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Protest voting in plurality elections: a theory of voter signaling

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Abstract This paper develops a model of *protest voting* in which unsatisfied voters may abandon their most-preferred candidate even though he or she has a good chance of winning, in the hope that this signal of disaffection will lead to downstream improvements in that candidate's performance. We use a spatial model to identify voters whose ideological profile makes protest voting an option, and an expected utility model to identify the conditions under which potential protest voters will in fact use their vote as a signaling device. Aggregate-level data provide suggestive evidence in the argument's favor.

Keywords Strategic voting · Protest voting · Electoral signaling · Spatial theory · Calculus of voting

1 Introduction

One of the most prominent empirical literatures in the study of voter behavior analyzes *strategic voting*. This work finds theoretical inspiration in Duverger's famous argument that voters in single-member district systems will avoid wasting their vote on candidates with little chance of being elected. Beginning with research by Black (1978) and Cain (1978), a series of papers derive formal theoretic statements of the conditions under which abandoning one's most-preferred party for a more viable party is optimal (Gutowski and Georges 1993; Kselman and Niou 2010).[1] Such studies focus on the *short-term instrumental* logic behind

[1] Empirical research has found supportive evidence of strategic voting at the aggregate as well as individual level. See, for example, Black (1978), Cain (1978), Abramson et al. (1992), Blais and Nadeau (1996), Ordeshook and Zeng (1997), Alvarez and Nagler (2000).

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strategic voting: voters abandon their most-preferred candidate because he or she has little chance of winning the current election. By relaxing the short-term rationality assumption, this paper uncovers a distinct motivation for abandoning a most-preferred candidate, even when this candidate has a good chance of winning the current election: the desire to cast a *protest vote, i.e., a targeted signal of disaffection to one's most-preferred political party*.

Our model is grounded in two seminal approaches to the study of electoral behavior: (a) the *spatial* approach in which voters choose candidates based on considerations of ideological proximity to their individual preferences (Downs 1957); and (b) the *expected utility* approach, often dubbed the 'calculus of voting' (Downs 1957; Riker and Ordeshook 1968; McKelvey and Ordeshook 1972). Section 3 incorporates both voters' spatial preferences and their electoral expectations into a unified analytic framework for the study of tactical choice. In turn, Sect. 4 employs this framework to identify voters whose strategic circumstances make protest voting a viable alternative. The formal argument therein demonstrates that electoral signaling should be unlikely among voters whose ideal points lie to the left of the party system's leftmost party or the right of its rightmost party (labeled the *party system exterior*).

Having identified the subset of potential protest voters, Sects. 5 and 6 develop an expected utility model of voter choice and voter signaling to establish the conditions under which such individuals should indeed abandon their most-preferred candidate. The analysis generates a series of comparative static hypotheses as to the relationship between protest voting and important systemic characteristics, including parties' *ideological distribution* in policy space, the extent of electoral *competitiveness*, and voters' relative levels of *indifference* between the competing candidates; Sect. 7 presents preliminary empirical evidence in these hypotheses' favor. Finally, Sect. 8 discusses introducing this paper's decision-theoretic analysis into a game theoretic environment. Indeed, just as the afore-mentioned 'calculus of voting' model provides the decision-theoretic foundation for later strategic models by Palfrey (1989) and Cox (1994), we conceive of this paper as an important step in subsequent game theoretic analyses of parties' optimal responses to vote shifts in the electorate.[2]

2 Literature review: dissatisfied voters and electoral signaling

Voters dissatisfied with their usual party of choice have a variety of options, the most obvious of which is simply not turning out to vote. Riker and Ordeshook (1968) first developed the calculus of voting to study turnout in plurality elections. In addition to the exogenous costs and benefits of voting, their results demonstrate that voters will more likely abstain in *non-competitive* elections when they are largely *indifferent* between an election's various candidates. Similarly, while most spatial models in the Downsian tradition assume that all voters turnout and choose the candidate 'closest' to their ideal point, some argue that abstention will occur when voters are either indifferent between or *alienated* from the available candidates (Hinich and Ordeshook 1970; Adams et al. 2006).

In these formulations voters do not receive utility from the act of abstention in itself. Rather, abstention occurs when some combination of indifference, non-competitiveness, and alienation renders the benefit of voting lower than its costs. One study moves beyond this passive conceptualization to investigate whether abstention might in and of itself have an instrumental logic (Rosenthal and Sen 1973). The authors note that in France, at every legislative election, some small number of ballots are *purposely spoiled*, marked with a vulgar

[2]We would like to thank an anonymous reviewer for encouraging us to develop this line of reasoning.

expletive to express dissatisfaction with some aspect of the political status quo. They investigate whether or not 'protest' behavior varies systematically with variables such as a district's competitiveness, the number of parties competing in a district, and the compatibility between party ideologies and a district's aggregate preference profile.

The protest behavior identified by Rosenthal and Sen is relatively blunt: though it communicates to political leaders that some portion of the electorate is dissatisfied with the status quo, it provides them little specific information as to what behavioral modifications would satisfy disaffected voters. Studies of third parties in American politics demonstrate how voters might communicate a more precise message to political actors. These studies' unifying theme is that individuals may *use their votes* to send a specific and targeted signal of dissatisfaction to one or more important political actors at time 't', in the hopes that this message will in some way contribute to a modification of elite behavior at time '$t+1$'. For example, in their study of third parties in American politics Rosenstone et al. argue (1996: 126; italics added):

> A third party vote is a vote against the major parties. Nevertheless, minor-party voting is an *instrumental* act. Citizens who cast a third party ballot do so to advance the same policy goals they were precluded from achieving from within the major parties.

When voters perceive the major parties to be mutually ignoring an important issue they will cast a third party vote to voice this displeasure. According to the authors, the major parties often recognize the signal, and position themselves appropriately on the neglected policy dimension at the next electoral cycle.[3]

In an important recent contribution, Kang (2004) aims to generate precise theoretical statements of the conditions under which using one's vote as a signal is in fact more valuable than voting sincerely or abstaining. According to the argument, citizens cast 'protest votes' when two conditions obtain: (a) when they receive higher 'quality' from a party other than the one they usually prefer; and (b) when choosing this other party is likely to induce 'quality recuperation' in their usual party of choice. However, a party's expected quality recuperation is in Kang's model an exogenous parameter. Without an explicit theoretical argument as to the manner in which protest votes may actually affect a political party's public behavior, it is not possible to gain a genuine understanding of the conditions that make protest voting a more likely alternative. Furthermore, the role of 'competitiveness' in Kang's model is inconsistent: while in the empirical analysis he argues that protest voting should be more likely in uncompetitive districts marked by one-party dominance, in his theoretical model protest voting becomes likely only when a 'viable' alternative to one's usual choice exists.

3 Party systems, preferences, and expectations

The model of voter signaling presented in Sects. 3–6 addresses each of these issues, providing a comprehensive framework for understanding the multi-faceted nature of voter choice and voter turnout in plurality elections. Consider a uni-dimensional ideological space $x \sim [0, 1]$ on which three parties $P \in \{L, M, R\}$ are located at policy positions x_L, x_M, and x_R, such that $x_L < x_M < x_R$.[4] Define x_{LM} and x_{MR} as the policy positions of voters who are

[3] In a related paper, Kellermann (2010) investigates the possibility that vote outcomes in sub-national elections might serve as signals of disaffection to a governing party. Similarly, Meirowitz and Tucker (2007) argue that voters may use their vote in elections to a less important institution (e.g., the Parliament) so as to alter elite behavior in subsequent elections to a more important institution (e.g., the Presidency).

[4] As the model is decision theoretic, it can be generalized to $N > 1$ dimensions and $N > 3$ political parties.

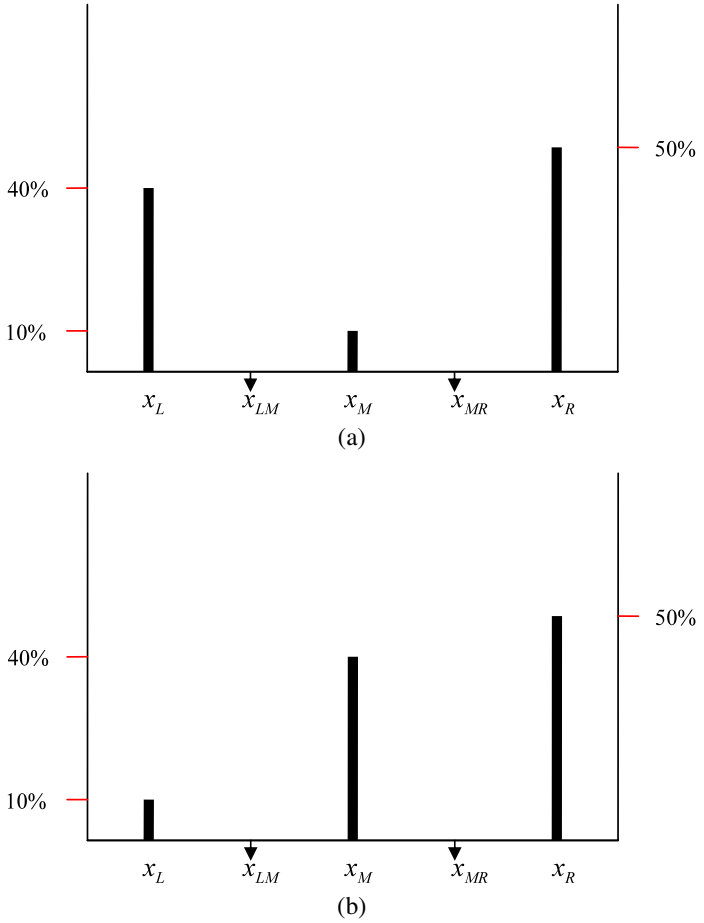

Fig. 1 Balanced and unbalanced party systems

perfectly indifferent between the two subscripted parties. As long as preferences are single-peaked, then $x_{LM} \in [x_L, x_M]$ and $x_{MR} \in [x_M, x_R]$, and voters with ideal points $x_i \in [0, x_{LM}]$ prefer L to M and M to R, while voters with ideal points $x_i \in [x_{MR}, 1]$ prefer R to M and M to L. Voters with ideal points in the range $[x_{LM}, x_{MR}]$ will most prefer the centrist party M; the identity of their second most-preferred party will depend on the relative extremism of L and R's ideological positions.

Figures 1a and 1b embed voters' expectations over electoral outcomes in this spatial framework. The horizontal axis represents the ideological space $x \sim [0, 1]$, and the vertical axis represents parties' expected electoral vote shares. For example, in Fig. 1a party L is located at position x_L and expected to receive 40% of the vote, party M is located at x_M and expected to receive 10% of the vote, and party R is located at x_R and expected to receive 50% of the vote.[5] Naturally, the expected vote shares sum to 1. Figure 1b differs in the ideological location of the least competitive party. While in Fig. 1a the center party is expected to place

[5]The specific vote shares chosen in Figs. 1a and 1b have no bearing on the model's results.

last in the election, in Fig. 1b the left party is least competitive. Label the former *balanced* party systems in which two larger parties are located around a smaller centrist party, and the latter *unbalanced* party systems in which the 'weight' of expected vote shares lies on either the left or right.[6]

We begin by demonstrating the variance in voters' *expectation profiles* which exists across different party systems. A voter's expectation profile is simply a ranking of his or her most-preferred, second-most-preferred, and least-preferred candidates according to their expected success in the election. Consider a voter with expectations identical to those expressed in Fig. 1a, and whose ideal point is in the range $[x_{MR}, 1]$. This voter's most-preferred candidate R is expected to place 1st in the race (50%), her second-most-preferred candidate M is expected to place 3rd (10%), and her least-preferred candidate L is expected to place 2nd (40%). Similarly, consider a voter with expectations identical to those in Fig. 1b, and whose ideal point is in the range $[x_{MR}, 1]$. This voter's most-preferred candidate R is expected to place 1st in the race (50%), her second-most-preferred candidate M is expected to place 2nd (40%), and her least-preferred candidate L is expected to place last (10%). Voters with similar ideological positions thus face very different strategic circumstances depending on their party system format.

This classification can be generalized to create an exhaustive typology of voter choice in 3-candidate plurality elections:[7]

Table 1 A typology of voter choice

Expectations						
	C1	C2	C3	C4	C5	C6
	1	1	2	3	2	3
	2	3	1	1	3	2
	3	2	3	2	1	1
Vote for 1	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote/ Sincere Vote	Straight-Forward Vote/ Sincere Vote
Vote for 2	Protest Vote	Protest Vote	Strategic Vote/ Protest Vote	Protest Vote	Strategic Vote	Strategic Vote
Vote for 3	X	Protest Vote	X	Protest Vote	X	X
Abstention	Abstention	Abstention	Abstention	Abstention	Abstention	Abstention

X ≡ no current theoretical explanation

[6]The model's implications are not affected by whether this weight is disproportionally concentrated on the right, as in Fig. 1b, or the left.

[7]In fact, the typology as presented here is not fully exhaustive as it does include the possibility that voters may be perfectly indifferent between two or more candidates, or may think that two or more candidates share the same probability of winning. Presenting these additional possibilities would not change our argument, but would make the presentation significantly more cumbersome.

Beginning in Table 1's top row, notice the set of column markers {C1, C2, ..., C6}. Each of these markers is associated with a particular voter profile. Label a voter's most-preferred candidate 1, a voter's second-most preferred candidate 2, and a voter's least-preferred candidate 3. For each profile, the candidate listed at the top is expected to place 1st, the candidate listed in the middle is expected to place 2nd, and the candidate listed at the bottom is expected to place 3rd. For example, voters have strategic profile C1 if they believe 1 will place 1st, 2 will place 2nd, and 3 will place 3rd: $\binom{1}{2}{}_{3}$. This is the case for voters with ideal points in the range $[x_{MR}, 1]$ from Fig. 1b. Similarly, voters have profile C2 if they believe 1 will place 1st, 3 will place 2nd, and 2 will place 3rd $\binom{1}{3}{}_{2}$. This is the case for voters in the range $[x_{MR}, 1]$ in Fig. 1a.

The table's left-hand side classifies decisions to vote for one's j^{th} preference or to abstain. Coupling these two dimensions as shown yields a 24 cell typology which exhaustively classifies voters according to their vote choice and expectation profile. A voter with expectation profile C6 who chooses her second-most preferred candidate 2 would occupy the cell labeled 'strategic vote' in the table's rightmost column; a voter with expectation profile C1 who chooses her most-preferred candidate 1 would fall into the cell labeled 'straightforward vote/sincere vote' in column C1; and so on. Sections 4–6 develop a theoretical model which helps to identify the distribution of expectation profiles in distinct party system environments, as well as the individual-level factors which impel citizens with particular profiles to cast protest votes.

4 Communicating ideological discontent

Define protest voting as *choosing a party other than one's most-preferred to send that most preferred party a signal of dissatisfaction*.[8] This behavior is qualitatively similar to that described by Albert Hirschman in his study of Exit, Voice, and Loyalty in consumer markets (Hirschman 1970). One option available to consumers dissatisfied with the quality of their most-preferred brand is 'Exit' to a substitute product. Consumer Exit reduces the most-preferred brand's market share, potentially inducing managers to improve quality so as to recapture its lost consumers. If this quality improvement occurs, many previously dissatisfied consumers will return to their usual brand. If managers do not react to the loss of market-share with quality improvements, dissatisfied consumers may eventually switch brand loyalties altogether.

Traditional studies of strategic voting address voters' choice to abandon their most-preferred party when this party is not a viable competitor, and in particular those with expectation profiles C5 and C6 from Table 1, who believe their *most-preferred candidate 1 will place last in the election*.[9] Among these voters we will not consider the possibility of protest voting. This implies that voters in balanced party systems with ideal points in the range $[x_{LM}, x_{MR}]$ will not be considered potential protest voters, nor will voters in unbalanced party systems with ideal points in the range $[0, x_{LM}]$. Qualitatively, this implies that

[8] Another form of protest voting may not have as its target a particular political party, but the political system more generally. In such instances voters may signal global disaffection by choosing 'anti-system' parties on the extreme right or left. We discuss this type of protest voting in the Conclusion.

[9] In fact, when three candidates find themselves in a very close three-way race, voters with expectation profile C3 may also cast strategic votes (Kselman and Niou 2010).

we will not consider protest voting a viable option for supporters of smaller 'third' parties.[10] As such, voters in C5–C6 from Table 1 may cast *strategic votes* of the traditional variety, i.e., choose 2 to prevent 3 from winning; they may cast *straightforward votes*, which occurs when choosing 1 is the utility maximizing choice despite the fact that she is in last place; or they may cast *sincere votes*, i.e., choose 1 for 'expressive' (i.e., non-instrumental) reasons despite the fact that strategically choosing 2 would maximize their expected utility (on expressive voting see Schuessler 2000).

The fact that within traditional frameworks it is primarily voters in C5 and C6 that should abandon their most-preferred party poses an interesting theoretical and empirical puzzle: a series of recent studies have provided evidence that many voters in three-candidate elections do indeed choose a candidate other than their most-preferred, despite the fact that this most-preferred candidate is expected to place first or second in the election (e.g., Franklin et al. 1994). The following model provides one possible explanation as to why and under what conditions voters with strategic profiles C1–C4 might abandon viable most-preferred candidates.

The argument is grounded in an understanding of political parties as organizations designed to win elections. Like firms in Hirschman's framework, when parties lose electoral support, and in particular when this loss of support threatens to result in a loss of office, *given the opportunity* they may adjust their behavior in the hopes of recapturing lost voters. The italicized qualifier 'given the opportunity' is paramount. Indeed, the optimal response of political party organizations to shifts in the aggregate distribution of votes is a strategic question addressed in Sect. 8. In the following Sections we will solve a decision theoretic model in which voters assume that parties respond to the loss of electoral market share with recuperative efforts, a notion formalized with *Signaling Assumption* 1 (immediately below). In a fully strategic model Signaling Assumption 1 will be *endogenized*, and voter choice will depend also on equilibrium organizational responses to electoral signals. As argued in Sect. 8, the decision-theoretic results derived here represent a useful starting point for this eventual game theoretic analysis.[11]

How might parties respond to vote losses on their ideological right or left? Consider a balanced party system in which party L, used to receiving an electoral plurality of 50%, in a particular election receives only 40% of the vote; and furthermore that the center party M increases its usual vote-share from 15% to 25%, leaving party R with the remaining 35%. An office-seeking party L, used to receiving a comfortable plurality, who wins the election over R by only 5 percentage points (i.e., 40% to 35%), has reason to assume a more centrist ideological position in the hopes of recapturing supporters lost to party M. We formalize this notion with the first of two *signaling assumptions*:

Signaling Assumption 1 Given the opportunity, parties will respond to vote losses on their right or left by adjusting their ideological position in the relevant direction.

[10] Among supporters of smaller parties protest voting is an unlikely option for a variety of reasons. Firstly, leaders of such parties are unlikely to 'receive' electoral signals in any meaningful way: leaders of small parties who witness a loss of votes will find it difficult to know whether this loss of support occurs due to strategic voting of the traditional 'wasted-vote' variety or the dissatisfaction of core supporters. Perhaps more basically, using Exit as a strategy is predicated upon the possibility of eventually 'consuming' one's most-preferred product if quality recuperation occurs. Among supporters of smaller third parties this eventual consumption is unlikely, as these parties have little medium- or long-term prospect for political incumbency. As such, protest voting among supporters of such parties will serve little downstream purpose.

[11] Evidence as to parties' responsiveness to vote losses is mixed, with some papers finding that parties react to vote losses (e.g., Budge 1994) and others that parties are more resistant to ideological shifts (e.g., Adams et al. 2004). Endogenizing organizational responsiveness will be an important goal of future theoretical research.

This assumption allows us to: (a) identify those voters with strategic profiles C1–C4 for whom protest voting is in fact a viable option; and (b) identify, among potential protest voters, which of their less-preferred parties is in fact a viable signaling option. Regarding (a), we establish the following Proposition:

Proposition 1 *Regardless of the party system format, voters with ideal points* $x_i < x_L$ *or* $x_i > x_R$ *will not cast protest votes.*

The voters identified in this Proposition have ideal points on the party system *exterior*, either to the left of the party system's leftmost party or to the right of the party system's rightmost party.[12] Among such preference types, there exists no political party for whom to cast a signaling vote; in fact any choice other than one's most-preferred party 1 will send that most-preferred party the *wrong ideological signal*. For example, if a voter with preference $x_i < x_L$ chooses either M or R, party L will witness a vote loss to its ideological right. By Signaling Assumption 1, given the opportunity L may react to this vote loss by moving to the right in hope of recapturing lost support. But this move is in precisely the opposite direction desired by voters with ideal point $x_i < x_L$, who find themselves on the far left. As such, voters with strategic profiles C1–C4 on the party system exterior have no viable party for whom to cast protest votes.

Citizens with ideal points in the range $[x_L, x_R]$ lie on the party system *interior*, and as such have at least one viable option for engaging in signaling behavior. However, their strategic flexibility may be constrained by the logic of Signaling Assumption 1. In balanced party systems, citizens with ideal points in the ranges $[x_L, x_{LM}]$ and $[x_{MR}, x_R]$ fit the profile of potential protest voters: their most-preferred party 1 is a viable competitor, and their ideal point is on the party system interior. Furthermore, they may choose either of their less-preferred parties when casting protest votes: among voters in the range $[x_L, x_{LM}]$ ($[x_{MR}, x_R]$), choosing either M or R (M or L) should impel their most-preferred party L (R) to alter its policy position in the 'proper' direction, since voters in $[x_L, x_{LM}]$ ($[x_{MR}, x_R]$) are located to the right (left) of their most-preferred party. In unbalanced systems the range of potential protest voters is (*ceteris paribus*) larger, as all those with ideal points in the range $[x_{LM}, x_R]$ fit the profile of potential protest voters. On the other hand, the strategic flexibility which characterizes potential protest voters in balanced systems does not exist. Those in the range $[x_{LM}, x_M]$ may protest vote only for L, since choosing R would send their first-preference M the wrong ideological signal; and those in the range $[x_M, x_{MR}]$ may protest vote only for R, since choosing L would, once again, send their most-preferred party M the wrong ideological signal.

This Section's analysis allows us to identify precisely the choice options available (and unavailable…) to voters in a decision theoretic framework. Its results are summarized visually in Figs. 2a and 2b. As demonstrated in these Figures, we have identified three distinct groups of voters, each of which faces a significantly different strategic situation than the others:

(i) those with profiles C5 and C6: range $[x_{LM}, x_{MR}]$ in Fig. 2a, and range $[0, x_{LM}]$ in Fig. 2b.

[12] Proposition 1 follows immediately from Signaling Assumption 1. In turn, its generalizability will depend on the extent to which this assumption can be endogenized in a fully strategic model. The electoral options available to voters on the party system exterior thus constitute a predominant subject of discussion in Sect. 8 below.

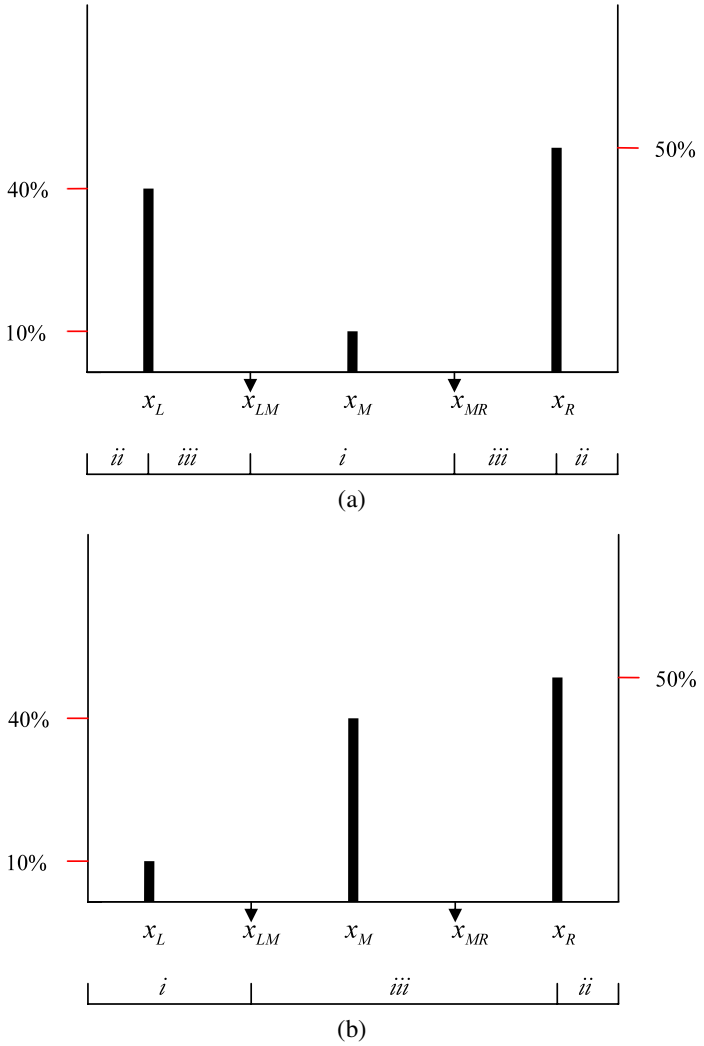

Fig. 2 Voting options in balanced and unbalanced systems

(ii) those with profiles C1–C4 on the party system exterior: ranges $[0, x_L]$ and $[x_R, 1]$ in Fig. 2a, and range $[x_R, 1]$ in Fig. 2b.
(iii) those with profiles C1–C4 on the party system interior: ranges $[x_L, x_{LM}]$ and $[x_{MR}, x_R]$ in Fig. 2a, and range $[x_{LM}, x_R]$ in Fig. 2b.

Voters in group (i) most prefer the system's minor party, and under certain circumstances may cast strategic votes for a more viable candidate; this behavior has been studied extensively in work cited above. Among members of group (ii) electoral choice is most constrained, as their only options tend to be choosing their most-preferred party or abstaining. Finally, voters in group (iii) will under certain circumstances cast protest votes for a party other than their most-preferred to send this party a signal of disaffection. Among those voters, Sects. 5 and 6 identify the circumstances which make protest voting more likely.

5 Voter signaling and the calculus of voting

We begin by adopting the expected utility framework first explored by Downs (1957) and Riker and Ordeshook (1968). Recalling from above that we label a voter's most-preferred candidate 1, second-most-preferred candidate 2, and least-preferred candidate 3, if only $j \in \{1, 2, 3\}$ compete in a winner-take all plurality contest there are seven possible outcomes in the outcome set θ:

$$\theta \in \begin{cases} 1 & 1\_wins \\ 2 & 2\_wins \\ 3 & 3\_wins \\ 12 & 1, 2\_tie \\ 13 & 1, 3\_tie \\ 23 & 2, 3\_tie \\ 123 & 1, 2, 3\_tie \end{cases} \tag{1}$$

Define q_θ^j as the probability that outcome θ occurs given that i votes for candidate j, while q_θ^0 is the probability that outcome θ occurs if she abstains. Further define U_θ as the utility i receives given outcome θ, where $U_1 > U_2 > U_3$ and ties are decided by a coin flip (or three-way perfect randomization if $\theta = 123$). In the framework developed in Sect. 3 above U_θ is determined by a voter's spatial proximity to the candidate or candidates in question. For the paper's remainder we employ the notation U_θ rather than an explicit loss function. Within the traditional calculus of voting these parameters are sufficient to state E_i^j, i's expected utility for choosing $j \in \{1, 2, 3\}$:

$$E_i^j = q_1^j \cdot U_1 + q_2^j \cdot U_2 + q_3^j \cdot U_3 + q_{12}^j \cdot U_{12} + q_{13}^j \cdot U_{13} + q_{23}^j \cdot U_{23} + q_{123}^j \cdot U_{123}. \tag{2}$$

Each of the terms in (2) represents the utility i receives from an outcome (U_θ) multiplied by the probability that outcome occurs given a vote for j.

The literature on electoral signaling reviewed above suggests that the traditional calculus of voting utility function is incomplete. This work's common theme is that voters consider not only their vote's short-term impact on the current election, but also its impact on the electoral signal received by key political actors. To capture signaling incentives, restate i's expected utility for choosing candidate j in the following manner:

$$\begin{aligned} E_i^j = {} & \alpha_i \cdot \{q_1^j \cdot U_1 + q_2^j \cdot U_2 + q_3^j \cdot U_3 + q_{12}^j \cdot U_{12} + q_{13}^j \cdot U_{13} + q_{23}^j \cdot U_{23} + q_{123} \cdot U_{123}\} \\ & + \beta_i \cdot \{U_1 + E(\sigma)\}. \end{aligned} \tag{3}$$

The first term represents the expected utility of choosing j as posited in the traditional COV, weighted by the constant α_i. The second term represents i's *expected satisfaction* with her most-preferred party 1 in future rounds of electoral competition, weighted by the constant β_i. Within the second term we find i's satisfaction for 1 during the current election U_1, which serves as a baseline from which i forms expectations concerning future electoral rounds; and $E(\sigma)$, the expected change in i's satisfaction between the current election and future electoral rounds, which can be positive or negative. Though a random variable subject to a variety of exogenous forces, in keeping with the notion of electoral signaling reviewed above $E(\sigma)$ will also be influenced by the current election's outcome, and as such by i's vote choice. Formally, we will let $E(\sigma) = \sigma^E + \sigma_i^j$, where σ^E represents the exogenous component and σ_i^j the component which is systematically related to voter i's choice for candidate j.

The parameters α_i and β_i capture the relative weight that voter i attaches to affecting the current election's outcome as compared to signaling disaffection with her most-preferred candidate. In his treatment of mass product markets, Hirschman identified this relative weighting as consumers' 'marginal sensitivity to quality decline.' There is some evidence that protest voting seems to occur among more educated and interested voters (Bowler and Lanoue 1992). Perhaps more basically α_i will be higher and β_i lower among voters with short *time horizons*, for whom the current election's outcome matters much more than a party's future ideological evolution. The individual-level determinants of α_i and β_i are obviously of central importance, and their identification comprises an essential object of our ongoing research.

The second of two signaling assumptions allows us to generate a more precise statement of (3). Consider a party which in most elections receives 70% of the district's vote share, with the remaining 30% split between its two competitors. A party with this type of electoral security is unlikely to be phased by a slight loss of votes to its ideological right or left. On the other hand, a party which usually receives 40% of the vote is more likely to react to marginal losses of support, since such losses genuinely influence its probability of being elected. Signaling Assumption 2 codifies this dynamic:

Signaling Assumption 2 A party's marginal reaction to vote losses will increase as it becomes more vulnerable to its electoral competitors.

To incorporate Signaling Assumption 2, substitute for σ_i^j as follows:

$$E(\sigma) = \sigma^E + \{q_2^j + q_3^j\}. \tag{4}$$

As such, the greater is the sum of q_2^j and q_3^j, that is, the greater the probability that candidate 2 wins or 3 wins when i votes for candidate j, the more likely one's most-preferred party will react to the loss of electoral support with ideological shifts. Since by definition $q_2^2 > q_2^1$ and $q_3^3 > q_3^1$ (i.e., the probability that 2 wins is higher if you vote for 2 than if you vote for 1, and similarly for the probability that 3 wins), expected recuperation will always be higher when a voter chooses a party other than his or her most-preferred. To identify the conditions under which abandoning one's most-preferred candidate 1 for one's second-most-preferred candidate 2 is a utility maximizing option, we engage in an explicit comparison of the expected utilities associated with both choices. After substituting (4) into (3) we can write the utility differential associated with choosing 1 as opposed to choosing 2 as follows:

$$\begin{aligned} E_i^1 - E_i^2 = \alpha_i \cdot \{&(q_1^1 - q_1^2) \cdot U_1 + (q_2^1 - q_2^2) \cdot U_2 + (q_3^1 - q_3^2) \cdot U_3 + (q_{12}^1 - q_{12}^2) \cdot U_{12} \\ &+ (q_{13}^1 - q_{13}^2) \cdot U_{13} + (q_{23}^1 - q_{23}^2) \cdot U_{23} + (q_{123}^1 - q_{123}^2) \cdot U_{123}\} \\ &+ \beta_i \cdot \{(q_2^1 - q_2^2) + (q_3^1 - q_3^2)\}. \end{aligned} \tag{5}$$

Voter i will have the incentive to protest vote for 2 over 1 if (5) < 0, i.e., the expected utility from choosing 2 is greater than that accrued from choosing 1. Each of the probability differentials $(q_\theta^1 - q_\theta^2)$ represents the likelihood that i is *pivotal* for the outcome θ, i.e., that switching her vote from 1 to 2 (or vice versa) will be *sufficient to upset or create the outcome* θ. To arrive at a more parsimonious statement of (5) we employ the following Lemma, proven exhaustively elsewhere:[13]

[13] Kselman and Niou (2010) derive this result.

Lemma 1

(a) $q_1^1 - q_1^2 = q_{12}^0 + q_{12}^2 + q_{13}^2 + q_{123}^0 + q_{123}^2$;
(b) $q_2^1 - q_2^2 = -(q_{12}^0 + q_{12}^1 + q_{23}^1 + q_{123}^0 + q_{123}^1)$;
(c) $q_3^1 - q_3^2 = q_{23}^2 - q_{13}^1$.

From here on assume for expository purposes that $q_{123}^j = 0$, i.e., that voters discount the possibility of three-way ties.[14] Substituting into (5) using Lemma 1 and rearranging results in the following comparison:

$$\begin{aligned} E_i^1 - E_i^2 = \alpha_i \cdot \{&(2q_{12}^0 + q_{12}^1 + q_{12}^2) \cdot (U_1 - U_2) \\ &+ (q_{13}^1 + q_{13}^2) \cdot (U_1 - U_3) - (q_{23}^1 + q_{23}^2) \cdot (U_2 - U_3)\} \\ &- \beta_i \cdot \{(2q_{12}^0 + 2q_{12}^1 + 2q_{23}^1) + (2q_{13}^1 - 2q_{23}^2)\}. \end{aligned} \tag{6}$$

The first term in (6) captures the short-term utility benefit i receives by choosing 1 and increasing her chances of winning the current election. The second term contains the signaling value of choosing 2 over 1. This second term is negative, since voters expect a higher future utility for 1 when they cast a protest vote for 2, thus reducing 1's market share and potentially inducing ideological shifts. Note the competing effects that competitive elections have on voters' incentives. On the one hand as the race between 1 and her competitors tightens (i.e., as q_{12} and q_{13} increase) the *cost* of signaling increases (first term in (6)), since choosing another candidate is more likely to prevent 1 from gaining office. On the other hand, the *benefits* of signaling simultaneously increase (second term in (6)), since parties are more likely to respond to vote losses in environments where such losses genuinely threaten their chances of winning.

Rearranging (6) yields following Proposition, specifying the conditions under which voters will prefer casting a protest vote for 2 to voting for their most-preferred party 1 (algebra omitted):

Proposition 2 *Voter i will prefer choosing her most-preferred candidate* 1 *over* 2 *as long as the following condition obtains*:

$$\begin{aligned} &\frac{\alpha_i}{\beta_i} > \Re_{12} \\ &\equiv \frac{\{(2q_{12}^0 + 2q_{12}^1 + 2q_{23}^1) + (2q_{13}^1 - 2q_{23}^2)\}}{\{(2q_{12}^0 + q_{12}^1 + q_{12}^2) \cdot (U_1 - U_2) + (q_{13}^1 + q_{13}^2) \cdot (U_1 - U_3) - (q_{23}^1 + q_{23}^2) \cdot (U_2 - U_3)\}}. \end{aligned} \tag{7}$$

Naturally, when the importance voters attach to affecting the current election greatly outweighs their signaling motivations (i.e., when α_i is high relative to β_i), voters will generally stay loyal to 1. As well, when the benefits to signaling ($\Re_{12}$'s numerator) increase in proportion to the costs ($\Re_{12}$'s denominator), it becomes more difficult to satisfy (7), and choosing 2 over 1 becomes more likely. Define $\Re_{12}$ as the *signaling ratio* for choosing between candidates 1 and 2.

[14] The present results are qualitatively identical, though less parsimonious, if we allow for three-way ties.

6 Protest voting, competitiveness, and indifference

While in the traditional calculus of voting choosing one's least-preferred candidate 3 is never a utility-maximizing choice, in the signaling model developed above some voters on the party system interior may be able to send 1 the 'proper' ideological signal by choosing 3. As such, to fully capture the calculus which informs voter choice we must also derive $\Re_{13}$ and $\Re_{23}$, signaling ratios that define the conditions under which candidate 3 might be preferred to both candidates 1 and 2. In turn, we must consider the possibility that some voters may prefer candidate 1 to candidate 2 in a head-to-head utility comparison (criterion in Proposition 2 is satisfied), but in turn prefer 3 to either 1 or 2 in head-to-head utility comparisons. As a result, Proposition 2 is not in itself sufficient to definitively identify when voters will or will not cast protest votes: voters might cast protest votes for 3 even if they prefer candidate 1 to candidate 2 in a head-to-head comparison.

Though a hypothetically plausible scenario, Proposition 3 in the Appendix demonstrates that it is theoretically impossible. In words, the Proposition tells us that, as long as a voter prefers 1 over 2 in a head-to-head utility comparison, he or she also always prefers 1 over 3 in a head-to-head utility comparison. The proof is grounded in a second theoretical Lemma, which states that the choice of 3 will always be strictly dominated among voters with expectation profiles C1 and C3, i.e., among voters who believe 2 will place ahead of 3 in the election (Lemma 2 in the Appendix). Proposition 3 implies that the statistic $\Re_{12}$ is sufficient for determining whether or not individuals engage in protest voting for either of their less-preferred parties. Regardless of party system type and expectation profile, protest voting becomes more likely when $\Re_{12}$ is high (making (7) harder to satisfy) and less likely when it is low. This simple fact allows us to generate a series of individual-level hypotheses about the strategic conditions which make electoral signaling a more palatable option. For example, by analyzing the consequences of transferring votes from the 1st place candidate to the 2nd place candidate, thus decreasing the front-runner's lead over her nearest competitor, we can study the relationship between protest voting and an election's *competitiveness*.

Such a vote transfer has three immediate effects: (a) it *increases* the probability of a tie between the two most viable candidates, by reducing the 1st place candidate's margin; (b) for the same reason it *increases* the probability of a tie between the 1st and 3rd place candidates; and (c) it *decreases* the probability of a tie between the 2nd and 3rd place candidates, by increasing the 2nd place candidate's margin. These effects have different implications for the probability parameters q_θ^j depending on a voter's strategic profile. For example, among voters with profiles C1 and C2, who believe 1 to be in first place, (a) implies an increase in q_{12}, (b) implies an increase in q_{13}, and (c) implies a decrease in q_{23}. For each profile C1–C4, Table 2 summarizes the effects of increasing the election's competitiveness on the probability parameters q_θ^j. In turn, Lemma 3 in the Appendix derives the comparative static effect of changes in q_θ^j on $\Re_{12}$.

Taken together, Table 2 and Lemma 3 thus allow us to undertake a profile by profile analysis of the impact of increasing competitiveness on protest voting:

Proposition 4A *Among voters with profiles* C1, C2, *and* C4, *increases in competitiveness will lead to decreases in* $\Re_{12}$, *making* (7) *easier to satisfy and protest voting* less likely.

Proposition 4B *Among voters with profile* C3, *increases in competitiveness will lead to increases in* $\Re_{12}$, *making* (7) *harder to satisfy and protest voting* more likely.

Table 2 Increasing electoral competitiveness

	Effects of increased competitiveness among the two leading candidates
C1	$+\Delta q_{12}, +\Delta q_{13}, -\Delta q_{23}$
C2	$+\Delta q_{12}, +\Delta q_{13}, -\Delta q_{23}$
C3	$+\Delta q_{12}, -\Delta q_{13}, +\Delta q_{23}$
C4	$-\Delta q_{12}, +\Delta q_{13}, +\Delta q_{23}$

Proof of Proposition 4A Among voters with profile C1 increased competitiveness leads to an increase in q_{12}, an increase in q_{13}, and a decrease in q_{23}. The latter two dynamics both decrease the size of $\Re_{12}$ and thus reduce the likelihood of protest voting. Changes in q_{12} have an indeterminate effect depending on the relative sizes of q_{13} and q_{23}. However, it is straightforward to show that the combined effects of changes in q_{13} and q_{23} outweigh the single effect of q_{12} regardless of q_{13} and q_{23}'s relative sizes (algebra omitted), such that protest voting becomes less likely as the election becomes more competitive.

Among voters with profile C2 increased competitiveness leads to an increase in q_{12}, an increase in q_{13}, and a decrease in q_{23}. The latter two dynamics both decrease the size of $\Re_{12}$ and thus reduce the likelihood of protest voting. Among voters with profile C2, increases in q_{12} make protest voting more likely (since $q_{13} > q_{23}$; see Appendix). However, it is nonetheless straightforward to show that the combined effects of changes in q_{13} and q_{23} outweigh the single effect of q_{12} (algebra omitted), such that protest voting becomes less likely as the election becomes more competitive.

Among voters with profile C4 increased competitiveness leads to a decrease in q_{12}, an increase in q_{13}, and an increase in q_{23}. The latter dynamic increases the size of $\Re_{12}$, making protest voting more likely. But, among voters with profile C4 both of the former dynamics decrease the size of $\Re_{12}$ (since $q_{13} > q_{23}$; see Appendix), making protest voting less likely. It is straightforward to show that the combined effects of changes in q_{13} and q_{12} outweigh the single effect of q_{23} (algebra omitted), such that protest voting becomes less likely as the election becomes more competitive. □

Proof of Proposition 4B Among voters with profile C3 increased competitiveness leads to an increase in q_{12}, a decrease in q_{13}, and an increase in q_{23}. The latter two dynamics both increase the size of $\Re_{12}$ and thus increase the likelihood of protest voting. Changes in q_{12} have an indeterminate effect depending on the relative sizes of q_{13} and q_{23}. However, it is straightforward to show that the combined effects of changes in q_{13} and q_{23} outweigh the single effect of q_{12} regardless of q_{13} and q_{23}'s relative sizes (algebra omitted), such that protest voting becomes more likely as the election becomes more competitive. □

In words, for most voters, when the election's leader is expected to win in a landslide, the risk of protest voting is decreased since protest voting does not significantly effect 1's chances of being elected. On the other hand, as competitiveness increases protest voting constitutes a more genuine threat to 1's chances of winning the election at hand, and thus becomes less likely. This basic implication does not apply to voters with profile C3, who believe 2 is leading the race, 1 is expected to place 2nd, and 3 is expected to place last. Among such voters, increases in competitiveness lead to a decrease in q_{13} and an increase in q_{23}, both of which have the effect of increasing $\Re_{12}$, thus increasing range of α_i and β_i for which protest voting is optimal. In words, by decreasing the likelihood that 3 catches

up to 1, but increasing the likelihood that 3 catches up to the expected plurality winner 2, increases in electoral competitiveness raise the value of protest voting for 2.

The parameter q_{13} is central in generating Propositions 4A and 4B. Among potential protest voters of all strategic profiles, increases in q_{13} always lead to a decrease in $\Re_{12}$, making (7) easier to satisfy and protest voting less likely (Lemma 3). In words, when voters perceive their most and least-preferred candidates to be engaged in a tight race, the short-term costs of protest voting will tend to become prohibitive for all but a small minority, and electoral signaling should be unlikely. Note from Table 2 that increases in competitiveness lead to increases in q_{13} among voters with profiles C1, C2, and C4, but not among those with profile C3. The particular significance of expectations concerning one's least-preferred party is thus responsible for generating the counter-intuitive relationship between protest voting and competitiveness among voters with expectation profile C3.

The following Proposition demonstrates a related result regarding voters' *utility* over the election's various candidates and the likelihood of protest voting:

Proposition 5 *Among all potential protest voters, as the utility differential separating one's first preference from the remaining two parties decreases protest voting will be more likely.*

It is straightforward to see that, as $(U_1 - U_2)$ and $(U_1 - U_3)$ decrease, $\Re_{12}$ will increase since its denominator decreases, making (7) more difficult to satisfy. In the language of previous rational choice studies, as voters perceive *less difference* between the elections' candidates, they become more likely to cast protest votes.

7 Suggestive empirical results

A full scale, individual-level empirical test of the preceding results would account not only for voters' expectation profiles, but also voters' ideological positions as well as those of the system's various parties. This information would allow us to distinguish between voters on the party system's 'interior' and its 'exterior', and also to identify more precisely the particular parties which constitute viable signaling options. The aggregate data analysis that follows thus does not constitute definitive evidence; but its implications present suggestive support for some of the model's basic implications. The Conclusion addresses in greater detail both the opportunities and challenges involved in accurately operationalizing and testing this and other models of tactical voter choice.

Table 3 classifies respondents from the 1988 Canadian National Election Survey[15] according to their expectation profile and vote choice, as motivated by the typology presented in Table 1.[16] Respondents from Quebec are omitted to remain consistent with the three-party

[15] Fieldwork for the study was undertaken by the Institute for Social Research at York University. The 1988 Canadian Election Study had three distinct modules: a pre-election telephone survey with a representative sample of 3609 Canadians; a post-election telephone survey with 2922 of the respondents who responded to the pre-election survey; and a mail-return questionnaire with 2115 of the total post-election survey respondents. We use here only data from the pre-election survey. See Johnston et al. (1992) for further information on the survey.

[16] In fact, Table 3 is more complicated than Table 1 in that many respondents assigned two or more candidates an equal probability of winning, thus falling into Table 3b which groups them according to their proper strategic profile. A significant number of voters also reported being indifferent between one or more of the election's candidates. Though the data are available, as the current analysis is meant only to be suggestive we omit such voters for reasons of space and parsimony: the number of categories in Table 3 would multiply

Table 3 Distribution of voters in the 1988 Canadian NES

a: Respondents with strict expectations

	C1	C2	C3	C4	C5	C6	Total
	1	1	2	3	2	3	
	2	3	1	1	3	2	
	3	2	3	2	1	1	
$V = P_1$	427	231	103	81	28	34	904
$V = P_2$	20	8	28	5	13	8	82
$V = P_3$	7	5	0	2	1	7	22
A	20	10	6	4	4	5	49
Total	474	254	137	92	46	54	$N = 1057$

b: Respondents with weak expectations

	C7	C8	C9	C10	C11	C12	C13	Total
	12	13	1	2	3	23	123	
	3	2	23	13	12	1		
$V = P_1$	93	67	133	33	37	15	13	391
$V = P_2$	11	1	6	11	4	6	0	39
$V = P_3$	1	2	2	0	3	0	0	8
A	10	1	7	5	3	2	2	30
Total	115	71	148	49	47	23	15	$N = 468$

In Table 3b, respondents assign two or more candidates an equal probability of winning. A respondent in column C7 assigns 1 and 2 an equal likelihood of winning, and assigns 3 a lesser likelihood. A respondent in column C10 believes 2 to be the front-runner, and believes that the two trailing candidates 1 and 3 have the same probability of winning. A respondent in column C13 believes all 3 candidates have an equal likelihood of winning

case developed above. To measure voters relative preferences for the various candidates we use feeling thermometer ratings; to capture their expectations we employ normalized assessments of each candidates' probability of winning; and to capture their vote choice we use their 'intended' pre-election vote choice to avoid well-known biases in post-election responses (Alvarez and Nagler 2000). Recall from above that we exclude from consideration voters whose most-preferred party is expected to place last in the election, i.e., those who believe 2 and 3 are the election's most viable candidates. As such, the following analysis excludes columns C5, C6 and C12 from Table 3.

Among the remaining voters, we first classify respondents in two groups: (a) columns C1, C3, C7, and C10 whose members see candidates 1 and 2 as at least tied for the election's 1st and 2nd place positions, such that 1 and 3 are by definition not the election's two most viable candidates; and (b) columns C2, C4, C8, C9, C11, and C13, whose members believe that 1 and 3 are at least tied for the election's two most competitive spots. Recall from above the importance of q_{13} in individuals' decision process: high values of q_{13} invariably make protest voting less likely. Voters from group (a) should thus be more likely than their counterparts

combinatorially to capture the many indifference profiles associated with each of its 13 strategic profiles. Such an analysis is beyond our current scope, and reserved for future empirical work which tests the model more exhaustively.

in group (b) to choose a candidate other than their most-preferred. In fact, voters whose expectation profiles correspond to group (a) abandon their most-preferred party 10.1% of the time (78 of 775), while those from group (b) do so at only a 6.1% clip (38 of 627). It is thus suggestive that, among potential protest voters, respondents who do not perceive 1 and 3 as the most viable candidates are nearly 60% more likely to abandon their most-preferred party.

In most Canadian districts voters are faced with unbalanced party system configurations: the National Democratic Party is both the system's smallest and furthest to the Left.[17] Proposition 5 above demonstrates the paucity of voters in unbalanced systems who ever even consider choosing their least-preferred candidate. The total number of potential protest voters that choose 3 in this sample is indeed a miniscule 1.5% (22 of 1,402). That said, recall from the above discussion of Proposition 3 that the choice for 3 over 2 should be more likely among voters who believe that 3 will place higher than 2 in the election (Lemma 2 Appendix). The above classification scheme can be used to investigate this hypothesis: respondents in group (a) believe that 2 is ahead of 3, while those in group (b) believe that 3 is either ahead of or at least tied with 2. Strikingly, the ratio of voters choosing 3 rather than 2 is 8/70 among the first group and 14/24 among the second. Put otherwise, among voters who believe 3 is either ahead of or tied with 2, nearly 1 out every 2.5 protest voters chooses 3; while barely 1 of 10 does so when 2 is perceived to be ahead of 3. To summarize the empirical analysis, when 3 is perceived as a viable candidate, choosing 1 becomes much more likely; but among those who do protest vote despite the fact that 3 is a viable candidate, the choice to protest vote for 3 becomes nearly as common as the choice to protest vote for 2. While far from definitive evidence, both dynamics are consistent with our model's basic implications.

8 Moving to a strategic environment

This paper introduces downstream considerations into the 'calculus of voting', and derives individual voters' optimal choice in this more complicated decision-theoretic context. Just as papers by Palfrey (1989) and Cox (1997) extend the traditional calculus of voting to a game theoretic environment, future research should allow voters' signaling behavior to take into account other voters' decisions, as well as the optimal response of political party organizations to aggregate electoral returns. A full game theoretic treatment is well beyond this paper's scope, and indeed would present the analyst numerous technical challenges.[18] However, it is useful here to outline the contours of such a model, which in turn will help to identify the conditions under which this paper's decision-theoretic hypotheses are most likely applicable.

[17] In the small number of districts where the NDP has traditionally competitive, the party system configuration will be balanced.

[18] For example, the above-cited models by Palfrey and Cox look only at strategic interaction among voters. Political parties' ideological positions are, on the other hand, assumed to be fixed. Ideally a general equilibrium in the current context would include strategic parties, as parties' equilibrium response to electoral signals is an object of particular theoretical interest. However, a model with strategic party organizations may generate meaningful results only if voters are assumed to be non-strategic, as is the case in most spatial models of electoral competition. The prospects of generating meaningful results for a model in which both voters and political parties are fully strategic are left to be identified in future research. Here, we provide a series of preliminary intuitions which might usefully guide such a modeling effort.

A defining dynamic of any such game theoretic model will be the informational assumptions used to model political parties. For example, it may be that political parties observe only aggregate-level vote returns, and have *no* individual-level information as to the choices of particular voters. On the other hand, one might consider a model in which parties are *fully informed* about the choices of all voters in an electorate, i.e., in which they directly observe all individual voting decisions. Finally, parties might be characterized as having *incomplete information* as to individual voters' behavior, i.e., as having beliefs about voter choice which must be updated in accordance with Bayes' rule as the game unfolds. The informational assumptions used to model political parties will define the conditions under which parties should (or perhaps should not ...) behave in accordance with Signaling Assumption 1 above.

Stated as a question, under what conditions will parties have the information necessary to identify vote losses on their ideological right or left as signals of disaffection from voters in the relevant preference ranges? Crucial to this question is the behavior of 'extremist' voters whose most-preferred policy position places them on the party system exterior. Proposition 1 above tells us that such voters should never protest vote, because such a vote would always send their most-preferred party organization the 'wrong' ideological signal. However, consider a model in which fully informed political parties directly observe the electoral choices of all individual voters. In this situation, extremist voters dissatisfied with their most-preferred party might choose to protest vote for a moderate party: fully informed party leaders will be able to identify the portion of a shift in aggregate vote shares which comes from disaffected extremists; in turn, they may ultimately adopt a more polarized ideological position *despite* observing the increased success of a more moderate party (in direct contradiction to Signaling Assumption 1).

We expect the key obstacle to such equilibria will be the *possibility of abstention*. Note that, while for reasons of space we do not undertake a full analysis of abstention above, the act itself sends an electoral signal by reducing the customary vote share of one's first-preference. However, this signal will not be as 'loud' as a protest vote, since it reduces one's most-preferred party's vote share *without* increasing the vote share of a competing organization (Signaling Assumption 2 above). Now, unlike voters on the party system interior, extremist voters on the party system exterior have a strong distaste for both centrist parties and, especially, parties at the opposite ideological extreme. Thus, while voters on the party system interior may be willing to risk increasing a competing party's vote share so as to send a 'loud' electoral signal, voters on the party system exterior should generally prefer abstention, i.e., prefer sending a 'soft' electoral signal which does not increase the vote share of a highly distasteful party organization.

As a result, even when political parties are fully informed about individual vote choices, Proposition 1's prediction should hold. This Proposition would be violated only given certain irregular values of α_i and β_i. For example what if $\alpha_i = 0$, i.e., voter i completely discounts the current election's outcome?[19] In this case, an extremist voter would be no less offended by the short-term policy positions of competing party organizations than a moderate voter, and may thus choose to protest vote for a centrist party. On the other hand, the logic of *party entry* in such situations may in fact make the point moot, as such situations may give rise to extremist parties which drive to '0' the number of voters actually on the party system exterior.[20]

[19]The parameter α_i might, for example, approach '0' in elections to the European Union's *European Parliament* (EP), which historically has had little legislative prerogative.

[20]Van der Eijk and Franklin (1996) find that smaller organizations on the political extreme experience increased success in EP elections. In fact, it may be that radical parties compete more intensely in these elec-

The preceding discussion applies to cases in which political parties are fully informed as to the choices of individual voters. Things become slightly more complicated when parties are either incompletely informed or completely uninformed about individual-level behavior. In both cases it may become possible for extremist voters to 'cloud' party organizations' ability to accurately interpret electoral signals. For example, if one's most-preferred party has incomplete information as to its supporters' choices, and extremists credibly threaten to protest vote for a moderate party, the most-preferred party may (in equilibrium…) be unable to identify aggregate electoral shifts to a more centrist party as clear messages for or against ideological moderation. A full equilibrium treatment of the conditions which may generate such confounding behavior by voters on the party-system exterior must naturally await future research. Suffice it so say here that such strategic jockeying will be risky, as attempts to cloud parties' interpretative capacity entail a risk of handing the election to parties of diametrically opposed ideological convictions. Outside of cases in which α_i approaches '0', one could envision such risk-taking as being strictly-dominated by the less risky choice to abstain.

Under a wide range of informational contexts we would thus expect voters on the party system exterior to prefer sending 'soft' signals via abstention rather than 'louder' but riskier signals via the protest vote. This implies an expectation that Assumption 1 and the accompanying Proposition 1 will be robust to a wide range of strategic specifications. Of course, the possibility that voters on the party system exterior might abstain raises another strategic issue: although parties may be able to confidently interpret the increased success of moderate parties as signals of disaffection, they may be reluctant to adopt more moderate policies for fear of driving their more extreme supporters to abstention. In a game theoretic setting, voters on the party system interior would take this reluctance into account and have little incentive to protest vote. The questions then become: (a) under what conditions will parties be able to tolerate abstention by extremists? (b) under what conditions will extremist supporters turn out despite seeing their parties move to the ideological center?

Answers to both questions should depend crucially on an election's *competitiveness*. For example, in highly uncompetitive elections political parties should be able to discount short-term abstention by a segment of their base, since this abstention ought not sway the electoral outcome one way or another. In contrast, if greater moderation induces abstention from the ideological base, party leaders in competitive elections may be reluctant to react in the way suggested by Signaling Assumption 1. That said, in competitive elections extremists themselves will find it more difficult to credibly abstain when faced with moderate platforms, as this abstention might tilt the electoral balance of power in favor of a distasteful organization. We look forward to investigating this possibility and others in future theoretical work, which endogenizes the conditions under which Signaling Assumption 1 and Proposition 1 are in fact viable. We hope the present discussion establishes, fairly convincingly, the fact that this paper's premises and results should hold in a substantial range of strategic scenarios, and should comprise a useful starting point for future game theoretic research.

tions precisely because they understand voters' reduced concern for the contests' short-term outcomes. Put otherwise, in cases when α_i approaches '0' we might often see the 'Entry' of extremist organizations into the political marketplace, which in turn 'moves' extremist voters from the party system exterior to the party system interior. If this logic has any purchase, then Proposition 1's hypothesis would continue to hold even as α_i approaches '0': the very situations which might make extremists protest vote for moderate parties (thus violating Proposition 1) will *also* lead to the appearance of smaller radical organizations, which in turn nullifies the very presence of a party system exterior.

9 Concluding discussion

Though protest voters like strategic voters of the 'traditional' variety choose a party other than their most-preferred, they do so under very different circumstances and for very different reasons. While strategic voters abandon their most-preferred party in order to affect the current election's outcome, protest voters do so for the sake of downstream quality improvements. Section 4 demonstrates that electoral signaling of the variety modeled here should not occur among voters on the party system exterior, i.e., whose ideal points are to the left of the leftmost party or the right of the rightmost party. Among voters on the party system interior, Sects. 5 and 6 then demonstrate that the likelihood of protest voting should increase when the current election's outcome is largely predetermined and/or of little personal consequence.

As discussed in Sect. 8, abstention can be thought of as a less risky signal of disaffection insofar as it reduces 1's vote share without increasing the vote share of 1's competitors. Indeed, while protest voting might be optimal among disaffected voters when β_i is high and/or α_i is low, among those for whom α_i and β_i assume intermediate values abstaining may constitute a more palatable if less effective expression of dissatisfaction. As well, among voters on the party system exterior, abstention is the *only* available option aside from choosing 1 (Proposition 1). Rather than a sign of political apathy, abstention may thus constitute a perfectly instrumental response to particular systemic stimuli. Future empirical work on abstention in distinct party system environments should help to unravel the complex and mutually determined relationship between voter turnout and voter signaling.

That said, empirical work testing this and other theories of tactical voter choice faces distinct challenges, arising primarily from the multiplicity of incentives capable of motivating the choice for a candidate other than one's most-preferred. We have already noted that past empirical research has often conflated strategic voters and protest voters. Beyond these two forms of tactical choice, Schuessler (2000) and others have studied the conditions under which voters might engage in *bandwagoning*, i.e., vote for the election's frontrunner not because he or she is one's first-preference, but rather to accrue the 'expressive' value of 'choosing the winning team'. As well, a different form of protest voting may arise among voters who wish to send a signal of dissatisfaction to the entire political status quo rather than simply their most-preferred party. We might expect this behavior when voters feel 'alienated' from the political-party system, for example when a system's main parties are either completely polarized (thus alienating centrist voters) or completely convergent on the median voter's ideal point (thus alienating extremist voters on either side of the political spectrum).

Like protest voting and strategic voting of the traditional variety, each of these behaviors will be associated with particular party system environments, as well as with particular expectation profiles within individual party systems. Furthermore, it is often the case that an observed tactical choice has many possible explanations. For example, voters with expectation profile C3 in Table 1, who believe their second-most preferred candidate 2 to be in 1st place, may choose 2 as a result of *strategic, signaling, or bandwagoning incentives*. An exhaustive empirical treatment of tactical voter choice would carefully catalogue both the parallel and divergent hypotheses associated with each of these theoretical arguments. When statistical evidence is consistent with more than one behavioral motivation, additional measures must be developed which help to parse between the multiple interpretations. Although operationalizing and testing theories of tactical behavior may thus represent a greater challenge than previously acknowledged, it also represents an opportunity to develop more fine-grained survey instruments, capable of distinguishing between the manifold motivations which inform voter behavior.

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Appendix

In addition to $\Re_{12}$ a complete analysis of voter choice in 3-candidate plurality elections requires the derivation of the statistics $\Re_{13}$ and $\Re_{23}$, the signaling ratios which govern the choice between 1 and 3 and 2 and 3 respectively. Employing slight variations of Lemma 1, and undertaking the identical algebraic procedure used to derive $\Re_{12}$ in the text, we see that individual i will choose 1 over 3 as long as:

$$\frac{\alpha_i}{\beta_i} > \Re_{13} = \frac{\{(2q_{13}^0 + 2q_{12}^1 + 2q_{23}^1) + (2q_{13}^1 - 2q_{23}^2)\}}{\{(2q_{13}^0 + q_{13}^1 + q_{13}^3) \cdot (U_1 - U_3) + (q_{12}^1 + q_{12}^3) \cdot (U_1 - U_2) + (q_{23}^1 + q_{23}^3) \cdot (U_2 - U_3)\}}. \quad (8)$$

Again, employing a slight variation of Lemma 1 yields the signaling ratio for choosing 2 over 3:

$$\frac{\alpha_i}{\beta_i}\Re_{23} = \frac{\{(2q_{13}^2 + 2q_{23}^2) - (2q_{12}^3 + 2q_{23}^3)\}}{\{(2q_{23}^0 + q_{23}^2 + q_{23}^3) \cdot (U_2 - U_3) + (q_{13}^2 + q_{13}^3) \cdot (U_1 - U_3) - (q_{12}^2 + q_{12}^3) \cdot (U_1 - U_2)\}}. \quad (9)$$

Among voters with expectation profiles C2 and C4, $\Re_{23}$'s numerator will be greater than 0, since $q_{13}^2 > q_{12}^3$ and $q_{23}^2 > q_{23}^3$ when voters perceive 1 and 3 as the two most viable parties. It is also straightforward to see that $\Re_{23}$'s denominator will be greater than 0 among voters with profile C2 and C4, making $\Re_{23} > 0$.

Among voters with expectation profiles C1 and C3, $\Re_{23}$'s numerator will be less than 0, since $q_{13}^2 < q_{12}^3$ and $q_{23}^2 < q_{23}^3$ when voters perceive 1 and 2 are the two most viable candidates. As well, among *most* such voters $\Re_{23}$'s denominator will be greater than 0, making $\Re_{23} < 0$. Among a small subset of voters with C1 and C3, $\Re_{23}$'s denominator will be less than 0, making $\Re_{23} > 0$.[21] We can now prove the following result:

Lemma 2 *Among voters with expectation profiles* C1 *and* C3 *the choice to vote for* 3 *is always strictly-dominated by the choice to vote for* 2.

Proof of Lemma 2 Among voters with profiles C1 and C3 for whom $\Re_{23} < 0$, choosing 2 over 3 is a dominant strategy if the following condition holds:

$$\frac{\alpha_i}{\beta_i} > \Re_{23} = \frac{\{(2q_{13}^2 + 2q_{23}^2) - (2q_{12}^3 + 2q_{23}^3)\}}{\{(2q_{23}^0 + q_{23}^2 + q_{23}^3) \cdot (U_2 - U_3) + (q_{13}^2 + q_{13}^3) \cdot (U_1 - U_3) - (q_{12}^2 + q_{12}^3) \cdot (U_1 - U_2)\}}. \quad (10)$$

Since $\Re_{23} < 0$ and $\alpha_i, \beta_i > 0$, this condition is always met, and choosing 3 over 2 can thus never be optimal.

[21] This occurs when the parameters q_{12}^j and U_{12} are especially large relative to q_{13}^j. In words, when 1 is greatly preferred to 2, and 2 is in a very close race with 1 while 3 lags far behind, it is possible that i in fact suffers a short-term *cost* from choosing 2 rather than 3: a vote for 3 is much less damaging to 1's chances of winning than a vote for 2, 1's closest competitor.

Among the small number of voters with profiles C1 and C3 for whom $\Re_{23} > 0$, the condition for choosing 2 over 3 is as follows:

$$\frac{\alpha_i}{\beta_i} < \Re_{23} = \frac{\{(2q_{13}^2 + 2q_{23}^2) - (2q_{12}^3 + 2q_{23}^3)\}}{\{(2q_{23}^0 + q_{23}^2 + q_{23}^3) \cdot (U_2 - U_3) + (q_{13}^2 + q_{13}^3) \cdot (U_1 - U_3) - (q_{12}^2 + q_{12}^3) \cdot (U_1 - U_2)\}}. \tag{11}$$

Condition (11) is identical to condition (10) except that the inequality is reversed: among this subset of voters $\Re_{23}$'s denominator is negative, such that in deriving (11) we were forced to divide the entire inequality by a negative quantity, in turn forcing us to reverse the inequality. The choice for 3 will be optimal if and only if both conditions (8) and (11) are satisfied, such that i prefers 3 to both 1 and 2. Among this subset of voters it is straight-forward to show that $\Re_{23} > \Re_{13}$ (algebra omitted), which in turn implies that whenever condition (8) is satisfied condition (11) will not be satisfied, and vice versa. As such, choosing 3 can thus never be optimal. □

Lemma 2 helps to derive the next result:

Proposition 3 *Among all voters with expectation profiles* C1–C4 *protest voting will occur if* $(\alpha_i/\beta_i) > \Re_{12}$, *and will not occur otherwise.*

Proof of Proposition 3 Among voters with expectation profiles C1 and C3 this is trivially the case, since the choice for 3 is always strictly-dominated. Among voters with expectation profiles C2 and C4, it is straight-forward to show that $\Re_{12} > \Re_{13} > \Re_{23} > 0$ (algebra omitted). As such, as long $(\alpha_i/\beta_i) > \Re_{12}$ individuals will choose (1), since both condition (7) from the text and condition (8) are satisfied. The parameters $\Re_{13}$ and $\Re_{23}$ are necessary to analyze the choice between 2 and 3; but the parameter $\Re_{12}$ itself is sufficient to determine whether or not individuals choose 1 or choose to protest vote. □

By calculating the relevant first derivatives we can study the effect of changes in the probability parameters q_θ^j on the signaling ratio $\Re_{12}$:

Lemma 3

(a) $\frac{\Delta\Re_{12}}{\Delta q_{13}} < 0$;
(b) $\frac{\Delta\Re_{12}}{\Delta q_{23}} > 0$;
(c) $\frac{\Delta\Re_{12}}{\Delta q_{12}} > 0$ *iff* $q_{13} > q_{23}$

Proof of Lemma 3 In proving Lemma 3 it helps to make the following expository assumption: $q_\theta^j = q_\theta^k$ for any candidates j and k.[22] This allows us to rewrite $\Re_{12}$ as follows:

$$\Re_{12} \equiv \frac{4q_{12} + 2q_{13}}{\{4q_{12} \cdot (U_1 - U_2) + 2q_{13} \cdot (U_1 - U_3) - 2q_{23} \cdot (U_2 - U_3)\}}. \tag{12}$$

[22] McKelvey and Ordeshook utilize this assumption throughout their original treatment, justifying by reference to fact the outcome sets it equates are *adjacent sets* in the outcome topology. While we prefer to maintain the distinction between q_θ^j and q_θ^k in the basic derivation for the sake of theoretical precision, when examining first derivatives it is natural that changes in q_θ^j should be almost perfectly replicated by changes in q_θ^k.

From (12) we immediately that increases in q_{23} lead to increases in $\Re_{12}$, establishing part (b). As well, application of the divisor rule establishes that the first derivative of $\Re_{12}$ w/r/t q_{13} is less than 0 (calculus omitted), thus establishing part (a). Finally, the derivative of $\Re_{12}$ w/r/t q_{12} is equal to $(q_{13} - q_{23}) \cdot (U_2 - U_3)$, which is greater than 0 as long as $q_{13} > q_{23}$, establishing part (c). □

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