1 A New Thought

As I mentioned in the previous paper, there are two different versions of the potential for the uniformly charged disk with \( r < a \). The first one is

\[
\Phi (r,\theta) = \frac{2Q}{a^2} \sum_n P_{2n} (0) \left( \frac{(4n+1)r}{(2n+2)(2n-1)} - \frac{r^{2n}}{(2n-1)a^{2n-1}} \right) P_{2n} (\cos \theta),
\]

(1.1)

and the second one is

\[
\Phi (r,\theta) = \frac{2Q}{a^2} \left( -r |P_1 (\cos \theta)| - \sum_n \left( \frac{(-1)^n (2n-3)!!}{(2n)!!} \right) \frac{r^{2n}}{a^{2n-1}} P_{2n} (\cos \theta) \right).
\]

(1.2)

Note that the absolute value appears in the first term, because now I include both \( z > 0 \) and \( z < 0 \) cases rather than just considering the former one. We shall see that it is important.

In fact, one can easily verify that the second term in eq.(1.1) and the second term in eq.(1.2) are identical. So we can ask the following question: whether this equality holds or not?

\[
\sum_n \frac{(4n+1)P_{2n} (0) P_{2n} (\cos \theta)}{(2n+2)(2n-1)} = -|P_1 (\cos \theta)|
\]

(1.3)

If it holds, then the problem is solved! No one is wrong. We just adopt different approaches. And I am so happy to inform you that it really holds!

Actually eq.(1.3) is quite subtle. While it is not easy to be verified directly via algebra manipulation, it indeed can be shown that if one expands \(|P_1 (x)|\) by Legendre polynomials, the correspond coefficients just satisfy eq.(1.3)! That is,

\[
|P_1 (x)| = \sum_l A_l P_l (x)
\]

\[
A_l = \frac{2l+1}{2} \int_{-1}^{1} |P_1 (x)| P_l (x) dx = \begin{cases} \frac{(2l+1)P_l (0)}{(l+2)(l-1)}, & \text{even } l \\ 0, & \text{odd } l. \end{cases}
\]

Ergo eq.(1.3) is verified! It is very simple to see why only terms with even \( l \) survive: because \(|P_1 (x)| = |x|\) is an even function, the integral can be nonzero only if another function in the integrand \( P_l (x) \) is also an even function. However, it is true that eq.(1.3) is somehow difficult to be verified directly. (I mean, at least for me, without the help of Mathematica I would have no idea to prove it.)

Consequently, eq.(1.1) and eq.(1.2) are equivalent to each other. The reason that I cannot find out what makes the inconsistency is that there is no inconsistency at all! If people got points deducted in the second midterm because you also did it in the way exactly what I did in the previous paper, it’s time to ask for more points back! Have fun and enjoy physics as well!