The Political Economy of Epidemic Management

David McAdams and Troy Day*

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Abstract

During an infectious-disease epidemic, a political leader imposes "stay-at-home orders" (limiting activity) or "go-out orders" (mandating activity) whenever preferred by the majority of the citizenry over the no-intervention status quo. We characterize the resulting equilibrium epidemic trajectory in an economic-epidemiological model that allows for asymptomatic infection and social-economic complementarities of activity, assuming that citizens are myopic optimizers. We find that the qualitative features of equilibrium policy dynamics hinge critically on whether the pathogen is transmitted before or after infected people have developed symptoms. If transmission only occurs symptomatically, then the leader never imposes stay-at-home orders on the healthy but may impose go-out orders during some phases of the epidemic. However, if transmission occurs asymptomatically, the leader never imposes go-out orders on the healthy.

Keywords: Epidemic management, asymptomatic transmission, stay-at-home orders, go-out orders, yo-yo dynamics

JEL Classification: C73, I12, I18

*McAdams: Fuqua School of Business and Economics Department, Duke University, Durham, North Carolina, USA. Email: david.mcadams@duke.edu. Day: Department of Mathematics and Statistics and Department of Biology, Queens University, Kingston, Ontario, Canada. Email: day@queensu.ca. McAdams acknowledges support from NSF Grant DMS-2230074.

The COVID-19 pandemic has provided a stark example of how changes in behavior can profoundly alter the course of a disease outbreak. Widespread physical-distancing measures were initially effective at curbing the spread of the disease (Lau et al. (2020)), but the resulting disruption to normal life came with enormous economic and social costs. For instance, in early April 2020, the Wall Street Journal reported that "at least one quarter of the U.S. economy has suddenly gone idle ... an unprecedented shutdown that economists say has never occurred on such a wide scale" (Mitchell (2020)). Many willingly endured these costs to reap the epidemiological benefits that come from mitigating disease spread (Faust et al. (2021)),[1] but early during the pandemic, many governments, firms, schools, and other institutions imposed mandatory activity restrictions (e.g., lockdowns and "going remote") as a means to suppress disease transmission (Talic et al. (2021)). As the pandemic progressed, however, many of these constraints were relaxed (Han et al. (2020)) and some firms even began requiring employees to return to in-person work (King (2022)).

In this paper, we study the course of an infectious-disease epidemic in which policies that limit or mandate the transmissive activity of a host population (e.g., citizens of a country, workers in a firm) emerge endogenously depending on what sort of policy is most popular among that population at each point in time. We consider a standard Susceptible-Carriage-Infected-Recovered epidemiological model with asymptomatic infection and acquired immunity, augmented by (i) an economic model in which each individual agent chooses their level of activity, trading off the current risk of infection against the current social-economic benefits associated with activity and (ii) a simple political model in which a policy that limits activity ("stay-at-home orders") or mandates activity ("go-out orders") is put in place whenever the majority of the population prefers that policy over the no-intervention status quo.

Our epidemiological model emphasizes the role of asymptomatic carriage, that agents can become infected prior to the onset of symptoms, and that pathogens may be more or less transmissible during this asymptomatic phase. Such variable transmissibility is an important factor for many real-world pathogens. For instance, the SARS-CoV-1 virus that caused the 2003 SARS epidemic is capable of establishing asymptomatic infection but most transmission occurs from those who are experiencing severe illness (Anderson et al. (2004)). On the other hand, the HIV virus is 8-10 times more transmissible during the initial "acute phase" when HIV infection causes cold-like symptoms, compared to later phases of infection when AIDS-specific symptoms begin to emerge (Pilcher et al. (2004)).

[1] Whether individuals chose to distance from others early during the pandemic depended on many factors that we abstract from in this paper's analysis, such as whether they knew someone with direct personal experience of the disease (Charoenwong et al. (2020)) and their ideological affiliation (Hsiehchen et al. (2020)).

Our key finding is that the qualitative features of equilibrium behavior during an epidemic hinge on whether transmission occurs mainly before or during sickness. If transmission only occurs symptomatically, then the political leader never imposes stay-at-home orders on those who have not yet gotten sick but may impose go-out orders during some phases of the epidemic. The reason for this, intuitively, is that activity by not-yet-sick agents creates a positive externality for others when transmission only occurs from the sick. A go-out order compelling not-yet-sick agents to be fully active could therefore sometimes be collectively preferred by all not-yet-skc agents even if, individually, they would choose to minimize their own activity.

By contrast, if transmission only occurs asymptomatically, then the leader never imposes go-out orders on the not-yet-sick but may sometimes require them to stay home. The intuition behind this finding is more nuanced, since not-yet-sick activity may create a positive or negative externality for other not-yet-sick agents, depending on the state of the epidemic. But as we show, not-yet-sick agents have a dominant strategy to go out whenever the epidemic state is such that going out creates a positive externality. By contrast, there may be periods of time during the epidemic when not-yet-sick agents have an individual incentive to go out but collectively prefer to all stay home.

Equilibrium epidemics in our model have some other interesting features as well, three of which we highlight here. First, equilibrium policy dynamics can induce a "yo-yo effect" in infection dynamics, whereby rapid oscillations in the equilibrium policy (repeatedly enforcing and then relaxing constraints on agent behavior) cause the prevalence of infection to rapidly rise and fall like a yo-yo for an extended period of time; see Figure 4. Second and similarly, the policy-maker may in equilibrium need to randomize among policies over an extended period; see Figure 6. Finally and perhaps most intriguingly, we find in a numerical example that the overall social-welfare loss due to the disease is roughly linear in the severity of the disease. This means that, at least in our example, the equilibrium benefits associated with reduced activity due to preventing infection are roughly "cancelled out" by the social-economic costs of this reduced activity; see Figure 8 and surrounding discussion.

Relation to the literature. The recent resurgence of interest in economic-epidemiological models of infectious-disease epidemics[2] builds on classic work by Philipson and Posner (1993), Geoffard and Philipson (1996), Kremer (1996), Reluga (2010), Quercioli and Smith (2024), and others. Like many papers in this literature, we derive an equilibrium epidemic trajectory consistent with individual optimization at each point in time; some notable examples include

[2]Surveys of the recent COVID-inspired literature include Avery et al. (2020), McAdams (2021), and Bloom et al. (2022). Influential earlier surveys include Philipson (2000), Gersovitz and Hammer (2003), and Fenichel et al. (2011).

Toxvaerd (2019), Farboodi et al. (2021), Keppo et al. (2020), Carnehl et al. (2023), and McAdams et al. (2023). However, our modeling approach and motivation is distinctive from the existing literature.

Most importantly, we consider a setting in which public-health-related policies at each point in time emerge endogenously based on the will of the majority of the population and hence are not necessarily optimal for the population as a whole. By contrast, there is a large existing literature on lockdowns and other public-health interventions from a social-welfare maximizing perspective, see e.g., Alvarez et al. (2021), Bethune and Korinek (2020), Forsyth (2020), Jones et al. (2021), Rowthorn and Maciejowski (2020), Sims and Finnoff (2020), Balazs Egert and Turner (2021), Budish (2024), and Rachel (2024).

If all agents were perfectly forward looking and sought to maximize overall social welfare, then our model would generate the trivial outcome in which the policy-maker carries out whatever policy is socially optimal. To focus on the novel implications of our approach, we consider a world in which agents are neither forward looking nor altruistic. In particular, the agents in our model are selfish myopic optimizers, trading off the current risk of becoming infected against the current social-economic costs to themselves of reducing their own contact with others. In this regard, the paper is closely related to Keppo et al. (2020) who also have myopic-optimizing agents.

There are advantages and disadvantages to models with myopic-optimizing agents, compared to the traditional approach of assuming perfect forward-looking optimization. The most obvious disadvantage is that myopic-optimizing agents' choices do not maximize their overall lifetime welfare. The most obvious advantage is tractability, since one needs to consider only the current state of the epidemic in order to determine agents' incentives at that time. This enhanced tractability makes it possible to characterize the equilibrium trajectory of an epidemic when the pathogen has more complex biology. In particular, both Keppo et al. (2020) and this paper are notable in being able to accommodate asymptomatic infection, whereas most prior work has focused on the simplest epidemiological models in which infected agents immediately display symptoms. Finally, it is unclear whether actual agent behavior is better captured by models with myopic optimization or forward-looking optimization. Consequently, there is value in having both modeling approaches represented in the literature

Another difference with most of the existing literature is that we follow McAdams (2020) and McAdams et al. (2023) in highlighting the complementarities associated with social activity.[3] Following Geoffard and Philipson (1996), most recent work considers an especially

[3]Other notable works such as Philipson and Posner (1993), Toxvaerd (2017) and Toxvaerd (2021) have analyzed complementarities in static or myopic-optimization frameworks.

simple economic model with private benefits from activity, i.e., how much benefit an agent gets from being active does not depend on the activity of others. Assuming that activity only generates private benefits is technically convenient, since there can be a vast multiplicity of equilibrium epidemic trajectories once complementarities are allowed, akin to how there can be multiple equilibria in a standard coordination game. However, complementarities are an essential aspect of much human activity and, as shown in McAdams et al. (2023), some qualitative and quantitative findings about equilibrium epidemics derived in traditional models without social-economic complementarities are not robust. Moreover, given that the agents in our model are myopic, the political analysis here becomes trivial if there are no complementarities. In particular, if the complementarity parameter (b_2) equals zero so that activity only generates private benefits, myopic not-yet-sick agents never perceive themselves as facing a collective-action problem and hence never want the policy-maker to impose any sort of restriction on their behavior.

The rest of the paper is organized as follows. Section 1 presents the model. Sections 2-3 provide the main analysis and equilibrium construction in the general case with both symptomatic and asymptomatic transmission. Sections 4-5 then focus on the special cases with symptomatic-only or asymptomatic-only transmission, respectively, highlighting distinctive features of the equilibrium trajectory in each case and considering some equilibrium comparative statics. Section 6 concludes with some discussion of future research directions.

1 Model

This paper's model combines a Susceptible-Carriage-Infected-Recovered (SCIR) epidemiological model allowing for asymptomatic infection, an economic model with myopic decision-makers that allows for social-economic complementarities of activity, and a simple political model of epidemic-management policy dynamics. An epidemic is formally described as the combination of an *epidemiological process* specifying the state of the epidemic at each point in time $t \geq 0$, an *activity process* specifying agents' behavior at each time, and a *policy process* specifying any behavioral constraints in effect at each time. These processes are intertwined, with (i) epidemiological dynamics determined by agents' behavior, (ii) agents' behavior depending on the state of the epidemic and constrained by epidemic-management policy, and (iii) epidemic-management policy depending on the state of the epidemic and how agents would choose to behave if left free to choose what to do.

Epidemiological model. There is a fixed unit-mass host population with no births and no deaths. At each point in time, each agent is in one of five epidemiological states (Figure 1):

susceptible, S; infectious but asymptomatic, C (for "carriage"); infectious and symptomatic, i.e., "sick," I; recovered from carriage without ever having been symptomatic, R_C; and recovered from sickness, R_I.[4] We assume for simplicity that all recovered individuals are henceforth immune to re-infection. Let $S(t)$, $C(t)$, $I(t)$, $R_C(t)$, and $R_I(t)$ denote the mass of agents in each of the five epidemiological states at any given time $t \geq 0$. Since the overall population has unit mass, these variables also capture the fraction of the population in each epidemiological state.

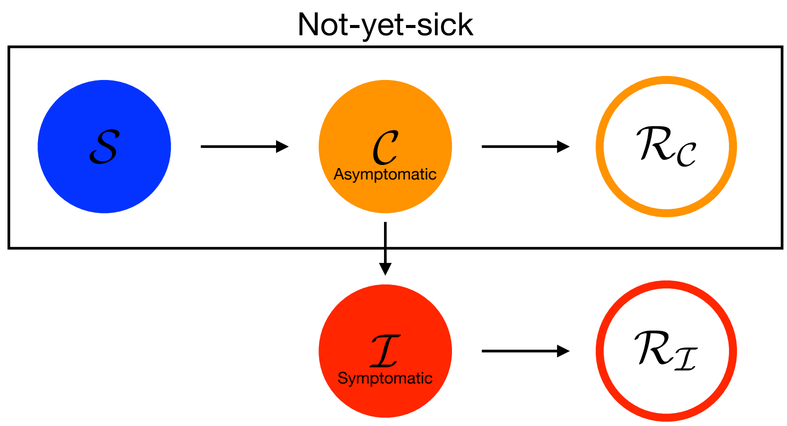


Figure 1: Flow diagram of epidemiological states. Box indicates all states with individuals who have not yet displayed symptoms.

Each agent i chooses activity level $a_i(t) \in [\alpha, 1]$ at each point in time, where $0 \leq \alpha < 1$. $a_i(t) = 1$ is referred to as "going out" while $a_i(t) = \alpha$ is referred to as "staying home." Let $a_\omega(t)$ be the average activity level across all agents in state $\omega \in \{S, C, I, R_C, R_I\}$. Agent i *encounters* an agent in state ω at rate $a_i(t)a_\omega(t)\omega(t)$. Encounters generate economic benefits (discussed below) but also create opportunities for disease transmission. In particular, any agent i who is currently susceptible becomes asymptomatically infected at rate

$$\lambda_i(t) = a_i(t)\left(\beta_C a_C(t)C(t) + \beta_I a_I(t)I(t)\right), \tag{1}$$

where $\beta_C \in [0, 1]$ and $\beta_I \in [0, 1]$ are parameters capturing the transmissibility of the pathogen during the asymptomatic and symptomatic phases of infection, respectively. Let $\lambda(t)$ be the average rate at which susceptible agents become infected, referred to as "the force of infection".

$$\lambda(t) = a_S(t)\left(\beta_C a_C(t)C(t) + \beta_I a_I(t)I(t)\right). \tag{2}$$

[4]The model can be easily extended to allow for innate immunity, with innately-immune hosts in state R_I at time $t = 0$ if they know that they are immune or in state R_C if they do not know that they are immune.

Let σ be the rate at which asymptomatic infection progresses to sickness. Let γ be the rate of recovery, assumed for simplicity to be the same in both infected states.

Epidemiological dynamics are governed by the system of differential equations

$$S'(t) = -S(t)\lambda(t) \tag{3}$$
$$C'(t) = S(t)\lambda(t) - \sigma C(t) - \gamma C(t) \tag{4}$$
$$I'(t) = \sigma C(t) - \gamma I(t) \tag{5}$$
$$R_C'(t) = \gamma C(t) \tag{6}$$
$$R_I'(t) = \gamma I(t) \tag{7}$$

plus the adding-up condition $S(t) + C(t) + I(t) + R_C(t) + R_I(t) = 1$. The initial condition has $C(0) \approx 0$ and $I(0) = R_C(0) = R_I(0) = 0$, meaning that very few people are initially infected and none of them has yet progressed to sickness. We refer to $\mathcal{E}(t) \equiv (S(t), C(t), I(t), R_C(t), R_I(t))$ as the "epidemic state" and $\mathcal{E} = \{\mathcal{E}(t) : t \geq 0\}$ as the "epidemic trajectory."

Agents know when they are sick and when they have recovered from sickness, but those who have not yet been sick do not know whether they are susceptible to infection, asymptomatically infected, or recovered from asymptomatic infection. The states $\{S, C, R_C\}$ are therefore in the same information class, denoted N and referred to collectively as "not-yet-sick" (Figure 1). Let $a_N(t)$ denote the average activity level of all N-agents. $a_N(t) = a_S(t) = a_C(t) = a_{R_C}(t)$ since all agents in the same information class must necessarily behave in the same way on average.

Let $N(t) = S(t) + C(t) + R_C(t)$ be the mass of not-yet-sick agents. Let $q(t)$ be the likelihood that a not-yet-sick agent is susceptible:

$$q(t) \equiv \frac{S(t)}{N(t)}. \tag{8}$$

So long as all N-agents have behaved symmetrically up to time t, each N-agent believes that they remain susceptible with probability $q(t)$.[5] By equations (2-4,6) and the fact that

[5] If some N-agents have been more active in the past, then these agents would assess that they are more likely to have been exposed in the past and hence less likely to be susceptible in the present. Such asymmetric behavior does not arise in equilibrium in our model.

$\frac{d\ln(q(t))}{dt} = \frac{q'(t)}{q(t)} = \frac{S'(t)}{S(t)} - \frac{N'(t)}{N(t)}$, we have

$$N'(t) = -\sigma C(t) \tag{9}$$

$$\frac{q'(t)}{q(t)} = -\lambda(t) + \frac{\sigma C(t)}{N(t)} \tag{10}$$

Economic model. Each agent i seeks to maximize a "time-t payoff" $\pi_{i,t}$ that depends only on current epidemic conditions, their current activity, and the current activity of others.

Benefits of social-economic activity. Each agent gets a flow social-economic payoff consisting of three parts: (i) a baseline level b_0, capturing the value of physically-isolated activity, such as reading a book or working and socializing online, which we can set without loss to $b_0 = 0$; (ii) an additional $b_1 a_i(t)$, with $b_1 > 0$ capturing the value of individual activity that requires entering spaces where transmissive encounters can occur but whose value does not derive from social interaction, such as going shopping; and (iii) an additional $b_2 a_i(t)A(t)$ capturing social-economic benefits, such as being able to chat with colleagues at work, where $A(t)$ is the average activity level across the population:

$$A(t) \equiv a_N(t)N(t) + a_I(t)I(t) + a_{R_I}(t)R_I(t). \tag{11}$$

$a_i(t)A(t)$ is the rate at which agent i encounters someone else, i.e., their "social activity." Overall, each agent i's social-economic flow payoff at time t is $a_i(t)(b_1 + b_2 A(t))$.

Harm from becoming infected. Sick agents incur flow "disease cost" $d > 0$ so long as they remain in the I state. Each newly-infected agent progresses to sickness with probability $\frac{\sigma}{\gamma+\sigma}$ and, once sick, remains sick for average duration $\frac{1}{\gamma}$. The expected total disease cost associated with becoming infected ("harm of infection") is

$$H \equiv \frac{d\sigma}{\gamma(\gamma+\sigma)}. \tag{12}$$

Time-t payoff of sick and previously-sick agents. Any agent who is currently sick (state I) or recovered from sickness (state R_I) is at no risk of becoming infected. For such agents, their time-t payoff is simply their social-economic flow benefit:

$$\pi_{i,t} = a_i(t)(b_1 + b_2 A(t)) \text{ for all } i \in \{I, R_I\}. \tag{13}$$

Time-t payoff of not-yet-sick agents. For N-agents, $\pi_{i,t}$ includes both the social-economic benefit from current activity and the expected harm due to potentially becoming infected from that activity. Each N-agent believes they are susceptible with probability $q(t) = \frac{S(t)}{N(t)}$ and, if susceptible, becomes infected at rate $\lambda_i(t)$. Agent i's expected rate of becoming infected is therefore $q(t)\lambda_i(t)$. This gives us

$$\pi_{i,t} = a_i(t)\,(b_1 + b_2 A(t)) - \lambda_i(t) q(t) H \text{ for all } i \in N. \tag{14}$$

Political model. A policy-maker (also referred to as "political leader") decides at each point in time whether to constrain the activity of agents in each information class $\omega \in \{N, I, R_I\}$. The policy-maker has three options for each class ω: (i) leave all ω-agents free to choose, in which case their activity choices will be determined in Nash equilibrium given agents' time-t payoffs; (ii) force all ω-agents to choose the minimal activity level, $a_i(t) = \alpha$, referred to as a "stay-at-home order" for N-agents and as "isolation of the sick" for I-agents;[6] or (iii) force all ω-agents to choose the maximal activity level, $a_i(t) = 1$, referred to as a "go-out-home order".

Agents left unconstrained seek to maximize their own time-t payoff. The policy-maker evaluates each policy option based on what Nash equilibrium will be played by those left unconstrained, and chooses a policy option that is most preferred by the majority of the population. (We show in Section 2 that this "will of the majority" is well-defined.) Without loss, suppose that the policy-maker only imposes constraints on agents that change their behavior. In particular, a go-out (or stay-at-home) order for N-agents is never issued if they would have chosen to be maximally (or minimally) active without any constraints in place.

Equilibrium concept. At each point in time t, a "time-t equilibrium" specifies (i) a policy option that is politically optimal for the policy-maker and (ii) activity levels $a_i(t)$ that are individually optimal for each agent i left free to choose. An equilibrium epidemic trajectory is one generated by time-t equilibrium play at each point in time.

Welfare loss due to the pathogen. The pathogen harms people in the population directly due to suffering from the disease and indirectly due to lost social-economic benefits. In a world in which the pathogen did not exist, all agents would earn flow payoff $b_1 + b_2$ at all times. By contrast, for each information class $\omega \in \{N, I, R_I\}$, type-ω agents' average social-economic flow payoff equals $b_1 a_\omega(t) + b_2 a_\omega(t) A(t)$. Moreover, mass $I(t)$ of agents incur

[6] Our analysis can be easily extended to allow isolation of the sick to be more or less effective than stay-at-home orders for the not-yet-sick.

flow disease cost d. Let $W(t)$ denote the flow of population-wide welfare losses at time t:

$$W(t) = dI(t) + \sum_{\omega \in \{N,I,R_I\}} \omega(t) \left(b_1(1 - a_\omega(t)) + b_2(1 - a_\omega(t)A(t))\right). \tag{15}$$

The (undiscounted) cumulative welfare loss due to the pathogen is $\int_0^\infty W(t)\mathrm{d}t$.

1.1 Discussion of modeling assumptions

Here we discuss several of the model's key simplifying assumptions. Some additional simplifying features of the model, such as the lack of agent heterogeneity and the lack of diagnostic tests, are discussed in the concluding remarks in the context of directions for future research.

Myopic agents. Sophisticated forward-looking agents understand that becoming infected has a time-varying impact on lifetime welfare. For such agents, the "harm of infection" H varies over time and depends endogenously on how the epidemic will unfold in the future. By contrast, agents in our model treat the harm of infection as a fixed constant. This assumption has substantial qualitative implications, as forward-looking agents will have different political demands. For instance, suppose that the policy-maker can commit at time $t = 0$ to a full policy path.[7] Forward-looking agents will demand that the policy-maker commit to the socially-optimal policy path. By contrast, a feature of our model is that agents may myopically demand that the policy-maker take actions that ultimately reduce social welfare.

Frictionless policy-making to please the majority. The policy-maker in our model imposes whatever policy is preferred by the majority, no matter how small that preference may be. Policy-makers in practice may resist myopic demands and/or may only enact a new policy once demand for that policy is sufficiently strong. Such considerations could lead to less equilibrium policy-switching than we see in our model, especially in examples like in Figure 4 in which "yo-yo dynamics" of infection arise from repeated and rapid changes to equilibrium policy. While interesting, we abstract from such nuances for the sake of tractability and to make the novel aspects of our analysis as clear as possible.

Policy options. The policy-maker in our model has two policy levers, stay-at-home orders and go-out orders, which can be targeted at agents according to their information class (not-yet-sick, sick, or previously sick). This modeling choice allows us to highlight how the policies that arise in equilibrium are qualitatively different depending on whether the pathogen transmits mainly during sickness or mainly during the asymptomatic phase. Our

[7] In the case when agents are forward-looking and the policy-maker cannot commit, the problem of characterizing the equilibrium policy path is much more complex and potentially intractable.

analysis can be adapted to other settings where the policy-maker has more or fewer policy options, but the resulting analysis may be much less clean. Here we provide three examples of such alternative policy settings.[8]

First, suppose that stay-at-home orders are the only feasible policy option, perhaps because compelling people to be more active is not possible. As we show later, the time-t game among N-agents may during some phases of the epidemic be a coordination game in which N-agents collectively prefer to all go out; see Figure 2(c). If go-out orders cannot be issued, there are two time-t equilibria: one in which all N-agents go out and another in which they all stay home. How the epidemic progresses will depend on which time-t equilibrium is played and, since such multiplicity can persist for an extended period, there may be a vast multiplicity of epidemic trajectories that can arise in equilibrium. By contrast, in our model, the policy-maker's ability to impose go-out and/or stay-at-home orders pins down a unique time-t equilibrium in all but certain boundary-case situations.

Second, suppose that the political leader is unable to target its policies to agents in any specific information class, perhaps because information about agents' health status is unavailable. This affects N-agents' policy preferences and thereby impacts what the political leader will do throughout the epidemic. For instance, consider the case with symptomatic-only transmission. When policies can be targeted, we show in Section 5 that N-agents may prefer during certain phases of the epidemic for themselves to be subjected to a go-out order while I-agents are isolated and R_I-agents are left free to choose. If policy targeting is not possible, then N-agents might support a general lockdown because that is the only feasible way to constrain the activity of I-agents, or might support a general go-out order because that is the only way to get their fellow N-agents to be more active.

Finally, suppose that it is only possible to implement a "test free" policy in which agents are subjected to (continual) diagnostic testing that reveals their current health status and are forced to stay home if and only if they are currently infected. Such testing changes the information structure of the model, which in turn impacts agents' individual incentives and policy preferences. In particular, S-agents now know that they remain susceptible, which increases their desire to avoid being exposed to infection and hence increases their support for policies that isolate C- and I-agents. On the other hand, C-agents now know that they are at no risk of infection, aligning them politically with I- and R_I-agents and thereby increasing political support for policies that increase activity.

Linear time-t payoffs. In our model, each agent i's time-t payoff is linear in their own activity level a_i. In alternative models with decreasing marginal returns to activity, the time-t game

[8]We thank an anonymous referee for suggesting some of these alternative policy settings.

that N-agents play may have Nash equilibria with intermediate levels of activity that are collectively preferred by N-agents over either extreme scenario. If so, the policy-maker *strictly* prefers not to impose any constraint on N-agents, something that never happens in our model (proof of Prop 2), and the epidemic trajectory will be determined for a period of time by how Nash equilibrium activity evolves during that period.

2 Equilibrium behavior in the "time-t game"

Because agents and the policy-maker are myopic decision-makers, we can focus on each moment of time during the epidemic as its own "time-t game." In this time-t game, (i) the policy-maker chooses a politically-optimal policy, imposing a most-preferred policy of whatever group of agents is currently in the majority, and (ii) agents who are left free to choose make individually-optimal choices about how active to be. In this section, we characterize the agent behavior that can potentially arise in equilibrium at each point in time t. In Section 3, we then apply these findings to construct all equilibrium epidemic trajectories that can arise from any given initial condition.

2.1 Once not-yet-sick agents are in the minority

Suppose that $I(t) + R_I(t) > \frac{1}{2}$. Because I- and R_I-agents prefer for everyone to be fully active, we must have $a_N(t) = a_I(t) = a_{R_I}(t) = 1$ in time-t equilibrium. Why? If some class of agents were not fully active, the policy-maker could increase the time-t payoff of I- and R_I-agents by imposing a go-out order on all agents. Thus, any policy that does not result in all agents being fully active is politically sub-optimal. Moreover, because agents do not return to the susceptible state, $I(t) + R_I(t)$ is monotonically increasing and these agents must remain in the majority at all future times as well. We conclude that, once a time t^* is reached at which $I(t^*) + R_I(t^*) = \frac{1}{2}$, all subsequent equilibrium dynamics are trivially determined by (2-7) and agent behavior $a_N(t) = a_I(t) = a_{R_I}(t) = 1$ for all $t > t^*$.

2.2 While not-yet-sick agents remain in the majority

Now we turn to the less trivial case when $N(t) > \frac{1}{2}$, considering the potential time-t equilibrium behavior of each class of agents.

R_I-agents who have recovered from sickness. Because N-agents prefer for R_I-agents to be as active as possible, we must have $a_{R_I} = 1$ in any time-t equilibrium at every time

t. Since R_I-agents also prefer to be fully active, this can be achieved simply by leaving R_I-agents free to choose.

I-agents who are currently sick. N-agents may or may not prefer for I-agents to be isolated. Differentiating equation (14), we have

$$\frac{\partial \pi_i}{\partial a_I} = a_i(t)I(t)(b_2 - q(t)\beta_I H). \tag{16}$$

The positive term $a_i(t)I(t)b_2$ in (16) reflects the social-economic benefit that N-agents get from interacting with I-agents, while the negative term $-a_i(t)I(t)q(t)\beta_I H$ reflects the expected harm from potentially being exposed to infection.

Let q_I^* denote the level of $q(t)$ so that the right-hand-side of (16) is zero:

$$q_I^* \equiv \frac{b_2}{\beta_I H}. \tag{17}$$

If $q(t) > q_I^*$, then N-agents prefer for I-agents to be less active. The policy-maker must therefore isolate the sick. On the other hand, if $q(t) < q_I^*$, then N-agents prefer for I-agents to be more active and I-agents must be fully active in equilibrium.

Overall, we conclude that (i) $a_I(t) = \alpha$ at all times t when $q(t) > q_I^*$ and (ii) $a_I(t) = 1$ at all times t when $q(t) < q_I^*$. In the last possibility that $q(t) = q_I^*$, the policy-maker may potentially impose a mixed policy in which I-agents are subjected to a stay-at-home order with probability $p_I^{stay}(t) \in [0, 1]$ and all go out with probability $1 - p_I^{stay}(t)$.

Proposition 1 summarizes what we have shown about I-agent equilibrium activity.

Proposition 1 (I-agent activity at time t)**.** *Suppose that $N(t) > \frac{1}{2}$. (i) If $q(t) > q_I^*$, then $a_I(t) = \alpha$ in any time-t equilibrium. (ii) If $q(t) < q_I^*$, then $a_I(t) = 1$ in any time-t equilibrium. (iii) If $q(t) = q_I^*$, then N-agents get the same time-t payoff regardless of what I-agents do.*

N-agents who have not yet gotten sick. Early and late during the epidemic when $C(t), I(t) \approx 0$, N-agents are at negligible risk of infection and the time-t game among N-agents is trivial: each N-agent has a dominant strategy to go out; N-agent activity generates a positive externality for other N-agents; and the game among N-agents has strategic complements. Consequently, N-agents all go out when free to choose and have no desire for the policy-maker to impose any constraint on their ability to do so. But as the epidemic progresses, the time-t game among N-agents may change in several ways: their individual incentives may change, so that they no longer have a dominant strategy or have a dominant strategy to stay home; the externality on other N-agents from going out may flip to negative; and the game itself may flip to having strategic substitutes.

Let $\Delta\pi_{a_N}(t)$ be the incremental benefit to any individual N-agent from going out rather than staying home, given average activity a_N across all N-agents:

$$\Delta\pi_{a_N}(t) = \pi_{i,t}(a_i = 1, a_N) - \pi_{i,t}(a_i = \alpha, a_N). \tag{18}$$

N-agents have a dominant strategy to go out if $\Delta\pi_1(t)$ and $\Delta\pi_\alpha(t)$ are both positive, or a dominant strategy to stay home if both are negative. The time-t game among N-agents has strategic complements if $\Delta\pi_1(t) > \Delta\pi_\alpha(t)$, or strategic substitutes if $\Delta\pi_1(t) < \Delta\pi_\alpha(t)$.

What about the externalities associated with N-agent activity? Differentiating (14),

$$\frac{\partial \pi_{i,t}}{\partial a_N} = a_i(t)(b_2 N(t) - q(t)\beta_C C(t) H). \tag{19}$$

Let $q_N^*(t)$ denote the time-varying level of $q(t)$ so that $\frac{\partial \pi_{i,t}}{\partial a_N} = 0$:

$$q_N^*(t) \equiv \frac{b_2 N(t)}{\beta_C C(t) H}. \tag{20}$$

N-agent activity generates a positive externality for other N-agents iff $q(t) < q_N^*(t)$.

When deciding what N-agent policy to put in place at each point in time, the policy-maker considers how N-agents rank three time-t possibilities:

1. *no constraints:* N-agents play a Nash equilibrium (NE) of their time-t game;

2. *go-out order:* N-agents all go out; or

3. *stay-at-home order:* N-agents all stay home.

If the NE that they would play is strictly preferred over all going out and all staying home, then placing no constraints on N-agents is the unique politically-optimal policy. On the other hand, a go-out order or a stay-at-home order is (weakly) politically optimal so long as *either* the NE they would play entails all N-agents going out or all staying home *or* that NE generates (weakly) lower time-t payoffs for N-agents than all going out or all staying home.

Let $\chi(t)$ denote the incremental time-t payoff that N-agents get when all of them go out, relative to all staying home:

$$\chi(t) = \pi_{i,t}(a_i = 1, a_N = 1) - \pi_{i,t}(a_i = \alpha, a_N = \alpha). \tag{21}$$

If $\chi(t) > 0$, then N-agents strictly prefer a go-out order over a stay-at-home order; so, a stay-at-home order cannot be politically optimal. Similarly, if $\chi(t) < 0$, then a go-out order cannot be politically optimal. What about being left unconstrained? Could the time-t game have a Nash equilibrium that N-agents prefer over both a stay-at-home order and a go-out order? Proposition 2 establishes that this never happens in any of the time-t games that can potentially arise during the epidemic. Consequently, N-agents' equilibrium behavior is uniquely determined at any given point in time when $\chi(t)$ is positive or negative.

Proposition 2 (N-agent activity at time t)**.** *Along any equilibrium trajectory: (i) If $\chi(t) > 0$, then $a_N(t) = 1$. (ii) If $\chi(t) < 0$, then $a_N(t) = \alpha$. (iii) If $\chi(t) = 0$ and $q(t) \neq q_N^*$, then N-agents all stay home with probability $p_N^{stay}(t)$ and all go out with probability $1 - p_N^{stay}(t)$.*[9]

The rest of this section provides the proof of Proposition 2, beginning with a useful preliminary result.

Lemma 1. *(i) $q(t) < q_N^*(t)$ if and only if $\frac{\partial \pi_{i,t}}{\partial a_N} > 0$ if and only if $\chi(t) > \Delta\pi_1(t) > \Delta\pi_\alpha(t)$. (ii) $q(t) > q_N^*(t)$ if and only if $\frac{\partial \pi_{i,t}}{\partial a_N} < 0$ if and only if $\chi(t) < \Delta\pi_1(t) < \Delta\pi_\alpha(t)$.*

Discussion of Lemma 1. The basic nature of the game among N-agents at any given time depends on which of four key terms are positive or negative:

(i) $\Delta\pi_1(t) \equiv \pi_{i,t}(1,1) - \pi_{i,t}(\alpha,1)$: determines whether N-agents have an individual incentive to go out or stay home when all other N-agents go out, represented in the schematic game diagrams of Figure 2 by an up-down arrow in the "out" column (this arrow points up if $\Delta\pi_1(t) > 0$ and down if $\Delta\pi_1(t) < 0$);

(ii) $\Delta\pi_\alpha(t) \equiv \pi_{i,t}(1,\alpha) - \pi_{i,t}(\alpha,\alpha)$: determines whether N-agents have an individual incentive to go out or stay home when all other N-agents stay home, represented by an up-down arrow in the "home" column;

(iii) $\Delta\pi_1(t) - \Delta\pi_\alpha(t)$: determines whether the time-t game among N-agents has strategic complements (if $\Delta\pi_1(t) - \Delta\pi_\alpha(t) > 0$) or strategic substitutes (if $\Delta\pi_1(t) - \Delta\pi_\alpha(t) < 0$), shown by the direction and relative magnitudes of the up-down arrows; and

(iv) $\chi(t) \equiv \pi_{i,t}(1,1) - \pi_{i,t}(\alpha,\alpha)$: determines whether N-agents collectively prefer to all go out or all stay home, represented by "+/-" in the (out, out) and (home, home) boxes.

Each of these four quantities can be positive or negative, so there are 16 basic types of games that N-agents could conceivably play at different times. Lemma 1 implies that, in

[9] In the last possibility that $\chi(t) = 0$ and $q(t) = q_N^*$, the time-t game is a trivial one in which N-agents get the same expected payoff no matter what they or other N-agents choose to do.

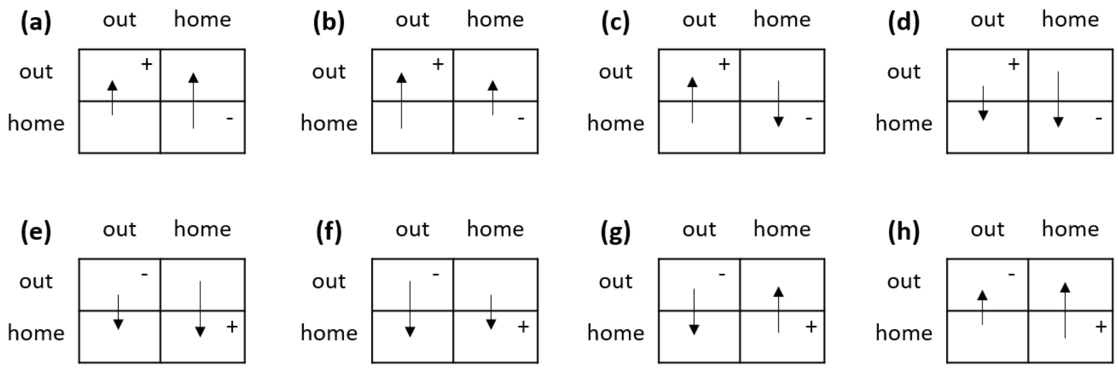


Figure 2: Schematic diagrams showing the possibilities for the time-t game among N-agents.

fact, only 8 of these 16 possibilities can ever arise: three in which N-agents have a dominant strategy to go out, including one with strategic complements (Fig 2(b)) and two with strategic substitutes (Fig 2(a,h)); three with a dominant strategy to stay home, including one with strategic substitutes (Fig 2(f)) and two with strategic complements (Fig 2(d,e)); one with two pure-strategy Nash equilibria (PSNE) and a collective preference to all go out (Fig 2(c)); and one with no PSNE and a collective preference to all stay home (Fig 2(g)).

Proof of Lemma 1. The fact that $\frac{\partial \pi_{i,t}}{\partial a_N} \gtreqless 0$ iff $q(t) \lesseqgtr q_N^*(t)$ is immediate from (20). By equations (1,11,14,21,18), the expressions $\Delta\pi_1(t)$, $\Delta\pi_\alpha(t)$, and $\chi(t)$ can be written as

$$\Delta\pi_1(t) = Z(t) + (1-\alpha)(b_2 N(t) - q(t)\beta_C C(t) H) \tag{22}$$

$$\Delta\pi_\alpha(t) = Z(t) + \alpha(1-\alpha)(b_2 N(t) - q(t)\beta_C C(t) H) \tag{23}$$

$$\chi(t) = Z(t) + (1-\alpha^2)(b_2 N(t) - q(t)\beta_C C(t) H) \tag{24}$$

where

$$Z(t) = (1-\alpha)(b_1 + b_2(a_I(t)I(t) + R_I(t)) - q(t)a_I(t)\beta_I I(t) H) \tag{25}$$

To complete the proof, note that (a) $b_2 N(t) - q(t)\beta_C C(t) H \gtreqless 0$ iff $q(t) \lesseqgtr q_N^*(t)$ and (b) $1-\alpha^2 > 1-\alpha > \alpha(1-\alpha) > 0$. $\square$

Proof of Proposition 2(i). Suppose that $\chi(t) > 0$, so that N-agents collectively prefer all going out over all staying home. There are four possibilities for what the time-t game among N-agents may be like, as shown in Figure 2(a-d). (The up-down arrows in each figure capture N-agents' individual incentives, while the $\pm$ notation captures their collective preference for all going out.) In each case, we will show that all N-agents must go out in equilibrium.

Case 1: $\Delta\pi_1(t) \geq \chi(t)$ (Fig 2(a)). By Lemma 1, $\Delta\pi_1(t) \geq \chi(t)$ implies that $\Delta\pi_\alpha(t) \geq$

$\Delta\pi_1(t)$; so, $\Delta\pi_\alpha(t) \geq \Delta\pi_1(t) \geq \chi(t) > 0$ and N-agents have a strictly dominant strategy to go out. The policy-maker's choice is therefore between all N-agents going out (by leaving them unconstrained to play the unique NE or by imposing a go-out order) or all staying home (by imposing a stay-at-home order). Since $\chi(t) > 0$ and N-agents are in the majority, any politically-optimal policy must induce them all to go out. We conclude as desired that $a_N(t) = 1$.

Case 2: $\chi(t) > \Delta\pi_1(t)$ and $\Delta\pi_\alpha(t) \geq 0$ (Fig 2(b)). By Lemma 1, $\chi(t) > \Delta\pi_1(t)$ implies that $q(t) < q_N^*(t)$ and $\chi(t) > \Delta\pi_1(t) > \Delta\pi_\alpha(t)$. Since $\Delta\pi_\alpha(t) \geq 0$, N-agents have a weakly dominant strategy to go out and the time-t game has a NE in which all N-agents go out. If $\Delta\pi_\alpha(t) > 0$, this is the unique NE and we conclude that $a_N(t) = 1$ by the same argument as in Case 1. If $\Delta\pi_\alpha(t) = 0$, there is also a NE in which all N-agents stay home but no NE in strictly mixed strategies. The policy-maker's choice is once again between all N-agents going out and all staying home, of which all going out is politically preferred since $\chi(t) > 0$.

Case 3: $\chi(t) > \Delta\pi_1(t) > 0 > \Delta\pi_\alpha(t)$ (Fig 2(c)). The time-t game is now a coordination game with three NE: one in which all go out, another in which all stay home, and a mixed-strategy NE in which N-agents have average activity $a_N^* \in (\alpha, 1)$. In this mixed-strategy NE, N-agents are indifferent whether to be active; so, $\pi_{i,t}(a_N^*, a_N^*) = \pi_{i,t}(1, a_N^*) = \pi_{i,t}(\alpha, a_N^*)$. However, by Lemma 1, $\chi(t) > \Delta\pi_1(t)$ implies that $\frac{\partial\pi_{i,t}(a_i,a_N)}{\partial a_N} > 0$; thus, $\pi_{i,t}(1, a_N^*) < \pi_{i,t}(1,1)$. Overall, $\pi_{i,t}(a_N^*, a_N^*) < \pi_{i,t}(1,1)$, meaning that N-agents strictly prefer being subjected to a go-out order rather than playing the mixed-strategy equilibrium. We conclude that, under any politically-optimal policy, N-agents must all go out.

Case 4: $\chi(t) > 0 \geq \Delta\pi_1(t) > \Delta\pi_\alpha(t)$ (Fig 2(d)). N-agents now have a weakly dominant strategy to stay home and the time-t game has a NE in which all N-agents stay home. If $\Delta\pi_1(t) < 0$, this is the unique NE but N-agents collectively prefer being subjected a go-out order rather than being left free to play this NE. If $\Delta\pi_1(t) = 0$, there is also a NE in which all N-agents go out but no NE in strictly mixed strategies. Overall, then, the policy-maker's choice between all N-agents going out and all staying home, of which all going out is politically preferred since $\chi(t) > 0$. □

Proof of Proposition 2(ii). Suppose that $\chi(t) < 0$. There are again four possibilities for the time-t game among N-agents, in each of which it must be that $a_N(t) = \alpha$ in equilibrium:

> *Case 1:* $\Delta\pi_1(t) \leq \chi(t)$, with $\Delta\pi_\alpha(t) \leq \Delta\pi_1(t) \leq \chi(t) < 0$ by Lemma 1 (Fig 2(e)).
>
> *Case 2:* $\chi(t) < \Delta\pi_1(t)$ and $\Delta\pi_\alpha(t) \leq 0$, with $\chi(t) < \Delta\pi_1(t) < \Delta\pi_\alpha(t) \leq 0$ by Lemma 1 (Fig 2(f))

Case 3: $\chi(t) < \Delta\pi_1(t) < 0 < \Delta\pi_\alpha(t)$ (Fig 2(g))

Case 4: $\chi(t) < 0 \leq \Delta\pi_1(t) < \Delta\pi_\alpha(t)$ (Fig 2(h))

To save space, we omit the proofs that $a_N(t) = \alpha$ in Cases 1,2,4, which exactly mirror the proofs that $a_N(t) = 1$ in Cases 1,2,4 in the proof of Prop 2(i). It remains to examine Case 3.

Case 3: $\chi(t) < \Delta\pi_1(t) < 0 < \Delta\pi_\alpha(t)$ (Fig 2(g)). The time-t game is a chicken game with a unique mixed-strategy NE in which N-agents have average activity $a_N^* \in (\alpha, 1)$. In this NE, N-agents are indifferent whether to be active; so, $\pi_{i,t}(a_N^*, a_N^*) = \pi_{i,t}(1, a_N^*) = \pi_{i,t}(\alpha, a_N^*)$. However, by Lemma 1, $\chi(t) < \Delta\pi_1(t)$ implies that $\frac{\partial \pi_{i,t}(a_i, a_N)}{\partial a_N} < 0$; thus, $\pi_{i,t}(\alpha, a_N^*) < \pi_{i,t}(\alpha, \alpha)$. Overall, $\pi_{i,t}(a_N^*, a_N^*) < \pi_{i,t}(\alpha, \alpha)$, meaning that N-agents strictly prefer being subjected to a stay-at-home order rather than playing the mixed-strategy equilibrium. We conclude that, under any politically-optimal policy, N-agents must all stay home. $\square$

Proof of Proposition 2(iii). Suppose finally that $\chi(t) = 0$. If $q(t) > q_N^*(t)$, then $\Delta\pi_\alpha(t) > \Delta\pi_1(t) > 0$ and N-agents have a strictly dominant strategy to go out. In this case, the policy-maker's choice is between all N-agents going out (by leaving them unconstrained to play the unique NE or by imposing a go-out order) or all staying home (by imposing a stay-at-home order). Since $\chi(t) = 0$, the policy-maker is indifferent between these outcomes and may mix between them. Similarly, if $q(t) < q_N^*(t)$, then $\Delta\pi_\alpha(t) < \Delta\pi_1(t) < 0$ and N-agents have a strictly dominant strategy to stay home. In this case as well, the policy-maker's choice is between all N-agents going out (by imposing a go-out order) or all staying home (by leaving them unconstrained or by imposing a stay-at-home order) and may mix due to indifference.

3 Equilibrium Epidemic Analysis

This section characterizes how politically-optimal policy and agents' resulting equilibrium behavior change throughout the epidemic in the general case with both symptomatic and asymptomatic transmission.

3.1 At the beginning of the epidemic

At the beginning of the epidemic, nearly everyone remains susceptible. Absent any restrictions on activity, each newly-infected person will infect R_0 others on average before recovery, where R_0 is the pathogen's basic reproduction number:[10]

$$R_0 = \frac{\beta_C}{\sigma + \gamma} + \frac{\sigma \beta_I}{(\sigma + \gamma)\gamma}. \tag{26}$$

Not-yet-sick all go out. When an N-agent i encounters another N-agent, i gets social-economic benefit b_2 and becomes infected with probability $\beta_C q(t) \frac{C(t)}{N(t)}$, where $q(t) = \frac{S(t)}{N(t)}$ is i's probability of remaining susceptible. Because $S(0) \approx 1$ and $C(0) \approx 0$, we have $q(0) \approx 1$ and $\beta_C q(0) \frac{C(0)}{N(0)} \approx 0$. Thus, N-agents benefit when encountering other N-agents, and so the policy-maker finds it politically optimal not to impose a stay-at-home order on N-agents.

Sick may or may not be isolated. Each time an N-agent encounters an I-agent, the N-agent gets social-economic benefit b_2 and becomes infected with probability $\beta_I q(t)$, for overall net expected benefit $b_2 - \beta_I q(t) H$. Because $q(0) \approx 1$, N-agents prefer for the sick to be isolated at the beginning of the epidemic if the harm of infection $H > \frac{b_2}{\beta_I}$, but not if $H < \frac{b_2}{\beta_I}$.

3.2 At the end of the epidemic

In the limit as $t \to \infty$, each agent is either still susceptible (i.e., never infected), recovered from sickness, or recovered from asymptomatic infection. Let $S^\infty \equiv \lim_{t\to\infty} S(t)$ be the share of agents who are never infected, let $R_I^\infty \equiv \lim_{t\to\infty} R_I(t)$ be the share who became sick at some point, and similarly for other notation.

If $R_I^\infty > \frac{1}{2}$, then N-agents will be in the minority at the end of the epidemic and the policy-maker will find it politically optimal to ensure that everyone is fully active.

If $R_I^\infty \leq \frac{1}{2}$, then N-agents remain in the majority throughout the epidemic. Because C-agents transition to state I at rate σ and to state R_C at rate γ, we have

$$\frac{R_I^\infty}{\sigma} = \frac{R_C^\infty}{\gamma}. \tag{27}$$

Using the facts that $1 - S^\infty = R_C^\infty + R_I^\infty$ and $N^\infty = S^\infty + R_C^\infty$, we have $R_C^\infty = \frac{\gamma}{\sigma+\gamma}\left(1 - S^\infty\right)$

[10] Since infected agents recover at rate γ and asymptomatic infection progresses to sickness at rate σ, the asymptomatic phase has average duration $\frac{1}{\sigma+\gamma}$, infection progresses to sickness with probability $\frac{\sigma}{\sigma+\gamma}$, and sickness has average duration $\frac{1}{\gamma}$. Thus, assuming that everyone is fully active, each newly-infected agent will on average encounter $\frac{1}{\sigma+\gamma}$ others while asymptomatic and $\frac{\sigma}{(\sigma+\gamma)\gamma}$ others while sick. At the start of the epidemic, essentially all of these encounters are with susceptible agents, resulting on average in $\frac{\beta_C}{\sigma+\gamma}$ new infections resulting from asymptomatic transmission and $\frac{\sigma\beta_I}{(\sigma+\gamma)\gamma}$ from symptomatic transmission.

and hence

$$q^{\infty}\left(S^{\infty}\right)=\frac{S^{\infty}}{\frac{\sigma}{\sigma+\gamma}S^{\infty}+\frac{\gamma}{\sigma+\gamma}}\in(0,1). \tag{28}$$

Note that $q^{\infty}\left(S^{\infty}\right)$ is strictly increasing in S^{∞} and hence decreasing in the overall number of people who get infected during the epidemic, with $q^{\infty}(1)=1$ and $q^{\infty}(0)=0$.

Not-yet-sick are not subjected to stay-at-home orders. Because $C^{\infty}=0$, N-agents are at no risk from other N-agents in the $t\to\infty$ limit. Consequently, the policy-maker must eventually leave N-agents free from any stay-at-home orders.

Political incentive to isolate the sick is weaker at the end than at the beginning. Each N-agent gets net expected benefit $b_2-\beta_I q^{\infty}\left(S^{\infty}\right)H$ when encountering a sick agent. This is higher than at the beginning of the epidemic, because each N-agent is only susceptible with probability $q^{\infty}\left(S^{\infty}\right)$. Thus, depending on the harm of infection H and how many people were previously infected, the policy-maker may find it politically optimal to isolate the sick early during the epidemic but not at the end. This can create an unfortunate feedback effect, whereby more infection early in the epidemic weakens the (myopic) political incentive to isolate the sick, which in turn can potentially result in much more overall infection.

Example: Let $\beta_C=0$, $\beta_I=1$, $\gamma=\frac{1}{10}$, $\sigma=\frac{1}{10}$, $\alpha=\frac{1}{3}$, $b_2=3$, and $H=4$. Instead of starting from time $t=0$ when $S(0)\approx 1$ and $C(0)\approx I(0)=0$, supoose that we start from a time $\hat{t}$ at which $S(\hat{t})=\frac{1}{2}$ and $C(\hat{t})\approx I(\hat{t})\approx 0$. This could arise if the population has already endured a wave of disease in which half of the population was infected but infection has now died down—and is kept down through a continued policy of isolating the sick. Because transmission in this example only occurs during sickness, isolating the sick is an effective way to curtail transmission.[11]

Will this policy-maker continue isolating the sick? At the beginning of the epidemic, it would have been politically optimal to do so, since the harm $H=4$ to N-agents from being exposed to an I-agent is less than the social-economic benefit $b_2=3$. However, isolating the sick is now no longer politically optimal. Because $\gamma=\sigma$, half of all infected people become sick before recovery. With $S(\hat{t})=\frac{1}{2}$ and $C(\hat{t})\approx I(\hat{t})\approx 0$, this implies that $R_C(\hat{t})\approx R_I(\hat{t})\approx\frac{1}{4}$. Thus, (i) N-agents are in the majority with about three-quarters of the

[11]If the sick are not isolated, the pathogen has basic reproductive number $R_0=5$ by equation (26) and herd immunity (meaning that each new infection causes at most one new infection) will not be achieved until at least $1-\frac{1}{R_0}=80\%$ of the population has been infected. By contrast, if the sick are isolated in perpetuity, the pathogen's basic reproductive number is only $5\alpha=\frac{5}{3}$ and herd immunity is reached once 40% of the population has been infected. If only half of the population is susceptible, each new infection results in $\frac{5}{6}=\frac{1}{2}\times\frac{5}{3}$ new infections.

population and (ii) each N-agent remains susceptible with probability $q(\hat{t}) = \frac{2}{3}$. When an N-agent encounters an I-agent, the social-economic benefit $b_2 = 3$ outweighs the expected harm $\frac{2}{3} * 4 < 3$ from the encounter. Consequently, there is political support for no longer isolating the sick, even though this will ultimately result in a large second wave of infection.

3.3 In the middle of the epidemic

The epidemic trajectory is governed by the system of differential equations (3-7). Agent behavior enters this system solely through the force-of-infection term $\lambda(t)$, which itself depends only on $(a_S(t), a_C(t), a_I(t))$, the activity levels of susceptible, asymptomatically infected, and sick agents. Because S- and C-agents are each in the not-yet-sick information class, we have $a_S(t) = a_C(t) = a_N(t)$. Moreover, because R_I-agents cannot be infected or infect anyone, they must all go out at all times in any equilibrium. It remains to characterize what I-agent and N-agent activity can arise in equilibrium.

3.3.1 *I*-agent activity

There are three possibilities for I-agent behavior at each point in time, depending on the likelihood $q(t)$ that each N-agent remains susceptible (Prop 1):

Full activity: If $q(t) < q_I^* = \frac{b_2}{\beta_I H}$, then I-agents all go out and $a_I(t) = 1$.

Full isolation: If $q(t) > q_I^*$, then I-agents are isolated and $a_I(t) = \alpha$.

Mixed isolation: If $q(t) = q_I^*$, then I-agents are isolated with some probability and $a_I(t) \in [\alpha, 1]$.

While the possibilities are fairly complex in general, there are important special cases in which I-agent behavior is very simple to describe.

1. *Transmissibility during sickness β_I is sufficiently small.*

When $\beta_I < \frac{b_2}{H}$, the threshold $q_I^* > 1$. N-agents therefore prefer for I-agents to be fully active no matter what the epidemic state, and we must have $a_I(t) = 1$ for all t.

2. *Harm of infection H is sufficiently large.*

Let $\mathcal{E}_0$ denote the epidemic trajectory that would result if all agents always went out, and let $S_0^\infty \equiv \lim_{t\to\infty} S_0(t)$ be the mass of agents who escape infection along this full-activity trajectory. As can be easily checked (straightforward details omitted), S_0^∞ is a uniform lower bound on $S(t)$ at any time t along any epidemic trajectory. Since $N(t) < 1$, we conclude that $q(t) = \frac{S(t)}{N(t)} > S_0^\infty$ at all times no matter what agents' behavior may be. Now, let H_I be the level of H so that $q_I^* = S_0^\infty$, i.e.,

$$H_I \equiv \frac{b_2}{\beta_I S_0^\infty}. \tag{29}$$

So long as $H > H_I$, $q(t)$ must remain above q_I^* throughout the entire epidemic, ensuring that I-agents are always isolated in equilibrium, i.e., $a_I(t) = \alpha$ for all t.

3. *Sickness is sufficiently rare, i.e., $\frac{\sigma}{\gamma}$ is sufficiently small.*

When $\frac{\sigma}{\gamma} \approx 0$, very few infections result in sickness. Almost all infected agents pass directly from the C state to the R_C state, with very few progressing to the I or R_I states. Consequently, $N(t) \approx 1$ throughout the epidemic and therefore $q(t) = \frac{S(t)}{N(t)} \approx S(t)$ declines monotonically over time as more people are infected. In equilibrium, I-agents remain subjected to isolation until (if) a time t^* is reached at which $q(t^*) = q_I^*$, after which the sick are no longer isolated; so, $a_I(t) = \alpha$ for all $t < t^*$ and $a_I(t) = 1$ for all $t > t^*$.

Having discussed some of the nuances of equilibrium isolation of the sick, and how it can change over time, we will henceforth shut down this source of complexity in order to focus more clearly on the equilibrium dynamics of N-agent behavior, which is our main interest. In particular, we henceforth assume that I-agent behavior is constant throughout the epidemic, either $a_I(t) = \alpha$ for all t (as when $H > H_I$) or $a_I(t) = 1$ for all t (as when $\beta_I < \frac{b_2}{H}$). Under this simplification, the epidemic trajectory is controlled entirely by N-agent activity.

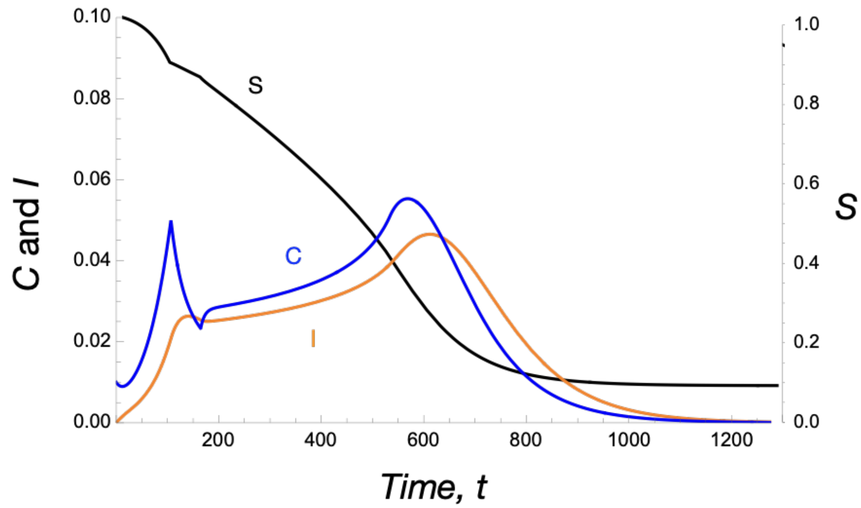
3.3.2 N-agent activity

There are three possibilities for N-agent behavior at each point in time depending on N-agents collective preference for all going out versus all staying home, as captured by $\chi(t) = \pi_{i,t}(1,1) - \pi_{i,t}(\alpha.\alpha)$ (Prop 2).

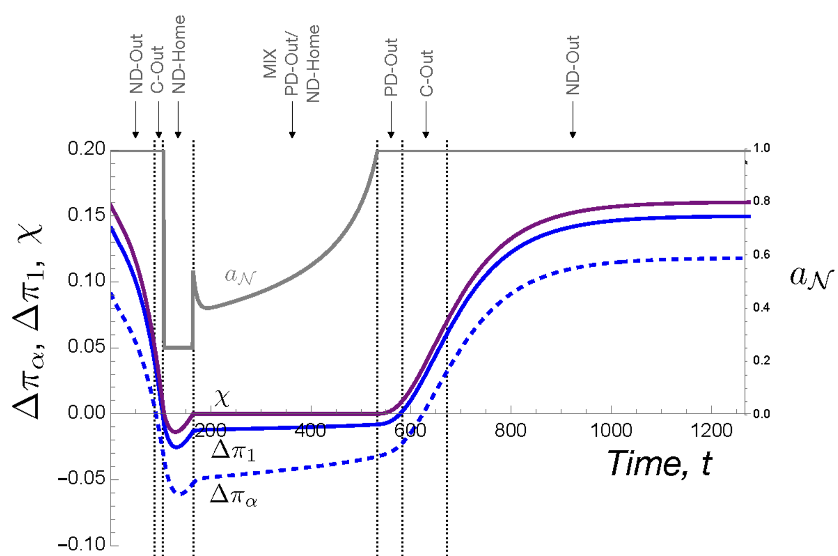
All Go Out: If $\chi(t) > 0$, then N-agents must all go out, voluntarily if that is a Nash equilibrium of their time-t game or subject to a go-out order, and $a_N(t) = 1$.

All Stay Home: If $\chi(t) < 0$, then N-agents must all stay home, voluntarily or subject to a stay-at-home order, and $a_N(t) = \alpha$.

Collective Mixing: If $\chi(t) = 0$, then N-agents either all go out or all stay home, each with some probability, and $a_N(t) \in [\alpha, 1]$.



(a) Infection dynamics



(b) Activity/game dynamics

Figure 3: Illustration of the equilibrium epidemic trajectory in an example with both symptomatic and asymptomatic transmission. Model parameters here are $C(0) = 0.01$, $\alpha = 0.25$, $\beta_C = 0.0143$, $\beta_I = 0.431$, $\sigma = 0.0157$, $\gamma = 0.0171$, $b_1 = 0.1$, $b_2 = 0.1$, $d = 2.8$.

Figure 3 illustrates the equilibrium epidemic trajectory in a numerical example with mostly symptomatic transmission: β_C and β_I are both positive but the ratio $\frac{\beta_I}{\beta_C} \approx 30$. Panel (a) shows how the mass of susceptible S-agents and infected C- and I-agents varies throughout the epidemic. ($S(t)$ is shown on a scale from 0 to 1, while $C(t)$ and $I(t)$ are shown on a scale from 0 to 0.1.) Panel (b) shows how $\chi(t)$, $\Delta\pi_1(t)$, and $\Delta\pi_\alpha(t)$ vary throughout the epidemic, as well as N-agents' equilibrium activity. This allows us to track how the time-t game that N-agents play changes over time. For convenience, we have given names to each of the possible game types documented earlier in Figure 2:

1. **PD-Out:** $\chi > 0$, $\Delta\pi_1 < 0$, $\Delta\pi_\alpha < 0$ (Fig 2(d)). Prisoners' Dilemma game (PD) in which N-agents collectively prefer all going out.

2. **PD-Home:** $\chi < 0$, $\Delta\pi_1 > 0$, $\Delta\pi_\alpha > 0$ (Fig 2(h)). PD in which N-agents collectively prefer all staying home.

3. **ND-Out:** $\chi > 0$, $\Delta\pi_1 > 0$, $\Delta\pi_\alpha > 0$ (Fig 2(a-b)). "No Dilemma" game (ND) in which N-agents each have a dominant strategy to go out and collectively prefer to do so.

4. **ND-Home:** $\chi < 0$, $\Delta\pi_1 < 0$, $\Delta\pi_\alpha < 0$ (Fig 2(e-f)). ND in which N-agents each have a dominant strategy to stay home and collectively prefer to do so.

5. **C-Out:** $\chi > 0$, $\Delta\pi_1 > 0$, $\Delta\pi_\alpha < 0$ (Fig 2(c)). Coordination game ("C") in which N-agents do not have a dominant strategy, all going out and all staying out are both pure-strategy Nash equilibria, and N-agents collectively prefer to go out.

6. **D-Home:** $\chi < 0$, $\Delta\pi_1 < 0$, $\Delta\pi_\alpha > 0$ (Fig 2(g)). Diversification game ("D"), i.e. a Chicken game, in which N-agents do not have a dominant strategy, the unique Nash equilibrium is in mixed strategies, and N-agents collectively prefer to stay home.

Panel (b) also shows how the time-t game transitions over time in this example through several phases: (i) ND-Out, (ii) C-Out, (iii) ND-Home, (iv) PD-Out/ND-Home (meaning that the time-t game remains on the boundary of these two game types, namely $\chi = 0$, $\Delta\pi_1 < 0$, and $\Delta\pi_\alpha < 0$), (v) PD-Out, (vi) C-Out, and (vii) ND-Out. Along the equilibrium trajectory, the policy-maker imposes a *pure go-out order* during the PD-Out phase, enabling N-agents to overcome their individual incentives to stay home and solve a collective-action problem. During the "MIX" phase (iv), the policy-maker imposes a *mixed go-out order*, meaning that go-out orders are imposed at each point in time with some probability between zero and one. The policy-maker never imposes stay-at-home orders on N-agents.

3.3.3 Constructing the equilibrium epidemic trajectory.

Next, we show how to construct the equilibrium trajectory in general, highlighting when and how the epidemic must transition from one behavioral regime to the next.

All Go Out initially. At the beginning of the epidemic, $S(0) \approx 1$ and hence $\chi(0) \approx (1-\alpha)b_1 + (1-\alpha^2)b_2 > 0$ by equation (24); so, $a_N(0) = 1$. Let $\mathcal{E}_1$ denote the epidemic trajectory generated by N-agent activity $a_N(t) = 1$ for all t, and let $\chi(t; \mathcal{E}_1)$ be the collective benefit of all going out under this trajectory. By construction, $\chi(t; \mathcal{E}_1)$ is continuously differentiable. If $\chi(t; \mathcal{E}_1) > 0$ for all t, then we are done: $\mathcal{E}_1$ is the unique equilibrium trajectory. Otherwise, let $t_1 \equiv \inf\{t > 0 : \chi(t; \mathcal{E}_1) < 0\}$ be the first time at which $\chi(t; \mathcal{E}_1)$ crosses to become negative. Since $\chi(t; \mathcal{E}_1)$ crosses zero from above at t_1, it must be that $\chi'(t_1; \mathcal{E}_1) \leq 0$.

Discussion: equilibrium uniqueness up to time t_1. Let $t_0 \equiv \min\{t > 0 : \chi(t; \mathcal{E}_1) = 0\}$. By construction, $t_0 \leq t_1$, with $t_0 = t_1$ so long as $\chi(t; \mathcal{E}_1)$ crosses zero the first time that it touches zero. Up until time t_0, $\chi(t) > 0$ and all N-agents must go out in equilibrium; so, any equilibrium trajectory must coincide with $\mathcal{E}_1$ up to time t_0. But what if $t_0 < t_1$? The policy-maker is indifferent at t_0 between all going out and all staying home. In our equilibrium construction, the policy-maker breaks such ties in favor of having N-agents continue to all go out. However, if switching to staying home induces a downward kink in $\chi(t)$ at t_0 (see the discussion of kinks below), then there also are equilibrium trajectories in which N-agents' behavior switches at time t_0 from All Go Out to All Stay Home. However, this requires the model parameters to be "just right" so that $\chi'(t_0; \mathcal{E}_1) = 0$ (and $\chi''(t_0; \mathcal{E}_1) > 0$). As can be easily shown (details omitted), this only occurs for parameters in a zero-measure subset of the parameter space.

Transition from All Go Out ... If N-agents were to continue to all go out after time t_1, then $\chi(t)$ would become negative and the policy-maker would find it optimal to impose a stay-at-home order immediately after t_1, a contradiction. Thus, N-agents cannot all continue going out. What exactly happens next depends on properties of the function $\chi(t)$. Recall that $\chi(t)$ is the extra time-t payoff that N-agents get if they all go out versus all staying home. $\chi(t)$ does not depend on N-agents' *actual* activity choices at time t, but is rather a statement about payoffs in the time-t game. That said, N-agent activity affects how $\chi(t)$ changes over time by influencing the overall course of the epidemic. Expanding equation

(24), we have

$$\chi(t) = (1-\alpha)(b_1 + b_2(a_I I(t) + R_I(t)) - q(t) a_I \beta_I I(t) H) + (1-\alpha^2)(b_2 N(t) - q(t)\beta_C C(t) H) \tag{30}$$

Switching N-agents at time t_1 to all stay home decreases the encounter rate between susceptible and infected agents and hence decreases the flow of new infections. So, $S'(t)$ jumps upward at time t_1 (as $S'(t)$ becomes less negative) while $C'(t)$ jumps downward by the same amount; see (2,3). This in turn causes a downward kink in $I'(t)$ and $R'_C(t)$ at time t_1; see (5,6). On the other hand, $N'(t)$ and $R'_I(t)$ do not have a jump or kink. Together this implies that $q'(t) = \frac{S'(t)N(t) - S(t)N'(t)}{N(t)^2}$ jumps upwards at time t_1.

Let $\Delta S'(t) > 0$, $\Delta C'(t) = -\Delta S'(t) < 0$, $\Delta q'(t) > 0$, and $\Delta\chi'(t)$ be the amount that the derivatives $S'(t)$, $C'(t)$, $q'(t)$, and $\chi'(t)$ each would jump if N-agents were to switch from all going out to all staying home at time t. We have

$$\Delta\chi'(t) = -\Delta q'(t)\left((1-\alpha)a_I\beta_I I(t)H + (1-\alpha^2)\beta_C C(t)H\right) - \Delta C'(t)(1-\alpha^2)q(t)\beta_C H. \tag{31}$$

Since $\Delta q'(t) > 0 > \Delta C'(t)$, $\Delta\chi'(t)$ could potentially be positive or negative.

Let $\chi'(t; p^{out})$ be the slope of $\chi(t)$ to the right of t if N-agents all go out with probability $p^{out} \in [0,1]$ and all stay home with probability $1 - p^{out}$:

$$\chi'(t; p^{out}) = \chi'(t-) + p^{out}\Delta\chi'(t). \tag{32}$$

Because $\chi(t; \mathcal{E}_1)$ becomes negative immediately after t_1, we know that $\chi'(t_1; 1) \leq 0$. What happens immediately after time t_1 depends on whether $\chi'(t_1; 0)$ is positive or negative.

... into All Stay Home. Suppose first that $\chi'(t_1; 0) < 0$. Because $\chi'(t_1; 1) < 0$ as well, $\chi'(t_1; p^{out}) < 0$ for all $p^{out} > 0$. Thus, $\chi(t)$ must fall below zero and the epidemic must transition immediately to an All Stay Home phase with $a_N(t) = \alpha$.

... into Collective Mixing. Suppose next that $\chi'(t_1; 0) > 0$.[12] N-agents cannot switch to All Stay Home because then $\chi(t)$ would rise above zero and N-agents would all have to go out, a contradiction. So, the epidemic must immediately transition to a Collective Mixing phase. Since $\chi'(t_1; p^{out})$ is linearly increasing in p^{out}, there is a unique $p^{out}(t_1)$ so that $\chi'(t_1; p^{out}(t)) = 0$. Thus, the equilibrium probability that N-agents all stay out at t_1 is uniquely determined.

[12] Details for the case when $\chi'(t_1; 0) = 0$ are similar, except that the Collective Mixing phase may potentially have zero length, and omitted to save space.

Transition from Collective Mixing ... Once in the Collective Mixing phase, $\chi(t)$ must remain equal to zero in order for N-agents to remain collectively indifferent between all going out and all staying home. Moreover, we must have $\chi'(t;1) \leq 0 \leq \chi'(t;0)$. Why? Suppose first that $\chi'(t;1)$ and $\chi'(t;0)$ were both positive (or non-negative and not both zero) during a Collective Mixing phase. Then $\chi'(t;p^{out}) > 0$ for all $p^{out} \in (0,1)$, meaning that $\chi(t)$ must become positive and N-agents strictly prefer to all go out immediately after time t, a contradiction. Similarly, if $\chi'(t;1)$ and $\chi'(t;0)$ were both negative, $\chi(t)$ would become negative and N-agents would strictly prefer all staying home, another contradiction. Finally, so long as these inequalities are satisfied with at least one of $\chi'(t;1), \chi'(t;0)$ being non-zero,[13] there is a unique mixing probability $p^{out}(t)$ such that $\chi'(t;p^{out}(t)) = 0$; this pins down N-agents' equilibrium behavior throughout the Collective Mixing phase.

... into All Stay Home. The Collective Mixing phase must continue until one of the inequality conditions (a) $\chi'(t;1) \leq 0$ or (b) $\chi'(t;0) \geq 0$ holds with equality. Let $\tilde{t}$ denote the first time at which either $\chi'(t;1) = 0$ or $\chi'(t;0) = 0$. Suppose first that $\chi'(\tilde{t};0) = 0$ and $\chi'(\tilde{t};1) < 0$. N-agents cannot all go out with positive probability, since then $\chi(t)$ would fall and N-agents would strictly prefer staying home, a contradiction. So, N-agents must all stay home at time $\tilde{t}$, i.e., $p^{out}(\tilde{t}) = 0$. What happens next depends on whether this causes $\chi(t)$ to increase or decrease after time $\tilde{t}$. If $\chi(t)$ would fall below zero after $\tilde{t}$ as N-agents all stay home, then N-agents strictly prefer all staying home and the epidemic enters an All Stay Home phase. On the other hand, if $\chi(t)$ would rise above zero after $\tilde{t}$ as N-agents all stay home, N-agents would then strictly prefer all going out after $\tilde{t}$, a contradiction. The period of collective mixing must then continue beyond time $\tilde{t}$, until the next time (call it $\tilde{t}_2$) at which either $\chi'(\tilde{t}_2;0) = 0$ and $\chi'(\tilde{t}_2;1) = 0$, and so on until the epidemic eventually exits the Collective Mixing phase.

... into All Go Out. Suppose next that $\chi'(\tilde{t};1) = 0$ and $\chi'(\tilde{t};0) > 0$. N-agents cannot stay home with positive probability, since that would create a contradiction whereby they all strictly prefer going out. So, it must be that $p^{out}(\tilde{t}) = 1$, inducing $\chi(t)$ to have zero slope to the right of $\tilde{t}$. If $\chi(t)$ would rise above zero after $\tilde{t}$ as a result of N-agents all going out, then N-agents strictly prefer all going out and the epidemic enters an All Go Out phase. On the other hand, if $\chi(t)$ would fall below zero after $\tilde{t}$, N-agents cannot continue all going out, since this would cause them to strictly prefer all staying home immediately after $\tilde{t}$, a

[13] If $\chi'(t;1) = \chi'(t;0) = 0$ for some t, the equilibrium mixing probability $p^{out}(t)$ is not uniquely determined. Moreover, there may be multiple continuation trajectories that can arise in equilibrium. For instance, the Collective Mixing phase might continue past time t along one equilibrium continuation trajectory, while the epidemic transitions immediately to an All Go Out or All Stay Home phase along another.

contradiction. In that case, the period of collective mixing must continue beyond time $\tilde{t}$, until the next time $\tilde{t}_2$ at which either $\chi'(\tilde{t}_2;0) = 0$ and $\chi'(\tilde{t}_2;1) = 0$, and so on, until the Collective Mixing phase eventually ends.

Transition from All Stay Home. Let t_2 be the first time after a Stay At Home phase begins at which $\chi(t)$ would become positive if N-agents continued to all stay at home. N-agents cannot all continue staying home after t_2 since, if they did, $\chi(t)$ would become positive and they would prefer to all go out. This leaves two possibilities: N-agents *either* switch to All Go Out *or* switch to Collective Mixing. Which of these occurs depends on what sort of kink switching to all going out causes in $\chi(t)$ at t_2, much as in our previous discussion of the transition from All Go Out: If there is an upward kink or sufficiently small downward kink in $\chi(t)$ at t_2, then the epidemic must transition to All Go Out. But if there is a sufficiently large downward kink that switching to all going out would cause $\chi(t)$ to fall, then the epidemic must transition to Collective Mixing.

End of the epidemic: All Go Out. Late in the epidemic when infection is rare, $\chi(t) \approx (1-\alpha)(b_1 + b_2 R_I(t)) + (1-\alpha^2)b_2 N(t) > 0$; so, the epidemic always ends in All Go Out.

4 Symptomatic Transmission

This section considers the special case in which the pathogen is transmitted only while infected hosts are symptomatic, i.e., $\beta_I > 0$ and $\beta_C = 0$. Proposition 3 highlights distinctive features of the equilibrium epidemic trajectory in this case.

Proposition 3. *Suppose that $\beta_I > 0$ and $\beta_C = 0$. Along the equilibrium epidemic trajectory: (i) the time-t game always exhibits strategic complements; (ii) N-agents are never subjected to a stay-at-home order; and (iii) there is no interval of time with collective mixing.*

Proof. (i-ii) When $\beta_C = 0$, equations (22-24) simplify to

$$\begin{aligned}
\Delta\pi_1(t) &= Z(t) + (1-\alpha)b_2 N(t) \\
\Delta\pi_\alpha(t) &= Z(t) + \alpha(1-\alpha)b_2 N(t) \\
\chi(t) &= Z(t) + (1-\alpha^2)b_2 N(t)
\end{aligned}$$

where $Z(t)$ is defined in (25). Since $b_2 N(t) > 0$ and $1-\alpha^2 > 1-\alpha > \alpha(1-\alpha)$, it must be that $\chi(t) > \Delta\pi_1(t) > \Delta\pi_\alpha(t)$ at all times. In particular: every time-t game has strategic complements and the policy-maker only finds it politically optimal to impose a stay-at-home order when N-agents already have a dominant strategy to stay home.

(iii) When $\beta_C = 0$, equation (31) simplifies to

$$\Delta\chi'(t) = -\Delta q'(t)(1-\alpha)a_I\beta_I I(t)H. \tag{33}$$

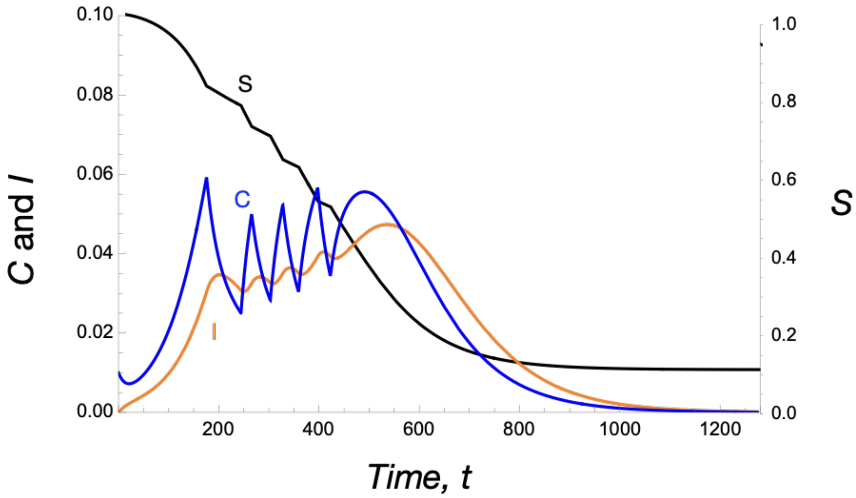
Because $\Delta q'(t) > 0$ (see the discussion after equation (30)), we have $\Delta\chi'(t) < 0$ and hence $\chi'(t;0) < \chi'(t;1)$. But that is inconsistent with the inequalities $\chi'(t;1) \leq 0 \leq \chi'(t;0)$ that must hold throughout any interval of time with collective mixing. $\square$

Figure 4 illustrates a numerical example with the same model parameters as Figure 3, except that the asymptomatic-transmission rate β_C is reduced from 0.0143 to zero. The equilibrium epidemic trajectories is broadly similar as in Figure 3, which is unsurprising since the parameters have only changed a little, but with a noticeable qualitative difference. In Figure 3, there is a period during which the time-t game remains *exactly* on the boundary between the Prisoners' Dilemma game PD-Out and the No Dilemma game ND-Home, and the prevalence of infection increases smoothly over time. By contrast, here in Figure 4, there is a period during which the prevalence of infection rises and falls rapidly several times, generating "yo-yo dynamics" of infection. During this period, N-agents have a dominant strategy to stay home and the policy-maker alternates several times between letting them stay home and forcing them to go out, resulting in a jagged infection trajectory.

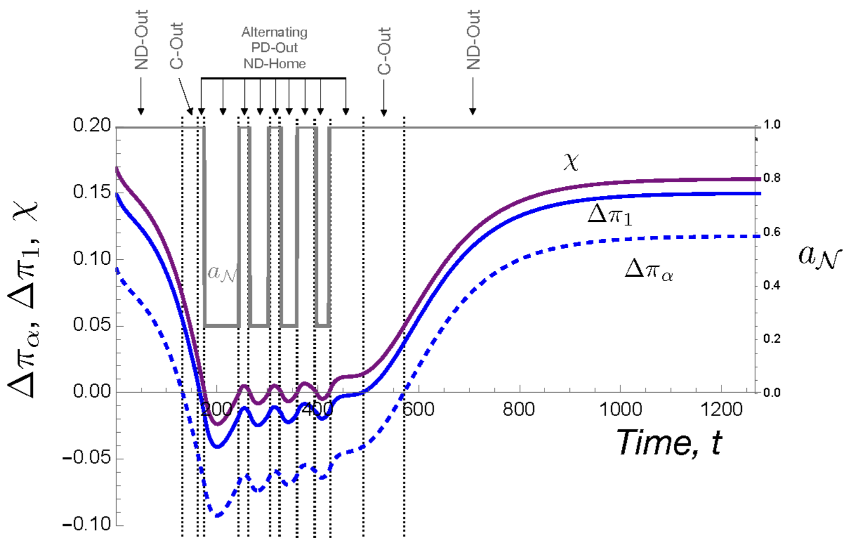
4.1 Equilibrium comparative statics: progression to sickness.

Here we consider how varying the progression-to-sickness parameter σ affects the equilibrium epidemic trajectory in the special case when all transmission is symptomatic. Increasing σ has three distinct effects. *First,* infected agents spend less time in the asymptomatic state C and more time in the symptomatic state I. Since transmission is symptomatic only, increasing σ increases disease transmission holding behavior fixed. *Second,* because a larger share of infected agents develop symptoms, not-yet-sick agents assess that they and other N-agents are more likely to remain susceptible, changing their incentives. *Finally,* because infected agents are more likely to get sick, the harm of infection H is increasing in σ; see equation (21).

Figure 5(a) illustrates how the indirect "incentive effects" of higher σ on agent behavior can be bigger than the direct "epidemiological effects" of higher σ on increased pathogen transmissibility. In particular, consider the mass S^∞ of agents who escape infection altogether. If agent activity were fixed, S^∞ would be unambiguously decreasing in σ. In this

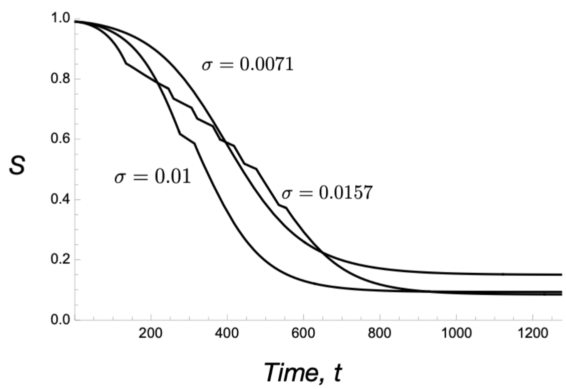


(a) Infection dynamics

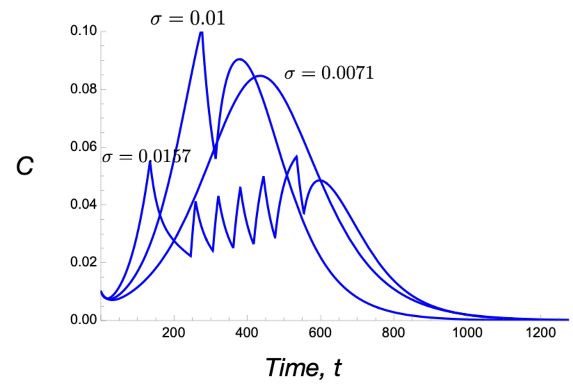


(b) Activity/game dynamics

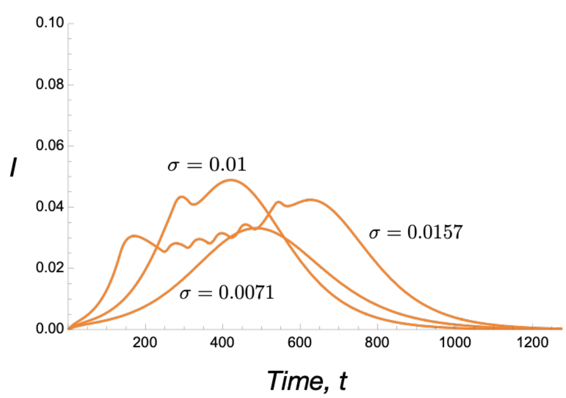
Figure 4: Illustration of the equilibrium epidemic trajectory in an example with symptomatic-only transmission, featuring a period of "yo-yo dynamics" during the middle of the epidemic. Model parameters are the same as in Figure 3 except that β_C is reduced from 0.0143 to 0.



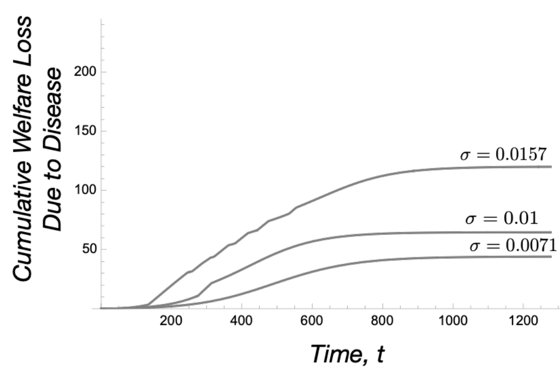
(a) Mass of susceptibles



(b) Mass of asymtomatically infected



(c) Mass of symptomatically infected



(d) Cumulative welfare loss

Figure 5: Illustration of the equilibrium epidemic trajectory in an example with symptomatic-only transmission and varying sickness-progression rate $\sigma = 0.0071, 0.01, 0.0157$. Other model parameters are $C(0) = .01$, $b_1 = 0.1$, $b_2 = 0.1$, $\alpha = 0.25$, $d = 2.8$, $\beta_I = 0.517$, $\beta_C = 0$, and $\gamma = 0.0171$.

example, however, S^∞ is higher when $\sigma = 0.01$ than when $\sigma = 0.0157$ because of incentive effects. Moreover, as shown in panels (b-c), the qualitative features of the equilibrium epidemic trajectory are quite different depending on the level of σ. In particular, there is a conventional single-humped trajectory in the case with $\sigma = 0.0071$, a two-waved epidemic in the case with $\sigma = 0.01$, and a lengthy "yo-yo period" in the middle of epidemic when $\sigma = 0.0157$, during which time N-agents repeatedly switch back and forth between all going out and all staying home.

5 Asymptomatic Transmission

This section considers the special case in which the pathogen is transmitted only while infected hosts are asymptomatic, i.e., $\beta_C > 0$ and $\beta_I = 0$. Proposition 4 highlights distinctive features of the equilibrium epidemic trajectory in this case.

Proposition 4. *Suppose that $\beta_C > 0$ and $\beta_I = 0$. Along the equilibrium epidemic trajectory: (i) the time-t game has strategic complements sufficiently early and late during the epidemic; (ii) N-agents are left free to choose whenever the time-t game has strategic complements; and (iii) N-agents are never subjected to a go-out order.*

Proof. (i) $C(t) \approx 0$ early and late during the epidemic; so, $q_N^*(t) \approx \infty$ by equation (20), implying that $\Delta\pi_1(t) > \Delta\pi_\alpha(t)$ by Lemma 1.

(ii) Given strategic complements, $q(t) < q_N^*(t)$ by Lemma 1 and the term $b_2 N(t) - q(t)\beta_C C(t) H$ is positive. Moreover, because $\beta_I = 0$, the term $Z(t)$ in equations (22-24) simplifies to

$$Z(t) = (1-\alpha)(b_1 + b_2(a_I I(t) + R_I(t))),$$

which is positive and does not depend on $q(t)$. By equations (22-24), we conclude that $\chi(t) > \Delta\pi_1(t) > \Delta\pi_\alpha(t) > Z(t) > 0$. In particular, the time-t game must be "No Dilemma: Out," in which the unique Nash equilibrium is for all N-agents to go out and the policy-maker finds it politically optimal not to intervene.

(iii) If $\Delta\pi_1 \geq 0$ and $\Delta\pi_\alpha > 0$, then the time-t game has a unique NE in which all N-agents go out and there is no need to impose a go-out order. So, suppose that $\Delta\pi_1 < 0$ and/or $\Delta\pi_\alpha \leq 0$. Because $Z(t) > 0$, each of these conditions is only possible when $q(t) > q_N^*(t)$, so that the term $b_2 N(t) - q(t)\beta_C C(t) H$ is negative. But then $\chi(t) < \Delta\pi_1(t) < \Delta\pi_\alpha(t)$ by Lemma 1, implying that $\chi(t) < 0$ and hence that a go-out order is not politically optimal. □

Figure 6 illustrates the equilibrium epidemic trajectory in a numerical example with asymptomatic transmission. Along the epidemic trajectory, N-agents transition directly from the initial All Go Out phase to a Collective Mixing phase and then to a final All Go Out phase.

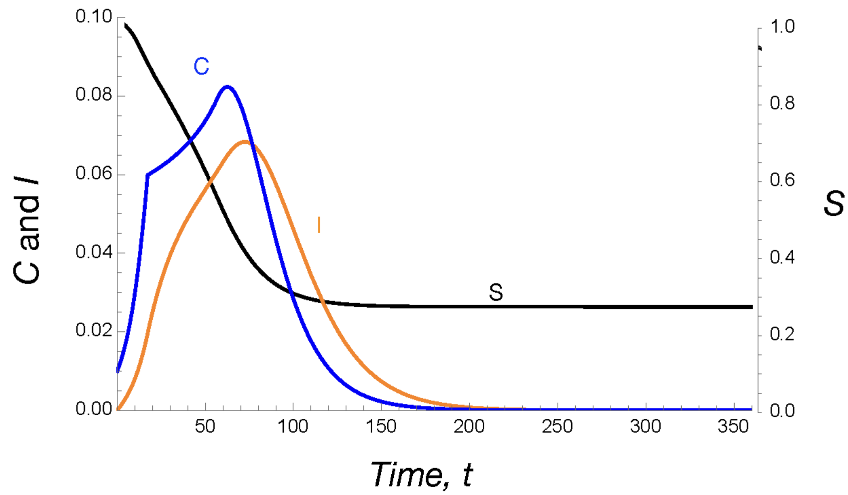
5.1 Equilibrium comparative statics: disease severity.

Figure 7 illustrates the equilibrium impact of increasing the disease-severity parameter d. As disease symptoms grow more severe, agents have more incentive to avoid becoming infected. This results in a lower and delayed peak in infections (panels (b-c)) as well as fewer cumulative infections over the course of the entire epidemic (panel (a)). What about the disease's overall welfare impact? Because there are fewer infections when d is higher, the health-associated welfare loss due to the disease is concave in d. However, as panel (d) shows, the overall welfare loss approximately doubles when d doubles from 1 to 2. What this means is that, as people's equilibrium behavior changes due to the disease being more severe, the health benefits associated with that behavioral change of having fewer infections are approximately "canceled out" by the extra social-economic cost associated with these more intensive efforts to avoid infection.

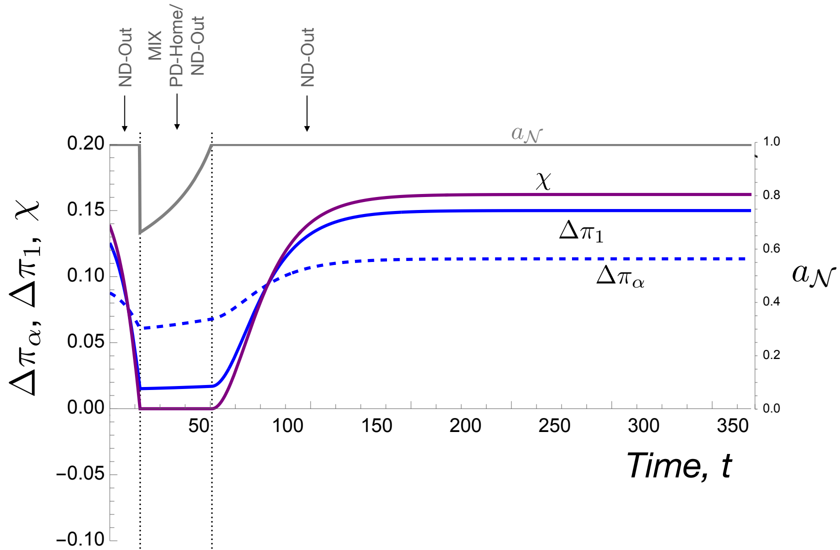
To explore this intriguing observation, we considered a wider range of disease severities. See Figure 8, which plots disease severity on the x-axis against cumulative welfare loss over the entire epidemic on the y-axis, holding all other parameters the same. We find that the approximate linearity of the cumulative welfare loss extends over a substantially wider range of disease severities in this numerical example.

Cumulative welfare loss being (approximately) linear in disease severity has rather profound implications.[14] When $d \approx 0$, there is zero social distancing in equilibrium and the epidemic runs unchecked through the host population. Equilibrium cumulative welfare loss being approximately linear in disease severity d means that the social-economic harm that people endure throughout the epidemic in equilibrium is so great that people's overall suffering is approximately the same as if everyone remained fully active throughout the entire epidemic—as if nothing were done at all to slow disease spread. Intuitively, the reason why this can happen is that people's efforts to avoid a more serious disease slow down transmission, which in turn drags out the epidemic and increases the window of time in which people need to make social-economic sacrifices to protect themselves. Thus, even though the social-economic cost of protecting oneself at any given time remains the same, people incur these costs for longer in equilibrium when the disease is more serious. See Atkeson (2022) for a related quantitative analysis in the context of COVID-19.

[14] In future work, it would be worthwhile to conduct further numerical explorations of this phenomenon under parametric conditions calibrated to SARS-CoV-2 or other diseases of specific interest.

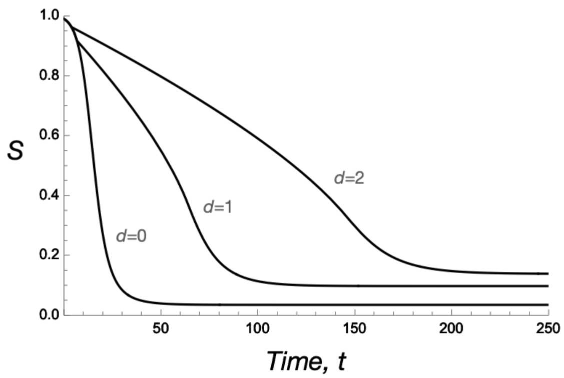


(a) Infection dynamics

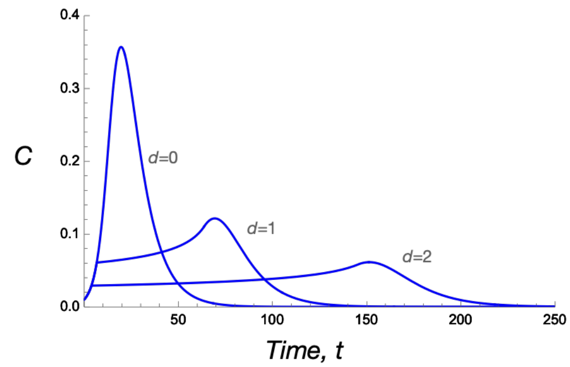


(b) Activity/game dynamics

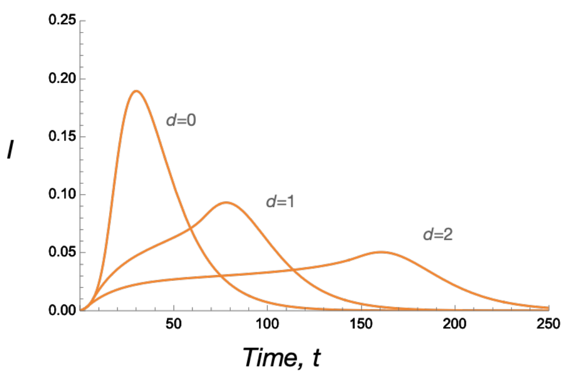
Figure 6: Illustration of the equilibrium epidemic trajectory in an example with asymptomatic-only transmission. Model parameters here are $C(0) = 0.01$, $\alpha = 0.25$, $\beta_C = 0.23$, $\beta_I = 0$, $\sigma = 0.055$, $\gamma = 0.06$, $b_1 = 0.1$, $b_2 = 0.1$, $d = 1.8$.



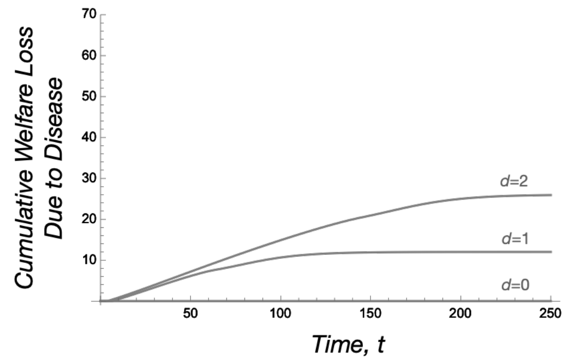
(a) Mass of susceptibles



(b) Mass of asymtomatically infected



(c) Mass of symptomatically infected



(d) Cumulative welfare loss

Figure 7: Illustration of the equilibrium epidemic trajectory in an example with asymptomatic-only transmission and varying disease severity $d = 0, 1, 2$. Other model parameters are $C(0) = 0.01$, $\alpha = 0.25$, $\beta_C = 0.4$, $\beta_I = 0$, $\sigma = 0.055$, $\gamma = 0.06$, $b_1 = 0.1$, $b_2 = 0.1$.

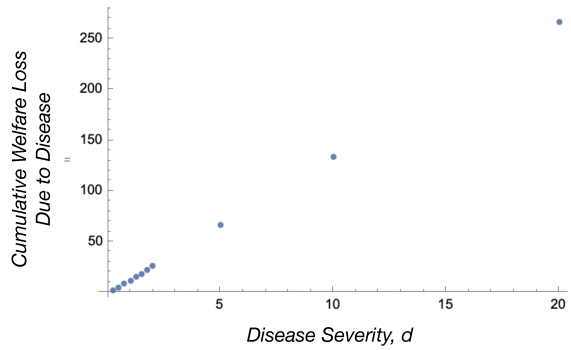


Figure 8: Cumulative welfare loss as a function of disease severity d, holding all other parameters fixed as in Figure 7.

6 Concluding remarks

This paper has introduced and analyzed a tractable model of endogenous collective action during an infectious-disease epidemic, in a setting with myopic-optimizing agents, social-economic complementarities from activity, and public-health policies that are responsive to the will of the majority. We characterize the unique equilibrium trajectory of the epidemic in this context, showing how the epidemic can progress through several qualitatively distinct phases—and how the nature of these epidemic phases depends critically on whether disease transmission occurs during the symptomatic or asymptomatic phases of infection.

For pathogens such as SARS-CoV-2 that spread mostly while people are not yet sick,[15] we find that policy-makers may find it politically optimal to impose constraints that limit healthy people's ability to be fully active ("stay-at-home orders") during some phases of the epidemic, but will never require people to be more active than they would otherwise choose to be ("go-out orders"). This is consistent with what has happened, as the vast majority of government interventions during the Covid pandemic have been to limit activity.

On the other hand, for pathogens such as SARS-CoV-1 that spread mostly while people are sick, our analysis makes the opposite prediction: policy-makers may sometimes find it politically optimal to compel healthy people to be *more* active than they would otherwise choose to be, but will never force people to limit their activity. This also appears to be consistent with actual events during the 2003 SARS-CoV-1 outbreak. Public-health officials in Toronto and other places actively sought to isolate the sick (and quarantine their close

[15] The findings summarized here apply to hypothetical pathogens that transmit only while asymptomatic or only while symptomatic. SARS-CoV-1 had low transmissibility prior to the onset of symptoms (Anderson et al. (2004)) and so came close to our symptomatic-only ideal. For SARS-CoV-2, the majority of transmission occurs during the pre-symptomatic phase (Johansson et al. (2021)), but there is also a substantial amount of transmission during sickness.

contacts), but no efforts were made to compel healthy people in general to limit their activity. In fact, the opposite was true, as the government in Toronto tried (unsuccessfully) to soothe the public's panic and encourage them to reengage in social-economic life (Blendon et al. (2004)).[16]

There are several natural and interesting directions for future work that might build on this paper's analysis. First, agents in our model are ex ante identical, differing only during the epidemic due to changes in their health status. In future work, it would be interesting to explore richer models that account for sources of agent heterogeneity as in Ellison (2024). Such heterogeneity could impact people's public-health policy preferences. For instance, the young and old may differ in their epidemic-management policy preferences because of differing health risks, differing need for social activity, and so on; see Anderson et al. (2012), Brotherhood et al. (2020) and Acemoglu et al. (2021). Similarly, because poorer people have less flexibility to socially distance early during an epidemic (Basu et al. (2021)), their likelihood of remaining susceptible may be much lower later in the epidemic, leading to a divergence between the policy preferences of the rich and the poor within a society.

Second, agents in our model have only one way to protect themselves from becoming infected, by "staying home." However, people in practice may also have access to *treatments* to reduce disease severity or duration and *vaccination* to reduce the likelihood of becoming infected. The impact of treatment and vaccination on equilibrium behavior during an epidemic has been previously studied, see e.g., Chen (2006) and Chen and Toxvaerd (2014) on vaccination, Rowthorn and Toxvaerd (2012) on treatment together with non-vaccine prevention, and Avery et al. (2023) on vaccination together with social distancing. However, being able to access a treatment and/or vaccine also impacts an agent's support for public-health interventions that impact social-economic life, since they are at less personal risk of being infected. If public-health measures are determined politically, as in our model, inequitable access to treatments and vaccines could drive subsequent inequitable policies.

Third, each agent i's social-economic benefit in our model is linear in i's activity level and in the population-average activity level. However, in many social situations, social-economic benefits are more naturally concave in overall activity. For instance, employees benefit from being able to chat with colleagues as soon as even a few people are returning to the office. Doubling the number of others present might double the number of chance encounters that agent i has, approximately doubling i's likelihood of being exposed,[17] but is unlikely to

[16]Most people infected with SARS-CoV-1 were hospitalized before they could transmit the pathogen widely. This allowed SARS-CoV-1 to be effectively contained, so much so that fewer than 10,000 people became sick worldwide. Despite the low likelihood of being exposed, people in many affected countries dramatically curtailed their activity, resulting in billions of dollars in economic losses (Popescu (2022)).

[17]Chen (2012) has examined the impact of congestion in transmission, where each agent's rate of trans-

double their social-economic benefits. Such *social congestion* could have substantial impacts on healthy agents' political demands. In particular, so long as there is enough N-agent activity to meet most of N-agents' social-economic needs, there could be much stronger political demand to isolate the sick, especially late during the epidemic when isolation would only impact a small number of sick people.

Finally, our analysis assumes that there are no diagnostics available to determine whether one has asymptomatic infection or acquired immunity to re-infection. Such tests, which are now possible due to recent advances in molecular biology, change agents' behavior by changing their beliefs about their own likelihood of being susceptible. Previous work has highlighted how having better information about one's health status need not necessarily improve epidemic outcomes, since people who learn that they are infected have a self-interested incentive to increase their own activity; see Acemoglu et al. (2023) and Deb et al. (2022).[18] Future work building on our analysis could also investigate the political-economic implications of diagnostics.

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missive encounters is concave in others' overall activity.

[18] Other notable analyses featuring testing and uncertainty about health status include Brotherhood et al. (2020), Phelan and Toda (2022), Troger (2024), and Gans (2022).

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