Security Design in a Production Economy with Flexible Information Acquisition

Ming Yang & Yao Zeng

Duke & Harvard

June 2013
Production Economy

- Entrepreneur: undertakes risky project if he gets financing
- Investor: capable of acquiring costly info about project and thus helps screen project through financing decision (Bond-Edmans-Goldstein 12, Da Rin-Hellmann-Puri 11, etc.)

Friction

- Dependence and separation of real production and info production

Question

- What is the optimal financing given such friction?
Entrepreneur’s trade-off

- Increase own share vs. induce favorable info acquisition

Results

- Standard debt
- Convertible preferred stock (Kaplan-Stromberg 03)
- Friction $\uparrow \implies$ debt $\rightarrow$ convertible preferred stock

Model Highlights

- A unified framework for distinct securities
- Over continuous states, without distributional assumptions
- Flexible info acquisition / rational inattention
Financing Entrepreneurial Production

- Two dates: \( t = 0, 1 \).
- Two risk neutral agents \( u = c_0 + c_1 \)
  - Entrepreneur: zero initial wealth
  - Investor: deep pocket

Technology

- Investment \( k \) at \( t = 0 \) generates uncertain cash flow \( \theta \) at \( t = 1 \)
  (common prior \( \Pi \))
- Entrepreneur’s human capital inalienable (relaxed later)
- Entrepreneur needs to raise \( k \) by selling \( s(\theta) \in [0, \theta] \) to investor

Timeline

<table>
<thead>
<tr>
<th>Entrepreneur proposes the security as a take-it-or-leave-it offer</th>
<th>Investor acquires information and decides whether to finance the project</th>
<th>Cash flow realized and agents consume</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t=0 )</td>
<td>( t=1 )</td>
<td></td>
</tr>
</tbody>
</table>
Investor’s Flexible Information Acquisition

- Facing offer \( s(\theta) - k \), investor chooses
  - Info structure \( f(x|\theta) \)
  - Decision rule \( \sigma : x \rightarrow [0, 1] \)

- Innovation
  - Conventional: parametric restrictions on \( f(x|\theta) \)
  - Flexible: any info structure feasible, incurs a cost proportional to the amount of info (rational inattention, Sims 03, etc.)

- A 2-state example: let \( m(\theta) = f(x = 1|\theta) \), then

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( x = 1 )</th>
<th>( x = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta = 1 )</td>
<td>( m(1) )</td>
<td>( 1 - m(1) )</td>
</tr>
<tr>
<td>( \theta = 0 )</td>
<td>( m(0) )</td>
<td>( 1 - m(0) )</td>
</tr>
</tbody>
</table>

and \( |m(1) - m(0)| \) reflects the informativeness.
Rational inattention with binary choice (Woodford 08, Yang 12)

- Actions \( \{ \text{reject} = 0, \text{accept} = 1 \} \) and signals \( x \in \{0, 1\} \)
- Binary info. structure \( m : \mathbb{R}_+ \to [0, 1] \)
  - \( m(\theta) = \) prob. of observing signal 1 if true state is \( \theta \)
  - choice variable of investor
  - incurs cost \( c(m) = \mu \cdot I(m) \)
  - \( |dm/d\theta| \) reflects allocation of attention

Amount of info \( I(m) \) (Shannon 48)

- Reduction of uncertainty through observing \( x \) generated by \( m \)

\[
I(m) = H(\text{prior}) - H(\text{posterior}) \\
= E g(m(\theta)) - g(E m(\theta)) ,
\]

where \( H(\cdot) \) is Shannon’s entropy and

\[
g(m) = m \ln m + (1 - m) \ln (1 - m)
\]
Flexible information Acquisition–Example 1: Debt

![Graphs showing price and security payment](image)

- Price: $k$
- Security payment: $s(\theta)$

![Graph showing strategy](image)

- Strategy: $m(\theta)$

![Graph showing attention](image)

- Attention: $|\frac{dm}{d\theta}|$
Solving Binary Decision Problem with Flexible Information Acquisition

**Decision problem:**

\[
\max_{m \in \Omega} V^* (m) \triangleq E \left[ m(\theta) \cdot (s(\theta) - k) \right] - \mu \cdot l(m),
\]

where

\[
\Omega \triangleq \{ m \text{ measurable} : m(\theta) \in [0, 1] \text{ for all } \theta \in \Theta \}.
\]

**Proposition 1:** Let \( m \in \Omega \) be an optimal strategy and

\[
\bar{p} = E m(\theta)
\]

be the corresponding unconditional probability of taking action 1. Then, i) the optimal strategy is unique;
ii) there are three possibilities for the optimal strategy $m(\cdot)$:

a) if

$$\mathbb{E}\exp\left(-\frac{s(\theta) - k}{\mu}\right) \leq 1 ,$$

then accept for sure without info acquisition, i.e., $m(\theta) = 1 \ a.s.;$

b) if

$$\mathbb{E}\exp\left(\frac{s(\theta) - k}{\mu}\right) \leq 1 ,$$

then reject for sure without info acquisition, i.e., $m(\theta) = 0 \ a.s.;$

c) otherwise, $m(\theta) \in (0, 1) \ a.s. \ and \ is \ characterized \ by$

$$\frac{s(\theta) - k}{\mu} = \ln\left(\frac{m(\theta)}{1 - m(\theta)}\right) - \ln\left(\frac{\bar{p}}{1 - \bar{p}}\right) .$$
Entrepreneur’s Security Design Problem

- Entrepreneur chooses $s : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ to maximize

$$
E [ m_s (\theta) \cdot (\theta - s (\theta)) ]
$$

subject to
- $s (\theta) \in [0, \theta]$
- investor’s best response $m_s (\cdot)$ given by Proposition 1
Proposition 2: If the optimal security \( s^* (\theta) \) induces the investor to accept the offer without acquiring information, then it takes the form of standard debt:

\[
 s^* (\theta) = \min (\theta, D^*)
\]

Intuition: as flat as possible to deter info acquisition ("ignorance is bliss"—Holmstrom 12)

Consistent with the conventional wisdom of info insensitiveness

- Myers-Majluf 84; Gorton-Pennacchi 90; DeMarzo-Duffie 99
Proposition 3: If the optimal security $s^*(\theta)$ induces the investor to acquire information, then it takes the following form:

$$s^*(\theta) = \begin{cases} 
\theta & \text{if } 0 \leq \theta < \hat{\theta} \\
\hat{s}(\theta) & \text{if } \theta \geq \hat{\theta}
\end{cases}$$

where $\hat{\theta} > 0$, $d\hat{s}(\theta)/d\theta \in (0,1)$, ($\hat{s}(\theta)$ is given in paper).
Optimal Security Inducing Information Acquisition (cont’d)

- (Participating) convertible preferred stock

Intuition: ignorance is no longer bliss
- Friction $\implies$ deviate from first-best ($45^\circ$ line)
- Screening/info acquisition makes sense only if $m(\theta) \uparrow$
- Entrepreneur’s payoff is $E[m(\theta) \cdot (\theta - s(\theta))]$
- Better to deviate at high states

Empirical evidence: 80% VC contracts are convertible preferred stock and half of them are participating
**Proposition 4**: When the project is financed (with positive prob.):

1. if $E[\theta] \leq k$, $s^*(\theta)$ is convertible preferred stock; or
2. if $E[\theta] > k$, $s^*(\theta)$ is either convertible preferred stock or standard debt.

**Intuition**

- Negative NPV $\implies$ no investment unless info updating
  - Use convertible preferred stock to *screen in* good projects
- Positive NPV
  - Modest prior $\implies$ invest as status quo, but info still valuable
    - Use convertible preferred stock to *screen out* bad projects
  - Good prior $\implies$ always invest, not worth to acquire info
    - Use debt to deter info acquisition, *ignorance is bliss*

A *unified framework* for debt and convertible preferred stock
Change of required investment $k$ (with $\mathbb{E}[\theta] = 0.5$)

- $k = 0.525$ (screen in);
- $k = 0.475$ (screen out);
- $k = 0.4$ (ignorance is bliss);

Intuition: $k \uparrow \implies$ info acquisition more relevant $\implies$ friction $\uparrow \implies$ convertible preferred stock
Welfare Analysis

- Efficiency benchmark: real production & info production centralized

**Proposition 5:** When the project is financed (with positive prob.):

1. Standard debt is optimal iff the friction is not severe in the sense that the optimal security achieves social efficiency;
2. Convertible preferred stock is optimal iff the friction is severe in the sense that even the optimal security cannot achieve social efficiency.

**Remark**

- Friction (in)significant $\Rightarrow$ real production (doesn’t) depend on info production $\Rightarrow$ info acquisition (un)desired $\Rightarrow$ calls for (debt) convertible preferred stock

**Implication**

- Debt / convertible preferred stock popular in industries with less / more friction
Welfare Analysis–Comparative Statics

- Change of prior

- Intuition: uncertainty $\uparrow \implies$ info acquisition more relevant $\implies$ friction $\uparrow \implies$ convertible preferred stock

- More comparative static analysis in paper...

\begin{itemize}
  \item \[ \pi(\theta) \]
  \begin{itemize}
    \item $\text{std.} = 0.125$
    \item $\text{std.} = 0.25$
    \item extre. dist.
  \end{itemize}
  \begin{itemize}
    \item $\theta$
    \item $1$
  \end{itemize}

  \begin{itemize}
    \item $s(\theta)$
    \item $1$
    \item $0$
    \item $\theta$
    \item $1$
  \end{itemize}

  \begin{itemize}
    \item extre. dist.
    \item $\text{std.} = 0.25$
    \item $\text{std.} = 0.125$
  \end{itemize}
\end{itemize}
Exchange Economy vs. Production Economy

Exchange economy

- Aggregate cash flow independent of financial decision
- Info acquisition is costly causes adverse selection \( \implies \) always reduce social surplus
- Ignorance is always bliss, debt is always optimal (Yang 12)

Production economy

- Aggregate cash flow depends on financial decision
- Info acquisition is still costly but helps screen projects \( \implies \) may or may not reduce social surplus
- Depending on which force dominates
- Debt or convertible preferred stock could be optimal
Conclusion

Summary

- Friction: dependence and separation of real and info production
- Different projects financed by different security design
- Production economy vs. exchange economy

Contributions

- Flexible info acquisition: security design over continuous states without distributional assumptions
- A unified framework for distinct securities
Proposition 6: To transfer the project at any fixed price is not optimal for the entrepreneur.

Argument:

Intuition

- Screening/info acquisition makes sense only if \( m(\theta) \uparrow \)
- Not good to deviate uniformly over all states, better to deviate at high states
Proposition 7: The multiple of the optimal convertible preferred stock is greater than one, i.e., \( \hat{\theta} > k \).

- Natural result, consistent with empirical evidence
Amount of information and information Cost

- Mutual info $I(m)$ (Shannon 48)
- Reduction of uncertainty through observing $x$ generated by $m$

$$I(m) = H(prior) - H(posterior) = E_g(m(\theta)) - g(E_m(\theta)),$$

where $H(\cdot)$ is Shannon’s entropy and

$$g(m) = m \ln m + (1 - m) \ln (1 - m)$$

- Info cost:

$$c(m) = \mu \cdot I(m)$$
Flexible information Acquisition—Example 1: Debt
Flexible information Acquisition–Example 2: Equity

![Graph 1: Security Payoff vs Fundamental](image1)

![Graph 2: Strategy vs Fundamental](image2)

![Graph 3: Attention vs Fundamental](image3)
**Proposition A1:** The project is financed with positive prob. iff

$$E \left[ \exp \left( \frac{\theta - k}{\mu} \right) \right] > 1.$$ 

- A new investment criterion: different from NPV
- Nests NPV criterion by taking $\mu \to \infty$
**Definition:** The sequential equilibrium is defined as a collection of the entrepreneur’s optimal security $s^* (\theta)$ and the investor’s optimal decision rule of information acquisition $m^*_s (\theta)$ based on which:

1. Given $\Pi, k$ and $\mu$, $s^* (\theta)$ and $m^*_s (\theta)$ maximize the expected payoffs of the entrepreneur and the investor, respectively;

2. Both agents use the Bayes’ rule to update their beliefs about the fundamental $\theta$, and follow sequential rationality.
Change of Information Cost: Positive vs. Negative NPV

Positive NPV

Negative NPV

\[ s(\theta) \]

\[ \theta \]

\[ 0 \]

\[ 1 \]

\[ \mu = 1 \]

\[ \mu = 0.4 \]

\[ \mu = 0.2 \]

\[ \mu = 0.225 \]

\[ \mu = 0.125 \]

\[ \mu = 0.075 \]
Change of Uncertainty: Positive vs. Negative NPV

Positive NPV

Negative NPV