Game Theory Problem Set 4

1. Two players need to divide a cake of size 1. Player 1 moves first, cutting the cake into two pieces, one with size \(x\) and the other with size \(1-x\). Then player 2 chooses one of the pieces, leaving the remaining piece to player 1. Both players prefer to have a piece as large as possible. In a subgame perfect equilibrium, how will player 1 cut the cake? Argue directly. No need to draw a game tree.

2. Three politicians, A, B and C, vote in that order on whether to give themselves a pay-raise of \(r\). The bill is passed if and only if two or more politicians vote yes. Each politician that votes yes, however, incurs citizen displeasure at a cost of \(c\), with \(c < r\), whether the bill is passed or not. The game is represented in the diagram below, with the payoffs respectively belonging to A, B, and C, in that order.
   a. How many possible strategies does B have? What are they?
   b. Find the subgame perfect Nash equilibrium. Remember an equilibrium should be written in the form of (A’s strategy, B’s strategy, C’s strategy).

3. The “Agenda control” problem (application of the ultimatum game) in lecture 4.

4. Exercise 221.2 in the textbook (just design the agenda. Don’t worry about the top cycle set.)

5. (This problem is not about extensive-form games but rather a review of the strategic-form game and mixed strategies that we learned in the beginning of the week.) A small town has two citizens and a ruling despot. A citizen can choose to rebel or not rebel. If both citizens rebel simultaneously, they can overthrow the despot and each gets a payoff of 5. If only one citizen rebels, the rebellion will fail, and each citizen gets nothing. Whether the rebellion
succeeds or not, a citizen that rebels must pay a rebellion cost of 2. Model the situation as a strategic game (i.e., representing the game in a table), and find all its Nash equilibrium (equilibria), including the strictly mixed strategy equilibrium if it exists.