

Introduction to Game Theory

Lecture 1: Strategic Game and Nash Equilibrium

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Rational Choice and Preference Relations

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- Rational choice: the action chosen by a decision maker is better or at least as good as every other available action, according to her preferences and given her constraints (e.g., resources, information, etc.).
- Preferences (偏好) are rational if they satisfy
 - ▷ **Completeness** (完备性): between any x and y in a set, $x \succ y$ (x is preferred to y), $y \succ x$, or $x \sim y$ (indifferent)
 - ▷ **Transitivity**(传递性): $x \succeq y$ and $y \succeq z \Rightarrow x \succeq z$ (\succeq means \succ or \sim)
 - \Rightarrow Say apple \succ banana, and banana \succ orange, then apple \succ orange

Preferences and payoff functions (utility functions)

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- Payoff function/utility function (支付函数/效用函数):
 $u(x) \geq u(y)$ iff $x \succeq y$

Types of games

- Games with complete information
 - ▷ Static games
 - ▷ Dynamic games
- Games with incomplete information
 - ▷ Static games (Bayesian games)
 - ▷ Dynamic games (dynamic Bayesian games)

Static game of complete information

- Static games: simultaneous-move, single-shot games
- complete information (完全信息): a player knows other players' utility functions
- We use the strategic form/normal form (策略型/正规型) to represent a static game of complete information
- Definition: A strategic-form game consists of
 - ① a set of players
 - ② for each player, a set of actions (i.e., strategies)
 - ③ for each player, preferences over the set of action/strategy profiles

Static game of complete information

- **Strategy profile** (策略组合): a list of all the player's strategies
 - ▷ E.g, my strategies: left or right; Your strategies: up or down
 - ▷ Strategy/action profiles: (left, up), (left, down), any other?
- Preferences are over strategy profiles rather than one's own actions/strategies
- In single-shot games, actions are equivalent to strategies

Illustration: Prisoner's Dilemma

- Players: two suspects, 1 and 2
- Actions: {stay silent, confess}
- Preferences:
 - $u_1(\text{confess, silent}) > u_1(\text{silent, silent}) > u_1(\text{confess, confess})$
 $> u_1(\text{silent, confess})$
 - $u_2(\text{silent, confess}) > u_2(\text{silent, silent}) > u_2(\text{confess, confess})$
 $> u_2(\text{confess, silent})$
- Game representation

		Suspect 2	
		silent	confess
Suspect 1	silent	0, 0	-2, 1
	confess	1, -2	-1, -1

Nash Equilibrium

- Definition: A strategy profile a^* is a **Nash equilibrium** if, for every player i and every strategy a_i of player i , a^* is at least as good for player i as the strategy profile (a_i, a_{-i}^*) in which player i chooses a_i while every other player j chooses a_j^* .
- In other words: $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for every strategy a_i of every player i .
- In plain English: no one can do better by unilaterally deviating from the strategy profile.

Prisoner's Dilemma (囚徒困境)

- What's the Nash equilibrium in PD?

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- Only the strategy profile (confess, confess) is a NE.
- In PD each player has an **dominant strategy** (恒优策略): a strategy that is better for a player regardless of what other players do.

Prisoner's Dilemma cont.

- Tragedy of the PD game: there is an outcome that is better for BOTH players, but they just cannot achieve it.
- Would communication between the two players help them?
 - Watch a real game: http://www.youtube.com/watch?v=p3Uos2fzIJ0&feature=player_embedded
- Applications: tragedy of commons; arms race

Battle of Sexes (两性之战)

- He wants to watch soccer, she wants to watch ballet, but they would rather be together than separate.

		She	
		soccer	ballet
He	soccer	2, 1	0, 0
	ballet	0, 0	1, 2

- What are the Nash equilibria?

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- 2 Nash equilibria: (soccer, soccer); (ballet, ballet)

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- 2 Nash equilibria: (soccer, soccer); (ballet, ballet)
- BoS models situations in which two parties want to cooperate, but differ on which point to cooperate (Obama & Clinton, or McCain & Palin, want to make a stand on an issue)

Matching Pennies (翻硬币游戏)

- A purely conflictual game (PD and BoS have elements of cooperation)

		Player 2	
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

- Player 1 wants to take the same action as player 2, but player 2 wants to take the opposite action
- Any (pure-strategy) Nash equilibrium?

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- Player 1 wants to take the same action as player 2, but player 2 wants to take the opposite action
- Any (pure-strategy) Nash equilibrium?
⇒ No.

Stag Hunt (猎鹿游戏)

- Two hunters can succeed in catching a stag if they all exert efforts, but each can catch a hare alone

		Hunter 2	
		stag	hare
Hunter 1	stag	2, 2	0, 1
	hare	1, 0	1, 1

- What are the Nash equilibria?

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⇒ (stag, stag) and (hare, hare)

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- What are the Nash equilibria?
 - \Rightarrow (stag, stag) and (hare, hare)
- Application: cooperative project; security dilemma

The Chicken Game (Hawk-Dove) (斗鸡博弈)

- Two drivers drive towards each other on a single lane. If neither swerves, they crash and may die; if one swerves while the other does not, the one who swerves loses face while the other gains respect.

		Driver 2	
		straight	swerve
Driver 1	straight	-10, -10	1, -1
	swerve	-1, 1	0, 0

- Application: brinksmanship
- Reducing options in a chicken game: throwing away the steering wheel? Burning the bridge after crossing the river?

Coordination and the Focal Point

- A coordination game: choosing a restaurant

		She	
		Italian	Japanese
He	Italian	1, 1	0, 0
	Japanese	0, 0	1, 1

Coordination and the Focal Point

- A coordination game: choosing a restaurant

		She	
		Italian	Japanese
He	Italian	1, 1	0, 0
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- NE: (Italian, Italian); (Japanese, Japanese)

Coordination and the Focal Point

- A coordination game: choosing a restaurant

		She	
		Italian	Japanese
He	Italian	1, 1	0, 0
	Japanese	0, 0	1, 1

- NE: (Italian, Italian); (Japanese, Japanese)
- **Focal point**: in some real-life situations players may be able to coordinate on a particular equilibrium, in a multiple equilibria game, by using information that is abstracted away by the strategic form.
 - Shelling's experiment about meeting in New York

Public good provision

- Exercise 33.1: Each of n people chooses whether to contribute a fixed amount toward the provision of a public good. The good is provided iff at least k people contribute, where $2 \leq k \leq n$; if it is not provided, contributions are not refunded. Each person ranks outcomes from best to worst as follows:
 - (a) any outcome in which the good is provided and she does not contribute;
 - (b) any outcome in which the good is provided and she contributes;
 - (c) any outcome in which the good is not provided and she does not contribute;
 - (d) any outcome in which the good is not provided and she contributes.Formulate this situation as a strategic game and find the NE.

Public good provision: strategic form

- Players: the n people
- Actions: each player's set of action is contribute, not contribute
- Preferences: $U_i(a) > U_i(b) > U_i(c) > U_i(d)$

Public good provision: NE

- Is there a NE in which more than k people contribute? One in which k people contribute? One in which fewer than k contribute?

Public good provision: NE

- Is there a NE in which more than k people contribute? One in which k people contribute? One in which fewer than k contribute?
- NE: k people contribute; none contributes

Strict and nonstrict equilibria

- If an action profile a^* is a NE, then $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for every action a_i of every player i .
- An equilibrium is **strict** if each player's equilibrium action is **better** than all her other actions. Or, $u_i(a^*) > u_i(a_i, a_{-i}^*)$ for every action $a_i \neq a_i^*$ of player i .

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- A variant of the prisoner's dilemma game

		Player 2	
		split	steal
Player 1	split	5, 5	0, 10
	steal	10, 0	0, 0

- How many Nash equilibria? Any strict NE?

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 \Rightarrow 3 and 0.