretic paper.

m (migrate to the city for a new productive opportunity). In a slight abuse of notation, because for any (f,g), we have $\alpha^1_{f,g}+\alpha^2_{f,g}=1$, we drop the superscript in this section and let $\alpha^1_{f,g}=\alpha_{f,g}$ and $\alpha^2_{f,g}=1-\alpha_{f,g}$.

Given the representation, migration is efficient if and only if $u(m+g)\geq u(v+g)$. However, Agent 1 will prefer to migrate if and only if

$$u(\alpha_{m,g}(m+g)) \geq u(\alpha_{v,g}(v+g)) \tag{4}$$

$$\alpha_{m,g}^{1-\eta}u\left(m+g\right)+u\left(\alpha_{m,g}\right) \geq \alpha_{v,g}^{1-\eta}u\left(v+g\right)+u\left(\alpha_{v,g}\right) \tag{5}$$

$$u\left(m+g\right) \geq u\left(v+g\right)\underbrace{\left(\frac{\alpha_{v,g}}{\alpha_{m,g}}\right)^{1-\eta}+\frac{\left(u\left(\alpha_{v,g}\right)-u\left(\alpha_{m,g}\right)\right)}{\alpha_{m,g}^{1-\eta}}}_{\text{effect of change in disagreement value}}, \tag{6}$$

where (5) follows from (4) by Lemma A.3 in the Appendix.

For the remainder of this section, suppose that $u(m+g)>u(v+g)$, so migration is efficient. The key is that, while migrating grows the size of the pie the agents can share, Agent 1 will also consider how migrating effects the share of the pie she will end up with. If her share is also higher by migrating (i.e., if $\alpha_{m,g}\geq\alpha_{v,g}$), then Agent 1 will indeed migrate because it gives her a greater share of a larger pie. However, if $\alpha_{m,g}<\alpha_{v,g}$, then she may decide to remain in the village at the expense of overall efficiency. If $\eta\leq 1$, then these forces are obvious by inspection of (6), and additional algebra confirms them for $\eta>1$ as well. For the remainder, if (6) fails, we say that there is *inefficient undermigration*.

Under any of the three prominent bargaining solutions from Section 4.1, the sharing rule is driven by the utilities of each agent's autarky option: the worse is an agent's autarky option, the lower is her share of the joint production. For example, if $u(v)=u(g)$ (i.e., the village options have the same expected utility in autarky), then $\alpha_{m,g}<\alpha_{v,g}$ if and only if $u(m)<u(v)$. Because agents are risk averse, this constellation of inequalities is easily achievable. In the following example, migrating is a high-risk/high-return endeavor.

Example 1 (Risky Migration) *State space $\Omega=\{\omega_1,\omega_2,\omega_3,\omega_4\}$, with $\mu(\omega_k)=1/4$ for all k. Acts $g(\omega_1)=g(\omega_3)=1$, $g(\omega_2)=g(\omega_4)=e$; $v(\omega_1)=v(\omega_2)=1$, $v(\omega_3)=v(\omega_4)=e$; $m(\omega_1)=m(\omega_2)=B$, $m(\omega_3)=m(\omega_4)=G$ where $0<B<1<e<G$. Utility $u(c)=\ln(c)$. The sharing rule adheres to the NBS. Migration is efficient if and only if*

$$G\geq G_E(B)=\frac{1}{2}\left(\sqrt{\frac{(e-1)^2B^2+(1+e)(e-1)^2B+e\left(17+30e+17e^2\right)}{(B+1)(B+e)}}-e-1\right).$$

However, there exists $G^*(B)>G_E(B)$ such that Agent 1 will migrate if and only if

$G \geq G^*(B)$. Hence, for all (B, G) such that $G \in (G_E(B), G^*(B))$, there will be inefficient undermigration.[27] *For instance, if $B = 1/2$, we have $G_E(B) = 3.79 < G^*(B) = 4.35$.*

In Example 1, migrating does not alter the correlation between the acts of the two agents. Instead, migrating is efficient when its mean production is sufficiently large, but it may still be unchosen because it is riskier. It would be natural to assume that migrating would decrease the correlation between the agents' acts, which could similarly produce inefficient undermigration if migrating is unattractive in autarky (i.e., low mean and/or high risk in production). Putting both forces together would only strengthen the results.

5.2 Subsidizing Migration

In order to address Empirical Observations (i)-(iv) above, we now consider the effect of a migration subsidy in our environment. We assume that any subsidy t paid to the migrator is incorporated into the resources she has available to share (i.e., if Agent 1 migrates, the agents will have $m+t+g$ to share). First, because sharing must respect disagreement values, a sufficiently generous subsidy will encourage migration as in Empirical Observation (i). It seems reasonable that if subsidy t is sufficient to induce migration, so too should subsidy $t' > t$. However, given the flexibility in the model regarding how the proportional shares may vary with the acts, this feature is not guaranteed without further assumption.

Fact 5.1 *Under any continuous and sensible sharing rule (Definition 4.1), subsidy level t induces migration if and only if $t \geq t_1 := \min\{t : u(\alpha_{m+t,g}(m+t+g)) \geq u(\alpha_{v,g}(v+g))\}$.*

For the remainder of this section we assume the sharing rule is continuous and sensible, such as rules generated by any of the three prominent bargaining solutions of Section 4.1 (Proposition 4.1).

Given that there is inefficient undermigration, we know that for any $t > 0$,

$$u(\alpha_{m,g}(m+g)) < u(\alpha_{v,g}(v+g)) < u(\alpha_{v,g}(m+g)) < u(\alpha_{v,g}(m+g+t)),$$

where the first inequality follows from migration being suboptimal for Agent 1, the second from migration being efficient, and the third from $u' > 0$. The first two of these quantities can be seen on the vertical axis of Figure 1. Along the horizontal axis, the subsidy increases and, therefore, so does $u(\alpha_{m+t,g}(m+t+g))$ by sensibility of the sharing rule.[28] We see that t_1 is the minimum subsidy level needed to switch Agent 1's preference

[27] The same is true of both the KSS and EBS.

[28] Both $u(m+t)$ and $u(m+g+t)$ are obviously increasing in t. Then apply Definition 4.1.

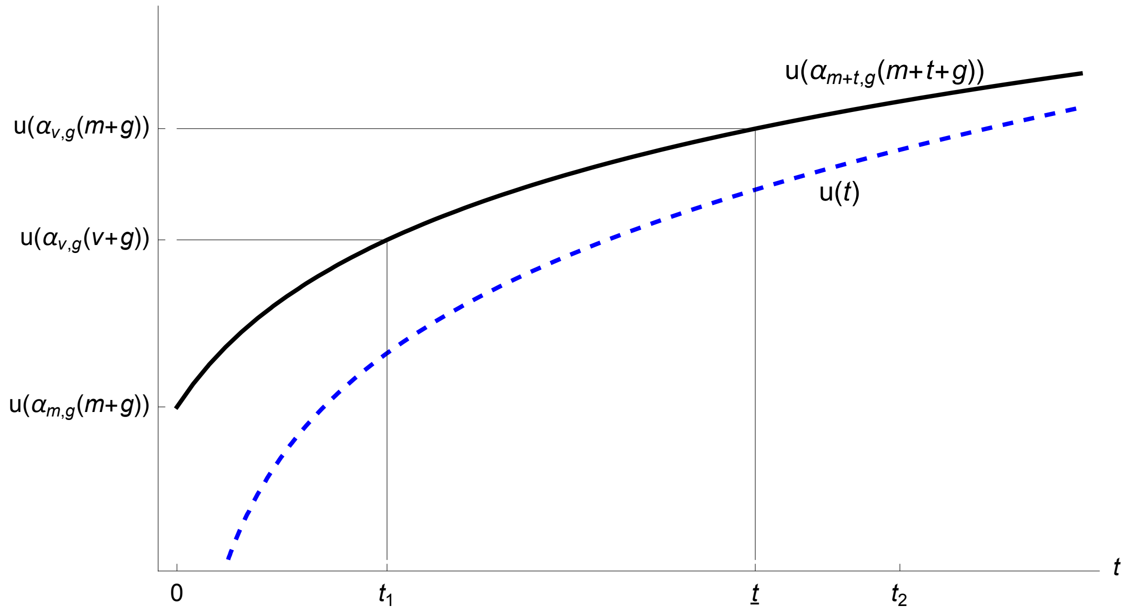

Figure 1: *Effect of a subsidy on incentive to migrate.* Solid curve depicts the expected utility from migrating as it continuously increases with the subsidy level, $u\left(\alpha_{m+t,g}(m+t+g)\right)$. As t increases, $u(\alpha_{m+t,g}(m+t+g))$ converges toward $u(t)$. Agent 1 will migrate if and only if $t \geq t_1$. At subsidy level t_2, the agents' shares are $(\alpha_{v,g}, 1-\alpha_{v,g})$ whether Agent 1 migrates or not (note that t_2 is always greater than the subsidy level labeled $\underline{t}$). Any subsidy level in $(t_1, t_2]$ generates a Pareto improvement over no subsidy.

from remaining in the village to migrating. In addition, while $\alpha_{m+t,g}$ is not necessarily monotonic as t increases, it must limit to one by Voluntary Participation (Axiom 3). Let $t_2 := \min\{t : \alpha_{m+t,g} \geq \alpha_{v,g}\}$, that is, the minimum subsidy such that Agent 1 gets the same share of the total output as she would have gotten had she remained in the village. Hence, migrating under t_2 gives the same shares to each agent as remaining in the village, but now these shares are of a larger pie and both agents are better off.

Fact 5.2 *If there is inefficient undermigration, then $t_1 < t_2$. Moreover, any subsidy level $t \in (t_1, t_2]$ makes **both** agents strictly better off than no subsidy.*

In Fact 5.2, the resources for the migration subsidy come from outside the two agents, and therefore have the added benefit of just making the pair richer. In order to convincingly demonstrate substantial welfare gains as in Empirical Observation (ii), suppose the subsidy had to be budget balanced in that it is paid through a proportional tax on the agents' income. Let superscript "$\sim$" denote quantities that account for this taxation. Then again there exists $\tilde{t}_2 := \min\{\tilde{\alpha}_{m+t,g} \geq \alpha_{v,g}\}$.[29]

[29] Under taxation we still have $\tilde{\alpha}_{m+t,g} \to 1$ as t grows large by Voluntary Participation (Axiom 3).

Fact 5.3 *If there is inefficient undermigration, a migration subsidy of $\tilde{t}_2$, paid through proportional taxation, will make both agents strictly better off than no subsidy.*

Returning to Example 1, we have $t_2 = \tilde{t}_2 = \frac{1}{2}\left(\sqrt{(G-B)^2+4e}-(B+G)\right)$.[30] For instance, when $B = 1/2$ and $G = 4 \in (G_E(B), G^*(B))$, then $t_2 = \tilde{t}_2 = 0.154 > t_1 = 0.062$. (A closed-form solution for t_1 in general is intractable.)

Recall that in Meghir et al. (2020), the cost of migration is interpreted as a utility cost that is directly welfare relevant. In contrast, if individuals take into account the sharing rule at the time they decide whether or not to migrate, then they may effectively perceive a transfer of resources from the migrating agent to the village agent. In that case, a migration-contingent transfer in the opposite direction (that recovers efficient migration as in Fact 5.3) is a more appealing policy intervention: migration has a lower direct cost than it would appear under a model that ignores the effect of the sharing rule on the migration decision.

5.3 Effect of a Migration Subsidy on Risk Sharing

How could subsidizing migration increase risk sharing as in Empirical Observation (iii), and even increase sharing between pairs of agents who do not migrate as in Empirical Observation (iv)? One intuition could be that migration makes risk sharing more valuable, and that agents must invest to form and maintain risk sharing partnerships before knowing the precise acts that each will have available to them.

To capture this, extend our model by adding a pre-stage in which *a*) the agents do not know if migrating will be a productive opportunity or not, and *b*) creating the risk sharing partnership requires a pair-specific investment cost of c per agent.[31] We continue to focus on the case where, when migrating is a productive opportunity, we have $u(m+g) > u(v+g)$ but (6) fails (as before), meaning Agent 1 would not migrate even if the pair had invested to form a risk sharing partnership.

In this extended model, a subsidy that is overly generous can create perverse incentives. First, there now exists $t_3 > t_1$ such that, if the agents have formed a partnership but migration is unproductive, Agent 1 still migrates if and only if $t \geq t_3$. Second, there exists t_4 such that, if the agents have not formed a partnership and migration is productive, Agent 1 migrates if and only if $t \geq t_4$. Now that forming a partnership carries a cost, if

[30] Equivalence of t_2 and $\tilde{t}_2$ in Example 1 owes to $u(v) = u(g)$, the symmetry of the NBS, and $\eta = 1$.

[31] Assume that both agents will invest if and only if each will be better off if both agents invest. That is, they never miscoordinate these investment decisions either because the decisions are made in sequence or because the agents play the Pareto dominant Nash equilibrium if the decisions are simultaneous.

$t \geq t_4$, Agent 1 may decide to refrain from the partnership, saving the cost, and migrate due to the generous subsidy. The rankings of t_2, t_3, and t_4 depend on parameters, and since we will want a subsidy that incentivizes Agent 1 to migrate efficiently (i.e., migrate if and only if it increases social welfare) and does not overly disadvantage Agent 2, we define $\bar{t} = \min\{t_2, t_3, t_4\}$. In Example 1, we have $\bar{t} = t_2 = t_4 < 0.778 < t_3 = 1.698$.[32]

Fact 5.4 *Under any subsidy level $t \geq 0$, there exists a cutoff $\underline{c}(t)$, such that the agents form a risk sharing partnership if and only if $c \leq \underline{c}(t)$. Moreover, $\underline{c}(t) > \underline{c}(0)$ for any $t \in (t_1, \bar{t})$ meaning more risk sharing partnerships form as a result of the subsidy.*

To summarize, suppose there is inefficient undermigration. If the agents will form a partnership, incentivizing Agent 1 to migrate efficiently (i.e., migrate if and only if it is a productive opportunity) can be accomplished with an appropriate subsidy. Importantly, the subsidy increases the investment costs the agents are willing to bear to form partnerships, because it makes the ability to share more valuable by encouraging efficient migration.[33] As a consequence, the subsidy leads more individuals to migrate and then share with their partners who remain in the village. Finally, with positive probability, agents that form a partnership will end up not having a productive migration opportunity, but will still engage in risk sharing within their village since they have already invested in the partnership. Hence, the migration subsidy leads to increased risk sharing even among agent pairs where neither migrates.

6 Risk Sharing with Frictions

As previously mentioned, a large literature has tested the efficiency of informal risk sharing between households in developing and developed countries. Important papers include Altug and Miller (1990); Townsend (1994); Gertler and Gruber (2002); Ligon et al. (2002); Banerjee and Duflo (2007). Most often, full efficiency is rejected. Common explanations of the apparent incompleteness in risk sharing are based on frictions that are unobservable to the analyst, such as private information that constrains the feasibility of transfers (see, for instance, Rogerson, 1985; Ligon, 1998).[34]

A particularly tractable generalization of our model has frictions that can be summarized by a probability of "breakdown" of the sharing arrangement. One interpretation

[32] Equivalence of t_2 and t_4 in Example 1 owes to $u(v) = u(g)$ and the symmetry of the NBS.

[33] The increased number of partnerships reflects the finding in Meghir et al. (2020) that "households [...] became more likely to report that they receive help from family and friends in the village".

[34] Mazzocco and Saini (2012) offer an alternative explanation. See footnote 21.

is that this probability captures the strength of the social tie between the agents: the stronger their social tie, the more likely sharing will succeed. We now introduce π as a pair-specific parameter such that $1-\pi$ measures the probability of a breakdown. Specifically, if $\Gamma(f,g) = (a,b)$, then agents receive the respective allocations a and b with probability π only, and retain their respective acts f and g as the final allocation otherwise.

We generalize the definitions of a model and how a model explains data accordingly.

Definition 6.1 *A* ***two-stage model of proportional risk sharing with frictions*** *is a triple (η,α,π) where (η,α) is two-stage model of proportional (frictionless) risk sharing (Definition 3.1) and $(1-\pi)\in[0,1)$ is the breakdown probability. A model with frictions (η,α,π)* ***explains*** *$\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ for $i\in\{1,2\}$, if*

$$\langle f,o\rangle \succsim_g^i \langle f',o'\rangle \iff \pi o u\left(\alpha_{f,g}^i\left(f+g\right)\right)+(1-\pi o)u(f) \geq \pi o' u\left(\alpha_{f',g}^i\left(f'+g\right)\right)+(1-\pi o')u(f').$$

Supplement S.2 provides an axiomatic characterization of proportional risk sharing with frictions. Not surprisingly, this requires a relaxation of No Waste (Axiom 5): It would be possible to achieve a Pareto improvement if breakdowns could be avoided.

Like α and η, the new parameter π is unobservable to the analyst. Importantly, it can be identified from observable behavior.

Theorem 6.1 (Identification with Frictions) *If (η,α,π) and (η',α',π') both explain preferences $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ for $i\in\{1,2\}$, then $(\eta,\alpha,\pi)=(\eta',\alpha',\pi')$.*

The model is silent on whether the second-stage bargaining takes place before or after the agents learn if they will indeed be able to reallocate or if their tie has broken down. In general, the interpretation of the (uniquely identified) sharing arrangement in terms of the model may depend on this timing. However, for many bargaining solutions this distinction is irrelevant, in which case we say the solution is *consequentialist*. For example, the NBS applied to bargaining before learning about breakdown is:

$$\begin{aligned}
(a^*,b^*) &= \arg\max_{(a,b)\in A(f,g)} \left(\pi u_1(a)+(1-\pi)u_1(f)-u_1(f)\right)\left(\pi u_2(b)+(1-\pi)u_2(g)-u_2(g)\right)\\
&= \arg\max_{(a,b)\in A(f,g)} \left(\pi u_1(a)-\pi u_1(f)\right)\left(\pi u_2(b)-\pi u_2(g)\right)\\
&= \arg\max_{(a,b)\in A(f,g)} \left(u_1(a)-u_1(f)\right)\left(u_2(b)-u_2(g)\right),
\end{aligned}$$

which coincides with the NBS applied after learning that breakdown has not occurred. Intuitively, since only utility surpluses matter for the NBS, the event where sharing fails is of no consequence, as then the surplus is zero (and the maximizer of a function T also maximizes πT). Similar analyses reveal that the KSS and EBS are also consequentialist.

6.1 Comparative Statics

In this section, we compare pairs of agents in terms of the strength of their ties as measured by π. To that end, consider agents $1A$, $2A$, $1B$, and $2B$. Let $(\eta, \alpha, \pi)^A$ and $(\eta, \alpha, \pi)^B$ denote the models of proportional risk sharing with frictions that describe the behavior of the agents in pairs A and B, respectively.

Definition 6.2 *Pair A has a* ***stronger preference to share*** *than does pair B if for all $f, g, f', g' \in \mathcal{F}$: $\langle f, 1\rangle \succ_g^{1B} \langle f', 0\rangle$ implies $\langle f, 1\rangle \succ_g^{1A} \langle f', 0\rangle$, and $\langle g, 1\rangle \succ_f^{2B} \langle g', 0\rangle$ implies $\langle g, 1\rangle \succ_f^{2A} \langle g', 0\rangle$, and the converse is not true.*

Theorem 6.2 (Preference for Sharing) *If pair A has a stronger preference to share than does pair B, then $\pi^A > \pi^B$ and $\eta^A = \eta^B$. Conversely, if $\alpha^A = \alpha^B$ and $\eta^A = \eta^B$, then $\pi^A > \pi^B$ implies that pair A has a stronger preference to share than pair B.*

In words, a stronger preference to share implies a stronger tie and identical risk preferences. The converse is also true if both pairs use the same sharing rule.

Theorem 6.2 observes that stronger ties mean that the opportunity to share ($o = 1$) is more valuable. One might intuit that stronger ties should also induce agents to choose first-stage acts more efficiently: that is, to pay more attention to the value of the total pie, $u(f + g)$, and less to disagreement values, $u(f)$ and $u(g)$. This intuition holds for sensible sharing rules (Definition 4.1).

Definition 6.3 *In the context of our representation, Given act g, act f is* ***more selfish*** *than f' if $u(f) > u(f')$ and $u(f + g) \leq u(f' + g)$.*

Theorem 6.3 (Efficiency vs. Selfishness) *In the context of our representation, consider pairs A and B such $\eta^A = \eta^B = \eta$. A proportional sharing rule is sensible if and only if for any g and $\pi^B < \pi^A$:*

$$\langle f', 1\rangle \succsim_g^{1A} \langle f, 1\rangle \text{ and } \langle f, 1\rangle \succ_g^{1B} \langle f', 1\rangle \implies f \text{ is more selfish than } f', \text{ given } g.$$

That is, under a sensible rule, when there is a preference disagreement between Agents $1A$ and $1B$, it is indeed because the agent with the stronger (weaker) tie prefers the act with the larger sharing surplus (disagreement value).

7 Discussion

For applied context, this document is framed in terms of informal risk sharing between households, which is particularly important in developing societies. As discussed previously, existing tests of efficient risk sharing take risky endowments as exogenous. We consider these risky endowments instead as choices made in an initial, pre-sharing stage. The effects of particular sharing arrangements on the choice of risky actions (e.g., migration, the take-up of novel crops, etc.) have received little attention, but may be relevant when evaluating policies. For example, should new crops be made available to all farmers at the same time or only to a subset initially? How would an improvement in the provision of formal insurance affect the agents' production decisions? What are the anticipated effects of subsidizing geographic mobility as discussed in Section 5?

As previously mentioned, another instance of informal risk sharing is *within* households, where it is considered one important benefit of marriage, especially in developing societies where individual incomes can be highly volatile (Mazzocco, 2004; Browning, Chiappori, and Weiss, 2014). The "collective model of the household" assumes that decisions are made in a Pareto efficient manner, where Pareto weights depend on disagreement values (Chiappori, 1988). Indeed it is well documented that, for a given total household income, the relative incomes of spouses affect the pattern of household expenditures and other household decisions (for developing economies see Thomas, 1994; Hoddinott and Haddad, 1995; Duflo, 2003).[35] Baland and Ziparo (2018) pointed out that premarital decisions (e.g, regarding education, occupation, and/or the timing of marriage), affect one's disagreement value and hence the Pareto weights within the collective model. Forward-looking agents should therefore account for these effects. Our model captures this feature, as premarital choices of risky acts determine both the disagreement allocation and individual contributions to household income. Understanding the effect of risk sharing on these choices should be relevant for a number of pressing policy questions, for instance about the effects of policies governing divorce and contraception on female education and participation in the labor force, on reproductive choices, on gender roles, and on the effectiveness of marriage as a risk sharing arrangement.

For clarity of exposition, we focused on bilateral risk sharing. As mentioned at the outset, extending our model and its foundations to risk sharing groups/syndicates (Wilson, 1968) is conceptually straightforward. In that setting, there are no relationship-specific constraints on how risk can be reallocated. An alternative modeling approach for multi-

[35]Here too, frictions (e.g., due to lack of commitment) appear to be important (Mazzocco, 2007).

agent risk sharing arrangements is to assume bilateral bargaining and transfers along the ties of a network (e.g., Ambrus et al., 2014). In addition to the (unobservable) sharing rule, the analyst must then also determine the underlying network. One approach is the game-theoretical study of network formation (Bramoullé and Kranton, 2007; Bloch, Genicot, and Ray, 2008; Ambrus and Elliott, 2020). In empirical work, often the network is assumed based on non-behavioral data such as family ties, geographic or demographic proximity, or an otherwise documented network of social ties (e.g., Fafchamps and Lund, 2003; De Weerdt and Dercon, 2006). If the (strengths of) network ties that are relevant for risk sharing differ from the assumed social network, this presumption will lead to model misspecification.[36] In contrast, our results allow the identification of sharing frictions, which are the appropriate measure of tie strength in our model (Section 6), directly from the observable choice of actions. Of course, an extension of our model to sharing on a network would need to resolve how bilateral sharing arrangements between different pairs are sequenced and interact. We leave such an extension as a topic for future research.

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[36]For instance, in the context of rural India, Munshi and Rosenzweig (2016) argue that the social network based on caste is more relevant for risk sharing than village membership, which was often used to define social ties in previous studies.

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A Appendix

A.1 Notation and Preliminaries

For a finite collection of acts $\mathcal{F}' \subset \mathcal{F}$, let $\mathcal{P}(\mathcal{F}')$ be the coarsest partition of Ω such that all $f \in \mathcal{F}'$ are measurable: that is, $f(\omega) = f(\omega')$ for all $\omega, \omega' \in S \in \mathcal{P}(\mathcal{F}')$ and $f \in \mathcal{F}'$. Note that $\mathcal{P}(\mathcal{F}')$ is well defined because all acts $f \in \mathcal{F}$ are simple. Further, $\mathcal{P}(\mathcal{F}') \subset \mathcal{A}$ since all $f \in \mathcal{F}$ are measurable in $\mathcal{A}$, which is a σ-algebra. For $f \in \mathcal{F}'$ and $S \in \mathcal{P}(\mathcal{F}')$ we use $f(S)$ to denote the (constant) value of f on S.

For any particular proportional sharing rule, since $\alpha^1_{f,g} + \alpha^2_{f,g} = 1$, we drop the superscript and let $\alpha^1_{f,g} = \alpha_{f,g}$ and $\alpha^2_{f,g} = 1 - \alpha_{f,g}$.

Lemma A.1 *For expected utility maximizers with differentiable, concave consumption utility functions u_1, u_2, allocation (a, b) is Pareto efficient in $A(f, g)$ according to u_1, u_2 if and only if, $a + b = f + g$ and for all $S, T \in \mathcal{P}(\{f, g, a, b\})$,*

$$\frac{u_1'(a(S))}{u_2'(b(S))} = \frac{u_1'(a(T))}{u_2'(b(T))}. \tag{7}$$

Proof. See Back (2016, ch. 4). ■

Lemma A.2 *Differentiable u is CRRA if and only if for all $c, c' \in \mathbb{R}_{++}$ and $\alpha \in (0, 1)$,*

$$\frac{u'(\alpha c)}{u'(c)} = \frac{u'(\alpha c')}{u'(c')}.$$

Proof. Suppose u is CRRA. Then $u'(c) = k \cdot c^{-\eta}$, for some $k > 0$. Thus, for $\alpha \in (0, 1)$, we have $\frac{u'(\alpha c)}{u'(c)} = \alpha^{-\eta} = \frac{u'(\alpha c')}{u'(c')}$. Now suppose u is not CRRA. Then an expected utility maximizer with consumption utility function u does not exhibit constant relative risk aversion. So there exist $\hat{c}, l, h, \alpha \in \mathbb{R}_{++}$ with $l < \hat{c}$ and $p \in (0, 1)$ such that $pu(\hat{c} + h) + (1 - p)u(\hat{c} - l) \leq u(\hat{c})$, but $pu(\alpha(\hat{c} + h)) + (1 - p)u(\alpha(\hat{c} - l)) > u(\alpha\hat{c})$ (Nielsen, 2005). This implies

$$\frac{\frac{u(\alpha(\hat{c})) - u(\alpha(\hat{c} - l))}{l}}{\frac{u(\hat{c}) - u(\hat{c} - l)}{l}} < \frac{\frac{u(\alpha(\hat{c} + h)) - u(\alpha\hat{c})}{h}}{\frac{u(\hat{c} + h) - u(\hat{c})}{h}}.$$

Since u is differentiable, there exist $c \in (\hat{c} - l, \hat{c})$ and $c' \in (\hat{c}, \hat{c} + h)$ for which

$$\frac{u'(\alpha c)}{u'(c)} < \frac{u'(\alpha c')}{u'(c')},$$

establishing the result. ■

Lemma A.3 *If u is CRRA from (2), $f \in \mathcal{F}$, and $\alpha > 0$, then $u(\alpha f) = \alpha^{1-\eta} u(f) + u(\alpha)$.*

Proof. Using (2), we have

$$\begin{aligned}
\alpha^{1-\eta} u(f) + u(\alpha) &= \alpha^{1-\eta} \sum_{c \in \chi(f)} \left(\mu\left(f^{-1}(c)\right) \frac{c^{1-\eta} - 1}{1-\eta} \right) + \frac{\alpha^{1-\eta} - 1}{1-\eta} \\
&= \sum_{c \in \chi(f)} \left(\mu\left(f^{-1}(c)\right) \left(\frac{\alpha^{1-\eta} \cdot c^{1-\eta}}{1-\eta} - \frac{\alpha^{1-\eta}}{1-\eta} \right) \right) + \left(\frac{\alpha^{1-\eta}}{1-\eta} - \frac{1}{1-\eta} \right) \\
&= \sum_{c \in \chi(f)} \mu\left(f^{-1}(c)\right) \frac{(\alpha c)^{1-\eta} - 1}{1-\eta} = u(\alpha f).
\end{aligned}$$

■

A.2 Proofs

Proof Proposition 3.1.

For $(i) \Rightarrow (ii)$: For any $f, g \in \mathcal{F}$ and $\alpha \in (0,1)$, under the proportional reallocation $(a,b) = (\alpha(f+g), (1-\alpha)(f+g))$, we have: $a + b = f + g$ and $a(S)/b(S) = \alpha/(1-\alpha)$ for all $S \in \mathcal{P}(f,g,a,b)$. Hence, for all $S, T \in \mathcal{P}(f,g,a,b)$ and $\eta \geq 0$,

$$\left(\frac{a(S)}{b(S)} \right)^{\eta} = \left(\frac{a(T)}{b(T)} \right)^{\eta}. \tag{8}$$

If u_1, u_2 are CRRA with $\eta_1 = \eta_2 = \eta$, then, using the functional form in (2), we have (8) is equivalent to (7). Thus, Lemma A.1 establishes that (a,b) is Pareto efficient in $A(f,g)$.

For $(i) \Rightarrow (iii)$: Given $f, g, a, b \in \mathcal{F}$ and $S \in \mathcal{P}(f,g,a,b)$, assign $\alpha = \frac{a(S)}{f(S)+g(S)}$ and hence $1 - \alpha = \frac{b(S)}{f(S)+g(S)}$. Suppose (a,b) is Pareto efficient in $A(f,g)$ and, for the purpose of contradiction, that $a \neq \alpha(f+g)$. Then there exists $T \in \mathcal{P}(f,g,a,b)$ such that $\frac{a(T)}{f(T)+g(T)} \neq \alpha$. Then $\frac{a(S)}{b(S)} \neq \frac{a(T)}{b(T)}$, and hence for any $\eta > 0$,

$$\left(\frac{a(S)}{b(S)} \right)^{\eta} \neq \left(\frac{a(T)}{b(T)} \right)^{\eta}. \tag{9}$$

If u_1, u_2 are CRRA with $\eta_1 = \eta_2 = \eta$, then, using the functional form in (2), we have (9) implies a failure of (7). Thus, Lemma A.1 establishes that (a,b) is not Pareto efficient in $A(f,g)$, a contradiction.

For $(ii) \Rightarrow (i)$: Let $\succsim_{u_i}$ be the preference over allocations induced by u_i. We first argue that (ii) implies that $\succsim_{u_1} = \succsim_{u_2}$. To do so, for two events $S, T \in \mathcal{A}$ with $\mu(S) = \mu(T) = \frac{1}{2}$, and $c, c' \in \mathbb{R}_{++}$, let $\langle c; c' \rangle$ denote the act that delivers c on event S and

c' on T. If $\succsim_{u_1} \neq \succsim_{u_2}$, then there exist $c, c', \delta, \delta' \in \mathbb{R}_{++}$ with $\delta < c$ and $\delta' < c'$ such that $u_1(\langle c + \delta; c' - \delta' \rangle) > u_1(\langle c; c' \rangle)$ and $u_2(\langle c - \delta; c' + \delta' \rangle) > u_2(\langle c; c' \rangle)$. Then for $f + g = \langle 2c; 2c' \rangle$, the proportional reallocation $(\frac{1}{2}(f+g), \frac{1}{2}(f+g)) = (\langle c; c' \rangle, \langle c; c' \rangle)$ is Pareto dominated by $(\langle c+\delta; c'-\delta \rangle, \langle c-\delta; c'+\delta' \rangle) \in A(f,g)$. This contradicts all proportional reallocations being Pareto efficient; hence $\succsim_{u_1} = \succsim_{u_2}$, and it is without loss to specify a common $u = u_1 = u_2$.

Next we show that the common u is (weakly) concave. For the purpose of contradiction, suppose not. Then there exists $c, \delta \in \mathbb{R}_{++}$ with $\delta < c$, such that $u(\langle c+\delta; c-\delta \rangle) > u(\langle c; c \rangle)$. Because $\mu(S) = \mu(T) = \frac{1}{2}$, it must also be that $u(\langle c-\delta; c+\delta \rangle) = u(\langle c+\delta; c-\delta \rangle) > u(\langle c; c \rangle)$. Then for $f + g = \langle 2c; 2c \rangle$, the proportional reallocation $(\frac{1}{2}(f+g), \frac{1}{2}(f+g)) = (\langle c; c \rangle, \langle c; c \rangle)$ is Pareto dominated by $(\langle c+\delta; c-\delta \rangle, \langle c-\delta; c+\delta \rangle) \in A(f,g)$. This contradicts all proportional reallocations being Pareto efficient; hence u is concave on $\mathbb{R}_{++}$. As a consequence, left-hand and right-hand derivatives, u'_- and u'_+, exist everywhere on $\mathbb{R}_{++}$ (Rockafellar, 1970, Theorem 23.1). Furthermore, the derivative u'_i exists (u'_- and u'_+ coincide) almost everywhere (Rockafellar, 1970, Theorem 25).

We next show that u is differentiable everywhere. For the purpose of contradiction, suppose that there exists $c_1 \in \mathbb{R}_{++}$ with $u'_-(c_1) \neq u'_+(c_1)$. Now choose $c_2, c_3, c_4 > 0$ such that u is differentiable at each and

$$\frac{c_1}{c_2} = \rho = \frac{c_3}{c_4},$$

which is possible because u is differentiable almost everywhere. Then $\kappa := \frac{u'(c_3)}{u'(c_4)}$ is well defined. Next, for $\xi \in \mathbb{R}_{++}$, let $MRS(\xi) := \frac{u'(\xi c_1)}{u'(\xi c_2)}$ when these derivatives are well defined, which is (at least) almost everywhere.

Now define $\alpha := \frac{\xi c_1}{c_3 + \xi c_1}$, $c := c_3 + \xi c_1$, and $c' := \frac{c_3 + \xi c_1}{\rho}$, and verify that $\alpha c = \xi c_1$, $\alpha c' = \xi c_2$, $(1-\alpha)c = c_3$, and $(1-\alpha)c' = c_4$. It is straightforward to extend Lemma A.1 to show that Pareto efficiency implies (7) whenever the four derivatives therein are well-defined. Therefore, wherever defined, $MRS(\xi) = \kappa$. Let $\xi_n^- \nearrow 1$ and $\xi_n^+ \searrow 1$ be sequences such that $MRS(\xi_n^-)$ and $MRS(\xi_n^+)$ are well defined for all n. Then, $MRS(\xi_n^-) = MRS(\xi_n^+) = \kappa$ for all n. It follows that

$$\lim_{n\to\infty} \frac{u'(\xi_n^- c_1)}{u'(\xi_n^- c_2)} = \lim_{n\to\infty} \frac{u'(\xi_n^+ c_1)}{u'(\xi_n^+ c_2)} = \kappa.$$

At the same time, because u is concave and differentiable at c_2, from Theorem 24.1 in

Rockafellar (1970) we have

$$\lim_{n\to\infty} u'(\xi_n^- c_2) = \lim_{n\to\infty} u'(\xi_n^+ c_2) = u'(c_2).$$

It follows that

$$\lim_{n\to\infty} u'(\xi_n^- c_1) = \kappa u'(c_2) = \lim_{n\to\infty} u'(\xi_n^+ c_1)$$

and again by the same theorem,

$$u'_-(c_1) = \lim_{n\to\infty} u'(\xi_n^- c_1) \quad \text{and} \quad \lim_{n\to\infty} u'(\xi_n^+ c_1) = u'_+(c_1),$$

which contradicts the supposition that $u'_-(c_1) \neq u'_+(c_1)$. Hence u is differentiable everywhere as claimed.

Finally, suppose, contrary to (i), that u_1 is not CRRA. Then by Lemma A.2, there exist $c, c', \alpha, \alpha' \in \mathbb{R}_{++}$ such that

$$\frac{u_1'(\alpha c)}{u_1'(\alpha c')} \neq \frac{u_1'(\alpha' c)}{u_1'(\alpha' c')}.$$

Let τ and $\widetilde{\alpha}$ be such that $(1-\widetilde{\alpha})\tau = 1-\alpha$ and $\widetilde{\alpha}\tau = \alpha'$. Then,

$$\frac{u_1'(\widetilde{\alpha}\tau c)}{u_1'(\widetilde{\alpha}\tau c')} = \frac{u_1'(\alpha' c)}{u_1'(\alpha' c')} \neq \frac{u_1'(\alpha c)}{u_1'(\alpha c')} = \frac{u_2'((1-\alpha)c)}{u_2'((1-\alpha)c')} = \frac{u_2'((1-\widetilde{\alpha})\tau c)}{u_2'((1-\widetilde{\alpha})\tau c')}.$$

Hence, by Lemma A.1, for any $f', g' \in \mathcal{F}$ with $f' + g' = \tau(f+g)$, the reallocation $(\widetilde{\alpha}(f+g), (1-\widetilde{\alpha})(f+g))$ is not Pareto efficient in $A(f', g')$. This contradicts all proportional reallocations being Pareto efficient; hence $u_1 = u_2$ is a CRRA utility (i.e., $\eta_1 = \eta_2$). ■

Proof of Lemma 3.1. If $u_i = u = \ln$ for both agents, then u is CRRA from (2) with $\eta = 1$. Using Lemma A.3, for any $\alpha \in (0,1)$, we have $u(\alpha(f+g)) = u(f+g) + \ln(\alpha)$. Given (f,g) and using Proposition 3.1, the EBS then implies the proportional reallocation satisfying $u(f+g) + \ln(\alpha) - u(f) = u(f+g) + \ln(1-\alpha) - u(g)$. Substituting $\ln(ce(f))$ for $u(f)$ and $\ln(ce(g))$ for $u(g)$, we find the unique solution: $\alpha = ce(f)/(ce(f)+ce(g))$. ■

Proof of Theorem 3.1.

Model $\Rightarrow$ Axioms: Let the model (η, α) explain $\{\succsim_g^i\}_{g\in\mathcal{F}}$. For $o = 0$, the model implies a monotonic expected utility representation that is independent of g for each agent. Axioms 1 and 2 follow immediately. Axiom 3 follows because $u_1(\alpha_{f,g}(f+g)) \geq u_1(f)$ and $u_2((1-\alpha_{f,g})(f+g)) \geq u_2(g)$ for the proportional sharing rule α (Definition 3.1), with at least one inequality strict whenever $g \neq \beta f$ for any $\beta > 0$ since u_i are CRRA with the same η (Proposition 3.1). Moreover, Proposition 3.1 implies that proportional sharing of

$f+g$ is Pareto efficient, and hence Axioms 5 and 6 follow. It further implies that, when sharing f and βf for $\beta > 0$, the only proportional allocation that respects disagreement values is to maintain the original allocation, which implies Axiom 4.

<u>Axioms $\Rightarrow$ Model</u>: The result is established in several steps. Throughout, assume that the primitive, $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ for $i\in\{1,2\}$, satisfies Axioms 1-6.

Step 1 *For any $f,g\in\mathcal{F}$ and $(a,b)\in PS(f,g)$, there exists $\alpha\in(0,1)$ such that $\langle a,0\rangle\sim_g^1\langle\alpha(f+g),0\rangle$ and $\langle b,0\rangle\sim_f^2\langle(1-\alpha)(f+g),0\rangle$.*

$\triangleright$ Fix $f,g\in\mathcal{F}$ and $(a,b)\in\mathcal{A}(f,g)$. Define

$$\begin{aligned}\Theta_1 &:= \left\{\alpha\in(0,1)\,\middle|\,\langle a,0\rangle\succ_b^1\langle\alpha(f+g),0\rangle\right\}\\ \Theta_2 &:= \left\{\alpha\in(0,1)\,\middle|\,\langle b,0\rangle\succ_a^2\langle(1-\alpha)(f+g),0\rangle\right\},\end{aligned}$$

and, for the purpose of contradiction, suppose that $\Theta_1\cup\Theta_2=(0,1)$. Because $a<f+g$, by monotonicity of autarky preferences (Axiom 2), there exists $\alpha<1$ large enough such that $\langle\alpha(f+g),0\rangle\succsim_b^1\langle a,0\rangle$ and hence $\Theta_2\neq\emptyset$. By a symmetric argument, $\Theta_1\neq\emptyset$.

Continuity and monotonicity of autarky preferences in α (which follow immediately from Axiom 2) then imply $\Theta_1=(0,\overline{\alpha}_{f,g})$ and $\Theta_2=(\underline{\alpha}_{f,g},1)$. Hence, for $\Theta_1\cup\Theta_2=(0,1)$, it must be that $\underline{\alpha}_{f,g}<\overline{\alpha}_{f,g}$ and there exists $\alpha^*\in\Theta_1\cap\Theta_2$, meaning

$$\langle a,0\rangle\succ_b^1\langle\alpha^*(f+g),0\rangle \text{ and } \langle b,0\rangle\succ_a^2\langle(1-\alpha^*)(f+g),0\rangle.$$

By Axiom 2, $\succsim_h^i$ is independent of $h\in\mathcal{F}$ in autarky for $i\in\{1,2\}$. Therefore,

$$\begin{aligned}\langle a,0\rangle &\succ^1_{(1-\alpha^*)(f+g)} \langle\alpha^*(f+g),0\rangle\\ \langle b,0\rangle &\succ^2_{\alpha^*(f+g)} \langle(1-\alpha^*)(f+g),0\rangle.\end{aligned}$$

Separately, by Axiom 4,

$$\begin{aligned}\langle\alpha^*(f+g),0\rangle &\succsim^1_{(1-\alpha^*)(f+g)} \langle\alpha^*(f+g),1\rangle\\ \langle(1-\alpha^*)(f+g),0\rangle &\succsim^2_{\alpha^*(f+g)} \langle(1-\alpha^*)(f+g),1\rangle.\end{aligned}$$

Then, by transitivity of $\succsim_h^i$ for $i\in\{1,2\}$ and any $h\in\mathcal{F}$ (Axiom 1),

$$\begin{aligned}\langle a,0\rangle &\succ^1_{(1-\alpha^*)(f+g)} (\alpha^*(f+g),1\rangle\\ \langle b,0\rangle &\succ^2_{\alpha^*(f+g)} \langle(1-\alpha^*)(f+g),1\rangle.\end{aligned}$$

By Axiom 5, $(a,b) \notin A(\alpha^*(f+g),(1-\alpha^*)(f+g))$. However, $A(\alpha^*(f+g),(1-\alpha^*)(f+g)) = A(f,g)$, producing a contradiction. Hence for all $(a,b) \in A(f,g)$ there is $\alpha \in (0,1)$ such that

$$\langle \alpha\,(f+g)\,,0\rangle \succsim_b^1 \langle a,0\rangle \text{ and } \langle (1-\alpha)\,(f+g)\,,0\rangle \succsim_a^2 \langle b,0\rangle.$$

Finally, by the definition of $PS(f,g)$ and because $\succsim_h^i$ is independent of $h \in \mathcal{F}$ in autarky (Axiom 2), we then have that for any $(a,b) \in PS\,(f,g)$ there is $\alpha \in (0,1)$ such that

$$\langle a,0\rangle \sim_g^1 \langle \alpha\,(f+g)\,,0\rangle \text{ and } \langle b,0\rangle \sim_f^2 \langle (1-\alpha)\,(f+g)\,,0\rangle. \quad \lhd$$

Step 2 *For all $\alpha \in (0,1)$, we have $(\alpha\,(f+g)\,,(1-\alpha)\,(f+g)) \in PS\,(f,g)$.*

$\rhd$ Fix $f,g \in \mathcal{F}$. For the purpose of contradiction, suppose that there exists $\alpha \in (0,1)$ and $(a,b) \in A(f,g)$ that Pareto dominates $(\alpha\,(f+g)\,,(1-\alpha)\,(f+g))$ according to autarky preferences. By Step 1, there exists $\hat{\alpha} \neq \alpha$ such that $\langle \hat{\alpha}(f+g),0\rangle \sim_g^1 \langle a,0\rangle$ and $\langle (1-\hat{\alpha})(f+g),0\rangle \sim_f^2 \langle b,0\rangle$. However, monotonicity of autarky preferences in α (which follow from Axiom 2) implies that $(\hat{\alpha}(f+g),(1-\hat{\alpha})(f+g))$ does not Pareto dominate $(\alpha\,(f+g)\,,(1-\alpha)\,(f+g))$ according to autarky preferences, a contradiction. $\lhd$

Step 3 *For all $f,g \in \mathcal{F}$, there exists $\alpha^* \in (0,1)$ such that:*

$$\langle \alpha^*\,(f+g)\,,0\rangle \sim_g^1 \langle f,1\rangle \text{ and } \langle (1-\alpha^*)\,(f+g)\,,0\rangle \sim_f^2 (g,1)\,.$$

$\rhd$ Fix $f,g \in \mathcal{F}$, and define

$$\begin{aligned}\Theta_3 &:= \left\{\alpha \in (0,1)\,\middle|\,\langle \alpha\,(f+g)\,,0\rangle \succsim_g^1 \langle f,1\rangle\right\}\\ \Theta_4 &:= \left\{\alpha \in (0,1)\,\middle|\,\langle (1-\alpha)\,(f+g)\,,0\rangle \succsim_f^2 \langle g,1\rangle\right\}.\end{aligned}$$

Step 2 and Axiom 6 together imply that $\Theta_3 \cup \Theta_4 = (0,1)$. For the purpose of contradiction, suppose that $\langle g,1\rangle \succ_f^2 \langle (1-\alpha)(f+g),0\rangle$ for all $\alpha \in (0,1)$ (i.e., $\Theta_4 = \emptyset$). But then, for α small enough we have the contradiction:

$$\langle \alpha(f+g),0\rangle \succsim_g^1 \langle f,1\rangle \succsim_g^1 \langle f,0\rangle \succ_f^2 \langle \alpha(f+g),0\rangle,$$

where the first ranking follows from Step 2 and Axiom 6, the second is by Axiom 3, and the third is from Axiom 2. Hence, $\Theta_4 \neq \emptyset$, and by symmetric argument $\Theta_3 \neq \emptyset$. Continuity and monotonicity of autarky preferences in α (which follow immediately from Axiom 2) then imply $\Theta_3 = [\underline{\alpha},1)$ and $\Theta_2 = (0,\overline{\alpha}]$. Hence, for $\Theta_3 \cup \Theta_4 = (0,1)$, it must be that $\underline{\alpha} \leq \overline{\alpha}$ and there exists $\alpha^* \in \Theta_3 \cap \Theta_4$, meaning

$$\langle \alpha^*\,(f+g)\,,0\rangle \succsim_g^1 \langle f,1\rangle \text{ and } \langle (1-\alpha^*)\,(f+g)\,,0\rangle \succsim_f^2 \langle g,1\rangle.$$

Finally, since $(\alpha^*(f+g),(1-\alpha^*)(f+g)) \in A(f,g)$, by Axiom 5

$$\langle \alpha^*(f+g), 0\rangle \sim_g^1 \langle f, 1\rangle \text{ and } \langle (1-\alpha^*)\,(f+g), 0\rangle \sim_f^2 \langle g, 1\rangle. \quad \lhd$$

Step 4 *For $i = 1, 2$ and any $g \in \mathcal{F}$, there exists an expected utility representation (with consumption utility function u_i) for $\succsim_g^i$ when $o = 0$. Moreover, u_i is independent of g.*

$\rhd$ Immediate from Axiom 2. $\lhd$

Step 5 *There exists a proportional sharing rule, with shares $\alpha^i_{f,g}$, such that for any $f, f', g \in \mathcal{F}$ and $i \in \{1, 2\}$ we have:*

$$\langle f, o\rangle \succsim_g^i \langle f', o'\rangle \iff ou\left(\alpha^i_{f,g}\,(f+g)\right) + (1-o)u(f) \geq o'u\left(\alpha^i_{f',g}\,(f'+g)\right) + (1-o')u(f').$$

$\rhd$ Implied by Steps 3 and 4. $\lhd$

Step 6 *There exists a two-stage model of proportional risk sharing (η, α), that explains preferences $\left\{\succsim_g^i\right\}_{g\in\mathcal{F}}$ for $i \in \{1, 2\}$.*

$\rhd$ Given Step 5, all that remains to show is that u_1, u_2 must be CRRA with $\eta_1 = \eta_2$ and that $u_i(\alpha^i_{f,g}(f+g)) \geq u_i(f)$ for all $f, g \in \mathcal{F}$ and $i \in \{1, 2\}$. Given Step 5, the latter is implied by Axiom 3. For the former, Steps 2 and 5 imply that any proportional sharing rule is Pareto efficient for expected utility maximizers with consumption utility functions u_1, u_2 identified in Step 4. Hence, u_1, u_2 must be CRRA with $\eta_1 = \eta_2$ by the equivalence of (i) and (ii) in Proposition 3.1. $\lhd$
This completes the proof of Theorem 3.1. ∎

Proof of Theorem 4.1. The identification of η from autarky preferences, which satisfy the vNM axioms according to Axiom 2, follows standard arguments. To identify the proportional sharing rule α, let $ce(f,g) \in \mathbb{R}_{++}$ be the autarky certainty equivalent of sharing f with g. That is, if act $h(\omega) = ce(f,g)$ for all $\omega \in \Omega$, then $\langle h, 0\rangle \sim_g^1 \langle f, 1\rangle$. So, $u\left(ce(f,g)\right) = u\left(\alpha_{f,g}(f+g)\right)$. It follows immediately from the representation that $ce(f,g)$ is well defined and, from strict monotonicity of u, that $u\left(\alpha(f+g)\right)$ is strictly monotone in α. Hence, the share $\alpha_{f,g}$ is uniquely identified. ∎

Proof of Proposition 4.1. We need to show that for sharing according to each of the three bargaining solutions, NBS, KSS, and EBS, and for any act g held by Agent $j \neq i$,

$$u(f) > u(f') \text{ and } u(f+g) > u(f'+g) \implies u(\alpha^i_{f,g}(f+g)) > u(\alpha^i_{f',g}(f'+g)).$$

It is clear that all three bargaining solutions lead to symmetric sharing rules, and that increasing the disagreement value $u(f)$ while holding fixed $u(g)$ and $u(f+g)$ strictly increases the share alotted to Agent 1. It is, therefore, sufficient to establish that

$$u(f) = u(f') \text{ and } u(f+g) > u(f'+g) \implies u(\alpha^1_{f,g}(f+g)) \geq u(\alpha^1_{f',g}(f'+g)). \tag{10}$$

Recall from Lemma A.3 that $u(\alpha h) = \alpha^{1-\eta}u(h) + u(\alpha)$, which is increasing in $u(h)$. Hence, $PS(f,g)$ is shifted outward from $PS(f',g)$. Since both the KSS and the EBS are monotonic, in the sense that each agent gets a weakly higher surplus when the Pareto frontier shifts outwards, they must both be sensible.

It remains to establish that sharing based on the NBS satisfies Condition (10). Since u is CRRA with coefficient η, again using Lemma A.3 we have

$$\frac{\partial}{\partial \alpha} u(\alpha h) = (1-\eta)\alpha^{-\eta}u(h) + \alpha^{-\eta} = \alpha^{-\eta}E^{\mu}[h(\omega)^{-\eta}] > 0. \tag{11}$$

Fixing f and g, the NBS selects $\alpha^1_{f,g}$ to solve

$$\max_{\alpha} \left(u\left(\alpha(f+g)\right) - u(f)\right)\left(u\left((1-\alpha)(f+g)\right) - u(g)\right).$$

So $\alpha = \alpha^1_{f,g}$ must satisfy the first order condition

$$\begin{aligned}\frac{u\left(\alpha(f+g)\right) - u(f)}{u\left((1-\alpha)(f+g)\right) - u(g)} &= \frac{(1-\eta)\alpha^{-\eta}u(f+g) + \alpha^{-\eta}}{(1-\eta)(1-\alpha)^{-\eta}u(f+g) + (1-\alpha)^{-\eta}} \\ &= \left(\frac{1-\alpha}{\alpha}\right)^{\eta}. \end{aligned} \tag{12}$$

Consider now f, f' and g that satisfy the qualifier in Condition (10) and, for the purpose of contradiction, suppose that $u\left(\alpha^1_{f,g}(f+g)\right) < u\left(\alpha^1_{f',g}(f'+g)\right)$. Since $u(\alpha h)$ is increasing in $u(h)$ and since $u(f+g) > u(f'+g)$, it follows from (11) that $\alpha^1_{f,g} < \alpha^1_{f',g}$. Equivalently, $1-\alpha^1_{f,g} > 1-\alpha^1_{f',g}$ and hence $u\left((1-\alpha^1_{f,g})(f+g)\right) > u\left((1-\alpha^1_{f',g})(f'+g)\right)$. Hence, the LHS of (12) must be smaller under f and $\alpha^1_{f,g}$ than it is under f' and $\alpha^1_{f',g}$. Hence, the same must be true of the RHS of (12). But the RHS of (12) is clearly decreasing in α, so this implies $\alpha^1_{f,g} > \alpha^1_{f',g}$, which is a contradiction. Hence, $u\left(\alpha^1_{f,g}(f+g)\right) \geq u\left(\alpha^1_{f',g}(f'+g)\right)$ must hold, which is the implication in Condition (10).

This completes the proof of the proposition. ■

Analysis of Application to Migration (Section 5). Here we collect the analysis underpinning Example 1 and Facts 5.1-5.4 in Section 5. In Example 1, the closed-form expression for $G_E(B)$ is found by solving $u(v+g) = u(m+g)$,

$$\frac{1}{4}\left(\ln(2) + 2\ln(1+e) + \ln(2e)\right) = \frac{1}{4}\left(\ln(B+1) + \ln(B+e) + \ln(G+1) + \ln(G+e)\right),$$

for G, which is matter of algebra.

Next, for fixed B, as G increases so to do both $u(m)$ and $u(m+g)$. Since each of the bargaining solutions from Section 4.1 generate sensible sharing rules (Proposition 4.1), we have that $u(\alpha_{m,g}(m+g))$ is increasing in G. Hence, for each solution, there is a unique cutoff $G^*(B)$ such that $u(\alpha_{m,g}(m+g)) \geq u(\alpha_{v,g}(v+g))$ if and only if $G \geq G^*(B)$.

To see that $G^*(B) > G_E(B)$, begin with three observations. First, with ln utility, (6) simplifies to $u(m+g) \geq u(v+g) - \left(u\left(\alpha_{v,g}\right) - u\left(\alpha_{m,g}\right)\right)$. Second, by definition, $u(m+g) = u(v+g)$ when $G = G_E(B)$. Third, under any of the bargaining solutions from Section 4.1, with $u(v) = u(g)$, we have that $\alpha_{m,g} < \alpha_{v,g}$ if and only if $u(m) < u(v)$. Putting the three together, it is sufficient to show that $u(m) < u(g)$ when $G = G_E(B)$, which, using the closed-form expression for $G_E(B)$, is a matter of direct calculation.

To conclude the claims in Example 1, fix $B = 1/2$ and calculate that $G_E(B) = 3.79$. Now, write $m = m(G)$ to emphasize that m depends on G, and we look for the pair (G, α) that simultaneously solves the following two equations:

$$\begin{aligned}
&u(\alpha(m(G)+g)) = u(\frac{1}{2}(v+g)) \\
&\frac{d}{d\alpha}\left(u(\alpha(m(G)+g)) - u(m(G))\right)\left(u((1-\alpha)(m(G)+g)) - u(g)\right) = 0,
\end{aligned}$$

where the first guarantees that Agent 1 gets as much by choosing m as by choosing v (since $u(v) = u(g)$, we know $\alpha_{v,g} = 1/2$ by the symmetry of the NBS), and the second guarantees that α is the sharing rule implied by the NBS when Agent 1 migrates. In this system, only G and α do not have numerical values, and can be solved numerically. The unique solution is $(G, \alpha) = (4.35, 0.4762)$.

Once the subsidy is introduced, the key observation is that $u\left(m+t\right)$ and $u\left(m+g+t\right)$ are each increasing in t. Hence, under any continuous and sensible sharing rule, it must be that $u\left(\alpha_{m+t,g}(m+t+g)\right)$ is continuous and increasing in t. Fact 5.1 is now immediate: Agent 1 weakly prefers m over v if and only if $t \geq t_1$.

Moving to Fact 5.2, fix $t \in (t_1, t_2]$. That Agent 1 is better off follows from Fact 5.1. By definition of t_2, we have that $\alpha_{m+t,g} \leq \alpha_{v,g}$, and therefore, $1 - \alpha_{m+t,g} \geq 1 - \alpha_{v,g}$. Hence, Agent 2 gets a weakly larger share when Agent 1 migrates than when Agent 1 remains in the village. Since $u(m+t+g) > u(v+g)$, by Lemma A.3, Agent 2 is also made better off by the subsidy policy since it induces migration. In Example 1, $u(g) = u(v)$, so symmetry of the NBS implies $\alpha_{v,g} = 1/2$. Therefore, t_2 must solve $u(m+t) = u(g)$, which gives $t_2 = \frac{1}{2}\left(\sqrt{(G-B)^2+4e} - (B+G)\right)$.

If taxation is introduced to balance the budget, the tax rate must be $\frac{t}{m+t+g}$ if there is migration (and zero otherwise). If Agent 1 migrates, then Agent i's utility will be $u(\tilde{\alpha}^i_{m+t,g}(m+t+g)\frac{m+g}{m+t+g}) = u(\tilde{\alpha}^i_{m+t,g}(m+g))$. Therefore, if $t = \tilde{t}_2$, then $\tilde{\alpha}^i_{m+t,g} = \alpha^i_{v,g}$ and (using Lemma A.3) we have that $u(\tilde{\alpha}^i_{m+t,g}(m+t+g)\frac{m+g}{m+t+g}) > u(\alpha^i_{v,g}(v+g))$ since $u(m+g) > u(v+g)$. This establishes Fact 5.3. Again in Example 1, $u(g) = u(v)$, so symmetry of the NBS implies $\alpha_{v,g} = 1/2$. Therefore, $\tilde{t}_2$ must solve $u((m+t)r) = u(gr)$, where r is 1 minus the tax rate. With ln utility, $\tilde{t}_2$ solves

$$E\left[\ln((m(\omega)+t)r(\omega))\right] = E\left[\ln(g(\omega)r(\omega))\right]$$
$$E\left[\ln((m(\omega)+t)\right] + E\left[\ln(r(\omega))\right] = E\left[\ln(g(\omega))\right] + E\left[\ln(r(\omega))\right].$$

The $E\left[\ln(r(\omega))\right]$ terms cancel, and $\tilde{t}_2$ solves the same equation that t_2 solved.

If the pre-stage is introduced, t_3 is the analog of t_1 for the case where migrating generates zero instead of m. Subsidy level t_4 solves $u(m+t) = u(v)$ (so is the same as t_2 in the context of Example 1). Fact 5.4 is immediate from Fact 5.2 and definitions of t_1, t_3, t_4. ■

Proof of Theorem 6.1. The parameters (η, α) are determined analogously to the identification in Theorem 4.1. It remains to establish the uniqueness of π.

Consider $(f,g) \notin PS(f,g)$. Then $u(\alpha_{f,g}(f+g)) > u(f)$ or $u((1-\alpha_{f,g})(f+g)) > u(g)$. Suppose $u(\alpha_{f,g}(f+g)) > u(f)$ and let $h \in \mathcal{F}$ be such that $\langle h, 0\rangle \sim^1_g \langle f, 1\rangle$. Then $\pi u(\alpha_{f,g}(f+g)) + (1-\pi)u(f)$ is strictly increasing in π and hence the π that satisfies the requirement $\pi u(\alpha_{f,g}(f+g)) + (1-\pi)u(f) = u(h)$ is unique. The case where $u((1-\alpha_{f,g})(f+g)) > u(g)$ is analogous. ■

Proof of Theorem 6.2.

Claim (i): Assume that pair A has a stronger preference to share than does pair B. We first show that $\eta^A = \eta^B$ by establishing that Agents $1A$ and $1B$ have the same preferences in autarky and hence the same consumption utility function. To begin, suppose that $\langle f, 0\rangle \succsim^{1A}_g \langle f', 0\rangle$ for some $f, f', g \in \mathcal{F}$. From Axiom 2, autarky preferences are independent of g, so $\langle f, 0\rangle \succsim^{1A}_{\beta f'} \langle f', 0\rangle \sim^{1A}_{\beta f'} \langle f', 1\rangle$, where the indifference results from Axioms $3'$ and 4 together. We then have that $\langle f, 0\rangle \succsim^{1B}_{\beta f'} \langle f', 1\rangle \sim^{1B}_{\beta f'} \langle f', 0\rangle$, where the strict preference is by the theorem's hypothesis and the indifference again results from Axioms $3'$ and 4 together. It follows for the CRRA utilities u^A and u^B from the respective two-stage models of proportional risk sharing for pairs A and B that $u^A(f) \geq u^A(f')$ implies $u^B(f) \geq u^B(f')$ for all $f, f' \in \mathcal{F}$. Because CRRA utility functions are continuous and monotonic, it must be the case that $u^A = u^B$ or $\eta^A = \eta^B$.

For the purpose of contradiction, suppose now that $\pi^A \leq \pi^B$. The following three cases are exhaustive:

- $\alpha^A_{f,g} < \alpha^B_{f,g}$ for some pair of acts (f, g). Then Agent $1B$ benefits strictly more from sharing (f, g) than Agent $1A$ does. That is, there exist an act f' with $\langle f, 1\rangle \succ^{1B}_g \langle f', 0\rangle$ but $\langle f', 0\rangle \succ^{1A}_g \langle f, 1\rangle$.

- $\alpha^A_{f,g} > \alpha^B_{f,g}$ for some pair of acts (f, g). Then, symmetrically, Agent $2B$ benefits strictly more from sharing (f, g) than Agent $2A$ does. That is, there exist an act f' with $\langle f, 1\rangle \succ^{2B}_g \langle f', 0\rangle$ but $\langle f', 0\rangle \succ^{2A}_g \langle f, 1\rangle$.

- $\alpha^A_{f,g} = \alpha^B_{f,g}$ for all pairs of acts (f, g). Then both agents in pair B benefit weakly more from sharing than those in pair A do. Hence there are no three acts f, g, f' such that either $\langle f, 1\rangle \succ^{1A}_g \langle f', 0\rangle$ and $\langle f', 0\rangle \succsim^{1B}_g \langle f, 1\rangle$, or $\langle f, 1\rangle \succ^{2A}_g \langle f', 0\rangle$ and $\langle f', 0\rangle \succsim^{2B}_g \langle f, 1\rangle$.

In all three cases, pair A does not have a stronger preference to share than pair B, contradicting the hypothesis. Hence, it must be that $\pi^A > \pi^B$.

Claim (ii): Follows immediately from the representation.[37] ■

Proof of Theorem 6.3. Fix models A and B that differ only in $\pi^A > \pi^B$, with a sensible sharing rule for u. For any $f, g \in \mathcal{F}$, let $\Psi(f, g) := u(\alpha_{f,g}(f + g))$. For the hypothesis, assume that $\langle f', 1\rangle \succsim^{1A}_g \langle f, 1\rangle$ and $\langle f, 1\rangle \succ^{1B}_g \langle f', 1\rangle$. It follows that $\Psi(f', g) \geq \Psi(f, g)$ and $u(f) > u(f')$. For the purpose of contradiction, suppose that $u(f + g) > u(f' + g)$. Together with $u(f) > u(f')$, sensibility then requires that $\Psi(f, g) > \Psi(f', g)$, which is a contradiction. Hence, $u(f + g) \leq u(f' + g)$, and f is more selfish than f'.

For the other direction, fix a model up to π with a sharing rule that is not sensible for u. It is sufficient to find $\pi^A > \pi^B$ and f, f', g such that $\langle f', 1\rangle \succsim^{1A}_g \langle f, 1\rangle$ and $\langle f, 1\rangle \succ^{1B}_g \langle f', 1\rangle$ but f is not more selfish than f', given g. Because the rule is not sensible, there exists f, f', g such that

$$u(f) > u(f') \text{ and } u(f + g) > u(f' + g), \text{ but } \Psi(f, g)) \leq \Psi(f', g)). \tag{13}$$

Using these f, f', g, for π^A large enough and π^B small enough, we have that $\langle f', 1\rangle \succsim^{1A}_g \langle f, 1\rangle$ (due to $\Psi(f', g) \geq \Psi(f, g)$) and $\langle f, 1\rangle \succ^{1B}_g \langle f', 1\rangle$ (due to $u(f) > u(f')$). However, it is clear from (13) that, given g, f is not more selfish than f'. ■

[37]The requirement of $\alpha^A = \alpha^B$ is essential in (ii) even though it was not needed for Claim (i): if $\alpha^A \neq \alpha^B$, the argument in proof of Claim (i) can be applied to show that pair A will not have a stronger preference to share than pair B as long as π^A and π^B are not too different.