# Algebraic Topology I

## MATH 611 (4820)

## Course Info ——————

- ⚡ Prerequisite: MATH 411 & 501
- 📅 Tue & Thu
- 🕐 3:05 pm - 4:20 pm
- 📍 Room 227, Physics Building
- ℹ️ Course Website: Canvas

## Instructor Info ———

- 👤 Niny Arcila-Maya (she/ella)
- 🕐 Office Hours: 12pm - 2pm Fri
- 📍 Room 045, Physics Building
- @ niny.arcilamaya@duke.edu

## Overview

This course is a Qualifying Eligible (QE) course for PhD students in the mathematics program, and it can be extended to a yearlong course by adding MATH 612.

Algebraic topology is an *amazing* field of mathematics that has deep connections with other areas, such us, differential geometry, differential topology, algebraic geometry, and topological data analysis, just to mention a few.

We will associate algebraic objects such as vector spaces, groups, or rings with topological spaces, aiming to distinguish them based on their "shape". These algebraic structures capture essential features like "connectivity", "loops", and "higher dimensional voids" present within the spaces.

### Prerequisites

I am hoping that you already have some familiarity with point-set topology from MATH 411 (ability to work with topological spaces, continuous maps, connectedness, path-connectedness, subspace/quotient/product topology is required), and abstract algebra from MATH 501 (ability to work with groups, rings, homomorphisms and isomorphisms is required).

### Course Duration

The duration of this course will extend until the conclusion of undergraduate classes, despite the fact that graduate courses officially conclude a week prior. Consequently, our final class will take place on **Thursday**, **December 7th**.

### Learning Outcomes

- Students will be able to define, both intuitively and rigorously, basic concepts in homotopy theory, homology theory, and cohomology theory.
- Students will learn to compute fundamental groups, homology and cohomology groups of some topological spaces.
- Students will enhance their logical reasoning, problem-solving abilities, and proficiency in articulating mathematical proofs involving algebraic topology.
- Students will be able to recognize, understand, and analyse material related to algebraic topology from written sources.
- Students will develop presentation and collaboration skills.

### How can you address me?

You are welcome to address me as "Prof.", "Prof. AM", "Professor Arcila-Maya", or "Dr. Arcila-Maya".

I color-code my board work and lecture notes. If you have color blindness, please inform me of the specific colors that would work best for you.

## Recommended Texts

Lectures will correspond to particular sections of (1) and (2). However, please note that in class I will often present material in a different order. There will be reading assignments from (1) and (2).

1. Hatcher, A., *Algebraic Topology*, Cambridge University Press, Cambridge, 2002.
   https://pi.math.cornell.edu/~hatcher/AT/ATpage.html.
   (Free online version.)

2. Munkres, J. R., *Topology*, 2nd Edition, Pearson College Div., 2000.
   (In the library.)

* You do not need to buy (2). Lecture notes, and handouts for reading assignments will be posted to Canvas.

## Grading Scheme

Your final grade in this course will be roughly comprised of the following:

| | |
|---|---|
| **Participation** | 10 % |
| **Homework** | 20 % |
| **Midterm Exams** (2 x 15%) | 30 % |
| **Group Project** | 20 % |
| **Final Exam** | 20 % |

At the end of this course your numerical grade, as computed using the percentages described above, will be converted to a letter grade. If your final percentage is $\geq 90$ you are guaranteed at least an A-, $\geq 80$ at least a B-, $\geq 70$ at least a C-.

### Participation

To satisfy the class participation requirement, each student must ask one question or make one comment in class at least twice before each midterm. Questions and comments must be of mathematical nature. Acceptable questions/comments might be: "could you give us one example of that definition?", "those definitions are equivalent, right?" or "would you remind us what $M\#N$ means?". I will do everything I can to help everyone satisfy this requirement.

### Homework

A problem set will be due every two weeks on **Fridays** at **11:59 pm** starting on **Sep 8th**. Problem sets will be posted and collected on Gradescope. Problem sets must be prepared using LaTeX. Late homework will be accepted only with an exceptionally good excuse.

I encourage you to work cooperatively with your peers while doing your homework. However, each student must write up their solutions to the problems individually and in their own words, and must provide their collaborators' names on their written assignments. Copying from another student or any other source is prohibited (Btw, ChatGPT is not that good at writing proofs!).

### Midterm Exams

There will be two take-home midterm exams. They will open on **Saturdays** at **8:30 am** and will close on **Sundays** at **8:30 pm**. Exams will be posted on Gradescope, and are scheduled as follows:

| | Opens on | Closes on |
|---|---|---|
| **Midterm 1** | October 7th | October 8th |
| **Midterm 2** | November 11th | November 12th |

**Make-up Policy:** I reserve the right to offer a make-up exam or use another exam score to replace the missed exam. Make-up exams will only be allowed for students who have a substantiated excuse approved by the instructor *before the due date*.

### Group Project

This project consists of creating a sequence of videos on algebraic topology. The goal is to make complex concepts comprehensible to individuals who may not have a mathematical background. Detailed instructions will be found on our website, but here are key highlights:

- **Collaborative Effort:** As a class, comprising all 15 enrolled students, you will need to collaboratively devise an implementation strategy for this project.
- **Individual Contribution:** Every student is required to create at least one video to obtain credit.
- **Guidance & Structure:** I will be with you all throughout the entire procedure, aiding in project planning and structuring.
- **Sharing Your Work:** As a group you will decide whether you want to make your project public (math department community or YouTube or my academic website or somewhere else). I am personally enthusiastic about showcasing your work to benefit a wider audience!!! However, no content will be uploaded without your explicit consent.

## Final Exam

There will a take-home final exam. It will be released on **Friday**, **December 8th** at **8:30 am** and must be submitted by **Thursday**, **December 14th** at **8:30 pm**. This exam will be posted on Gradescope.

## Duke Community Standard

All students must adhere to the Duke Community Standard (DCS): Duke University is a community dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, and accountability. Citizens of this community commit to reflect upon these principles in all academic and non-academic endeavors, and to protect and promote a culture of integrity.

To uphold the Duke Community Standard, students agree:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

Regardless of course delivery format, it is the responsibility of all students to understand and follow all Duke policies, including academic integrity (e.g., completing one's own work, following proper citation of sources, adhering to guidance around group work projects, and more). Ignoring these requirements is a violation of the Duke Community Standard. Any questions and/or concerns regarding academic integrity can be directed to the Office of Student Conduct and Community Standards at conduct@duke.edu.

If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment. I reserve the right to further reduce the final grade for the course by up to two letter grades. If a student's admitted academic dishonesty is resolved directly through a faculty-student resolution agreement approved by the Office of Student Conduct, the terms of that agreement will dictate the grading response to the assignment at issue.

## Getting Help

I cannot emphasize enough how important it is seek help whenever you need it, and I hope you will reach out to me at those times. This course will present some challenging material, and I expect that you will have questions.

### Office Hours

> "At key times, it is more useful to take stock of what one knows than blindly march forward hoping for the best. *A difficulty at this time signals the need to reread the previous material carefully.* If the mystery persists, that's what office hours are there for. But typically you should be able to find your way out on your own, based on the information we have given you, and you will most likely learn more this way. You should give it your best try before seeking professional help."
>
> **Paolo Aluffi, Algebra: Chapter 0, I.3**

Please take full advantage of office hours to resolve any questions you may have about course material or homework. There are three options for my office hours:

1. I will usually stay after class for 30 mins in order to answer any questions you may have.
2. I will hold weekly office hours on Tue and Th from 1pm - 2pm (this is a provisional schedule; a poll will be sent to determine a better one).
3. You can make an appointment to meet with me. We can either meet in my office, or on Zoom. Either way, use this link to schedule.

### Ed Discussion

We will be using Ed Discussion this semester. Whenever you have a question pertaining to course material, please post it to Ed Discussion. This way, all students can benefit from questions and answers. Additionally, I encourage you to try and answer your peer's questions. This can be done by a collective students response, and I will have the ability to add to such responses.

### Email

You are welcome to email me your questions of non-mathematical nature. For those of mathematical nature, I prefer you use Ed Discussion (because it is more convenient to type symbols). There is an option in Ed Discussion where you can post a thread as "private", which means it will only be visible to you and me.

As my own work-life balance policy, I do not answer emails on weekends nor after 6pm on weekdays.

## Diversity and Inclusivity Statement

I would like my classroom to be a place where students will be treated with respect. I welcome students of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, capacity and other visible and invisible differences. Please join me in creating a welcoming and vibrant classroom climate!

If you ever have concerns in this course about harassment, discrimination, or any unequal treatment, or if you seek accommodations or resources, I invite you to share directly with me. I will take your communication seriously and seek mutually acceptable resolutions and accommodations. Reporting will never impact your course grade.

### Pronouns

Pronouns are meaningful tools to communicate identities and experiences, and using pronouns supports a campus environment where all community members can thrive. Please update your gender pronouns in Duke Hub. You can learn more at the Center for Sexual and Gender Diversity.

## Duke Neurodiversity Connections

Duke Neurodiversity Connections offers many resources and accommodations to support students.

## Student Disability Access Office (SDAO)

The goal of SDAO is to provide and coordinate accommodations, support services and programs that enable students with disabilities to have equal access to all Duke University programs and activities. The staff of the SDAO encourages and assists students in understanding the nature of their disability and it's impact on their living and learning at Duke University. Students are provided with opportunities to learn about their rights and responsibilities, procedures for requesting accommodations and about the wide range of programs and opportunities available at Duke University.

## Personal Wellbeing

### Attendance Policy Related to COVID Symptoms, Exposure, or Infection

Student health, safety, and well-being are the university's top priorities. To help ensure your well-being and the well-being of those around you, **please do not come to class if you have tested positive for COVID-19 or have possible symptoms and have not yet been tested**. If any of these situations apply to you, you must follow university guidance related to the ongoing COVID-19 pandemic and current health and safety protocols. If you are experiencing any COVID-19 symptoms, contact Student Health (dshcheckin@duke.edu, 919 681 9355). Learn more about current university policy related to COVID-19 at `https://coronavirus.duke.edu/`.

To keep the university community as safe and healthy as possible, you will be expected to follow these guidelines. Please reach out to me and your academic dean as soon as possible if you need to quarantine or isolate so that we can discuss arrangements for your continued participation in class.

### Mental Health and Wellness

If your mental health concerns and/or stressful events negatively affect your daily emotional state, academic performance, or ability to participate in your daily activities, many resources are available to help you through difficult times. Duke encourages all students to access these resources:

- **DukeReach** Provides comprehensive outreach services to identify and support students in managing all aspects of well-being. If you have concerns about a student's behavior or health visit the website for resources and assistance:
  `https://students.duke.edu/wellness/dukereach/`.
- **Counseling and Psychological Services (CAPS)** CAPS services include individual and group counseling services, psychiatric services, and workshops. CAPS also provides referral to off-campus resources for specialized care.
  919 660 1000 `https://students.duke.edu/wellness/caps/`.
- **TimelyCare (formerly known as Blue Devils Care)** An online platform that is a convenient, confidential, and free way for Duke students to receive 24/7 mental health support through TalkNow and scheduled counseling.
  `https://timely.md/schools/index.html?school=bluedevils&`.
- **Duke Student Wellness Center** `https://students.duke.edu/wellness/duwell/`
- **Student Health Services** `https://students.duke.edu/wellness/studenthealth/`

See Health and Wellbeing for more resources for graduate students.

**In case of an emergency call Duke Police at 919 684 2444 or 911.**

Gender Violence or Sexual Assault

If you are a victim-survivor of gender violence or sexual assault, you can call 24/7, Monday-Friday, 9am-5pm at 919 684 3897. See https://students.duke.edu/wellness/sexual-assault-gender-violence/ for more information.

Hate-Bias

If you experience a bias incident, please see https://students.duke.edu/get-assistance/hate-bias/ for more information.

## Syllabus

The following list of topics is aspirational and subject to change.

Module 1

- Categories and Functors
- Homotopy of Paths
- The Fundamental Group, Functoriality
- The Fundamental Group of the Circle
- Seifert–van Kampen Theorem
- Covering Spaces, Lifting Properties, Classification of Covering Spaces
- Deck Transformations and Group Actions

Module 2

- Singular Homology, Homotopy Invariance
- Relative Homology and The Long Exact Sequence
- Mayer–Vietoris Sequences
- Eilenberg–Steenrod Axioms of Homology
- Relation between $\pi_1$ and $H_1$
- Reduced Homology
- Cellular Homology
- Euler Characteristic

Module 3

- Singular Cohomology
- Relative Cohomology and The Long Exact Sequence
- Eilenberg–Steenrod Axioms of Cohomology
- Cellular Cohomology