



Introduction to Abstract Algebra

MATH 401 (4780)

Course Info

-  Prerequisite: MATH 221
-  Tue & Thu
-  3:30 pm - 4:45 pm
-  Room 259, Physics Building
-  Course Website: [Sakai](#)

Instructor Info

-  Niny Arcila-Maya (she/her)
-  Office Hours: 2pm - 3pm TuTh
-  Room 045, Physics Building
-  niny.arcilamaya@duke.edu

Overview

A more advanced version of our course is MATH 501 which can be extended to a yearlong course by adding MATH 502. However, you cannot take MATH 501 if you have already taken MATH 401, so please plan carefully.

In previous courses you have seen many kinds of “algebraic objects”: the integers, real and complex numbers, polynomials, functions, vectors, and matrices. Abstract algebra includes all of these objects and much more. Roughly speaking, abstract algebra studies the structure of sets with operations on them. We will study three basic kinds of “sets with operations on them”, called Groups, Rings, and Fields.

I am hoping that you already have some familiarity with proofs from MATH 221 or a similar course. If not, the following might be helpful: [Introduction to Mathematical Arguments](#) by Michael Hutchings, and [Suggestions for Writing Mathematics](#) by Ian Agol.

Learning Outcomes

- Students will be able to perform computations involving divisibility of integers.
- Students will be asked to identify group-theoretic and ring-theoretic properties and identify these properties in familiar rings and groups.
- Students will learn to write clear and correct mathematical proofs to simple assertions of group- and ring-theoretic principles.

How can you address me?

You are welcome to address me as “Professor Arcila-Maya”, “Dr. Arcila-Maya”, or “Prof.”.

Recommended Texts

Most of the lectures will correspond to particular sections of (1). However, please note that in class I will often present material in a different order. There will be reading assignments from (1) and (2). You might try skimming through (3) for an additional perspective, it has a wide range of exercises, both computational and theoretical, plus many nontrivial applications. A book that has been well liked by students is (4), it is very readable, contains lots of good exercises, examples, and figures.

1. Hungerford, T. W., *Abstract Algebra: An Introduction*, 3rd Edition, Brooks/Cole CENGAGE Learning, 2012.
(In the library.)
2. Gallian, J. A., *Contemporary Abstract Algebra*, 10th Edition, United States: CRC Press, 2021. <https://www.d.umn.edu/~jgallian/>
(Previous editions in the library.)
3. Judson, T. W., & Beezer, R. A., *Abstract Algebra: Theory and Applications*, Annual Edition 2022. <http://abstract.ups.edu/index.html>
(Free online version.)
4. Fraleigh, J. B., *A First Course in Abstract Algebra*, 7th Edition, Pearson, 2014.
(5th edition in the library.)

* You do not need to buy any of these books. Lecture notes, and handouts for reading assignments will be posted to Sakai.

Grading Scheme

Attendance: Even though it is incredibly important to attend class, and I strongly encourage you to do so, I will not take attendance. However, I do take it into account for borderline grades at the end of the term.

Your final grade in this course will be roughly comprised of the following:

Quizzes	10 %
Participation	10 %
Homework	20 %
Midterm Exams (2 × 20%)	40 %
Group Presentation	20 %

At the end of this course your numerical grade, as computed using the percentages described above, will be converted to a letter grade. If your final percentage is ≥ 90 you are guaranteed at least an A-, ≥ 80 at least a B-, ≥ 70 at least a C-.

Quizzes & Participation

There will be weekly Gradescope quizzes. They will open at 3:30pm on Mondays and close at 3:30pm on Tuesdays. Quizzes will be about reading assignments, and topics seen in the respective previous week.

Participation is mandatory, each student must ask one question/comment in class at least once before each midterm. Acceptable questions/comments might be: "could you give us one example of that theorem?", "those definitions are equivalent, right?" or "would you explain what σ is?".

Homework

A problem set will be due each week on **Wednesdays** at **6 pm**, with the exception of Aug 31st, Oct 19th, and Nov 30th. Problem sets will be posted and collected on Gradescope. At the end of the term, the lowest problem set grade will be dropped. Late homework will be accepted only with an exceptionally good excuse.

I encourage you to work cooperatively with your peers while doing your homework. However, each student must write up their solutions to the problems individually and in their own words, and must provide their collaborators' names on their written assignments. Copying from another student or any other source is prohibited.

Midterm Exams

There will be two in-class midterm exams. They are tentatively scheduled for **Tuesday, Oct 18th** and **Tuesday, Dec 6th**.

Make-up Policy: I reserve the right to offer a make-up exam or use another exam score to replace the missed exam. Make-up exams will only be allowed for students who have a substantiated excuse approved by the instructor *before the due date*. In case you are allowed to take a make-up exam, you must use the Testing Center to take it. See Testing Accommodations below.

Group Presentation

In lieu of a final exam, there will be a group presentation at the end of the term.

- *Audience:* Just Professor Arcila-Maya!
- *When?* You must book an appointment with me between **Nov 1st** and **Dec 9th**.
- *Group:* 2 or 3 people (I can only allow one group of 2 people).
- *Time:* Each person in the group must speak for 10 minutes on the topic (so, the presentation will be up to 30 minutes).
- *Topic:* A list will be given later. Let me know if you would like to choose a topic that is not in the list.

Testing Accommodations

This class will use the Testing Center to provide testing accommodations and temporary test-taking supports to undergraduates registered with and approved by the Student Disability Access Office (SDAO) and/or Academic Resource Center (ARC). The Testing Center operates by appointment only and **appointments must be made at least 7 consecutive days in advance**, but please schedule your appointments as far in advance as possible. You will not be able to make an appointment until you have submitted a Semester Request with the SDAO or completed screening at the ARC and your accommodations or supports have been approved. If you have not already done so, promptly submit a Semester Request to the SDAO or schedule your screening with the ARC in order to make your appointment in time. For instructions on how to register with SDAO, visit <https://access.duke.edu/requests>. For instructions on how to schedule a screening with the ARC, visit <https://arc.duke.edu/ld-adhd/overview>. For instructions on how to make an appointment at the Testing Center, visit <https://testingcenter.duke.edu>.

Duke Community Standard

All students must adhere to the Duke Community Standard (DCS): Duke University is a community dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, and accountability. Citizens of this community commit to reflect upon these principles in all academic and non-academic endeavors, and to protect and promote a culture of integrity.

To uphold the Duke Community Standard, students agree:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

Regardless of course delivery format, it is the responsibility of all students to understand and follow all Duke policies, including academic integrity (e.g., completing one's own work, following proper citation of sources, adhering to guidance around group work projects, and more). Ignoring these requirements is a violation of the Duke Community Standard. Any questions and/or concerns regarding academic integrity can be directed to the Office of Student Conduct and Community Standards at conduct@duke.edu.

If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment. I reserve the right to further reduce the final grade for the course by up to two letter grades. If a student's admitted academic dishonesty is resolved directly through a faculty-student resolution agreement approved by the Office of Student Conduct, the terms of that agreement will dictate the grading response to the assignment at issue.

Getting Help

I cannot emphasize enough how important it is seek help whenever you need it, and I hope you will reach out to me at those times. This course will present some challenging material, and I expect that you will have questions.

Office Hours

"At key times, it is more useful to take stock of what one knows than blindly march forward hoping for the best. *A difficulty at this time signals the need to reread the previous material carefully.* If the mystery persists, that's what office hours are there for. But typically you should be able to find your way out on your own, based on the information we have given you, and you will most likely learn more this way. You should give it your best try before seeking professional help."

Paolo Aluffi, Algebra: Chapter 0, 1.3

Please take full advantage of office hours to resolve any questions you may have about course material or homework. There are three options for my office hours:

1. I will usually stay after class for 30 mins in order to answer any questions you may have.
2. I will hold weekly office hours on Tuesdays at 2 pm - 3 pm, and Fridays at 4 pm - 5 pm.
3. You can make an appointment to meet with me. We can either meet in my office, or on [Zoom](#). Either way, use this [link](#) to schedule.

Ed Discussion

We will be using Ed Discussion this semester. Whenever you have a question pertaining to course material, please post it to Ed Discussion. This way, all students can benefit from questions and answers. Additionally, I encourage you to try and answer your peer's questions. This can be done by a collective students response, and I will have the ability to add to such responses.

Academic Resource Center

The Academic Resource Center (the ARC) offers services to support students academically during their undergraduate careers at Duke. The ARC can provide support with time management, academic skills and strategies, unique learning styles, peer tutoring, learning consultations, learning communities, and more. ARC services are available free to any Duke undergraduate student, in any year, studying in any discipline. 919 684 5917, theARC@duke.edu, or <https://arc.duke.edu/>

Diversity and Inclusivity Statement

I would like my classroom to be a place where students will be treated with respect. I welcome students of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, capacity and other visible and invisible differences. Please join me in creating a welcoming and vibrant classroom climate!

If you ever have concerns in this course about harassment, discrimination, or any unequal treatment, or if you seek accommodations or resources, I invite you to share directly with me. I will take your communication seriously and seek mutually acceptable resolutions and accommodations. Reporting will never impact your course grade.

Color-coded lectures and lectures notes

I tend to color-code my lectures and my lecture notes. Please let me know if you are colorblind so I can adjust the colors accordingly.

Pronouns

Pronouns are meaningful tools to communicate identities and experiences, and using pronouns supports a campus environment where all community members can thrive. Please update your gender pronouns in Duke Hub. You can learn more at the [Center for Sexual and Gender Diversity](#).

Personal Wellbeing

Attendance Policy Related to COVID Symptoms, Exposure, or Infection

Student health, safety, and well-being are the university's top priorities. To help ensure your well-being and the well-being of those around you, **please do not come to class if you have tested positive for COVID-19 or have possible symptoms and have not yet been tested.** If any of these situations apply to you, you must follow university guidance related to the ongoing COVID-19 pandemic and current health and safety protocols. If you are experiencing any COVID-19 symptoms, contact [Student Health](mailto:dshcheckin@duke.edu) (dshcheckin@duke.edu, 919 681 9355). Learn more about current university policy related to COVID-19 at <https://coronavirus.duke.edu/>.

To keep the university community as safe and healthy as possible, you will be expected to follow these guidelines. Please reach out to me and your academic dean as soon as possible if you need to quarantine or isolate so that we can discuss arrangements for your continued participation in class.

Mental Health and Wellness

If your mental health concerns and/or stressful events negatively affect your daily emotional state, academic performance, or ability to participate in your daily activities, many resources are available to help you through difficult times. Duke encourages all students to access these resources:

- **DukeReach** Provides comprehensive outreach services to identify and support students in managing all aspects of well-being. If you have concerns about a student's behavior or health visit the website for resources and assistance: <https://students.duke.edu/wellness/dukereach/>.
- **Counseling and Psychological Services (CAPS)** CAPS services include individual and group counseling services, psychiatric services, and workshops. CAPS also provides referral to off-campus resources for specialized care. 919 660 1000 <https://students.duke.edu/wellness/caps/>.
- **TimelyCare (formerly known as Blue Devils Care)** An online platform that is a convenient, confidential, and free way for Duke students to receive 24/7 mental health support through TalkNow and scheduled counseling. <https://timely.md/schools/index.html?school=bluedevils&>.
- **Two-Click Support** Duke Student Government and DukeReach partnership that connects students to help in just two clicks. <https://bit.ly/TwoClickSupport>.

In case of an emergency call Duke Police at 919 684 2444 or 911.

Gender Violence or Sexual Assault

If you are a victim-survivor of gender violence or sexual assault, you can call 24/7, Monday-Friday, 9am-5pm at 919 684 3897. See <https://students.duke.edu/wellness/sexual-assault-gender-violence/> for more information.

Hate-Bias

If you experience a bias incident, please see <https://students.duke.edu/get-assistance/hate-bias/> for more information.

Syllabus

The following list of topics is aspirational and subject to change.

Module 1

- The Division Algorithm
- Divisibility and the Euclidean algorithm
- Primes and Unique Factorization
- The Fundamental Theorem of Arithmetic
- Congruence in \mathbb{Z} and Modular Arithmetic

Module 2

- Groups, Subgroups, and Group Homomorphisms
- The Symmetric, Alternating and Dihedral Groups
- Congruence and Lagrange's theorem
- Normal Subgroups and Quotient Groups
- Theorems of Isomorphism for Groups

Module 3

- Rings, Integral Domains, Division Rings, Fields, Subrings and Ring Homomorphisms.
- Ideals, Prime Ideals, Maximal Ideals and Quotient Rings
- Theorems of Isomorphism for Rings
- Ring of Polynomials $R[X]$, and the Division Algorithm