Steady-State Social Distancing and Vaccination

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Abstract

This paper analyzes an economic-epidemiological model of infectious disease where it is possible to become infected more than once and individual agents make endogenous choices of social distancing and vaccine adoption. Protective actions adopted by any one person reduce future risks to other people. The positive externalities associated with these behaviors provide motivation for vaccine and social-distancing subsidies, but subsidizing one protective action reduces incentives for other protective actions. A vaccine subsidy increases vaccine adoption and reduces steady-state infection prevalence; a social-distancing subsidy can either increase or reduce steady-state infection prevalence.

1 Introduction

The COVID-19 pandemic sparked renewed interest in the economics of infectious disease. Given the prevailing initial view that prior infection provided lasting immunity against future infection, most COVID-related papers have focused on the "Susceptible-Infected-Recovered" (SIR) model. During the course of the pandemic, however, it has become apparent that it is possible to contract COVID more than once, both because of waning immunity and the development of new pathogen variants (Giannitsarou, Kissler, and Toxvaerd (2021)). In a recent editorial, Columbia Professor Jeffrey Shaman discussed the possible transition of COVID-19 from pandemic to endemic phase.[1]

Motivated by this background, we consider a "Susceptible-Infected-Recovered-Susceptible" (SIRS) epidemiological model in which recovered agents eventually become susceptible to re-infection. As suggested by Peltzman (1975), a central challenge for disease control is that incentivizing one protective action decreases incentives for other protective actions that reduce the same source of risk.[2] We therefore augment the SIRS epidemiological model with a game-theoretic model in which each individual has the opportunity to mitigate risk through social distancing or vaccination. Our theoretical framework yields a unique steady-state Nash equilibrium, which facilitates analysis of policy interventions such as vaccine subsidies or mandates and highlights the simple economics underlying the dynamics of infectious disease. In particular, individuals make cost-benefit tradeoffs for risk mitigation, the equilibrium level of vaccine adoption is determined by familiar supply and demand dynamics, and some comparative static results turn on the elasticity of demand for vaccination.

The paper is most closely related to several previous studies: Chen, Jiang, Rabidoux, and Robinson (2011), Chen (2012), and Toxvaerd (2019) conduct equilibrium analysis in an SIS model with endogenous social distancing; Reluga and Galvani (2011) study equilibrium adoption of vaccination in an SIS model; Chen (2006) and Chen and Cottrell (2009) study incentives for vaccination and abstinence to reduce the risk of contracting HIV; Rowthorn and Toxvaerd (2020) study the optimal timing and tradeoffs between treatment and vaccination in an SIS model.

The paper proceeds as follows. Section 2 describes the model. Section 3 provides steady-state equilibrium analysis with endogenous social distancing and exogenous vaccination, including the case when no vaccine is available. Section 4 expands the equilibrium analysis to allow for both endogenous vaccination and social distancing and also considers the effects of policy applications such as subsidies and vaccine mandates. Section 5 concludes.

[1] "What Will Our Covid Future Be Like? Here Are Two Signs to Look Out For," https://www.nytimes.com/2022/03/04/opinion/endemic-covid-future.html

[2] Similarly, Kremer (1996) observes that a reduction in the transmission rate can result in increased prevalence of infectious disease.

2 The Model

We consider an economic-epidemiological model of an endemic infectious disease, combining a Susceptible-Infected-Recovered-Susceptible (SIRS) model of epidemiological dynamics with an economic model in which agents make individually-optimal decisions regarding personal social distancing and whether to get vaccinated.

Epidemiological Framework. An endemic infectious disease circulates among a fixed population of agents having unit mass. At each point in time $t \in \mathbb{R}$, each agent is either susceptible (S), infected (I), recovered-and-immune (R), or vaccinated (V) and knows their health status. Let $S(t)$, $I(t)$, $R(t)$, and $V(t)$, respectively, be the mass of susceptible, infected, recovered-and-immune, and vaccinated agents at time t. We refer to $I(t)$ as the "infection prevalence" at time t. Each susceptible agent i becomes infected once exposed to an infected agent, which occurs at rate $\beta I(t)[1 - x_i(t)]$, where $\beta > 0$ is the transmission rate and $x_i(t) \in [0, 1]$ is agent i's chosen level of "social distancing." Infected agents recover at rate $\gamma > 0$ and then enjoy immunity from infection for known length of time $t_I \geq 0$, after which they return to the susceptible state. Similarly, a newly-vaccinated agent is immune for a known length of time $t_V > 0$, at which point they may choose to renew their vaccination.

For analytical simplicity, we focus on settings where the mass of vaccinated agents is constant over time, i.e., $V(t) = V$ for all t. In Section 3, V is treated as an exogenous parameter. In Section 4, V emerges endogenously as the mass of agents who choose to become and remain vaccinated in steady-state Nash equilibrium.

At each point in time, there is a flow of agents into the infected state as susceptible people are exposed and become infected, a flow into the temporarily-immune state as infected people recover, and a flow into the susceptible state as recovered people lose their temporary immunity. Epidemiological dynamics for the system as a whole are governed by two differential equations:

$$I'(t) = \beta I(t)[1 - x(t)]S(t) - \gamma I(t) \tag{1}$$

$$R'(t) = \gamma I(t) - \gamma I(t - t_I) \tag{2}$$

plus the adding-up condition that $S(t) + I(t) + R(t) = 1 - V$, where $x(t)$ is average social distancing of susceptible agents at time t.

If $\beta(1 - V) \leq \gamma$, then each infected person exposes less than one unvaccinated person on average even without social distancing and $I(t)$ necessarily falls toward zero over time. We therefore focus on the case when $\beta > \gamma$ and $V < 1 - \frac{\gamma}{\beta}$, creating the potential for persistent disease transmission.

Economic Model. Each agent seeks to minimize the expected present value of costs incurred during the rest of their lifetime, including the costs of sickness, social distancing, and vaccination. All agents discount future payoffs at discount rate $r > 0$. Infected agents incur flow cost $d > 0$ due to the disease. Susceptible agents who choose social distancing x incur flow cost $c(x)$ from foregone activity; we use principles of time-use optimization to restrict $c(x)$. First, $c(0) = c'(0) = 0$ since, absent any fear of infection, people engage in ordinary activity ($x = 0$) and are indifferent at the margin whether to increase or decrease activity. Second, $c'(x) > 0$ and $c''(x) > 0$ since people can prioritize activities according to benefit per unit time and hence implement social distancing by forgoing the least valuable activities first.[3]

Let $C_h(t)$ be the expected lifetime cost for unvaccinated agents upon entering health status $h \in \{S, I, R\}$.[4] Upon recovery from infection, an agent enjoys immunity and incurs no costs for length of time $t_I \geq 0$ before returning to the susceptible state. Thus,

$$C_R(t) = e^{-rt_I} C_S(t + t_I). \tag{3}$$

While infected, agents incur flow cost d from the disease and recover at rate γ, at which point their subsequent expected lifetime cost changes from $C_I(t)$ to $C_R(t)$. Thus,

$$C_I'(t) = -d + \gamma(C_I(t) - C_R(t)) + rC_I(t). \tag{4}$$

Voluntary social distancing. A susceptible agent i who chooses social distancing $x_i(t)$ incurs flow cost $c(x_i(t))$ and transitions to the infected state at rate $\beta I(t)(1-x_i(t))$. Given $C_S(t)$ and $C_I(t)$, such an agent chooses $x_i(t)$ to minimize $c(x_i(t)) + \beta I(t)(1 - x_i(t))(C_I(t) - C_S(t))$, trading off the current cost of social distancing versus the benefit of avoiding infection. Overall, a susceptible agent's dynamic-programming problem is given by (3-4) and

$$C_S'(t) = -\min_{x \in [0,1]} \{c(x) + \beta I(t)(1 - x)(C_I(t) - C_S(t))\} + rC_S(t). \tag{5}$$

Voluntary vaccination. In Section 4 with endogenous vaccination, each agent i is modeled as having a random cost of vaccination c_{iV}, drawn i.i.d. across agents from a distribution with support $[0, \overline{c}_V]$,[5] continuous p.d.f. $f(\cdot)$, and c.d.f. $F(\cdot)$. Vaccination provides full protection from infection for period of time t_V, after which agent i becomes susceptible and may be vaccinated again at additional cost c_{iV}.

[3] Toxvaerd (2019) analyzes a related model and produces results similar to Proposition 1 and Corollary 2 with linear rather than strictly-convex costs of social distancing.

[4] Agents' expected lifetime costs at time t depend on the subsequent epidemic trajectory. Our analysis is simplified by the fact that we focus on steady states where infection prevalence and susceptible-agent social distancing are constant.

[5] Our analysis with endogenous vaccination is easily extended to a richer setting in which some agents have negative vaccination cost. However, if $\Pr(c_{iV} \leq 0) \geq 1 - \frac{\gamma}{\beta}$, then so many agents automatically get vaccinated that the only equilibrium steady state is the trivial one with zero infection prevalence.

3 Steady-State Equilibrium with Exogenous Vaccination

In this section, we characterize the set of steady-state equilibria, taking the mass V of vaccinated agents as exogenous and assuming that V is small enough to allow for persistent disease transmission, i.e., $0 \leq V < 1 - \frac{\gamma}{\beta}$. We have three main findings. First, a steady-state equilibrium exists and is unique. Second, any policy that reduces the cost of social distancing or that increases the fraction of the population that is vaccinated will result in strictly fewer infections in the new steady-state equilibrium. Finally, we consider the effect of inducing susceptible agents to increase their social distancing on overall social welfare. Slightly increasing social distancing unambiguously increases social welfare in the special case without temporary immunity ($t_I = 0$), but not in general when $t_I > 0$.

In a steady-state equilibrium, the percentages of people in each state $\{S, I, R, V\}$ and the individually-optimal social distancing level for susceptible agents are constant over time.

Definition 1. *A steady-state equilibrium with exogenous vaccination level* $0 \leq V < 1 - \frac{\gamma}{\beta}$ *is characterized by infection prevalence* I^**, temporary-immunity level* $R^* = \gamma t_I I^*$*, and susceptible-agent social distancing* x^* *such that:*

1. (I^*, x^*, V) *satisfy the steady-state condition* $I'(t) = 0$*, which requires*

$$\beta(1 - x^*)(1 - I^* - \gamma t_I I^* - V) = \gamma. \tag{6}$$

2. x^* *is individually optimal for susceptible agents in this steady state.*

Let $x_{SS}(I)$ be the social-distancing level that supports steady-state infection prevalence I. By the steady-state condition, $x_{SS}(I) = 1 - \frac{\gamma}{\beta(1-I-t_I\gamma I-V)}$. Let $x^*(I)$ be the individually-optimal social-distancing level for susceptible agents in the steady state with infection prevalence I. A steady-state equilibrium exists with infection prevalence $I > 0$ if and only if $x_{SS}(I) = x^*(I)$.

Let $C_S^*(I)$ and $C_I^*(I)$, respectively, denote the steady-state values of $C_S(t)$ and $C_I(t)$ for an agent who chooses the individually-optimal social-distancing level $x^*(I)$ whenever susceptible in the steady state with infection prevalence I. By (3-5),

$$C_I^*(I) = \frac{d + \gamma e^{-rt_I} C_S^*(I)}{\gamma + r}, \tag{7}$$

$$C_S^*(I) = \frac{c(x^*(I)) + \beta I(1 - x^*(I)) C_I^*(I)}{\beta I(1 - x^*(I)) + r}, \tag{8}$$

with $x^*(I) \equiv \arg\min_{x \in [0,1]} \left\{ \frac{c(x) + \beta I(1-x) C_I^*(I)}{\beta I(1-x) + r} \right\}$. Combining (7) and (8) gives

$$C_S^*(I) = \frac{(\gamma + r)c(x^*(I)) + \beta I(1 - x^*(I))d}{\beta I(1 - x^*(I))(\gamma + r - \gamma e^{-rt_I}) + r(\gamma + r)}. \tag{9}$$

The first-order condition for individually-optimal social distancing $x^*(I)$ in a steady state with infection prevalence I is

$$c'(x^*(I)) = \beta I[C_I^*(I) - C_S^*(I)], \tag{10}$$

where $C_I^*(I) - C_S^*(I)$ can be interpreted as the "harm of becoming infected" for a susceptible agent. Replacing $x^*(I)$ with $x = 0$ in (8) gives the bound $C_S^*(I) \leq \frac{\beta I C_I^*(I)}{\beta I + r}$, so $C_S^*(I) < C_I^*(I)$, which implies $x^*(I) > 0$ for each I.

With no social distancing ($x^* = 0$), steady-state infection prevalence would be $1 - V - \frac{\gamma}{\beta} > 0$ and, because $c'(0) = 0$, susceptible agents would not choose $x = 0$, a contradiction. Similarly, complete social distancing $x^* = 1$ would eliminate the disease and induce susceptible agents to choose full activity, another contradiction. Thus, there must be partial social distancing ($0 < x^* < 1$) and the first-order condition (10) must hold with equality in any steady-state equilibrium.

An increase in infection prevalence directly increases the marginal benefit of distancing for susceptible agents. Yet, a susceptible agent who successfully avoids being infected at any given instant ("now") faces a higher risk of being infected later, so an increase in infection prevalence indirectly reduces the marginal benefit of distancing. If this indirect effect were stronger than the direct effect, an increase in infection prevalence would reduce the incentive of susceptible agents and create the potential for multiple equilibria. Proposition 1 rules out this possibility.

Proposition 1. *For each $V < 1 - \frac{\gamma}{\beta}$, there is a unique steady-state equilibrium with exogenous vaccination and associated infection rate $I^*(V)$.*

Proof. For convenience, define

$$C_S(I, x) \equiv \frac{(\gamma + r)c(x) + \beta I(1 - x)d}{\beta I(1 - x)\theta + r(\gamma + r)}, \tag{11}$$

where $\theta = \gamma + r - \gamma e^{-rt_I}$. $C_S(I, x)$ is the expected lifetime cost incurred by a susceptible agent in the steady state with infection prevalence I who chooses social-distancing level x whenever susceptible. First, we show that there is a unique x that minimizes $C_S(I, x)$ for any given I. Using the quotient rule,

$$\frac{\partial C_S(I, x)}{\partial x} = \frac{[(\gamma + r)c'(x) - \beta I d][\beta I(1 - x)\theta + r(\gamma + r)] + \beta I\theta[(\gamma + r)c(x) + \beta I(1 - x)d]}{[\beta I(1 - x)\theta + r(\gamma + r)]^2}. \tag{12}$$

Using (11) to substitute for the last term in the numerator,

$$\frac{\partial C_S(I, x)}{\partial x} = \frac{(\gamma + r)c'(x) - \beta I d + \beta I\theta C_S(I, x)}{\beta I(1 - x)\theta + r(\gamma + r)}. \tag{13}$$

If $\frac{\partial C_S(I,x)}{\partial x} = 0$, the numerator of (13) is zero and so, by the quotient rule, $\frac{\partial^2 C_S(I,x)}{\partial x^2}$ takes the same sign as $(\gamma + r)c''(x)$, which is positive since c is strictly convex. Thus, $\frac{\partial C_S(I,x)}{\partial x} = 0$ implies $\frac{\partial^2 C_S(I,x)}{\partial x^2} > 0$ and hence $C_S(I, x)$ has a unique minimum in x for each I. Moreover, the properties of $c(x)$ and $C_S(I, x)$

imply that this solution $x^*(I)$ is continuous in I.

Next, we show that $x^*(I)$ is increasing in I. The first-order condition $\frac{\partial C_S(I,x)}{\partial x} = 0$ can be rewritten as $c'(x) = \frac{\beta I(d-\theta C_S(I,x))}{\gamma+r}$. From (11),

$$\theta C_S(I,x) = \frac{(\gamma+r)c(x) + \beta I(1-x)d}{\beta I(1-x) + \frac{r}{\theta}(\gamma+r)}. \tag{14}$$

Thus,

$$d - \theta C_S(I,x) = \frac{(\gamma+r)[\frac{rd}{\theta} - c(x)]}{\beta I(1-x) + \frac{r}{\theta}(\gamma+r)}. \tag{15}$$

Multiplying both sides by βI,

$$\beta I\left(d - \theta C_S(I,x)\right) = \frac{(\gamma+r)[\frac{rd}{\theta} - c(x)]}{(1-x) + \frac{r}{\beta\theta}\frac{\gamma+r}{I}}. \tag{16}$$

We know from above that $x^*(I) > 0$, so $d - \theta C_S(I,x) > 0$ for any I and $x = x^*(I)$. Thus, the left-hand side of (16) is positive and the right-hand side of (16) must also be positive for each $[I, x^*(I)]$. The only term on the right-hand side of (16) that varies with I is $\frac{\gamma+r}{I}$. Holding x fixed at $x^*(I)$, a slight increase in I yields an increase in the marginal value of social distancing at $x = x^*(I)$, i.e. the marginal benefit of social distancing is now greater than its marginal cost at $x = x^*(I)$. Since we know from above that $C_S(I,x)$ has a unique minimum in x for each I, this means that $x^*(I)$ is increasing in I.

To complete the proof of Proposition 1, we show that there is a unique I that supports a steady-state equilibrium. Let $g(I)$ denote the net expected flow into the infected state per infected agent in the steady state with infection prevalence I. By (1), $g(I) = \beta(1 - x^*(I))S - \gamma$, where $S = 1 - I - R - V$ and $R = \gamma t_I I$. In any steady-state equilibrium, $g(I) = 0$. Given that $x^*(0) = 0$ and our maintained assumption that $V < 1 - \frac{\gamma}{\beta}$, we have $g(0) > 0$. On the other hand, $g\left(\frac{1-V}{1+\gamma t_I}\right) = -\gamma < 0$. Since $x^*(I)$ is continuous and increasing in I, $g(I)$ is strictly decreasing and continuous in I. Thus, there is a unique $I \in \left(0, \frac{1-V}{1+\gamma t_I}\right)$ that solves $g(I) = 0$. □

Example 1. *Suppose that* $V = 0, \beta = 0.4945, d = 0.3, \gamma = 0.2, t_I = 1, c(x) = 0.03761x^2$, *and* $r = .00519$.[6]

In a steady-state equilibrium, the flow of new infections (solid line in Figure 1a) must equal the flow of recoveries (dashed line in Figure 1a). Figure 1a compares these flows in Example 1 for any given steady-state infection prevalence I, accounting for how susceptible agents' optimal social-distancing intensity $x^*(I)$ varies with I.[7] The unique steady-state equilibrium corresponds to Point A, with stationary infection prevalence 8%.

[6] These parameter values have been chosen to yield steady-state equilibria in Figures 1, 2, 3a, and 3b at infection rates close to round numbers, and so that the per-unit-time discount factor of $e^{-r} = .95$,

[7] When social-distancing costs are quadratic as in Example 1, equations (7-10) imply that, in any steady-state equilibrium, (i) $C_I^*(I)$ and x^* are each linear functions of $C_S^*(I)$ and (ii) $C_S^*(I)$ is the solution to a quadratic equation. See the online Appendix for details.

Figure 1b illustrates an equilibrium epidemic trajectory in Example 1, starting from low infection prevalence 0.01% and eventually converging to the steady-state level of 8%.[8] One complication that arises from temporary immunity of recovered agents is that after the initial increase of infection prevalence to the steady-state level, there are fewer immune agents and more susceptible agents than in the steady state. Consequently, the infection trajectory typically "overshoots" the steady-state level before settling back to the steady-state level. However, such overshooting has been small in all the cases that we have explored, including Example 1 where it is so slight as to be visually undetectable.

Figure 1a suggests two corollaries that follow almost immediately from the proof of Proposition 1.

Corollary 1. *Any exogenous increase in (perfect) vaccination strictly reduces the infection prevalence in the unique steady-state equilibrium.*

Proof. Vaccination reduces the size of the susceptible population but has no effect on $x^*(I)$, the individually-optimal level of social distancing given stationary infection prevalence I. An exogenous increase in vaccine adoption therefore shifts the new-infection curve down, while having no effect on the new-recovery curve. The intersection point in Figure 1 shifts down and to the left, meaning that infection prevalence in the unique steady-state equilibrium must decline. □

Corollary 2. *A reduction in the cost of social distancing from $c(x)$ to $c_1(x)$ where $c_1(0) = c_1'(0) = 0$ and $c_1'(x) < c'(x)$ for each $x > 0$ reduces the steady-state equilibrium infection prevalence.*

Proof. By (15), the first-order condition $c'(x) = \frac{\beta I(d - rC_S(I,x))}{\gamma + r}$ for individually-optimal social distancing can be written as

$$c'(x) = \beta I \frac{(\gamma + r)[\frac{rd}{\theta} - c(x)]}{\beta I(1 - x) + \frac{r}{\theta}(\gamma + r)}. \tag{17}$$

Note that the marginal value of social distancing declines with $c(x)$ for each (x, I). Since the marginal cost of social distancing is lower and the marginal value of social distancing is greater with $c_1(x)$ than with $c(x)$, $x_1^*(I) > x^*(I)$ for each I. This shifts the new-infection curve down and to the left, yielding a reduction in equilibrium steady-state infection prevalence as in Corollary 1. □

3.1 Infection Prevalence and Social Welfare

Let $B(I)$ denote the "burden of the disease," the aggregate lifetime costs of the entire population, in the steady state with infection prevalence I. Expected lifetime costs are $C_I(I)$ for mass I of infected agents; $C_S(I)$ for mass $S = 1 - V - (1 + \gamma t_I)I$ of susceptible agents; $e^{-rT_I}C_S(I)$ for temporarily-immune agents who have $T_I \sim U[0, t_I]$ time remaining until they return to being susceptible again, where $R = \gamma t_I I$ is

[8] We used numerical analysis and a backward-shooting algorithm for value functions (as in Farboodi et al. (2021)) to approximate this equilibrium in a discrete-time version of Example 1 with 100 periods per unit time.

the total mass of temporarily-immune agents; and zero for mass V of vaccinated agents. Overall,

$$B(I) \equiv IC_I(I) + (1 - V - (1 + t_I)I)C_S(I) + \gamma I \int_0^{t_I} e^{-rs} ds C_S(I). \tag{18}$$

There is a positive externality associated with social distancing because preventing oneself from becoming infected also indirectly protects others. One might therefore expect $B(I)$ to be increasing in I in a neighborhood of I^*, the steady-state equilibrium infection prevalence. We show that this is indeed true in the special case in which recovered agents are immediately susceptible to reinfection (Proposition 2) but not in general with temporary immunity (Example 2).

Proposition 2. *Suppose that $t_I = 0$ so that recovered agents are not even temporarily immune. Then $B'(I^*) > 0$.*

Proof. Define $I(x)$ to be the prevalence I that satisfies the steady-state condition $\beta(1-x)(1-(1+\gamma t_I)I) = \gamma$. Note that $I^* = I(x^*)$, where x^* is susceptible agents' social distancing in the unique steady-state equilibrium. We can express the burden of the disease in (18) equivalently as a function of social distancing x:

$$B(x) \equiv \underbrace{I(x)\left(\frac{d + \gamma e^{-rt_I}}{\gamma + r} C_S\left(I(x), x\right)\right)}_{\text{infected agents}} + \underbrace{\left(1 - V - (1 + \gamma t_I)\, I(x)\right) C_S\left(I(x), x\right)}_{\text{susceptible agents}} + \underbrace{\gamma I(x)\left(\int_0^{t_I} e^{-rs} ds\right) C_S\left(I(x), x\right)}_{\text{currently immune agents}}.$$

Since $I'(x) < 0$, we need to show that $B'(x^*) < 0$ in the case when $t_I = 0$. The derivative $B'(x)$ can be usefully decomposed into four terms:

$$B'(x) = (A)\frac{\partial C_S\left(I(x), x\right)}{\partial x} \tag{19}$$

$$+ I'(x)\,(A)\frac{\partial C_S\left(I(x), x\right)}{\partial I} \tag{20}$$

$$+ I'(x)\left(\frac{d}{\gamma + r} + \frac{\gamma}{\gamma + r} e^{-rt_I} C_S\left(I(x), x\right) - C_S\left(I(x), x\right)\right) \tag{21}$$

$$+ I'(x)\left(\gamma C_S\left(I(x), x\right)\left(\int_0^{t_I} e^{-rs} ds - t_I\right)\right), \tag{22}$$

where

$$A \equiv I(x)\frac{\gamma}{\gamma + r} e^{-rt_I} + \left(1 - V - (1 + \gamma t_I)\, I(x)\right) + \gamma I(x)\left(\int_0^{t_I} e^{-rs} ds\right) > 0.$$

The term in (19) is zero at $x = x^*$ because $\frac{\partial C_S(I^*, x^*)}{\partial x} = 0$ due to susceptible agents' individual optimization in the equilibrium steady state. The term in (20) is negative for all x because $I'(x) < 0$ and $\frac{\partial C_S(I(x), x)}{\partial I} > 0$. The term in (21) is negative at $x = x^*$ because $\frac{d}{\gamma + r} + \frac{\gamma}{\gamma + r} e^{-rt_I} C_S\left(I(x), x\right)$ is the expected lifetime cost of an infected agent, which is greater than $C_S\left(I(x), x\right)$. Finally, the term

in (22) is zero if $t_I = 0$ (but positive if $t_I > 0$ because $\int_0^{t_I} e^{-rs} ds < t_I$). We conclude as desired that $B'(x^*) < 0$. □

More generally when $t_I > 0$ so that recovered agents enjoy temporary immunity, the burden of the disease can be lower in steady states with *higher* infection prevalence than in the equilibrium steady state. The reason, intuitively, is that susceptible agents discount the future benefit that they will get due to immunity (after eventually recovering from infection) when deciding how much to socially distance. Consequently, depending on the duration of temporary immunity and other model parameters, it is possible that susceptible agents may choose a level of social distancing *greater* than the level that minimizes the steady-state burden of the disease.

Example 2. *Suppose that* $V = 0, \beta = 0.9, d = 1, \gamma = 0.2, t_I = 1, c(x) = x^2$, *and* $r = 0.1$.

The unique equilibrium steady state in Example 2 has social distancing $x^* = 0.0415$ and infection prevalence $I^* = 0.0366$. The level of social distancing that minimizes the steady-state burden of the disease is $0.0241 < x^*$, resulting in a higher steady-state infection prevalence $0.0368 > I^*$. Details for these computations are provided in the Appendix.

4 Equilibrium with Endogenous Vaccination

This section extends the model to incorporate interactions between social distancing and vaccination decisions. We begin by extending the definition of steady-state equilibrium to require individually-optimal vaccination decisions. In a stationary setting, incentives for a susceptible agent do not change with time; so, each agent either never chooses to be vaccinated or gets vaccinated and revaccinated at the first moment that they become susceptible.

Definition 2. *A steady-state equilibrium with endogenous vaccination is characterized by infection level* I^*, *susceptible-agent social distancing* x^*, *and vaccination level* V^* *such that:*

1. (I^*, x^*, V^*) *satisfy the steady-state condition*

$$\beta(1 - x^*)(1 - I^* - \gamma t_I I^* - V^*) = \gamma. \tag{23}$$

2. x^* *is individually optimal for susceptible agents in this steady state, i.e.,*

$$x^* \in \arg\min_{x \in [0,1]} C_S\left(I^*, x\right). \tag{24}$$

3. *Fraction* V^* *of newly-susceptible agents find it individually-optimal to become vaccinated, i.e.,*

$$F\left(C_S^*(I^*)\right)\left(1 - e^{-rt_V}\right) = V^*. \tag{25}$$

where $C_S^*(I^*) \equiv C_S(I^*, x^*)$

Vaccination allows agents to avoid infection for t_V units of time without social distancing. In a steady state with infection prevalence I, a susceptible agent with vaccination cost c_{iV} benefits from adopting the vaccine if $c_{iV} + e^{-rt_V} C_S^*(I) < C_S^*(I)$, or $c_{iV} < C_S^*(I)\left(1 - e^{-rt_V}\right)$.[9] Given c.d.f. F for vaccination cost, the fraction of newborn agents who find it optimal to vaccinate ("vaccine demand") is $D_V(I) = F\left(C_S^*(I)\right)\left(1 - e^{-rt_V}\right)$, which is continuous and strictly increasing in I.

Proposition 3. *There is a unique steady-state equilibrium with endogenous vaccination.*

Proof. For any fixed $V < 1 - \frac{\gamma}{\beta}$, let $I(V) > 0$ be the infection level in the unique steady-state equilibrium with exogenous vaccination level V, and let $\overline{I} \equiv I(0)$. For all $I \in (0, \overline{I}]$, let $SS_V(I)$ be the vaccination level that induces equilibrium steady-state infection prevalence I, i.e., $I(SS_V(I)) = I$. Because $I(V)$ is continuous and strictly decreasing (Corollary 1), $SS_V(I)$ is also continuous and strictly decreasing.

A steady-state equilibrium with endogenous vaccination exists with infection prevalence I and vaccination level V if and only if $SS_V(I) = D_V(I) = V$. For all $I \approx 0$, we have $SS_V(I) \approx 1 - \frac{\gamma}{\beta} > D_V(I) \approx 0$. On the other hand, $SS_V(\overline{I}) = 0 < D_V(\overline{I})$. Since $SS_V(I) - D_V(I)$ is continuous and strictly decreasing, there is a unique I^* such that $SS_V(I^*) = D_V(I^*) \equiv V^*$, as desired.

□

4.1 Subsidizing Vaccination vs. Subsidizing Social Distancing

The externalities associated with vaccination and social distancing provide motivation for policy interventions.[10] There may also be societal benefits outside of our model associated with reducing the steady-state infection prevalence, e.g., workplace productivity gains and less-burdened health systems, and/or from increasing the steady-state vaccination level, e.g., blunting the severity of any new-variant outbreak. However, interventions that promote social distancing or vaccination can have quite different effects on steady-state infection prevalence, once one accounts for how these interventions impact endogenous vaccination. Most strikingly, we show that a social-distancing subsidy can sometimes have the ironic effect of *increasing* steady-state infection prevalence. In particular, consider a lump-sum subsidy S_V for susceptible agents who get vaccinated ("vaccine subsidy") and a flow subsidy $S(x)$ that reduces their social-distancing cost to $c_1(x) = c(x) - S(x)$ as in Corollary 2 ("social-distancing subsidy").

[9] With vaccination cost c_{iV}, the expected lifetime cost of adopting the vaccine and renewing it whenever immunity runs out is $c_{iV} + e^{-rt_V} c_{iV} + e^{-2rt_V} c_{iV} + ... = \frac{c_{iV}}{1 - e^{-rt_V}}$.

[10] As observed by Brito, Sheshinski, and Intriligator (1991), a universal vaccination requirement can reduce the equilibrium utility of susceptible agents who are affected by that rule. Similarly, Geoffard and Philipson (1997) suggest that infectious diseases tend to remain endemic because reductions in prevalence erode individual incentives for precautionary behavior.

Proposition 4. *A subsidy for vaccination increases vaccine adoption and reduces infection prevalence in the steady-state equilibrium. A subsidy for social distancing reduces vaccine adoption and could either increase or reduce infection prevalence in the steady-state equilibrium.*

Proof. Let I_V denote the infection prevalence in the steady-state equilibrium with endogenous vaccination and no subsidy. A vaccine subsidy increases the demand for vaccination but has no effect on the level of vaccination required for a steady-state equilibrium at infection prevalence I. Define the demand for vaccination with the subsidy as $D_{V,S}(I)$. Since $D_{V,S}(I) > D_V(I)$ for each I and $D_V(I_V) = SS_V(I_V)$, we know $D_{V,S}(I_V) > SS_V(I_V)$. Therefore, $D_{V,S}(I)$ and $SS_V(I)$ intersect at some $I < I_V$, proving the desired result.

By contrast, a subsidy for social distancing has two effects. First, following the logic of Corollary 2, the subsidy reduces the marginal cost of social distancing, and therefore reduces the number of new infections for any stationary infection prevalence I without vaccination. Because of this shift in the new-infection curve, the subsidy reduces $SS_V(I)$, the vaccination rate required to produce a steady-state equilibrium with infection prevalence I. Second, the subsidy reduces the expected future cost $C_S^*(I)$ for a susceptible person and thus reduces $D_V(I)$, the demand for vaccination given any stationary infection prevalence I. These changes each cause the steady-state vaccination level to fall but have opposite effects on the steady-state infection prevalence. Figures 3a-3b provide examples of both possibilities, that steady-state infection prevalence may rise or fall. □

Figure 2 depicts the effect of a vaccine subsidy on steady-state vaccine adoption and infection prevalence in Example 1, with agents' cost of vaccination c_{iV} uniformly distributed on $(0, .0204)$ and a subsidy of .00112 for susceptible agents for each vaccination. Under the original conditions with no subsidy, the level of vaccination required for a steady-state equilibrium is decreasing while demand for vaccination is increasing in infection prevalence. The intersection of these two curves at point A represents the baseline equilibrium with a stationary infection prevalence of 2% and approximately 50% of the population adopting the vaccine. For reasons explained in the proof of Proposition 4, the subsidy shifts the vaccine demand curve $D_V(I)$ up and to the left, which increases vaccination and reduces infection prevalence at the new steady-state equilibrium point SV.

Figures 3a and 3b illustrate the possible equilibrium impacts of a subsidy that reduces the cost of social distancing, in the context of Example 1 with distancing costs $0.01889x^2$ after the subsidy, but with different vaccination-cost distributions.[11]

[11] Figures 2, 3a, and 3b extend Example 1 to include a distribution of vaccination costs with $t_V = 1$. In Figures 2 and 3a, $c_{iV} \sim U(0, 0.0204)$ for Figures 2 and 3a. We use a much tighter distribution $c_{iV} \sim U(.0084, .0102)$ for Figure 3b, though this is not consistent with the assumption in the text that $F_{C_V}(x) > 0$ for values x near 0 (which simplifies exposition by ensuring that the marginal adopted is indifferent between vaccination and not vaccinating). It is possible to adapt the distribution of vaccination costs for Figure 3b to satisfy $F_{C_V}(x) > 0$ for values x near 0 by shifting mass ϵ of the distribution to the range $(0, .0084)$.

The social-distancing subsidy shifts the vaccine demand curve $D_V(I)$ down and to the right and shifts the level of vaccination required for equilibrium $SS_V(I)$ down and to the left. The steady-state equilibrium shifts from point A to point SD in both figures. As described in Proposition 4, a social-distancing subsidy unambiguously reduces steady-state vaccination but may decrease (Figure 3a) or increase (Figure 3b) steady-state infection prevalence. When demand for vaccination is relatively inelastic, as in Figure 3a, the effect of the shift in $SS_V(I)$ predominates and so a social distancing subsidy reduces steady-state infection prevalence.

By contrast, when demand for vaccination is relatively elastic, as in Figure 3b, the primary effect of a social-distancing subsidy is to reduce demand for vaccination. In this case, the subsidy leads to increased infection prevalence at the new steady-state equilibrium point $SD2$.

The difference we identify between the effects of vaccine and social-distancing subsidies arises because reducing the cost of vaccination only indirectly affects incentives for social distancing (by causing steady-state infection prevalence I to fall) while changes in the marginal cost of social distancing directly affect incentives for vaccination (by reducing $C_S^*(I)$ and hence the marginal value of vaccination). Thus, as suggested by Corollary 1, a vaccine subsidy has an unambiguous effect on steady-state infection prevalence, whereas a social-distancing subsidy has an ambiguous effect in the fashion described by Peltzman (1975).

4.2 Vaccine Mandates

We model a vaccine mandate as either imposing a financial penalty or prohibiting certain activities for unvaccinated people. For example, New York City required proof of vaccination for indoor restaurant dining from August 2021 through early March 2022. We model such limits on activity as imposing a minimum level of social distancing on unvaccinated people.

Proposition 5. *(a) A vaccine mandate that imposes financial penalties on unvaccinated people increases vaccination and reduces infection prevalence in the steady-state equilibrium.*
(b) A vaccine mandate that prohibits certain activities for unvaccinated people reduces infection prevalence and could either increase or reduce vaccination in the steady-state equilibrium.

Proof. A fine for unvaccinated people is analogous to a subsidy for vaccination. Specifically, a flow fine c_F for unvaccinated people increases $C_S^*(I)$ but has no effect on susceptible agents' social-distancing incentives, so long as unvaccinated infected people also pay the fine. Thus, such a fine increases the demand for vaccination $D_V(I)$ and has no effect on $SS_V(I)$, the level of vaccination required for steady-state equilibrium with infection prevalence I. Shifting the demand curve for vaccination shifts the intersection between D_V and SS_V up and to the left, resulting in a new steady-state equilibrium with

increased vaccination and lower infection prevalence than without the fine.

Barring all unvaccinated people from an activity has the same impact on transmission as barring only susceptible people, but also imposes costs on the infected and temporarily-immune unvaccinated people who are barred as well. Such an activity restriction therefore has two effects. First, so long as susceptible people are barred from some activities they would otherwise have wanted to do, susceptible-agent social distancing increases and the mandate reduces the number of new infections corresponding to any stationary infection prevalence I and vaccination level V. This results in a decline in $SS_V(I)$, the required level of vaccination for a steady-state equilibrium with infection prevalence I. Second, by imposing additional costs on infected and temporarily-immune people, the mandate increases $C_S^*(I)$, the expected future costs that susceptible people will face for any given I, thereby increasing $D_V(I)$, the demand for vaccination. Since D_V is increasing and SS_V is decreasing in I, steady-state infection prevalence must fall, but the impact on the steady-state vaccination level is ambiguous. □

In January 2022, France barred unvaccinated people not just from enclosed spaces such as long-distance trains but also open-air cafes and other public places where the risk of transmission is relatively low. The goal of this policy, in the words of President Emmanuel Macron, was to "annoy the unvaccinated."[12] Such restrictions are effectively equivalent to a financial penalty on unvaccinated people and, as such, have the unambiguous effect of both increasing steady-state vaccination and reducing steady-state infection prevalence (Proposition 5(a)). On the other hand, a policy that more judiciously restricts activity, only barring unvaccinated people from the highest-risk and lowest-value activities, might ironically reduce the steady-state level of vaccination (Proposition 5(b)).

5 Discussion and Conclusion

We have presented an economic SIRS model in which agents can engage in social distancing or choose to get vaccinated in order to reduce the chance of getting infected. The model allows us to consider the effects of various disease-control policies on the level of social distancing, infection prevalence, and the demand for vaccination during the endemic phase of an infectious disease. It also gives us insights into how social distancing and vaccination, two types of behavioral responses, interact to determine the steady-state level of infection. In addition, we have shown how the pillars of economic analysis—cost-benefit tradeoffs, supply-and-demand, comparative statics—can be applied to study the dynamics of infectious diseases.

Our basic model can be extended in several directions. One possibility is to examine the effects on agents' behavior and infection prevalence when vaccination reduces but does not eliminate the risk of

[12] "Les non-vaccinés, j'ai très envie de les emmerder. Et donc, on va continuer de le faire, jusqu'au bout".

infection. Chen (2006) and Chen & Cottrell (2009), show that multiple steady-state equilibria can arise when vaccines are imperfect since the benefit of vaccination may not be monotonic in disease prevalence. Similarly in our model, demand for vaccination could be increasing over low prevalences but decreasing over high prevalences if the efficacy of the vaccine is low. This implies that the D_V curve could intersect the SS_V curve at more than one point, resulting in multiple steady-state equilibria.

Although we have modeled the vaccine in our setting as a prophylactic one, that assumption could easily be replaced with one where the vaccine is disease-modifying, i.e., it does not prevent infection but reduces the severity of disease. In our framework, that means vaccination would have no effect on the transmission rate β, but it would decrease the cost of infection d. Once vaccinated with a disease-modifying vaccine, an agent would receive less benefit from social distancing. This leads to the possibility that when the rate of vaccination is high in the population—which would be expected when infection prevalence is high—the overall level of social distancing would be low, resulting in more infections. Thus, high prevalence begets high prevalence, creating a positive feedback loop, with significant policy implications that could be explored in future work.

We considered the impact of a vaccine subsidy and a social-distancing subsidy on the equilibrium steady state, but other sorts of interventions can also be analyzed within our framework. For instance, consider a policy whereby unvaccinated people are only allowed to enter a venue for economic activity (e.g., a restaurant) if they can show a recent negative test. By imposing an additional cost to enter the venue, a testing requirement reduces the cost associating with forgoing these activities and hence has the effect of reducing social-distancing cost. Moreover, by reducing the number of infected people who are able to enter the venue, such a requirement effectively reduces the transmission rate associated with activities within the venue. The overall effect of a testing requirement therefore combines the effects of a social-distancing subsidy and a transmission-reducing intervention.

A final extension we will mention, motivated by the notion of vaccine passports (Hall and Studdert (2021)),[13] is to consider the effects of making agents' vaccination status known to others. In our setting, there is random mixing in the population, meaning that the types of agents that one encounters is independent of one's vaccination status. Making people's vaccination status known to others could lead to assortative mixing whereby vaccinated and unvaccinated people are more likely to mix amongst themselves. Making information about people's vaccination status public could also change their incentive to get vaccinated by creating a social consequence for the vaccination decision. These issues and their implications merit consideration and thorough examination in the future.

[13]Diagnostic tests that provide information on a consumer's current health status appear to raise similar issues, but with additional nuances because people who test negative at one point can subsequently become infected and people have private information about their health status. See Phelan and Toda (2022) and Deb, Pai, Vohra, and Vohra (2022) for insightful analysis of the equilibrium impact of imperfect testing.

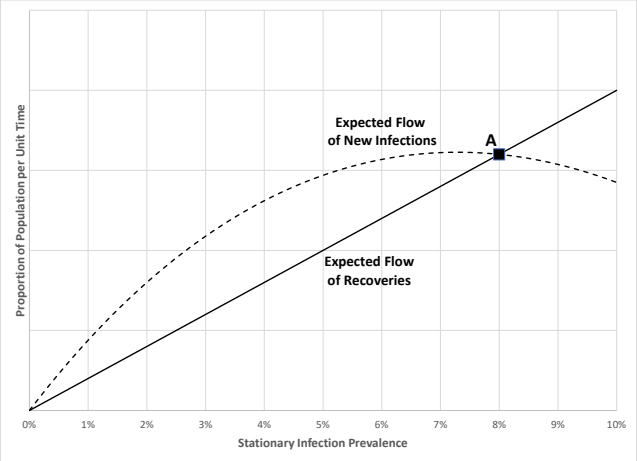

Figure 1a. *New Infections vs New Recoveries given any stationary infection prevalence*

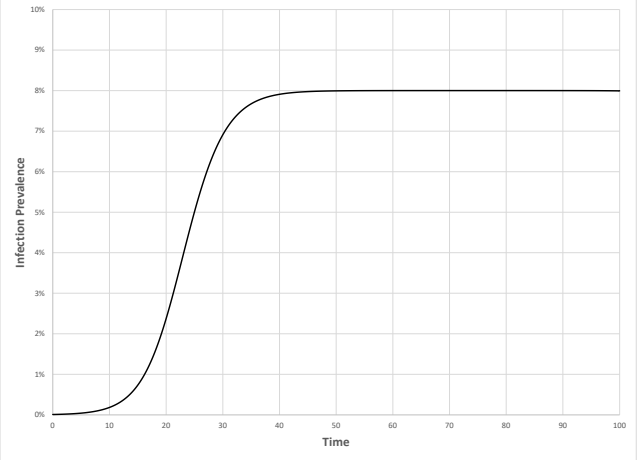

Figure 1b. *Infection-prevalence trajectory leading to the steady state*

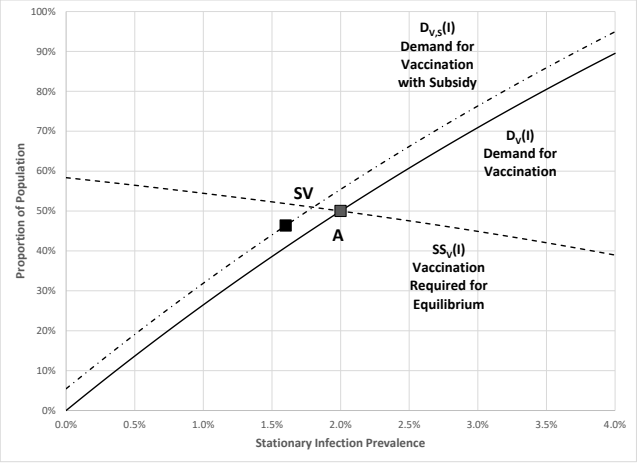

Figure 2. *The Effect of a Vaccine Subsidy*

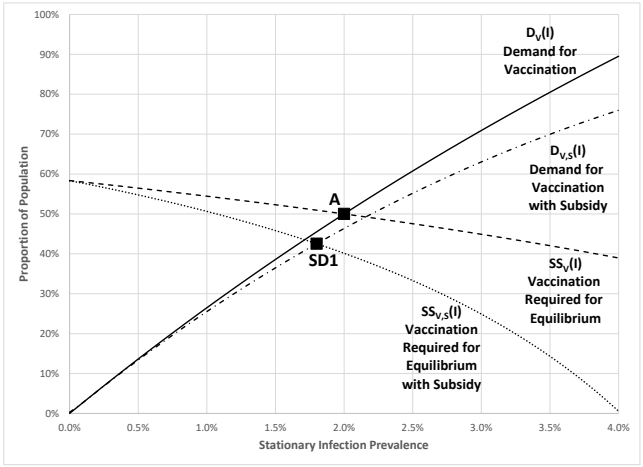

Figure 3a. *The Effect of a Social Distancing Subsidy Part 1*

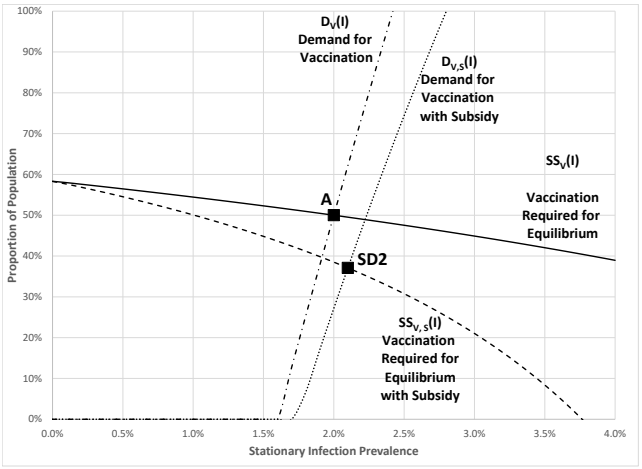

Figure 3b. *The Effect of a Social Distancing Subsidy Part 2*

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TECHNICAL APPENDIX (for online publication only)

A Quadratic Costs

Suppose that the flow cost of social distancing x is quadratic, $c(x) = \frac{\alpha x^2}{2}$, so that $c'(x) = \alpha x$. First, we verify that the steady-state continuation value for infected agents C_I and the individually-optimal social-distancing level x^* for susceptible agents are each linear functions of the steady-state continuation value for susceptible agents C_S, namely,

$$C_I = a_1 + b_1 C_S \tag{26}$$

$$x^* = a_2 + b_2 C_S. \tag{27}$$

(To simplify exposition, we suppress the dependence of C_I, C_S, and x^* on the steady-state infection prevalence, which we denote here simply as I.) Equation (26) follows directly from equation (7) in the main text, which states that $C_I = \frac{d+\gamma e^{-rt} C_S}{\gamma + r}$; so,

$$a_1 = \frac{d}{\gamma + r} \tag{28}$$

$$b_1 = \frac{\gamma e^{-rt}}{\gamma + r}. \tag{29}$$

By the first-order condition (10), $c'(x^*) = \alpha X = \beta I(C_I - C_S)$; so, $x^* = \frac{\beta I}{\alpha}(C_I - C_S)$. This verifies equation (27) with

$$a_2 = \frac{\beta I}{\alpha} a_1 \tag{30}$$

$$b_2 = \frac{-(1 - b_1)\beta I}{\alpha}. \tag{31}$$

Next, we verify that the steady-state value of C_S for any given infection prevalence I can be determined by solving a quadratic equation. By equation (8), $C_S = \frac{c(x) + \beta(1-x) I C_I}{\beta(1-x)I + r}$. Cross-multiplying and substituting $C_I = a_1 + b_1 C_S$ and $x = a_2 + b_2 C_S$ gives

$$C_S[\beta(1 - a_2 - b_2 C_S)I + r] = \frac{\alpha}{2}(a_2 + b_2 C_S)^2 + \beta I(1 - a_2 - b_2 C_S)(a_1 + b_1 C_S). \tag{32}$$

This yields a quadratic equation of form $X C_S^2 + Y C_S + Z = 0$ where

$$X = \beta I b_2 + \frac{\alpha}{2} b_2^2 - \beta I b_1 b_2 \tag{33}$$

$$Y = \alpha a_2 b_2 + \beta I(1 - a_2) b_1 - \beta I a_1 b_2 - r - \beta I(1 - a_2) \tag{34}$$

$$Z = \frac{\alpha}{2} a_2^2 + \beta I(1 - a_2) a_1. \tag{35}$$

Finally, we provide details on how to compute SS_V, the "supply of vaccination required for equilib-

rium" shown in Figures 2-3. In a steady-state equilibrium with infection rate I, there is a constant flow of agents from the infected to the recovered state at rate γI. Since recovered agents have temporary immunity for t_I units of time, the proportion of recently recovered / currently immune agents is $\gamma I t_I$ in this steady-state equilibrium. After accounting for infected and temporarily immune agents, and assuming that a stationary proportion SS_V of the population is vaccinated, the proportion of susceptible agents at each moment in the steady-state equilibrium is $S = 1 - I(1 + \gamma t_I) - SS_V$. Thus, the flow of agents from the susceptible to the infected state is $S\beta I(1 - x^*)$. In a steady-state equilibrium, the flow in and out of the infected state must be the same, so

$$\gamma I = (1 - I(1 + \gamma t_I) - SS_V)\beta I(1 - x^*). \tag{36}$$

Solving (36) allows us to compute SS_V.

B Details of Example 1

We set the parameters $d - 0.3$, $\gamma = 0.2$, $t_I = 1$ and chose $r = -\ln(0.95)$ to be consistent with a discount factor $\delta = 0.95$ per unit time. For Figures 1a and 1b, we assume that V is exogenously set to 0. For Figures 2, 3a, and 3b, we assume that $t_V = 1$ and describe our choices for the distribution of vaccination costs below.

We used a trial-and-error process with the aid of a spreadsheet to choose the remaining parameters for Example 1, targeting three desirable properties for graphical purposes: (1) steady-state infection rate of 8% with endogenous social distancing and no vaccination; (2) steady-state infection rate of 2% with endogenous social distancing and endogenous vaccination; (3) steady-state vaccination rate of 50% with endogenous social distancing and endogenous vaccination. Through this process, we identified the values $\beta = .480248$, $\alpha = .0750329$ for Example 1. These parameter values also apply for Figures 1a and 1b.

Equation (36) identifies the required supply of vaccination for a steady-state equilibrium at each infection rate. We worked backwards from this equation and our desired result that 50% of agents would choose to be vaccinated with a steady-state infection level of 2% to identify the baseline distribution for vaccination costs for Figures 2 and 3a. Given the assumption that vaccination provides immunity for one unit of time, the lifetime cost of vaccination at discount factor .95 per unit time is $\frac{c_V}{1-.95} = 20c_V$. The value of C_S with steady-state infection rate 2% is approximately 0.1987501, so it is optimal to adopt vaccination in the steady-state equilibrium, and to do so consistently over time whenever immunity from prior vaccination wanes, if $20c_V < .19875$. Since we targeted a vaccination rate of 50%, we chose a uniform distribution of costs on $(0, 0.019875)$.[14]

[14]The reported values in the text are approximations. See the associated spreadsheets for the more precise values that

For our policy applications, we chose parameters to yield steady-state equilibria at a new infection rate of 1.8% for a vaccine subsidy in Figure 2 and for a social distancing subsidy in Figure 3a. To compute the appropriate vaccine subsidy for Figure 2, we observed that the approximate value of C_S with steady-state infection rate 1.8% is 0.180926 and the required vaccination rate for a steady-state equilibrium at this infection rate is 50.94%. Working backwards from the uniform distribution identified above, this suggests a subsidy of .001077 for Figure 2. Similarly, we worked backwards from a desired equilibrium with stationary infection rate 1.8% to identify an alternate value $\alpha = .038452$ after accounting for a social distancing subsidy in Figure 3a.

We used trial and error to identify an alternate multiplier for the quadratic cost of social distancing to produce a steady-state equilibrium at infection rate 1.8%. The alternate value of α for this case is .0384152. Then to create a more elastic demand function for vaccination for Figure 3b, we tightened the range of the uniform distribution of values of C_V so that no one would adopt vaccination with a steady-state infection rate less than 1.6% and that once again 50% would adopt vaccination at a steady state infection rate of 2%. Working backwards from these desired results, we chose a uniform distribution on the range $[.00813, .01174]$ for vaccination costs for Example 3b and an alternate value $\alpha = 036063$ to produce an equilibrium with steady state infection rate of 2.1%.

B.1 Details of the Spreadsheet containing Computations

The computations for Figures 1a, 2, 3a, and 3b are provided in the Excel spreadsheet "Technical Appendix". There is one separate table of data and computations for each of Figure 1, Figure 2a, and Figure 3a and two separate tables for Figure 3b. Each of these tables follows the same format with one row per infection rate.

The entries in these spreadsheet tables are as follows. Column A contains the fixed parameter values, Column D lists the conjectured steady-state infection rate, Column E contains the the associated proportion of recently recovered and currently immunized agents is in Column E, and the right most columns find the roots of the quadratic equation given the parameters as defined by (33), (34), and (35). Column F identifies the relevant root of the quadratic equation as the steady-state cost value for susceptible agents C_S, and Columns G and H contain the associated values of costs C_R and C_I. Given these values, Column I contains the optimal steady-state level of social distancing for a susceptible agent, which follow from the fixed parameter values and the cost values in Columns F through H. Column J contains the flow rate of new infections and Column K contains the flow rate of recoveries, where these values correspond to the relevant portions of equations (1) and (2) from the main text. Column L uses the ratio of these values to compute the required level proportion V of susceptible agents who are

we used.

vaccinated according to the steady-state condition $(1-V)N = R$ or $V = 1 - \frac{R}{N}$. Column M converts the value for expected steady-state payoff for susceptible agents into the demand for vaccination (i.e. the proportion of susceptible agents who would adopt vaccination) at the given stationary infection rate.

The relevant equilibrium outcomes are highlighted in the top rows of each table. In the table labeled "Figure 1a", Row 1 highlights the (approximate) equality of the flow rate of new infections and recoveries at stationary infection rate 8% without vaccination, while Row 2 highlights the fact that a 50% vaccination rate is required for an equilibrium with stationary infection rate 2% given endogenous vaccination.

In the table labeled "Figure 2", Row 1 highlights the (approximate) equality of supply and demand for vaccination at stationary infection rate 2% given the parameters listed above, while Row 2 highlights the approximate equality of supply and demand for vaccination at stationary infection rate 1.8% with the vaccination subsidy identified above.

In the table labeled "Figure 3a", Row 1 highlights the (approximate) equality of supply and demand for vaccination at stationary infection rate 1.8% given the original parameters *except* for a change in the value of α to .038452 after accounting for a social distancing subsidy.

In the table labeled "Figure 3b, No Subsidy", Row 1 highlights the (approximate) equality of supply and demand for vaccination at stationary infection rate 2% given the original parameters *except* for a change in the lower and upper limits of the uniform distribution of vaccination costs.

In the table labeled "Figure 3b, Subsidy", Row 1 highlights the (approximate) equality of supply and demand for vaccination at stationary infection rate 2.1% given the original parameters *except* for a change in the lower and upper limits of the uniform distribution of vaccination costs and a reduction in the value of α to .036063.

B.2 Details of the Spreadsheets for Figures 1a, 2, 3a, and 3b

The Excel spreadsheet "Figures" uses results from the "Technical Appendix" spreadsheet to produce Figures 1, 2, 3a, and 3b. This spreadsheet includes one table labeled "Data" with the relevant values from the other spreadsheet and separate tabs for each of the figures themselves.

Columns C and D of the spreadsheet "Graphs / Data" contain data from Columns J and K of the spreadsheet "Technical Appendix / Figure 1".

Columns G through I of the spreadsheet "Graphs / Data" contain data from Columns L through N of the spreadsheet "Technical Appendix / Figure 2".

Columns L and M in the spreadsheet "Graphs / Data" repeat the data from Columns G and H in that same spreadsheet. Columns N and O in the spreadsheet "Graphs / Data" contain data from Columns L and M of the spreadsheet "Technical Appendix / FIgure 3a."

Columns R and S in the spreadsheet "Graphs / Data" contain data from Columns L and M of the spreadsheet "Technical Appendix / FIgure 3b No Subsidy." Columns T and U in the spreadsheet "Graphs / Data" contain data from Columns L and M of the spreadsheet "Technical Appendix / FIgure 3b Subsidy."

B.3 Details of the Spreadsheets for Figure 1b

We used a straightforward algorithm to estimate convergence of an epidemic to steady state equilibrium. Our computations are shown in a series of spreadsheets titled "SIS Convergence Part 1", "Part 2", "Part 3", and "Part 4". We consider a 100 period model where each period is divided into 100 equal length segments of time with prorated parameters and discounting structure derived from the parameters of Example 1.

In each iteration of the algorithm, we assume an initial infection rate of .01% and use a conjectured time series for C_S and C_I. Our analysis for each iteration proceeds in two steps. In the first step for a given iteration, we compute the infection trajectory for the current iteration with the following procedure. Starting in time $t = 0$, we compute the individually optimal level of social distancing for susceptible people at time t given the current infection rate and the anticipated values for C_S and C_I at time $t + .01$, then use that level of social distancing to compute the resulting infection rate at time $t + .01$. By iterating this process moving forward in time to time $t = 100$, we can trace the infection trajectory induced by the conjectured time series values for C_I and C_S.

In the second step of each iteration, we work backwards from period 100 to compute the realized time series values for C_S and C_I given the infection trajectory and social distancing pattern that was identified in the first step of analysis. We start in period 100 by assuming that the steady state values for C_S and C_I will be realized in period 100.01. Then at each time t, we compute realized expected costs for each state from the level of social distancing at time t and the **computed expected costs** for time $t + .01$. By iterating this process working backwards from time $t = 100$ to time $t = 0$, we identify the full time series of induced expected costs for this iteration of the algorithm.

The only difference between one iteration and the next of the algorithm is the choice of the conjectured time series for C_S and C_I. In the very first iteration, we assume constant values for C_S and C_I equal to half the steady state values for each quantity for each point in time from $t = 0$ to $t = 100$. Then for iteration $n + 1$, we take a weighted average with weight $\frac{2}{3}$ on the conjectured time series and weight $\frac{1}{3}$ on the realized time series values for C_S and C_I in iteration n. Given considerable weight to the previously conjectured time series helps to maintain slow progress of the algorithm towards (approximate) equilibrium values for social distancing and infection trajectory.

Starting in Column E of each spreadsheet, each iteration is represented in 12 columns of data and

a 13th blank column. We include separate iterations in (1) Columns E to P; (2) Columns R to AC; (3) Columns AE to AP; (4) Columns AR to BC; (5) Columns BE to BP; (6) Columns BR to CC; (7) Columns CE to CP; (8) Columns CR to DC; (9) Columns DE to DP; (10) Columns DR to EC. We compute the difference between conjectured and actual value for C_S for the last of these iterations in column ED, then identify the maximum and min difference for this last iteration in column EE. Finally, we compute the implied values for C_S and C_I for the next iteration in columns EG and EH.

Thus in all, each spreadsheet includes 10 iterations of the approximation algorithm. To move from one spreadsheet *Part k* to spreadsheet *Part* $k+1$, we simply use the final time series in Columns EG and EH of the previous spreadsheet as the conjectured values for C_S and C_I in Columns M and N for the first iteration in the next spreadsheet. In sum, the four spreadsheets include 40 iterations of the algorithm. As shown in Column EE, the process is near to complete convergence by the end of the 40th iteration, with changes in the estimated value functions on the order of 10^{-7} or 10^{-8} at that point. Figure 1b graphs the infection trajectory from this 40th iteration.

C Details of Example 2

We define two useful quantities to simplify the details of further derivations:

$$\alpha(x) = \frac{\beta I(1-x)}{r + \beta I(1-x)};$$

$$\theta = \frac{\gamma}{\gamma + r}.$$

Using these definitions, we can write

$$C_S^* = \min_x \{(1-\alpha(x))\frac{c(x)}{r} + \alpha(x) C_I^*\}, \tag{37}$$

where

$$C_I^* = (1-\theta)\frac{d}{r} + \theta e^{-rt_i} C_S^*. \tag{38}$$

Combining (37) and (38), we get

$$C_S^I = \min_x \{\frac{(1-\alpha(x))c(x) + \alpha(x)(1-\theta)d}{r(1-\alpha(x)\theta e^{-rt_I})}\} \tag{39}$$

A useful property of $\alpha(x)$ is that

$$\alpha'(x) = -\frac{\alpha(x)(1-\alpha(x))}{1-x}. \tag{40}$$

Define

$$C_S'(x) = \frac{1-\alpha(x)}{r(1-x)(1-\alpha(x)\theta e^{-rt_I})} \tag{41}$$

Taking the derivative of $C_S(x)$ with respect to x - treating I as fixed, and making use of (40) - yields

$$C_S'(x) = \frac{1-\alpha(x)}{r(1-x)(1-\alpha(x)\theta e^{-rt_I})}[(1-x)c'(x) - \alpha(x)((1-\theta)d - c(x) + \theta e^{-rt_I} r C_S(x))]. \tag{42}$$

For our purposes, the sign of $(1-x)c'(x) - \alpha(x)((1-\theta)d - c(x) + \theta e^{-rt_I} r C_S(x))$. is of primary interest.

C.1 Social Planner's Problem

For convenience, we define $R = \gamma t_I I$. Since we are assuming that vaccination level V is exogenous, we exclude vaccination costs from the social planner's problem. In a steady state with I infected agents and $1 - I - R - V$ susceptible agents, the social planner's problem is

$$\min_x \{I(x)\frac{d}{r} + (1 - I(X) - R - V)\frac{c(x)}{r}\} \tag{43}$$

Note that in this equation, $I(x)$ satisfies the steady state condition

$$\beta(1-x)(1-(1+\gamma t_I)I(x) - V) = \gamma. \tag{44}$$

So the planner's problem can be rewritten as

$$\min_x \{I(x)\frac{d}{r} + (1 - (1+\gamma r t_I)I(x) - V)\frac{c(x)}{r}\} \tag{45}$$

Denoting the solution to this optimization problem for the social planner as x^*, the FOC is

$$[1 - (1+\gamma t_I)I(x^*) - V]c'(x^*) + I'(x^*)(d - (1+\gamma t_I)c(x^*)) = 0. \tag{46}$$

From (46), we get

$$I'(x) = -\frac{[1 - (1+\gamma t_I)I(x) - V]}{(1-x)(1+\gamma t_I)} \tag{47}$$

Hence, (47) can be rewritten as

$$(1-x^*)c'(x^*) = \frac{d}{1+\gamma t_I} - c(x^*). \tag{48}$$

C.2 Comparison

We combine (42) and (48) to check the sign of $C_S'(x)$. Thus,

$$C_S'(x) = \frac{1-\alpha^*(x)}{r(1-x^*)(1-\alpha(x^*)\theta e^{-rt_I})}[(1-x^*)c'(x^*) - \alpha(x^*)((1-\theta)d - c(x^*) + \theta e^{-rt_I} r C_S(x^*))]. \tag{49}$$

We make use of (48) so that

$$(1-x^*)c'(x) - \alpha(x^*)[(1-\theta)d - c(x^*) + \theta e^{-rt_I} r C_S(x^*)] = \frac{d}{1+\gamma t_I} - c(x^*) - \alpha(x^*)[(1-\theta)d - c(x^*) + \theta e^{-rt_I} r C_S(x^*)].$$

Rearranging terms,

$$(1-x^*)c'(x)-\alpha(x^*)[(1-\theta)d-c(x^*)+\theta e^{-rt_I}rC_S(x^*)] = \frac{d}{1+\gamma t_I}-c(x^*)-\alpha(x^*)[d-c(x^*)-\theta(d-e^{-rt_I}rC_S(x^*))]. \tag{50}$$

Given that $\frac{d}{1+\gamma t_I} - c(x^*) < d - c(x^*)$, the sign of (50) is not obvious.

C.3 Details of Example 2

We chose $\beta = 0.9, \gamma = 0.2, r = 0.1, d = 1, t_I = 100, c(x) = x^2, V = 0$. The solution to the individual optimization problem is then

$$x(I) = -\frac{20e^{10} - 120I + 180e^{10}I - \sqrt{120e^{10}I(60I - 90e^{10}I) + (20e^{10} - 120I + 180e^{10}I)^2}}{(120 - 180e^{10})I}. \tag{51}$$

The steady state condition is

$$\beta(1 - x)(1 - I - \gamma t_I I) = \gamma \tag{52}$$

Solving for x gives

$$x = \frac{7(1 - 27I)}{9(1 - 21I)}. \tag{53}$$

The steady state equilibrium solves (51) and (53) simultaneously, which pinpoints the steady state infection level $I = 0.036579$ and associated level of social dstancing $x = 0.0414925$.

By contrast, the objective function for the social planner is

$$I(x)\frac{d}{r} + [1 - (1 + \gamma t_I)I(x)]\frac{x^2}{r}. \tag{54}$$

Solving (52) for I gives

$$I(x) = \frac{7 - 9x}{180(1 - x)} \tag{55}$$

Combining (54) and (55), the plannier's objective function (in expected cost) is minimized at $x =$ .0240999. That is, there is **too much** social distancing in the steady-state equilibrium relative to the social optimum.