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A PROBLEM WITH REFERENDUMS

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ABSTRACT

When some voters have nonseparable preferences across multiple binary issues, majority rule may not select a Condorcet winning set of outcomes when one exists, and the social choice may be a Condorcet loser or Pareto-dominated by every other set of outcomes. We present an empirical example of one such paradox from voting on the Internet. We evaluate potential solutions to the problem of nonseparable preferences in referendums, including set-wise voting, sequential voting, and vote-trading. Sequential voting and vote-trading prevent the selection of Condorcet losers and universally Pareto-dominated outcomes. Legislatures facilitate sequential voting and vote-trading better than referendums, suggesting that referendums increase the quantity of participants in democratic decision-making but decrease the quality of participation.

KEY WORDS • Condorcet winner • nonseparable preferences • referendums • sequential voting • strategic voting • vote-trading

The resurrection of direct democracy through referendums[1] is one of the clear trends of democratic politics. As Brian Beedham (1993: 5) writes: 'The difference between today's politics and the politics of the coming century is likely to be a change in what people mean by "democracy"...this overdue change is a shift from "representative democracy" to "direct democracy".' Beedham (1993: 7) welcomes the trend, reasoning that 'direct democracy...leaves no ambiguity about the answer to the question: What did the people want?'

Many social scientists have extolled the virtues of direct democracy through referendums and initiatives. Romer and Rosenthal (1979) argue that referendums restrain public agencies from overspending. Matsusaka (1995) demonstrates empirically that public spending is lower in states that have public initiatives than in states using only representative democracy. Matsusaka (1995: 618) further reasons that 'under the assumption that fiscal outcomes in initiative states

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1. Most dictionaries list both 'referendums' and 'referenda' as acceptable plural forms of 'referendum'. We adopt the convention of the *Oxford American Dictionary*: 'The Latin word *referendum* (= referring) has no plural in Latin, so careful writers prefer to use *referendums* as the plural in English.'

are close to median voter outcomes, the results in this paper can be interpreted as evidence that legislatures do not implement median voter outcomes'. Gerber develops a unidimensional model to show that the threat of popular initiative moves public policy closer to the position of the median voter in states that allow initiatives. Gerber (1996: 125) concludes that 'when laws are made by initiative, given the at-large counting procedures and the nature of majority rule, the resulting policies are expected to reflect the majority of voters in the population'. All of these papers assume that referendums produce outcomes preferred by a majority of voters.

Many people oppose direct democracy. Opponents of referendums are usually known for their distrust of the abilities of the average citizen to make political decisions. They argue that voters are often uninformed about important issues and unable to make reliable decisions. They worry about the susceptibility of the public to well-rehearsed advertisements and well-financed campaigns by special interests. They also worry about the impact of the initiative and referendum on other political institutions, such as parties and legislatures. While most opponents of referendums indict the ability of the average citizen to comprehend politics, few opponents indict the ability of referendums to represent the average citizen. In short, the problem is the people, not the referendum.

We offer a different indictment of referendums as a means to reflect accurately the preferences of voters. When multiple issues appear on a ballot, referendums as currently practiced force people to separate their votes on issues that may be linked in their minds. Our particular concern in this paper is that when voters have *nonseparable preferences* for issues on a referendum, the common practice of tallying votes one issue at a time can fail to select an overall Condorcet winner, when one exists. Worse yet, referendum voting can select an overall Condorcet loser or an outcome that is Pareto-dominated by every other possible outcome. This is indeed a perversion of majority rule, and it undermines claims that referendums reflect 'what the people want'. Our results extend beyond Arrovian arguments that no voting rule can guarantee a consistent and fair social welfare ordering (Arrow, 1951). Instead, we demonstrate that even in cases where voter preferences are such that a Condorcet-winning social choice exists, referendum voting may fail to select such an outcome.

After demonstrating several paradoxes of voting on multiple issues, we consider alternative methods for guaranteeing the selection of Condorcet-winning outcomes, when they exist, and for avoiding the selection of Condorcet losers and universally Pareto-dominated outcomes. Issue-by-issue voting, sophisticated voting, and vote-trading emerge as the best hope for selecting Condorcet winning outcomes. Legislatures are better able than referendums to facilitate issue-by-issue voting, sophisticated voting, and vote-trading. While referendums increase the *number* of participants in decision-making, they decrease the *quality* of participation by preventing voters from coordinating votes and voting issue-by-issue. Legislatures may decrease the *number* of

participants, but they increase the *quality* of participation by allowing vote-trading and issue-by-issue voting.

Definitions and Notation

We define a referendum as a vote on a set of ordered binary issues, 1,...,*n*, where the outcome of each issue is decided by majority vote. Any issue, *k*, on a referendum has two possible outcomes, $o_k = \{x_k, x'_k\}$ where x_k can be either passage or failure of issue *k*, and x'_k is its inverse. X is an *n*-tuple of outcomes, one on each of the (*n*) issues, where $X \in \mathbf{X} = \times_{k=1}^{n} o_k$, $|\mathbf{X}| = 2^n$. To simplify notation, we denote X_{-k} as an $(n-1)$ tuple of outcomes, one on each of the *n* issues, except issue *k*. There are *I* voters, 1,...,*I*, each of whom casts one vote for each *k*. Referendum outcomes are determined by majority rule: x_k is the social choice if more than one half of the voters vote for x_k over x'_k.[2]

To define formally the different types of preferences voters may have, we assume that every voter has a strict preference relation, f_i, over any pair of *n*-tuples of vote outcomes, $X, Y \in \mathbf{X}$.

DEFINITION 1: *i's preferences are* separable *if and only if for every issue k, if*

$$(x_k, X_{-k})\ \mathrm{f}_i\ (x'_k, X_{-k}) \textit{ for any } X \in \mathbf{X}, \textit{ then}$$

$$(x_k, Y_{-k})\ \mathrm{f}_i\ (x'_k, Y_{-k}) \textit{ for all } Y \in \mathbf{X}$$

DEFINITION 2: *i's preferences are* completely nonseparable *if and only if for every issue k there exists an X_{-k} and Y_{-k} such that*

$$(x_k, X_{-k})\ \mathrm{f}_i\ (x'_k, X_{-k}) \textit{ and } (x'_k, Y_{-k})\ \mathrm{f}_i\ (x_k, Y_{-k})$$

DEFINITION 3: *i's preferences are* partially nonseparable *if and only if for some but not all issues k there exists an X_{-k} and Y_{-k} such that*

$$(x_k, X_{-k})\ \mathrm{f}_i\ (x'_k, X_{-k}) \text{ and } (x'_k, Y_{-k})\ \mathrm{f}_i\ (x_k, Y_{-k})$$

If a person has separable preferences, then if s/he prefers x to x' as the outcome on some issue, *k*, s/he always prefers x to x' regardless of the outcome on other issues. If, however, a person's preference for the outcome of issue *k* depends on the outcome on other issues, then her/his preferences are nonseparable. If her/his preferences are partially nonseparable, then there are only some issues where her/his preference depends on the outcome on other issues. If her/his preferences are completely nonseparable, then on every issue her/his preference depends on the outcome on other issues.

We also define the rules governing voting.

2. In most referendums a proposal must earn $\frac{I}{2}+1$ votes in order to pass, though in some settings ties may be broken randomly.

DEFINITION 4: *A vote on multiple issues is* simultaneous *if the outcome of the vote on one issue is not revealed to voters before the vote on other issues.*

DEFINITION 5: *A vote on multiple issues is* sequential *if the outcome of the vote on one issue is revealed to voters before the vote on other issues.*

Sequential voting is also known as issue-by-issue voting (Schwartz, 1977; Ordeshook, 1986; Kramer, 1972), bill-by-bill voting (Miller, 1995) or a Plott–Levine agenda (Plott and Levine, 1978; Miller, 1995).

What Did the People Want?

The usual justification for referendum voting is to provide an answer to the question 'What did the people want?' When people vote on a single issue with three or more alternatives, the answer to this question may be unclear given the many well-known voting paradoxes. In particular, we know that a Condorcet winner may not exist (Condorcet, 1770 [1785]), and, even if one does, it may not be chosen under plurality rule or a runoff (Borda, 1781).[3]

When an issue has only two alternatives, majority rule has many attractive properties (May, 1952).[4] Majority voting on a binary issue eliminates Condorcet's paradox since the vote is decisive and the outcome is preferred to the alternative by at least half of the voters. If voter preferences are representable by single-peaked and symmetric utility functions in a one-dimensional space ($\Re^1$), then the outcome of majority rule will be the position of the median voter, which is the Condorcet winner (Black, 1958). When, as in a referendum, voters are presented with only two alternatives from $\Re^1$, the alternative closest to the median voter's position is the social choice. Most analyses of referendums base their results on single-dimensional models, implying that voters vote on a single issue (Romer and Rosenthal, 1979; Gerber, 1996).

Problems arise from majority voting on multiple issues. Most of the literature focuses on the improbable existence of a stable outcome (a majority rule equilibrium, Condorcet winner, or core) in multi-dimensional choice spaces ($\Re^2$ and higher). When preferences over issues are representable in Euclidean n-space, $n \geq 2$, then a Condorcet winner exists only for restrictive distributions of voter ideal points (Black and Newing, 1951; Davis et al., 1972; Enelow and Hinich, 1984; Plott, 1967). Worse yet, in the absence of a Condorcet winner, many outcomes may be attainable via some sequence of votes (McKelvey, 1976; Schofield, 1978), including Pareto-dominated outcomes.

3. See Nurmi (1998) for a discussion of these and other paradoxes of voting.

4. We distinguish majority rule on binary issues from majority rule in general. Majority rule may be applied to more than two alternatives in, for example, a run-off election in which the winner must have at least one-half plus one of all votes cast.

The lack of preference-induced equilibria in majority voting generated interest in the role of institutions for producing stability. A large literature demonstrates that even in the absence of a Condorcet winner, the outcome of majority voting may be stable under certain institutions, such as issue-by-issue voting (Kramer, 1972) or committee voting (Shepsle, 1979; Shepsle and Weingast, 1987; see Miller, 1995, or Ordeshook, 1986 for a review).

Even though Condorcet winners may be rare on multiple issues, voters should expect majoritarian voting rules to select them, when they exist, since the most common rationale for majority rule is to reveal 'what the people want'. A Condorcet winner is a stable outcome in the sense that no majority coalition of voters can be constructed to overturn the outcome in favor of another outcome that appeared on the same ballot. As Schwartz (1977: 1002) writes, 'the very oldest problem of social choice theory – the principal problem addressed by Borda, Condorcet, Dodgson, and Nanson – is precisely to find a voting rule that picks a stable outcome when one exists'. Voting rules should also avoid selecting Pareto-dominated (inefficient) outcomes. Throughout this paper, we focus on the conditions under which majority rule applied to multiple binary issues can be expected to choose Condorcet winners and avoid the selection of Condorcet losers or Pareto-dominated outcomes.

Several important papers describe the outcome of majority voting on multiple issues when all voters have separable preferences. Kadane (1972) proved that division of the question, or counting votes separately on every issue, will always select a Condorcet winner when one exists, assuming all voters have separable preferences. Schwartz (1977) proved that if a Condorcet winning set of outcomes exists on multiple binary issues, then majority rule will select the overall Condorcet winner regardless of whether voters are sophisticated or sincere, and regardless of whether voting is simultaneous or sequential.[5] Schwartz's result stands as an important justification for referendum voting. If a Condorcet winner exists, then presumably a referendum will select it, and we can rest assured that referendum voting reveals 'what the people want'.

Kramer (1972) also established that when votes are cast issue-by-issue, then the overall Condorcet winner is chosen under majority rule. Kramer's result specifically assumes sophisticated voters whose preferences are representable in a Euclidean space, though the result also applies to binary votes on multiple issues where preferences may not be spatially representable.

Thus far we have focused on results pertaining to the selection of Condorcet winners when they exist. Vote outcomes may be judged on other criteria, such as the selection of Pareto-optimal (efficient) outcomes. When voter preferences are single-peaked in $\mathfrak{R}^1$, the majority rule outcome is Pareto-optimal, though it is but

5. Schwartz (1977) applied his result to voting in legislatures, showing that if a Condorcet winner exists for a set of proposals, then the only stable vote trade is the trade that produces the Condorcet winner. Therefore, one should anticipate that vote trading in legislatures produces sub-optimal outcomes only when a Condorcet winner does not exist.

one of many Pareto-optimal outcomes.[6] In a multi-dimensional Euclidean space, if a Condorcet winner exists, it is chosen under majority rule and is one of many Pareto-optimal outcomes. In the absence of a Condorcet winner, the majority rule outcome may be Pareto-dominated (Kadane, 1972; McKelvey, 1976; Schofield, 1978).

An additional restriction on preferences is sufficient to guarantee that a Pareto-optimal outcome is selected by a referendum vote. Benoit and Kornhauser (1994: Theorem 5, p. 189) prove that if voter preferences are not only separable but also top lexicographic and indexed by the same issue – that is, all voters agree on the order of importance of the issues, though they may disagree on the best outcome – then a Pareto-optimal outcome will be selected by majority voting on binary issues. Assuming that all voters have top lexicographic preferences indexed by the same issue is a very restrictive assumption about voters in a referendum. Thus, referendums risk producing Pareto-dominated outcomes when all voters have separable preferences.

The Problem of Nonseparable Preferences

An important restriction on voter preferences appears in much of the literature on majority voting on multiple binary issues: all voters must have separable preferences across the outcomes of the issues (Kadane, 1972; Schwartz, 1977). When a person has nonseparable preferences, what s/he wants on one issue depends on the outcome of or the options available on another issue. Conversely, when a person has separable preferences, what s/he wants on each issue is independent of the outcome of any other issue.

Assuming that people have separable preferences eliminates *most* of the preference orderings that people could hold, including many that are intuitively plausible. For the case of two YES or NO issues, any of four outcomes is possible: YY, NY, YN, and NN. When we assume that voters have separable preferences, then for any first preference only two complete orderings are possible. For a first preference of YY, the remaining ordering must be [YN f NY f NN] or [NY f YN f NN]. For the first preference of NY, only [YY f NN f YN] or [NN f YY f YN] are possible complete orderings. Whatever a person's first preference, the inverse of it must be her/his last preference to maintain a separable ordering.[7] For two issues, only eight preference orderings are separable while 16 preference orderings are nonseparable. Examples of nonseparable orderings include all-or-nothing preferences such as [YY f NN f NY f YN], nothing-or-all preferences such as [NN f YY f NY f YN], and one-or-the-other

6. Any outcome contained in the set bounded by the left-most voter's ideal point and the right-most voter's ideal point is Pareto-undominated.

7. The inverse property is both a necessary and a sufficient condition for separable preferences in the case of two issues, but only a necessary condition with three or more issues.

preferences such as [NY f YN f YY f NN]. To assume separable preferences is to eliminate two-thirds of the possible preference orderings across two issues. Many of these preference orderings are intuitively plausible and probably common in politics.[8]

With three issues, eight different first preferences are possible, leaving 8! = 40,320 different strict preference orderings. Of the possible orderings, only 384, less than 0.01 percent, are separable.[9] With more than three issues, only a tiny fraction of a percent of preference orderings are separable.

The outcome of majority voting by voters with separable preferences is well understood. Kadane (1972) and Schwartz (1977) demonstrate that if an overall Condorcet winner exists, it will be the social choice. However, the outcome of majority voting may be Pareto-inefficient (Kadane, 1972), except in the case where all voters have separable and top lexicographic preferences indexed by the same issue (Benoit and Kornhauser, 1994).

The outcome of majority voting when some voters have nonseparable preferences is not well understood. In the remainder of this paper, we demonstrate how unstable and inefficient referendum-style voting may be when some voters have nonseparable preferences: a Condorcet winner may not be chosen when one exists, and a Condorcet loser may be the social choice. In Table 1, a referendum is held on two issues, where YN indicates approval of Issue 1 and failure of Issue 2. Voters can vote YES or NO on each issue, and their preference ranking is given for outcomes on both issues.

Table 1. A Referendum Produces an Outcome Ranked Last by a Majority

Rank	Voter 1	Voter 2	Voter 3
1	YN	NY	NN
2	YY	YY	YY
3	NY	YN	NY
4	NN	NN	YN

If each voter votes according to her/his first preference, then the outcome is NN. Voters 2 and 3 vote N on Issue 1 and Voters 1 and 3 vote N on Issue 2. This is the behavior we might expect in an election where voters do not know each

8. A large literature in economics demonstrates that people cannot be assumed to have separable preferences on public spending issues (Denzau and Parks, 1977, 1979, 1983; Diba and Feldman, 1984; Mackay and Weaver, 1983; Mackay and Whitney, 1980; Slutsky, 1977). Diba and Feldman (1984) demonstrate that even on a single-dimensional public spending issue, consumer utilities are unlikely to be strictly quasiconcave (single-peaked), thus upsetting the conditions for a one-dimensional majority-rule voting equilibrium (Black, 1958). Brams et al. (1997) present several examples of nonseparable preferences.

9. For n issues, 2^n different outcomes are possible, leaving $2^n!$ different strict preference orderings.

other's preferences. The problem is apparent: YY is the Condorcet winner, NN is the Condorcet loser. For majority voting to select a Condorcet loser over a Condorcet winner is certainly perverse. Referendum voting leaves an ambiguous, if not completely wrong, answer to the question 'What did the people want?' We offer this as Result l.

RESULT 1: *If some voters have nonseparable preferences for the outcomes of votes across multiple issues, if voters do not know each other's preferences, and if the issues are decided simultaneously by majority rule with binary votes on separate issues, then a Condorcet winner may not be chosen, and the outcome may be a Condorcet loser.*

While the conditions that establish Result l may seem extreme, they are precisely the conditions that govern most referendums. Voters have YES or NO votes across multiple issues, but they do not know the outcome of any one issue until all votes are cast. Due to the size of the electorate, voters do not know the preferences of others and thus do not have the information needed to vote strategically. Methods of scoring on referendums are based on the premise that *all* voters have separable preferences over multiple issues, or that when a person registers a YES vote on an issue, s/he wants that issue to pass regardless of the outcome on other issues. This paradox of referendums arises only when some voters have nonseparable preferences.

Brams et al. (1998) define a different paradox of voting on multiple issues. Their 'paradox of multiple elections' occurs when the winning overall outcome is supported by the fewest number of voters. In Table 1, the outcome of the election is not a paradox according to Brams et al. since the winning outcome, NN, is supported by the largest number of voters, though it ties YN and NY. Conversely, if YY (the Condorcet winner) were chosen, Brams et al. would call the result a paradox since YY is supported by fewer voters (none) than any other outcome.[10]

The outcome of a referendum may be even worse than Result l suggests. Consider the case of a bond referendum in Table 2, where three voters must vote on three bonds. All three voters impose a budget constraint of two bonds; they want two bonds to pass, but they disagree on which two. All voters prefer the passage of any two bonds to the passage of any one bond, and all voters rank last the passage of all three bonds. The voters may believe that passage of three bonds will increase state debt and raise taxes while passage of one or two bonds will not create an unreasonable financial burden on the state.

When voters do not know each other's preferences and vote for their most preferred outcome, the method of scoring results in the passage of all three issues even though no voter wants all three to pass. In other words, the referendum

10. Nurmi (1997) also describes a 'referendum paradox' when voters in a nonbinding consultative referendum choose an outcome that differs from the legislature's vote on the same issue.

Table 2. A Referendum Produces an Outcome Ranked Last by Everyone

Rank	Voter 1	Voter 2	Voter 3
1	YYN	YNY	NYY
2	YNY	NYY	YYN
3	NYY	YYN	YNY
4	NNY	NYN	YNN
5	YNN	YNN	NYN
6	NYN	NNY	NNY
7	NNN	NNN	NNN
8	YYY	YYY	YYY

selects an outcome that is Pareto-dominated by every other possible outcome, a result we offer as Result 2.

RESULT 2: *If some voters have nonseparable preferences for the outcomes of votes across multiple issues, if voters do not know each other's preferences, and if the issues are decided simultaneously by majority rule with binary votes on the separate issues, then the social choice may be Pareto-dominated by all other outcomes.*

Such an outcome is not possible when all voters have separable preferences.

RESULT 3: *If all voters have separable preferences, then the social choice will not be Pareto-dominated by all other outcomes*. [See Appendix I for proof]

Even though it is possible that a Pareto-inefficient outcome will be chosen when all voters have separable preferences (Kadane, 1972), it is not possible that a universally Pareto-dominated outcome will be chosen. However, if some voters have nonseparable preferences, then a universally Pareto-dominated outcome may be the social choice.

Results l and 2 demonstrate the failure of the referendum as a method of collective choice. Not only does a referendum not guarantee the selection of a Condorcet winner when one exists, it may also lead to the selection of a Condorcet loser or a universally Pareto-dominated outcome. A crucial practical question thus arises: When do voters have nonseparable preferences? Ballots from a real-world referendum cannot determine whether voter preferences are separable since ballots do not reveal complete preference rankings. To determine if preferences are separable, we need at least a partial ordering of voter preferences across the outcomes of multiple issues. An example from voting on the Internet provides the kind of information one needs to determine if voter preferences are separable.

In March 1994 a vote was conducted via the Internet to determine whether two newsgroups – one mediate and one unmediated – should be established for

fans of fantasy professional wrestling.[11] Interested individuals could record a YES or NO vote on each of the proposed newsgroups, thus every voter cast up to two votes. For a newsgroup to gain admission to Usenet, it must receive twice as many YES votes as NO votes.

Votes are recorded for all of the 160 persons who voted on this Internet referendum.[12] Both newsgroups failed to achieve the necessary margin of YES votes. The rejection of both newsgroups appears to have been a result of the inability of the voting procedure to capture nonseparable preferences. Of the votes cast, 78 were for both newsgroups to pass (YY) and 48 were for neither newsgroup to pass (NN). The problem for the voting procedure, and for interpreting this example, rests in the 34 other votes. Of these 34 votes, 15 were for the first newsgroup to pass and the second to fail (YN), nine were for the opposite (NY), five were abstentions on the first group and YES on the second (–Y), three were YES votes on the first group and abstentions on the second (Y–), and two were NO on the first group and abstentions on the second (N–). Table 3 summarizes the vote.

Table 3. Voting on Internet Newsgroups

Newsgroup	YES	NO	Outcome
rec.sport.pro-wrestling.fantasy	96	59	Failed
rec.sport.pro-wrestling.info	92	83	Failed

Usenet voting procedures require separate, simultaneous votes on questions that, at least in this example, may not be separable to voters. If all voters held separable preferences for the two questions, then all YY voters would have NN as their last preference and YN and NY as their second and third preferences, in either order. All NN voters would have YY as their last preference and YN and NY in second and third place. All NY voters would have YN as their last choice (NN and YY could be in any order as second and third preferences), and all YN voters would have the opposite ranking. To determine if voter preferences are

11. Information on Usenet votes appears in the news group news.groups and on the Internet at http://www.clark.net/pub/usenet-i/www/info-center-faq.html. In mediated newsgroups a mediator filters postings that are redundant with other postings, irrelevant to the group, or obscene. Unmediated newsgroups have no such screening of messages. The authors did not participate in the vote on these newsgroups. We uncovered it while searching news.groups for recent votes. Some readers may consider this example frivolous. However, the vote was the only recent vote on Usenet involving two related newsgroups. More generally, the Internet provides an ideal place to gather data about voting since it is costly, if not impossible, to gather ballots from a real-world referendum and then to survey voters about their preference on each issue conditional on the passage or failure of other issues.

12. The vote moderator counted 161 valid votes, but one of these was an abstention on both questions, which we do not count.

separable, we need a rank ordering of preferences from the voters who voted in the election. Votes on the Internet prove useful since the e-mail addresses of all voters are provided as public record along with their votes. We sent a short survey (see Appendix 2) to a sample of the voters to determine their full preference ranking for combinations of outcomes of the vote.[13]

The survey shows that many voters had nonseparable preferences for the adoption of the newsgroups. All NN and YY voters who responded to the survey had separable preferences for the outcomes. NN voters least preferred YY while YY voters held the reverse preference. Although neither of the two N– voters responded to the survey, it is probably safe to assume that these voters preferred YY least of the four possible outcomes of the vote. The interesting voters for present purposes are the NY, YN, Y–, and –Y voters. Eight of these voters responded to the survey, and all ranked NN last in their preference ordering. While their positioning of YY, YN, and NY varied, it is clear that these voters wanted at least one of the newsgroups to pass. Extrapolating the results of our survey to the remaining voters suggests that voter preferences can be summarized as in Table 4.

Table 4. Voter Preferences (Extrapolated from Survey)

Rank	78 Voters	50 Voters	18 Voters	14 Voters
1	YY	NN	YN	NY
2	YN/NY	NY/YN	YY	YY
3	NY/YN	YN/NY	NY	YN
4	NN	YY	NN	NN

According to the results of the survey, had the vote paired NN against YY, YY would have won with 110 votes to 50. In a paired comparison, YY beat NN by the two-thirds majority required by Usenet procedures. Were NN paired against YN (or NY), all YY, YN, NY, –Y, and Y– voters would vote for either YN (or NY) and NN would again lose by at least a two-to-one margin. Similar to Example 1 at the beginning of this paper, NN was the choice according to the voting procedure, but it was dominated by a two-to-one margin by every other

13. We sent the survey in July 1994 to a sample of 52 respondents, including all 32 respondents who voted YN, NY, –Y, or Y–. Our purpose was to determine the relative rankings of YY and NN across all respondents. We randomly selected 20 of the YY and NN voters, and we received five responses indicating, as we suspected, that all of these voters had separable preferences: if they ranked YY first, they ranked NN last, and vice versa. We received eight responses from the 32 voters who approved only one of the two newsgroups, and all of them ranked YY ahead of NN. The response rate to our survey is quite low since this was an e-mail survey and many of the respondents (probably college students on summer vacation) were not answering their e-mail. Some voters also objected to the survey since they believed their votes should not be publicly available.

set of outcomes.[14] And in the Internet vote we estimate that at most 32 of 160 voters had nonseparable preferences.

In the fantasy professional wrestling vote, YY failed to receive the necessary votes since its support was splintered. All of the YN and NY voters apparently wanted at least one of the newsgroups to pass. Had they known that their votes would cancel each other out and cause the failure of both newsgroups, they would likely have voted for YY and both newsgroups would have passed.

Several recent referendums have contained issues that were probably nonseparable to many voters. A 1978 Swiss national referendum contained one issue on the age of retirement and another on a revision of the old-age pension. Some voters may have preferred the passage of one issue only if the other also passed. Another Swiss referendum in 1990 contained one proposal to establish a 10-year moratorium on nuclear plant construction, while a separate proposal called for an end to the use of nuclear energy (see Koback, 1993: 76–7). Some voters may have preferred to continue the use of nuclear energy only under a 10-year moratorium on plant construction. A November 1988 California ballot contained five separate proposals on automobile insurance reform (see Lupia, 1994). Since the proposals addressed similar issues, many voters probably had nonseparable preferences across the issues. Any series of bond referendums is a likely place to find nonseparable voter preferences. Yet methods of voting on referendums are unable to address the problems caused by nonseparable preferences. To avoid these problems, we need other methods for making collective choices.

Solutions to the Problem of Nonseparable Preferences

We now consider possible solutions to the problem of nonseparable preferences in referendums. We focus on four factors in voting: divided questions, strategic (or sophisticated) voting, sequential votes, and vote-trading.[15] Several important papers examine the impact of each of these variables on voting outcomes when voter preferences are separable (Kadane, 1972; Kramer, 1972; Schwartz, 1977; see Ordeshook, 1986, and Miller, 1995, for reviews). Our purpose is to determine which voting mechanisms select a Condorcet winner when one exists and which mechanisms avoid selection of Condorcet losers and universally Pareto-dominated outcomes.

14. Votes on the Usenet also require that a newsgroup receive 100 more YES votes than NO votes in order to be established. None of the outcomes YY, YN, or NY would beat NN by more than 100 votes since NN had 48 supporters (50 including the N– voters) and only 110 other people cast votes. Therefore, this paper does not indict the outcome of the Usenet vote; it uses the vote as an example of the problem of referendum voting.

15. Brams et al. (1997) consider other methods for avoiding their ‘paradox of multiple elections’, including approval voting and counting abstentions on each issue.

Voting by Sets

Many of the mechanisms of social choice separate items for purposes of voting. According to common parliamentary procedure, 'If a series of independent resolutions relating to different subjects is included in one motion, it must be divided upon the request of a single member' (Robert's *Rules of Order* 13: 91). The key word is 'independent', which should imply that the rule holds only when members have separable preferences across the measures. But often *division of the question* enforces separability on questions that may not be separable in the minds of voters.

Many state constitutions require division of the question for referendums. Undivided issues are widely reviled since they allow strategic politicians to ensure the passage of unpopular measures by packaging them with popular measures. Opponents of this practice claim that division of the question leads to more accurate representations of voter preferences. This may not be true when voter preferences are nonseparable. We capture the distinction between divided and packaged issues as a difference between *item-wise* and *set-wise* voting.

DEFINITION 6: *A voting procedure is* item-wise *if every voter casts a separate vote for each issue.*

DEFINITION 7: *A voting procedure is* set-wise *if every voter casts a vote for groups of issues.*

A set-wise voting procedure reduces the alternatives to subsets over which all voters' preferences are separable. Under set-wise voting, voters could cast a single vote for their most preferred overall outcome, cast approval votes for each of several different outcomes (Brams and Fishburn, 1983, 1993), or rank-order all possible outcomes in order to produce a Condorcet winner or Borda winner. If voters cast a single set-wise vote, set-wise and item-wise voting might produce different outcomes even when only a few voters have nonseparable preferences. In Table 1, if each voter casts a vote for her/his most preferred set-wise alternative, the outcome is a three-way tie, with one vote each for NN, NY, and YN.

Tables 1 and 2 underscore a problem with set-wise voting when the number of possible outcomes exceeds the number of voters. In both examples, if each voter casts a vote for her/his most preferred set of outcomes, the outcome is indeterminate. Since both examples contain fewer voters than alternatives, no alternative receives more than one vote. Using ballots from the 1990 California state election, Brams et al. (1998) illustrate how quickly alternatives can outnumber voters. The California ballot contained 28 state issues, leaving 2^{28} or 268.4 million different sets of outcomes. In a sample of over one million ballots from Los Angeles County, no voter voted for the set of issues that won. The outcome of the California vote was approved in totality by no one in the Los Angeles area since the number of possible outcomes exceeds the number of

voters by such a wide margin that 99 percent of the possible outcomes did not receive a single vote.

While the likelihood of indeterminacy makes set-wise voting an unattractive method of social choice when the number of issues is large, the method has many advantages when the number of issues is sufficiently small, say three or less. Set-wise voting accurately reflects the wishes of voters whose preferences are nonseparable since their votes are counted as nonseparable. Set-wise voting does not obscure the preferences of voters whose preferences are separable since they can still vote for their most-preferred set of outcomes. Therefore, set-wise methods are more likely to uncover optimal social choices than item-wise methods when some voters have nonseparable preferences, especially if voters rank order the sets of outcomes under Condorcet's or Borda's rule.

An interesting implication of set-wise versus item-wise voting procedures is their impact on the likelihood that issues will pass. Divided issues are much more likely to pass than packaged issues. During 1967 and 1968, nine American states proposed to their voters changes in the state constitutions. The five states that offered the changes in one omnibus issue saw the proposals defeated. In the four states that divided the changes into multiple issues, nearly all of the changes passed. Hawaii offered 23 separate changes and 22 passed; New Hampshire offered six and five passed; Pennsylvania voters passed all five of their issues; Florida voters passed all three of theirs (Kadane, 1972: 51). Kadane proves that packaged issues fail when voters have separable preferences and when different groups of voters object to different issues. When each of these groups votes against the whole package due to its single objection, then the package fails, assuming that each group's dissatisfaction with the objectionable item is greater than its satisfaction with the remaining parts of the package. When voters face multiple issues, then each of the different groups votes against only the objectionable item, giving each item enough support to win.

While Kadane demonstrates why packaged issues are prone to failure, the more interesting phenomenon may be the other side of the equation: divided questions are prone to success. If we take a series of ballot measures that all pass and package them into an omnibus package that fails, which should we assume to be the 'better' social choice? If some voters have nonseparable preferences, then the packaged measure is the only way to gauge their preferences. Depending on the configuration of voter preferences, the success of divided questions may be an artifact of the voting process. Divided questions force voters to separate their votes when underlying preferences may not be separable.

The example of voting on the Internet illustrates a case where issues fail when divided but would likely pass when packaged. The fantasy professional wrestling newsgroups were substitutes to many voters. People who voted YN or NY wanted a fantasy professional wrestling newsgroup, but they could not agree on which one. Their votes canceled each other out. The implications for strategically combining ballot measures are clear: proposing several substitute measures on a

referendum runs the risk of splitting supporters and defeating the measures. To avoid this problem, proposers should find the single measure that is disliked by the fewest voters and make it the only issue on the ballot. To defeat a package of issues that are substitutes, one should break them into separate questions and hope that the proponents of the measures will be unable to coordinate voters. The November 1988 California ballot contained five separate issues on auto insurance reform, four of which were proposed by the insurance lobby. The four measures proposed by the insurance lobby differed slightly, suggesting that the insurance lobby may have hoped to split supporters of insurance reform and defeat all of the initiatives. Packaging and dividing issues on a referendum is ripe for strategic manipulation, something political scientists have known for years, but only in the context of separable voter preferences (Kadane, 1972). When preferences are nonseparable, the impact of packaging and dividing issues becomes more complex and potentially even more important to vote outcomes.

Strategic Voting

Thus far we have assumed that voters vote for their most preferred outcome because they do not know the preferences of other voters. But when voters have complete information about the preferences of other voters, strategic interaction may change their vote choice and thus change the outcome of the vote. 'Strategic voting' and 'sophisticated voting' are often used interchangeably in the literature, though convention has usually reserved the term 'sophisticated voting' for votes that are conducted sequentially (since Farquharson [1969] used this term). We define strategic voting as the iterated elimination of weakly dominated strategies in a simultaneous election with more than two alternatives, or with more than two issues, each of which has at least two alternatives. Strategic voting often results in a voter deserting a more preferred outcome with a poor chance of winning for a less preferred outcome with a better chance of winning. However, a voter who is voting strategically may actually vote for her/his most preferred outcome. Strategic and sophisticated voting are not electoral institutions, but they are important variables to consider in elections. The use of strategic or sophisticated voting may be increased by conducting elections in environments that enable voters to gain truthful information about each other's preferences.

As the example in Table 5 illustrates, in simultaneous elections, strategic voting does not guarantee the selection of a Condorcet winner when one exists.

To demonstrate the effect of strategic voting on referendums, we must first describe the behavior of voters who have separable preferences.

RESULT 4*: If a voter has separable preferences for the outcomes of votes across multiple issues, and if these issues are decided simultaneously by majority rule with binary votes on separate issues, then voting for one's most preferred outcome is the weakly dominant strategy.* (See Appendix 1 for proof.)

Table 5. Strategic Voting Fails to Choose a Condorcet Winner

Rank	Voter 1	Voter 2	Voter 3
1	NY	YN	NN
2	YY	YY	YY
3	NN	NN	NY
4	YN	NY	YN

In the example in Table 5, Voters 1 and 2 have separable preferences, Voter 3 has nonseparable preferences. The outcome is NN, but YY is the overall Condorcet winner. By Result 4, Voter 1 votes NY and Voter 2 votes YN. Voter 3 is pivotal for each issue, and her/his best response to the others' voting strategies is to vote NN. NN is the social choice even though YY is the Condorcet winner. In short, strategic voting alone proves insufficient to resolve the problem created by nonseparable preferences, an observation we offer as Result 5.

RESULT 5: *In a complete information preference profile, simultaneous voting by strategic voters, some of whom have nonseparable preferences, cannot guarantee the selection of a Condorcet winner when one exists.*

Result 5 suggests that even if voters can estimate the probability that each issue will pass by reading pre-election poll results, a simultaneous vote on multiple issues may not yield a Condorcet winner when one exists.

Sequential Voting

If we wish to retain divided questions on a referendum, one way to avoid the problems of nonseparable preferences may be to vote on issues sequentially. Sequential voting provides information to voters since voters know the outcome of the vote on one issue before voting on subsequent issues. Such information can be critical to voters who have nonseparable preferences since it is difficult for a voter with nonseparable preferences to express her/his true preference on one issue without knowing the outcome of votes on related issues. Consequently, voters with separable preferences might need to vote sophisticatedly in anticipation of the votes by voters with nonseparable preferences.

Most legislatures disclose the outcome of a vote immediately after it is taken rather than at the end of the day or at the end of the legislative session. One reason that legislatures vote sequentially may be to provide legislators with the information necessary to cast successive votes. When all voters' preferences are separable over multiple issues, the outcome on any issue is irrelevant to a voter's preference on other issues, and a voter does not need to know the outcome of the vote on one issue before casting her/his vote on the next.

Sequential voting offers a solution to the problem in Table 1, where NN is dominated by every other pair of outcomes. Suppose that every voter begins with

the simple act of voting for her/his most preferred outcome on the first issue, which produces N as the outcome. Once N is revealed as the outcome, Voters 1 and 2 will vote Y on the second issue, yielding NY as the outcome. If the order of the issues in the voting sequence is reversed, then on the second issue (voted on first), the outcome is N. On the first issue (voted on second), the outcome will be Y. With sequential voting, the outcome of the first vote will be N, and the outcome of the second will be Y, leaving YN or NY as possible outcomes. Sequential voting prevents the selection of Condorcet losers, a result we offer as Result 6.

RESULT 6: *If multiple issues are decided sequentially by majority rule with binary votes, then the social choice will not be a Condorcet loser or a universally Pareto-dominated outcome, regardless of whether voter preferences are separable or nonseparable.* (See Appendix 1 for proof.)

Result 6 shows that, at minimum, sequential voting prevents the selection of outcomes dominated by all other outcomes (whether by majority rule or unanimity), which is something that simultaneous voting cannot accomplish. Sequential voting works since voters know the outcome of all previous votes before voting on the last issue. Their vote on the last issue is a product of their preference over the outcome of the full set of issues. Under simultaneous voting procedures, voters do not know the outcome of previous votes and cannot state a preference for the last issue as a component of the full set of outcomes. When votes are simultaneous, voting outcomes can reflect voters' preferences accurately only if all voters have separable preferences.

Sequential voting prevents the selection of Condorcet losers, but it can also do more if voters are sophisticated. If voters are sophisticated, then sequential voting will select the Condorcet winner when one exists. Suppose all voters reason through the possible outcomes of sequential voting in Table 1. If Issue 1 is voted on first and the outcome is revealed to be Y, then Voters 2 and 3 will vote Y on Issue 2, and the outcome will be YY. If the outcome of Issue 1 is N, then Voters 1 and 2 will vote Y on Issue 2 and the outcome will be NY. All voters can look forward to discover that if Y wins on Issue 1, the outcome will be YY; if N wins, the outcome will be NY. Voters 1 and 3 prefer YY to NY, so they will vote Y on Issue 1, then Y on Issue 2, and the outcome will be YY. Similarly, suppose Issue 2 is voted on first and the outcome is revealed to be Y. Voters 1 and 3 will vote Y on Issue 1 and the outcome will be YY. If the outcome on Issue 2 is N, then Voters 1 and 2 will vote Y on Issue 1 and the outcome will be YN. Since all voters can reason through this sequence, they know that a Y vote on Issue 2 will produce YY while an N vote will lead to YN.[16] Since Voters 2 and 3 prefer YY to YN, they will vote Y on both issues in the sequence, producing YY as the

16. Voters have perfect-foresight expectations about the outcome of successive votes in the sequence (see Denzau and Mackay, 1981).

outcome. This illustrates that sequential voting with sophisticated voters selects the Condorcet winner when one exists. A general statement of this property of sophisticated sequential voting becomes Result 7.

RESULT 7: *Sophisticated sequential voting on multiple binary issues will produce an overall Condorcet winner when one exists, regardless of whether voters have separable or nonseparable preferences.* (See Appendix 1 for proof.)

Result 7, an extension of Farquharson's Theorem on sophisticated voting outcomes (1969), demonstrates that sophisticated sequential voting with nonseparable preferences produces a Condorcet winner when one exists even though the Condorcet winner would not survive with sincere voters or simultaneous votes.

Kramer (1972) proves that when issues are considered sequentially and all voters have separable preferences in an *n*-dimensional Euclidean space, then a unique strategic voting equilibrium exists. The significance of Kramer's result is that as long as voter preferences are separable, the outcome of sequential voting is the same regardless of whether voters are sincere or strategic: a unique stable outcome exists that is unaffected by the order of voting, and a Condorcet winner will be chosen if one exists. When voter preferences are nonseparable and voters are sincere, the outcome of sequential voting is slightly different: a unique stable outcome exists, but the specific outcome depends on the order of the vote. Again, if a Condorcet winning outcome exists, it will be chosen. If some voters have nonseparable preferences and voters are strategic, then the Condorcet winner will be chosen when one exists; if a Condorcet winner does not exist, stability breaks down (Ordeshook, 1986: 300). The significance of the result in our paper is that as long as a Condorcet winner exists, then strategic sequential voting will select it regardless of whether voter preferences are separable or not. When there is not an overall Condorcet winner, then strategic sequential voting cannot induce stability, as Kramer proves.

While sequential voting with strategic voters can solve the problem of nonseparable preferences, it is probably unreasonable to expect strategic voting in a referendum. Strategic voting requires that voters have a good idea about the preferences of all other voters. For voters in a state or national referendum to vote strategically is often difficult.

A problem inherent to any sequential voting scheme – whether voters are strategic or not – is that it takes too long. Sequential voting in mass publics would require that the outcome of each issue be announced before votes are cast on subsequent issues. Voters would know the outcome of previous votes before casting their next vote, which would allow voters with nonseparable preferences to reveal their preferences accurately. The cost to voters of going to the polls and the cost to governments of keeping polls open for several days will likely prevent the use of sequential voting schemes. However, California holds referendums in

June and November of most even-numbered years, which allows some sequential voting if related issues are kept off the same ballot.

New Zealand recently used sequential voting to choose a new electoral system. A September, 1992, referendum presented voters with two issues: (1) whether they preferred the status quo (plurality rule) or a change; and (2) if they preferred a change in the electoral system, which of four alternative systems they preferred. A year later, voters were confronted with a ballot that pitted the status quo against the plurality winner from the previous referendum (a German-style mixed member proportional system, MMP). MMP won the second vote. Many voters clearly preferred a change from the status quo *only if* the change were in favor of their most preferred alternative (see Boston et al., 1996). By holding the votes in sequence, New Zealand may have avoided the selection of a Condorcet loser and insured the selection of a Condorcet winner, assuming one existed.

Despite the costs of sequential voting and the limitations of voter sophistication in mass elections, sophisticated sequential voting guarantees the selection of stable, or Condorcet winning, outcomes when such outcomes exist and when voters have nonseparable preferences. Sophisticated sequential voting is not the only mechanism that guarantees selection of a Condorcet winner when one exists. Other devices, such as vote-trading, can also uncover stable social outcomes.

Voting in Legislatures and the Benefits of Vote-Trading

Strategic voting and coordination are virtually impossible in referendums since the number of participants is large and their interaction is minimal. Legislatures avoid the problem of too many participants in politics. Representative government has as its normative foundation the conviction that a small, select group of people – with responsibility to the rest of the public through elections – is better able to make social choices than the public writ-large. One advantage of representative government over direct democracy springs from the ability of voters (as legislators) to engage in pre-vote communication and to witness the outcome on the last vote before voting in the next one.

The trading of votes, or logrolling, is common in legislatures. Legislators often make pacts to support each other's preferred policies. Vote-trading is a form of sophisticated collusion by subsets of the voting body, often resulting in sub-optimal or collectively irrational outcomes (Bernholz, 1973, 1974, 1975; Riker and Brams, 1973; Koehler, 1975; Oppenheimer, 1975; Schwartz, 1977). Opponents of logrolling point to it as the impetus behind pork-barrel projects and rampant government spending (Riker and Brams, 1973). They also point to vote-trading as the reason why 'good' proposals get voted down by conspiring opponents. Logrolling is a form of conspiracy, which is anathema to any believer in clean government.

Schwartz (1977) proves that vote-trading is not as destructive as widely

believed. If a Condorcet winning outcome exists, and if legislators have *separable* preferences over the components of that outcome, then the trade that yields the Condorcet winner is the only stable trade. All other vote trades can be overturned by some coalition of legislators. In other words, Schwartz demonstrates that if a stable outcome exists and legislator preferences are separable, logrolling will not undermine that outcome.

Schwartz's result assumes that all legislators have separable preferences for the bills under consideration and that voting is issue-by-issue. When some legislators have nonseparable preferences, then bills must be evaluated as packages rather than as individual items. When comparing packages of issues under majority rule, the set of Condorcet winning outcomes is equivalent to the core of a coalitional voting game (Edgeworth, 1881; Scarf, 1967) if vote-trading is allowed. Any proposal that is not a Condorcet winner, and therefore not in the core, can be overturned by a majority coalition favoring a different outcome.

Once a Condorcet winner exists among packages of proposals, then vote trading is sufficient to select the Condorcet winner. Despite its reputation, logrolling may actually be good when some voters have nonseparable preferences.[17] Suppose that the overall Condorcet winner is proposed by a majority of voters. No other alternatives can beat that outcome. If a minority defect to trade votes, then that minority can be defeated by a majority who prefer the Condorcet winner and object to the trade. If the Condorcet winner is not proposed, then a majority who prefer the Condorcet winner to the proposal can coordinate their votes to ensure the selection of the Condorcet winner. In Table 1, if Voters 1 and 2 coordinate their votes, the outcome will be YY. In Table 5, the only stable vote trade produces YY, the Condorcet winner (Schwartz, 1977). When some voters have nonseparable preferences, then vote-trading is *not* simply *not a bad thing*; it can be *a good thing*. It is certainly an improvement over referendum-style voting.

Explicit vote-trading is not necessary to select a Condorcet winner since sequential voting by sophisticated voters will also produce a Condorcet winner when one exists. Sophisticated legislators who vote sequentially will act as though they are trading votes when, in fact, they are not (see Result 7). Without sequential votes, vote trades become unenforceable since one voter may be able to lure other voters into voting for a trade while shirking from the arrangement him/herself. If all voters recognize the potential for defection from vote trades, then cooperation will not be sustainable except under the conditions of the Folk Theorems, such as high valuation of future payoffs, repeated interaction, and an unforseeable time horizon. Kramer (1972: 166) recognized the importance of sequential voting as an enforcement mechanism: 'The sequential character of voting is not important in a regime where collusive agreements, such as vote trades, can be made binding and enforceable on those who enter into them'. We

17. The final example in Schwartz (1977) shows a beneficial vote-trade when some voters have nonseparable preferences.

reverse Kramer's argument: vote-trading is not important in a regime with sequential votes and sophisticated voters. Explicit coordination of votes is unnecessary to produce outcomes that appear to be vote trades.

Our discussion thus far has assumed that a Condorcet winner exists, or that the core is non-empty. When a Condorcet winner does not exist, then the outcome of any vote trade will cycle among the set of majority-undominated outcomes, regardless of whether voters have separable or nonseparable preferences.[18]

When some voters have nonseparable preferences, then vote trading is an effective means to produce Pareto-undominated or stable social outcomes, when such outcomes exist. Vote-trading requires communication, coordination, and credible commitments among voters, something that is highly unlikely in a mass public voting on a referendum. Without coordination and commitment, stable social outcomes are still possible if voters are sophisticated and votes are sequential. Mass publics are unable to engage effectively in any of these practices due to the sheer number of people involved.

If we could reduce to a manageable number the participants in democratic decision-making and allow them to communicate their preferences, enter agreements, and vote sequentially, then many of the problems of nonseparable preferences would be solved. Too often political commentators fall into the trap of assuming that political participation is one-dimensional, where the larger the number of participants in decision-making, the higher the level of participation. But participation should be measured on a second dimension: quality of participation. Political processes that allow or even require communication and coordination across participants are 'participatory' even if the number of actors involved is relatively small. Ideally, all political questions could be decided via direct democracy with sequential votes following extensive deliberation, but the costs in time and energy of such a process are unreasonable. If we admit that participation involves another dimension – quality of participation – then an increase in the number of participants does not directly translate into an increase in 'political participation'. Direct democracy scores high on the number of participants but low on the quality of participation; representative democracy scores low on the number of participants but high on the quality of participation.

Conclusions

We have identified in this paper a problem with referendums as currently practiced. When some voters have nonseparable preferences for the issues under consideration, referendums are unable to capture the complexity of those

18. Riker and Brams (1973) show that when voters have separable preferences and a Condorcet winner does not exist, vote trades may produce Pareto-inefficient outcomes.

preferences. Referendums are not the embodiment of majoritarian democracy; instead, referendums may produce unstable collective choices that are opposed by a majority of voters or by all voters.

Referendums are often touted for their ability to uncover what the people want. We know from an extensive literature on social choice that a stable collective choice, for our purposes a Condorcet winner, often does not exist (Arrow, 1951; Davis et al., 1972; Enelow and Hinich, 1985; Plott, 1967). But in the cases where there is a Condorcet winner, we should expect a referendum to uncover it. But as we have shown, a referendum may not select a Condorcet winner, and, in many cases, a referendum will select a Condorcet loser or a unanimous loser.

Solutions to the problem with referendums that retain the important characteristic of referendums – mass participation – bring with them a host of other problems. Voting by sets is too costly for voters when more than a few (two or three) issues are under consideration. Partitioning referendums into related issues that can be voted on by sets is one way to reduce the cost of set-wise voting, but any process of partitioning measures into related subsets is likely to become arbitrary. Packaging sets of issues, particularly on bond referendums, is commonplace in many states and localities, but it is often manipulated for strategic purposes (see Mackay and Weaver, 1983). Sequential voting is another solution to the problem of nonseparable preferences and referendums, but it is costly to both voters and governments.

Conferring important social choices on legislatures offers much more hope as a solution to the problems posed by nonseparable preferences. Sequential voting, even without sophisticated voters, can guarantee that outcomes will not be selected that are majority-dominated or Pareto-dominated by every other set of outcomes. However, sequential voting cannot guarantee selection of a Condorcet winner when one exists. Sophisticated issue-by-issue voting and vote-trading are both sufficient to produce Condorcet winning outcomes when they exist. In the absence of overall Condorcet winners, sophisticated issue-by-issue voting and vote-trading can eliminate potentially bad social choices that simple referendum-style voting cannot. The important advantage of legislatures over direct democracy springs from the ability of voters in a legislature to communicate their preferences and coordinate their votes. People voting on a referendum are forced to cast *blind votes* that consider neither the outcome of votes on related issues nor the preferences of other voters. Legislatures encourage communication and coordination, forms of political participation often overlooked by proponents of direct democracy, yet these forms of participation are crucial to the selection of optimal social outcomes when people hold nonseparable preferences.

The primary weakness of a legislature as a tool of social choice is the likelihood that legislators will vote their own preferences rather than represent the preferences of their constituents. Legislatures are a highly imperfect method

of revealing and aggregating social preferences. But as we demonstrate, so are referendums.

As the bandwagon for direct democracy grows and as referendum ballots lengthen in state and national politics, the problems posed by nonseparable preferences will grow in kind. Only in the special case where every voter has separable preferences does the referendum work well in the sense of choosing Condorcet winners and avoiding universally Pareto-dominated outcomes. When even a few voters have nonseparable preferences, the referendum is a Procrustean method of aggregating individual preferences into social choices, and it leaves ambiguous, if not completely wrong, the answer to the question: 'What did the people want?'

APPENDIX 1

Proofs of Results

Proof of Result 3: Suppose the social choice, $X = (x_1,x_2,...,x_k,...,x_n)$, is Pareto-dominated by all other outcomes. This implies that for every voter i and any issue k, $(x_1,x_2,...,x'_k,...,x_n)$ f_i $(x_1,x_2,...,x_k,...,x_n)$. By the definition of separable preferences, if (x'_k,X_{-k}) f_i (x_k,X_{-k}), then (x'_k,Y_{-k}) f_i (x_k,Y_{-k}) for every $Y \in \mathbf{X}$. Thus i's most preferred outcome on issue k must be x'_k, regardless of the outcome on all other issues. Since we assume everyone votes for their most preferred outcome, all voters vote for x'_k over x_k for all k. Under majority rule, the social choice cannot be $(x_1,x_2,...,x_k,...,x_n)$. ❑

Proof of Result 4: Suppose voter i's most preferred outcome is $x^i = (x_1,...,x_n)$. Given the vote choices of other voters, if i is not pivotal on any issue, voting x^i is as good as voting for any other outcome. If i is pivotal on k issues, we can reorder these issues as issues 1 through k. By the definition of separability,

$$(x_1,...,x_k,...,x_n) > (x_1,...,x'_k,...,x_n),\ k = 1,...,k, \text{ and}$$

$$(x_1,...,x_j,x'_k,...,x_n) > (x_1,...,x'_j,x'_k,...,x_n),\ j,k = 1,...,k$$

By transitivity, $(x_1,...,x_j,x_k,...,x_n) > (x_1,...,x'_j,x'_k,...,x_n)$, j,k=1,...,k. By the same reasoning, we can show that voting for $(x_1,...,x_k,...,x_n)$ is preferred to any other combination of outcomes on the k issues. ❑

Proof of Result 6: Suppose that the outcomes of votes on n–1 issues are accurately revealed to voters before the vote on the nth issue. Voters who vote in favor of n must hold the preference $(x_1,x_2,...,x_{n-1},x_n)$ f $(x_1,x_2,...,x_{n-1},x'_n)$. Any voter who holds this preference will vote x_n instead of x'_n since, in the last stage of the voting sequence, voting for her/his preferred outcome weakly dominates voting against it in majority voting with binary votes. If the number of voters for which this statement holds is greater than the number of voters for which it does not hold, then x_n will be approved and $(x_1,x_2,...,x_{n-1},x_n)$ will be the winning set of outcomes after the final vote. This set is not majority-dominated by all other outcomes since it was not defeated by $(x_1,x_2,...,x_{n-1},x'_n)$. ❑

Proof of Result 7: Suppose $(x_1,x_2,...,x_{n-1},x_n)$ is an overall Condorcet winner. On a vote of $(x_1,x_2,...,x_n)$ against $(x_1,x_2,...,x'_n)$, $(x_1,x_2,...,x_n)$ wins by definition of a Condorcet winner. On a vote of $(x_1,x_2,...,x_{n-1})$ versus $(x_1,x_2,...,x'_{n-1})$, $(x_1,x_2,...,x_{n-1})$ wins since a majority of voters prefer the strategic equivalent of the vote, $(x_1,x_2,...,x_{n-1},x_n)$, to any other outcome. The same holds true for each preceding component of the overall Condorcet winner. By reasoning forward along the sequence of votes, a majority of voters will vote for every outcome that produces $(x_1,x_2,...,x_{n-1},x_n)$, regardless of the ordering of the issues in the sequence. ❑

APPENDIX 2

Fantasy Professional Wrestling Survey

Dear Usenet Voter,

I would like to bother you for ONE (1) minute to solicit your opinion for a project on voting procedures. My work focuses on referendums, but few (if any) real-world referendum ballots provide the kind of information I need about people's preferences for the outcome of the vote. I have turned to the Internet for some examples, and I would greatly appreciate your help.

Over a month ago you voted on whether or not to include two new newsgroups on Usenet:

1. rec.sport.pro-wrestling.fantasy, an unmoderated newsgroup for discussion of fantasy professional wrestling, and
2. rec.sport.pro-wrestling.info, a moderated newsgroup for posting information about professional wrestling.

I am interested in what you wanted as the outcome of this vote. Could you please fill out the following survey and e-mail it back to me? All responses are strictly confidential (in fact, your name will be completely forgotten), and this survey will be used only for research. I do not intend to question the result of the vote or lobby for or against the newsgroups.

Please rank your preferences for the possible outcomes of the vote by typing the appropriate number in each of the ( )'s below:

1 ≡ your most preferred outcome (what you wanted)
2 ≡ your second choice
3 ≡ your third choice
4 ≡ your least preferred outcome (what you hoped would not happen)

The possible outcomes are:		**Your Rank**
YY ≡	both newsgroups PASS	( )
YN ≡	rec.sport.pro-wrestling.fantasy PASSES rec.sport.pro-wrestling.info FAILS	( )
NY ≡	rec.sport.pro-wrestling.fantasy FAILS rec.sport.pro-wrestling.info PASSES	( )
NN ≡	both newsgroups FAIL	( )

Thanks for your help.

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