

# **Preferences with Taste Shock Representations: Price Volatility and the Liquidity Premium\***

R. VIJAY KRISHNA†      PHILIPP SADOWSKI‡

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## **Abstract**

If price volatility is caused in some part by taste shocks, then it should be positively correlated with the liquidity premium. Our argument is based on Krishna and Sadowski (2014), who provide foundations for a representation of dynamic choice with taste shocks, and show that volatility in tastes corresponds to a desire to maintain flexibility. To formally connect volatile tastes to price volatility and preference for flexibility to the liquidity premium, we analyze a modified simple Lucas tree economy with two otherwise identical assets, where one provides more liquidity because its output can be traded on an auxiliary international market, and where the representative agent is uncertain about his degree of future risk aversion. We show that a representative agent with a higher degree of uncertainty about his future risk aversion implies a higher liquidity premium (ie, a higher price for the more liquid asset) and more price volatility.

## **1. Introduction**

Following Black (1987), a growing literature has argued that taste shocks are important for our understanding of business cycles and asset prices. For example,

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(†) Florida State University <rvk3570@gmail.com>

(‡) Duke University <p.sadowski@duke.edu>

Smith and Whitelaw (2009) find evidence that the largest component of changes in the equity risk premium is variation in risk aversion, rather than the amount of risk, and Bekaert, Engstrom and Grenadier (2010) show that stochastic risk aversion that is not driven by, or perfectly correlated with, the fundamentals of the economy can simultaneously explain a range of asset pricing phenomena as well as the behavior of bond and stock markets.

At the same time, a powerful critique of the use of taste shocks in empirical work notes that taste shocks are typically not directly observable, so that they become free parameters. As Nason (1997) writes, ‘... for taste shocks to have economically meaningful content, they must be grounded solidly in economic theory and tied to observable phenomena, which is not always easy.’

In this paper we build on the axiomatically founded model of taste shocks in Krishna and Sadowski (2014) (henceforth KS), in which those shocks can be uniquely identified from observable behavior. Based on this identification, KS provide comparative statics that link more volatile tastes to greater preference for flexibility, that is, a desire not to commit to future choice ahead of time. We argue that more volatile tastes should correspond to higher price volatility, and a desire to maintain flexibility should lead to a higher liquidity premium.

There is indeed strong evidence of a positive correlation between the liquidity premium and price volatility, for example in Nagel (2012). Existing explanations of this phenomenon understand the demand for liquidity as a reaction to increased volatility, for instance via the assumption that risk aversion increases when volatility is high.<sup>1</sup> We show instead that taste shocks, such as varying risk aversion, can directly drive both, price volatility and the liquidity premium.<sup>2</sup>

To formally make our argument in a very simple model, we consider a representative agent, who is modelled as in KS and receives iid shocks to his aversion to risk in current consumption. We then analyze a small Lucas tree economy with closed asset markets but partially open goods markets, that is, we enrich the most basic Lucas tree economy by adding a second productive asset. Output from the

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(1) See Honarvar (2016) for an extensive discussion of possible explanations.

(2) It has been argued that demand for liquidity may drive up the equity premium, for example by Bansal and Coleman (1996). To the extent that this is accurate, taste shocks would also indirectly drive up the equity premium, and thus provide an additional channel for explaining it.

two assets are perfect substitutes for domestic consumption, but only one of them can be traded on the international market, and so provides more liquidity. Our main set of results establishes that, indeed, more severe taste shocks correspond to higher price volatility (which taste shock models are often used to explain), and they also drive a larger liquidity premium (ie, a higher price of the liquid asset compared to the illiquid output and corresponding asset). This direct link between price volatility and the liquidity premium could provide discipline for the use of taste shock models in applied work.

The basic intuition for the relationship between volatility in tastes and in prices is as follows. The current realized utility (the taste for current consumption) obviously affects the representative agent's valuation of his rights to current output. At the same time, an agent with iid taste shocks values shares of the productive asset independently of the current realized utility. The price for trading shares of the asset against rights to current output should be determined by the difference in these valuations. Hence, more variation in utilities should imply more variation in prices. This intuition is incomplete, because the equilibrium price in the Lucas model depends not on the representative agent's realized utility, but on the realized *marginal* utility in the current output of the productive asset (from the first order condition that ensures that at the equilibrium price all output is consumed). We handle this gap between intuition and model with a simple trick: We interpret shares as probabilistic rights to the entire output, so that utility becomes linear in the share for any given level of output, and the marginal utility of additional probability of receiving the current output is precisely the realized utility of current output.<sup>3</sup>

Now suppose that one productive asset provides less liquid output than another productive asset, as suggested above. We expect preference for flexibility to be associated with a tendency to invest less in the asset with illiquid output and to value that output less. In order to capture this intuition in our simple Lucas tree economy, consider the following variant: There is a green tree that produces a perishable green fruit which is only for domestic consumption. There is also a red tree which produces a perishable red fruit. Both trees produce the same uncertain

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(3) Since our focus is the interplay between taste shocks and liquidity concerns, we chose to keep the model as simple as possible in all other respects. A less immediate, more common way to ensure that higher utility correspond to a higher price would be an appropriate single crossing property on the slope of  $u$ . The use of the linear structure provided by lotteries in equilibrium models has its first precedent in Prescott and Townsend (1984a, 1984b).

amount of fruit, and the two fruit are perfect substitutes domestically. The only difference is that probabilistic rights to some amount of red fruit can be traded internationally for rights to some other amount of red fruit. Therefore, ownership rights to red output provide the agent with more flexibility or ‘liquidity’ than ownership rights to green fruit, and the propensity to invest in the more liquid asset results in a greater price for the this asset — the liquidity premium — which is increasing in the desire for flexibility.

The remainder of the paper is structured as follows. Section 2 introduces the representation as well as the comparative statics from KS. Section 3 analyzes the standard Lucas tree economy without liquidity concerns and relates volatility in tastes to volatility in prices. Section 4 adds liquidity concerns to this economy and presents our main results.

## 2. Taste Shock representation

Let  $K$  be a finite set of prizes with typical member  $k$ . We follow Gul and Pesendorfer (2004) (henceforth GP) in defining an infinite horizon consumption problem (IHCP) as a collection of lotteries that yield a consumption prize in the present period and a new infinite horizon problem starting in the next period. Let  $Z$  be the collection of all IHCPs.<sup>4</sup> GP show that  $Z$  is a compact metric space, and that each  $z \in Z$  can be identified with a compact set of probability measures over  $K \times Z$ . For the compact metric space  $K \times Z$ , let  $\mathcal{P}(K \times Z)$  denote the space of probability measures endowed with the topology of weak convergence, so that  $\mathcal{P}(K \times Z)$  is compact and metrizable. Let  $\mathcal{F}(\mathcal{P}(K \times Z))$  denote the space of closed subsets of  $\mathcal{P}(K \times Z)$ , endowed with the Hausdorff metric, such that  $\mathcal{F}(\mathcal{P}(K \times Z))$  is a compact metric space. It can be shown that  $Z$  is linearly homeomorphic to  $\mathcal{F}(\mathcal{P}(K \times Z))$ . We shall denote this linear homeomorphism as  $Z \simeq \mathcal{F}(\mathcal{P}(K \times Z))$ . Typical elements  $x, y, z \in Z$  are interpreted as *menus* of lotteries over consumption and continuation problems.

KS provide representations of choice over  $Z$  with the understanding that DM will choose from the IHCP he faces in every subsequent period. Take  $p, q$  to be typical lotteries in  $\mathcal{P}(K \times Z)$ . By the recursive nature of  $Z$ , continuation problems are members of  $Z$ . When there is no risk of confusion, we identify prizes

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(4) See GP for the recursive construction of  $Z$ .

and continuation problems with degenerate lotteries and lotteries with singleton menus.

As in KS, for any  $q^* \in \mathcal{P}(K)$ , let  $\mathcal{U}_{q^*} := \{u \in \mathbb{R}^K : \sum u_i q_i^* = 0\}$  be the set of all vN-M utility functions over instantaneous consumption (ie, over  $K$ ) that are identified up to a constant.<sup>5</sup> (In what follows, for notational simplicity, we will drop the parameter  $q^*$  and simply denote the space by  $\mathcal{U}$ , because  $q^*$  will be clear from the context.) The subjective state space relevant for the taste shock representation is  $\mathcal{U}$ . To ensure that expected consumption utility under a measure  $\mu$  is well defined, the measure  $\mu$  must be *nice*, in the sense that the expected utility from each prize is finite, ie,  $\mu u_k := \int_{\mathcal{U}} u_k \, d\mu(u)$  is finite for each  $k \in K$ .

Subjective states  $u \in \mathcal{U}$  are naturally interpreted as consumption utilities, and the two terms are treated as synonyms. Similarly, in what follows, all probability measures are interpreted as subjective beliefs, and the two terms are used interchangeably. For  $p \in \mathcal{P}(K \times Z)$ , let  $p_k$  and  $p_z$  denote the marginal distributions on  $K$  and  $Z$  respectively.

**Definition 2.1.** Let  $\mathcal{U}$  be defined as above,  $\mu$  a nice probability measure on (the Borel  $\sigma$ -algebra of)  $\mathcal{U}$ , and  $\delta \in (0, 1)$ . We say that  $\succsim$  has a *taste shock* representation,  $(\mu, \delta)$ , if there exists a continuous function  $V : Z \rightarrow \mathbb{R}$ , linear on  $Z$ , that satisfies

$$[2.1] \quad V(x) = \int_{\mathcal{U}} \max_{p \in x} [u(p_k) + \delta V(p_z)] \, d\mu(u)$$

and represents  $\succsim$ .

In the representation above,  $u(p_k) = \sum_{k' \in K} p_k(k')u(k')$ , and  $V(p_z)$  is the extension of  $V$  to  $\mathcal{P}(Z)$  by linearity (and continuity), ie  $V(p_z) = \int_Z V(z') \, dp_z(z')$ . KS provide axioms on a preference relation,  $\succsim$ , that are equivalent to the existence of a taste shock representation,  $(\mu, \delta)$ , of  $\succsim$ . Moreover, they establish that  $\delta$  is unique and  $\mu$  is unique up to scaling (Theorem 1 in KS).

**Definition 2.2.** Two probability measures  $\mu$  and  $\mu'$  on  $\mathcal{U}$  are *identical up to scaling*, if there is  $\lambda > 0$  such that  $\mu(E) = \mu'(\lambda E)$  for all measurable  $E \subset \mathcal{U}$ , where  $\lambda E := \{\lambda u : u \in E\}$ .

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(5) KS take  $q^*$  to be the uniform lottery over  $K$ , in which case  $\sum_i u_i q_i^* = \sum_i u_i = 0$ . The particular choice of  $q^*$  is not important for their results. In particular, in the sequel, we will take  $K$  to be a linearly ordered set, and  $q^*$  to be the degenerate lottery over the smallest consumption level.

Preference for flexibility is the preference for non degenerate menus over singletons. Intuitively, one DM has more preference for flexibility than another if she has a stronger preference for menus over singletons. To make this precise requires us to consider the restricted domain  $L \subset Z$  of *Infinite Horizon Consumption Streams (IHCSs)*. This domain consists of lotteries that deliver consumption for the present period and an IHCS for the next period. It is easy to show that  $L$  is a closed and convex subset of  $Z$ . In a manner analogous to the characterization of risk aversion where lotteries are compared to certain amounts of money, characterizing preference for flexibility requires a comparison between IHCPs and IHCSs. This comparison is meaningful only if the preference restricted to  $L$  is non-trivial, ie, there exist  $\ell, \ell' \in L$  such that  $\ell \succ \ell'$ . We refer to this property of  $\succsim$  as *Consumption non-triviality*. If  $\succsim$  has a taste shock representation, KS show that  $\succsim$  satisfies Consumption non-triviality if, and only if,  $\mu u \neq 0$ .

**Definition 2.3.**  $\succsim^*$  has a *greater preference for flexibility* than  $\succsim$  if

$$x \succsim \ell \text{ implies } x \succsim^* \ell$$

for all  $\ell \in L$  and  $x \in Z$ .

Note that the comparison in the definition requires that  $\succsim$  and  $\succsim^*$  rank IHCSs the same; that is,  $\ell \succsim \ell'$  if, and only if,  $\ell \succsim^* \ell'$ .

**Definition 2.4** (Dilation). Let  $Q(u, D)$  be a Markov kernel from  $\mathcal{U}$  to itself.<sup>6</sup> Then  $Q(u, D)$  is a *dilation* if it is expectation preserving, ie, for each  $u \in \mathcal{U}$ ,  $\int_{\mathcal{U}} u' Q(u, du') = u$ . If  $\mu$  and  $\mu^*$  are probability measures on  $\mathcal{U}$ , then  $\mu^*$  is a *dilation* of  $\mu$  if there exists a dilation  $Q$ , such that  $\mu^* = Q\mu$ , ie,  $\mu^*(du') := \int Q(u, du') \mu(du)$ .

The taste shock representation  $(\mu, \delta)$  only identifies the measure  $\mu$  up to scaling. In order to facilitate a comparison of measures, we shall say that a taste shock representation  $(\mu, \delta)$  is *canonical* if  $\|\mu u\|_2 = 1$ . Obviously,  $\succsim$  admits a canonical taste shock representation if, and only if,  $\mu u \neq 0$  if, and only if,  $\succsim$  satisfies Consumption non-triviality.

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(6) That is,  $Q(u, D)$  is the probability of transitioning from the state  $u$  to the (measurable) set  $D \subset \mathcal{U}$ . For each  $u \in \mathcal{U}$ ,  $Q(u, \cdot)$  is a probability measure over  $\mathcal{U}$ , and for each measurable  $D \subset \mathcal{U}$ ,  $Q(\cdot, D)$  is a measurable function from  $\mathcal{U}$  to  $\mathbb{R}$ .

**Theorem 1** (Theorem 2 in KS). *Let  $\succsim$  and  $\succsim^*$  have canonical taste shock representations  $(\mu, \delta)$  and  $(\mu^*, \delta^*)$ , respectively. Then, the following are equivalent:*

- (a)  $\succsim^*$  has a greater preference for flexibility than  $\succsim$ .
- (b)  $\delta = \delta^*$  and  $\mu^*$  is a dilation of  $\mu$ .

Observe that if  $\mu^*$  is a dilation of  $\mu$ , then both have the same expected utility (function), ie,  $\mu u = \mu^* u$ .

### 3. A Lucas Tree Economy

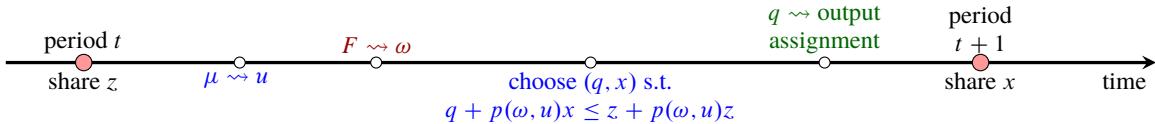
Consider a version of the Lucas tree after Lucas (1978). There is an economy with a representative agent and one productive asset. The asset produces  $\omega \geq 0$  units of perishable output, or dividends, in each period. For simplicity, we assume that output is distributed identically and independently over time, according to the distribution  $F(\omega)$  with finite support. In period 0 there is no production and no trade.

The agent has  $z \in [0, 1]$  shares in the asset, which gives him a proportional right to the output. Specifically, with probability  $z$  he gets all the output, and with complementary probability, he gets none of the output.

There is a market where, once output is known (ie,  $\omega$  is realized), the agent can trade the probabilistic right  $q$  of getting all of the current output and shares of the right to future output. This assumption makes preferences linear in shares and rights to output, which has the following convenient implications.

First, the marginal utility of an extra unit of probability is constant because the value function is linear in probabilities. This makes the model very tractable. Second, if we consider the representative agent to be an aggregation of identical individuals, and if trade is in probabilities of all the output, then individual preferences can be aggregated linearly and the representative agent has the same utility over outcomes as the individuals in the economy, each of which receives a proportional fraction of the total probability of an outcome. This makes it easy to connect individual choice data to the value function of the representative agent.

The price of a unit of  $q$  is normalized to 1 in each state  $(\omega, u)$ , while the price of a share is  $p(\omega, u)$ . The timing of events is illustrated in Figure 1.



**Figure 1: Timeline for the Lucas tree Economy**

Consider an agent whose period 0 preferences have a taste shock representation of choice over IHCPs, where the support of  $F$  is in  $K$ . The value of holding  $z$  shares in the asset is then

$$V(z, \omega, u) = \max_{q, x} \left[ u([q; \omega]) + \delta \iint V(x, \tilde{\omega}, \tilde{u}) dF(\tilde{\omega}) d\mu(\tilde{u}) \right]$$

subject to the budget constraint

$$q + p(\omega, u)x \leq z + p(\omega, u)z$$

where  $[q; \omega]$  is the lottery that gives  $\omega$  with probability  $q$  and 0 with probability  $1 - q$ . Then,  $u([q; \omega]) = qu(\omega) + (1 - q)u(0) = qu(\omega)$  (because  $u(0) = 0$  for all  $u \in \mathcal{U}$ ).

By following the arguments in Lucas, we can show that for each continuous  $p(\omega, u)$ , there exists a unique continuous, bounded, nonnegative function  $V(z, \omega, u)$  that satisfies the Bellman equation above, and which is concave in  $z$ .

We know that in equilibrium, we must have  $q = z + p(\omega, u)z - p(\omega, u)x$ , ie the budget constraint must bind, so that  $u([q; \omega]) = u(\omega)[z + p(\omega, u)z - p(\omega, u)x]$ .

In equilibrium, the optimal choice of  $q$  and  $x$  must satisfy

$$\text{[FOC]} \quad p(\omega, u)u(\omega) = \iint \delta V'(x, \tilde{\omega}, \tilde{u}) dF(\tilde{\omega}) d\mu(\tilde{u})$$

Thus, in equilibrium, the price  $p(\omega, u)$  is the marginal rate of transformation between shares in the productive asset and probabilistic shares of current consumption. Similarly, in equilibrium, the envelope condition is

$$\text{[Env]} \quad V'(z, \omega, u) = u(\omega)(1 + p(\omega, u))$$

Substituting [Env] in [FOC], we find that in equilibrium, the pricing function  $p$  must satisfy

$$\text{[3.1]} \quad p(\omega, u)u(\omega) = \delta \iint \tilde{u}(\tilde{\omega})(1 + p(\tilde{\omega}, \tilde{u})) dF(\tilde{\omega}) d\mu(\tilde{u})$$

Because output and taste shocks are iid, this reduces to

$$p(\omega, u)u(\omega) = \frac{\delta}{1-\delta} \iint \tilde{u}(\tilde{\omega}) dF(\tilde{\omega}) d\mu(\tilde{u}) = \frac{\delta}{1-\delta} \mathbf{E}[\tilde{u}(\tilde{\omega}); F, \mu] =: \Lambda(F, \mu)$$

In other words,  $\Lambda$  is precisely the discounted present value of the asset in utility terms. Because utility and output processes are iid,  $\Lambda$  is state-independent. It is useful to write the price as

$$[3.2] \quad p(\omega, u) = \frac{\delta}{1-\delta} \iint \frac{\tilde{u}(\tilde{\omega})}{u(\omega)} dF(\tilde{\omega}) d\mu(\tilde{u}) = \Lambda(F, \mu)/u(\omega)$$

as this facilitates a comparison with the usual case where the consumption good is traded.<sup>7</sup>

Note that one can just as easily normalize the price of a share to be 1 in each state  $(\omega, u)$ , in which case the price of a unit of  $q$  (the probability for immediate consumption) becomes  $\psi(\omega, u) = 1/p(\omega, u)$ . In what follows, it is more natural to work with the price  $\psi(\omega, u)$ . The solution to the pricing equation can thus be rewritten as

$$\psi(\omega, u) = \frac{u(\omega)}{\Lambda}$$

Suppose, now, that there are only three levels of output, 0,  $1/2$  and 1. We are now in a position to relate the distribution of prices to the distribution of utilities. Consider two exchange economies,  $A$  and  $B$ , with representative agents  $A$  and  $B$  respectively. We assume that both agents have period 0 preferences with a taste shock representation based on the state space  $\mathcal{U}^\dagger \subset \mathcal{U}$ , where  $\mathcal{U}^\dagger = \{u \in \mathcal{U} : u(0) = 0, u(\frac{1}{2}) \in [\frac{1}{2}, 1], u(1) = 1\}$ . Thus, agents are uncertain about their risk aversion, which is captured by  $u(1/2)$  being random in  $[\frac{1}{2}, 1]$ . We also assume that  $\succ^A$  and  $\succ^B$  agree on the intertemporal tradeoff for getting 1 instead of 0, which implies  $\delta_A = \delta_B$ .

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(7) If the consumption good were to be traded, the price process would be  $p(\omega, u) = \frac{\delta}{1-\delta} \iint \frac{\tilde{u}'(\omega')}{u'(\omega)} dF(\omega') d\mu(\tilde{u})$ , where  $\tilde{u}'$  and  $u'$  now represent the marginal utility for the utility functions  $\tilde{u}$  and  $u$  respectively.

Thus, for  $\omega \in \{0, 1/2, 1\}$ , prices are given by

$$\begin{aligned}\psi_i(0, u) &= 0 \\ \psi_i(1, u) &= \frac{1}{\Lambda_i} \\ \psi_i(1/2, u) &= \frac{u(1/2)}{\Lambda_i}\end{aligned}$$

where  $u(1/2) \in [\frac{1}{2}, 1]$ . In both economies the price of a unit (in probability) of consumption is constant across states if output is either 0 or 1. Since utilities are stochastic, we can say something about the distribution of prices in the two economies for the case where output is  $\omega = 1/2$ . We let  $H_i(\lambda) = \mathbf{P}(\psi_i(1/2, u) \leq \lambda)$  denote this distribution in economy  $i$ .

**Proposition 3.1.** For the two economies described above, statements (a) and (b) below are equivalent and imply (c).

- (a) Agent  $B$  has a greater preference for flexibility than agent  $A$ .
- (b)  $H_A$  second order stochastically dominates  $H_B$ .
- (c)  $\psi_A(1, 1) = \psi_B(1, 1)$ .

*Proof.* (a) implies (c): By Theorem 1, it follows that  $\mu^*$  is a dilation of  $\mu$ . But this means that  $\mu_A u = \mu_B u$ , which implies that  $\Lambda_A = \Lambda_B$ . This establishes (c) as claimed.

(a) iff (b): Recall that  $\Lambda_i = (\frac{\delta}{1-\delta}) \int [u(1/2) f_{1/2} + f_1] d\mu_i(u)$ . Then,

$$\mathbf{E}[\psi_i(1/2, u); F, \mu_i] = \frac{\int u(1/2) d\mu_i(u)}{f_1 + f_{1/2} \int u(1/2) d\mu_i(u)} \frac{1-\delta}{\delta}$$

Hence,  $\mathbf{E}[\psi_A(1/2, u); F, \mu_A] = \mathbf{E}[\psi_B(1/2, u); F, \mu_B]$  if, and only if,  $\int u(1/2) d\mu_A(u) = \int u(1/2) d\mu_B(u)$  if, and only if,  $\Lambda_A = \Lambda_B = \Lambda$ . It is then immediate that  $H_A$  second order stochastically dominates  $H_B$  if, and only if,  $\mu_A$  second order stochastically dominates  $\mu_B$ . Applying Theorem 1 to the one dimensional case implies (a) iff (b), which concludes the proof.  $\square$

The proposition says that in our asset pricing model, where preference for flexibility stems from uncertainty about future risk aversion, a greater preference for flexibility corresponds to higher price volatility.

#### 4. A Lucas Tree Economy with Investment

To study the impact of increased preference for flexibility on the liquidity premium, we now consider a Lucas tree economy with *two* trees. The *green tree* produces green fruit and is as previously described. The *red tree* is identical to the green, with the only difference that red fruit can be traded in an international market (neither green fruit nor shares in either of the trees can be traded internationally). On those markets, the probabilistic right to red fruit can be exchanged for probabilistic rights to a different amount of red fruit at a fixed price, so that the output of the red tree provides more liquidity than that of the green tree.

The *liquidity premium* can be measured as the ratio between the prices of the red and green trees. Proposition 4.1 shows that shares in the red tree are more expensive than shares in the green tree. An increase in preference for flexibility keeps the price of green shares fixed, conditional on the realization of  $u$  and  $\omega$ , but increases the price of red shares. At the same time, it increases (in the same sense as in Section 3 above) the volatility of the price of green shares and, after renormalizing the price distributions of red shares to have the same mean, also that of red shares. This is the sense in which an increase in preference for flexibility leads to an increased liquidity premium, and to increased volatility.

To simplify matters, we assume that the output of each tree is random and takes values in  $\{0, \frac{1}{2}, 1\}$  (as before), but that the output is the same across trees (ie, the random output is perfectly correlated across trees). We will also assume (for simplicity) that utility from the consumption of the two fruit is *additively separable*, ie, consuming  $x_c$  units of color  $c$  fruit provides utility  $u(x_c)$ . Moreover, we assume (as before) that it is only  $u(1/2)$  that is random, taking values in  $[\frac{1}{2}, 1]$ , and that in spite of this randomness the two types of fruit are perfect substitutes for domestic consumption, ie, utility is always the same for a particular amount of fruit of either color.

The mechanics of trade in this new environment are crucial. In any period, after output  $\omega$  and taste shock  $u(1/2)$  are realized, probabilistic rights to fruit from both trees can be traded domestically for shares in trees (just as with a single tree). However, only *red fruit* can be traded in an international market later in the day, where probabilistic rights to  $\frac{1}{2}$  unit of the red fruit can be exchanged for probabilistic rights to 1 unit of red fruit at a fixed price of  $\kappa \in [1, 2]$ , and the agent can participate

on either side of this market, depending on the value of  $\omega$ .

The utility from entering the international market with  $\omega$  units of output of red fruit and utility function  $u$  leads to a final (indirect) ‘utility’ of

$$v(\omega, u) = \begin{cases} 0 & \omega = 0 \\ \max[u(1/2), \frac{1}{\kappa}u(1)] & \omega = \frac{1}{2} \\ \max[\kappa u(1/2), u(1)] & \omega = 1 \end{cases}$$

Because  $\kappa > 1$ ,  $v$  cannot be directly interpreted as an expected utility. However, as argued in Section 3, because the representative agent may be thought of as an aggregation of  $N$  identical individuals who share the probabilistic rights to output proportionally, a large enough  $N$  will ensure that the probability with which each individual receives output is well defined. In particular, because the Bellman equation is linear in the probabilities, a very convenient feature of aggregating individuals probabilistically is that the representative agent has the same value function as each individual, up to a scaling which does not affect the solution. For ease of notation, we therefore allow ‘probabilities’ to exceed 1 in the Bellman equation below.

To set up the value function, let  $\mathbf{z} = (z_g, z_r)$  denote shares in the green and red trees respectively,  $\omega$  the current output (in both trees), and  $u$  the current taste shock (only  $u(1/2)$  is uncertain). Let  $p_c$  denote the price of a share in a tree of color  $c$ , and let  $a_r$  denote the price of red fruit (where all prices are functions of  $(\omega, u)$ ). Recall that the price of green fruit is normalized to 1. Standard arguments lead us to the Bellman equation

$$[\text{VF}_2] \quad V(\mathbf{z}, \omega, u) = \max_{\mathbf{q}, \mathbf{x}} \left[ q_g u(\omega) + q_r v(\omega, u) + \delta \iint V(\mathbf{x}, \omega', u') dF(\omega') d\mu(u') \right]$$

subject to the budget constraint

$$[4.1] \quad q_g + a_r q_r + p_r x_r + p_g x_g \leq z_g + a_r z_r + p_g z_g + p_r z_r$$

where  $\mathbf{q} = (q_g, q_r)$  and  $\mathbf{x} = (x_g, x_r)$  denote the vectors of shares in fruit and trees respectively.

Notice the extra  $q_r v(\omega, u)$  term in  $[\text{VF}_2]$ , which denotes the extra (flow) utility from being able to trade red fruit in the international market. This is the only difference from the one tree case.

Clearly, in equilibrium, the budget constraint [4.1] will bind with equality. Substituting for  $q_g$  in [VF<sub>2</sub>], we find the first order condition for  $q_r$  to be

$$[\text{FOC-}q_r] \quad a_r = \frac{v(\omega, u)}{u(\omega)}$$

The first order condition for shares  $x_c$  in a tree of color  $c$  is

$$[\text{FOC-}x_c] \quad p_c u(\omega) = \delta \iint \partial_c V(\mathbf{x}, \omega', u') dF(\omega') d\mu(u')$$

where  $\partial_c V(\mathbf{x}, \omega, u)$  denotes  $\partial V / \partial x_c$  for  $c = g, r$ . Similarly, the envelope conditions are

$$\begin{aligned} [\text{Env-}z_g] \quad \partial_g V(\mathbf{z}, \omega, u) &= u(\omega)(1 + p_g) \\ [\text{Env-}z_r] \quad \partial_r V(\mathbf{z}, \omega, u) &= u(\omega)(a_r + p_r) \end{aligned}$$

Combining the first order conditions for  $x_c$  and the envelope conditions, we find the pricing equation for  $p_g$  to be

$$p_g(\omega, u) u(\omega) = \delta \iint u'(\omega') [1 + p_g(\omega', u')] dF(\omega') d\mu(u')$$

which leads to

$$[4.2] \quad p_g(\omega, u) u(\omega) = \Lambda_g(F, \mu) := \frac{\delta}{1 - \delta} \mathbf{E}[u(\omega); F, \mu]$$

The pricing equation for  $p_r$  can be written as

$$p_r(\omega, u) u(\omega) = \delta \iint u'(\omega') [a_r(\omega', u') + p_r(\omega', u')] dF(\omega') d\mu(u')$$

which leads to

$$[4.3] \quad p_r(\omega, u) u(\omega) = \Lambda_r(F, \mu) := \frac{\delta}{1 - \delta} \mathbf{E}[v(\omega, u); F, \mu]$$

To study the effect of greater preference for flexibility, consider two economies  $A$  and  $B$ , with representative agents  $A$  and  $B$ , where  $B$  has a greater preference for flexibility than  $A$ . As in Section 3, we now appropriately define inverse asset prices  $\psi_{c,j}$  for color  $c \in \{g, r\}$  in economy  $j \in \{A, B\}$ , depending on the level of output

$\omega$  and utility  $u$ . In particular, we compare the distributions  $H_{c,j}(t) := \mathbf{P}_j(\psi_{c,j} \leq t)$  across the two economies for each  $c \in \{g, r\}$  in terms of second order stochastic dominance.

First for the green asset, which cannot be traded on international markets, we can define, exactly as before,

$$[4.4] \quad \psi_g(\omega, u) = \frac{1}{p_g(\omega, u)} = \frac{u(\omega)}{\Lambda_g}$$

Notice that the indirect utility from consuming red fruit is  $v(1/2, u) = \max[u(1/2), 1/\kappa]$  when  $\omega = \frac{1}{2}$  (recall that  $u(1) = 1$  with certainty). Thus, the distribution of future indirect utilities is truncated, and a mean preserving spread of  $u(1/2)$  will no longer leave the mean of the indirect utility  $v(\omega, u)$  unchanged.

The *price*,  $p_r(\omega, u)$ , of the red asset as noted in [4.3], depends on the distribution of expected indirect utility. As a consequence, the price distributions for  $\omega = \frac{1}{2}$  in the two economies generally have different means. We will compare relative volatility of the price as a fraction of the respective means, that is, we rescale the price distributions, so that for  $\omega = \frac{1}{2}$ , they have the same mean, and a comparison in terms of second order dominance now becomes possible. Towards that end, let

$$[4.5] \quad \psi_{r,j}(\omega, u) = \frac{1}{p_{r,j}} \frac{1}{\Lambda_{r,-j}} = \frac{u(\omega)}{\Lambda_{r,A}\Lambda_{r,B}}$$

and note that for  $\omega = \frac{1}{2}$  the mean of this random variable is

$$\int \psi_{r,j}(\frac{1}{2}, u') d\mu(u') = \frac{\int u'(\frac{1}{2}) d\mu_j(u')}{\Lambda_{r,A}\Lambda_{r,B}}$$

which is indeed independent of  $j \in \{A, B\}$  because  $\mu_B$  is a dilation (ie, mean preserving spread) of  $\mu_A$ , so that both have the same mean.

For  $c \in \{g, r\}$  we can now compare the cumulative distribution  $H_{c,j}(t) := \mathbf{P}_j(\psi_{c,j}(\frac{1}{2}, u) \leq t)$  for  $j = A$  to that for  $j = B$ .

**Proposition 4.1.** For the two economies described above, statements (a) and (b) below are equivalent and imply (c).

- (a) Agent  $B$  has a greater preference for flexibility than agent  $A$ .
- (b)  $H_{c,A}$  second order stochastically dominates  $H_{c,B}$  for  $c \in \{g, r\}$ .

(c)  $\psi_{g,A}(\omega, u) = \psi_{g,B}(\omega, u)$  and

$$\frac{p_{r,B}(\omega, u)}{p_{r,A}(\omega, u)} = \frac{\Lambda_{r,B}}{\Lambda_{r,A}} > 1 = \frac{\psi_{r,A}(\omega, u)}{\psi_{r,B}(\omega, u)}$$

Thus, as in the single tree case in Section 3, greater preference for flexibility corresponds to greater dispersion of (relative) price and greater volatility (in the sense of second order stochastic dominance). For green trees, this is the same result as in Proposition 3.1. For red trees the claim is more involved.

*Proof of Proposition 4.1.* As noted above, that (a) implies (c) is exactly as in the proof of Proposition 3.1, as is the claim of that (a) and (b) are equivalent for green trees. That  $\psi_{c,A}(\omega, u) = \psi_{c,B}(\omega, u)$  for  $c = g, r$  follows immediately from equations [4.4] and [4.5] respectively. In particular, *conditional on the realization of the taste shock and endowment*, the (appropriately scaled) price is the same in each economy.

To see that  $p_{r,A}/p_{r,B} = \Lambda_{r,A}/\Lambda_{r,B} < 1$ , notice that the equality follows immediately from [4.3], while the inequality amounts to  $\mathbf{E}[v(\omega, u); F, \mu_A] \leq \mathbf{E}[v(\omega, u); F, \mu_B]$ , which is true because  $\mu_B$  is a dilation of  $\mu_A$  and  $v$  is convex in  $u$ .  $\square$

Part (c) establishes that for every realization of  $u$  and  $\omega$ ,  $p_{r,B}(\omega, u) > p_{r,A}(\omega, u)$ , ie, the red tree is more expensive in economy  $B$  than  $A$  (recall that the price  $p_{c,j}$  is the ‘raw’ price, not the scaled version). Because rights to red fruit provide more liquidity than rights to green fruit (they can be traded on international markets) this ratio greater than unity can be understood as a liquidity premium.

The greater value for liquidity in economy  $B$  is also reflected in the expected price ratio between red and green fruit in each of the economies. The relative price of rights to the red compared to the green fruit is simply  $a_{r,j}$  in economy  $j \in \{A, B\}$ .

**Proposition 4.2.** The price  $a_{r,j}$  of red fruit relative to green fruit is weakly bigger than 1 for all  $u$  and  $\omega$ . Moreover, for  $\omega \neq 0$  greater preference for flexibility in economy  $B$  than in economy  $A$  means that, on average, the price of red fruit is higher in economy  $B$  than in  $A$ ,

$$\int a_{r,A}(\omega, u) d\mu_A(u) \leq \int a_{r,B}(\omega, u) d\mu_B(u)$$

for  $\omega \in \{1/2, 1\}$ .

*Proof.* As seen in [FOC- $q_r$ ],  $a_{r,j}(\omega, u) = v(\omega, u)/u(\omega)$ . As noted above, for  $\omega > 0$ ,  $v(\omega, u) \geq u(\omega)$  and  $v(\omega, u)$  is convex in  $u(1/2)$ . Notice that

$$a_{r,j}(1/2, u) = \frac{\max[u(1/2), u(1)/\kappa]}{u(1/2)} = \max[1, u(1)/\kappa u(1/2)]$$

But the function  $t \mapsto \max[1, 1/\kappa t]$  is the maximum of two convex functions, and hence is convex. This implies (recall that  $u(1) = 1$ )  $a_{r,j}(1/2, u)$  is convex in  $u$ . That  $\int a_{r,A}(\omega, u) d\mu_A(u) \leq \int a_{r,B}(\omega, u) d\mu_B(u)$  now follows immediately from the fact that  $\mu_B$  is a dilation (mean preserving spread) of  $\mu_A$ .  $\square$

In words, if agent  $B$  has more severe taste shocks, which is equivalent by Theorem 1 to a greater preference for flexibility, then by Proposition 4.1 there is more price volatility in economy  $B$  and the liquidity premium for red trees is larger in economy  $B$ . By Proposition 4.2, in both economies there is a non-trivial liquidity premium (whenever  $u(1/2) > 1/\kappa$ ) for the more liquid red fruit over green fruit, and this premium is higher in economy  $B$ . The two propositions illustrate how taste shocks drive not only the price volatility they are often employed to explain, but also a liquidity premium.

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