



# REDISTRICTING 101

## METRICS FOR GERRYMANDERING

---

*Moon Duchin*

# SETTING THE STAGE

# GERRYMANDERING: THE POWER OF THE PEN

---

- We will see that you can produce extremely skewed outcomes by drawing **designer districts**
- Throughout: seek to distinguish *neutral* vs *fair* and set some bounds on permissibility
- How do our gerrymandering rules and metrics line up with political values?
- How are the most vulnerable/marginal populations harmed or protected?



*Many lines to draw:*

*NC has 13 U.S. House districts*

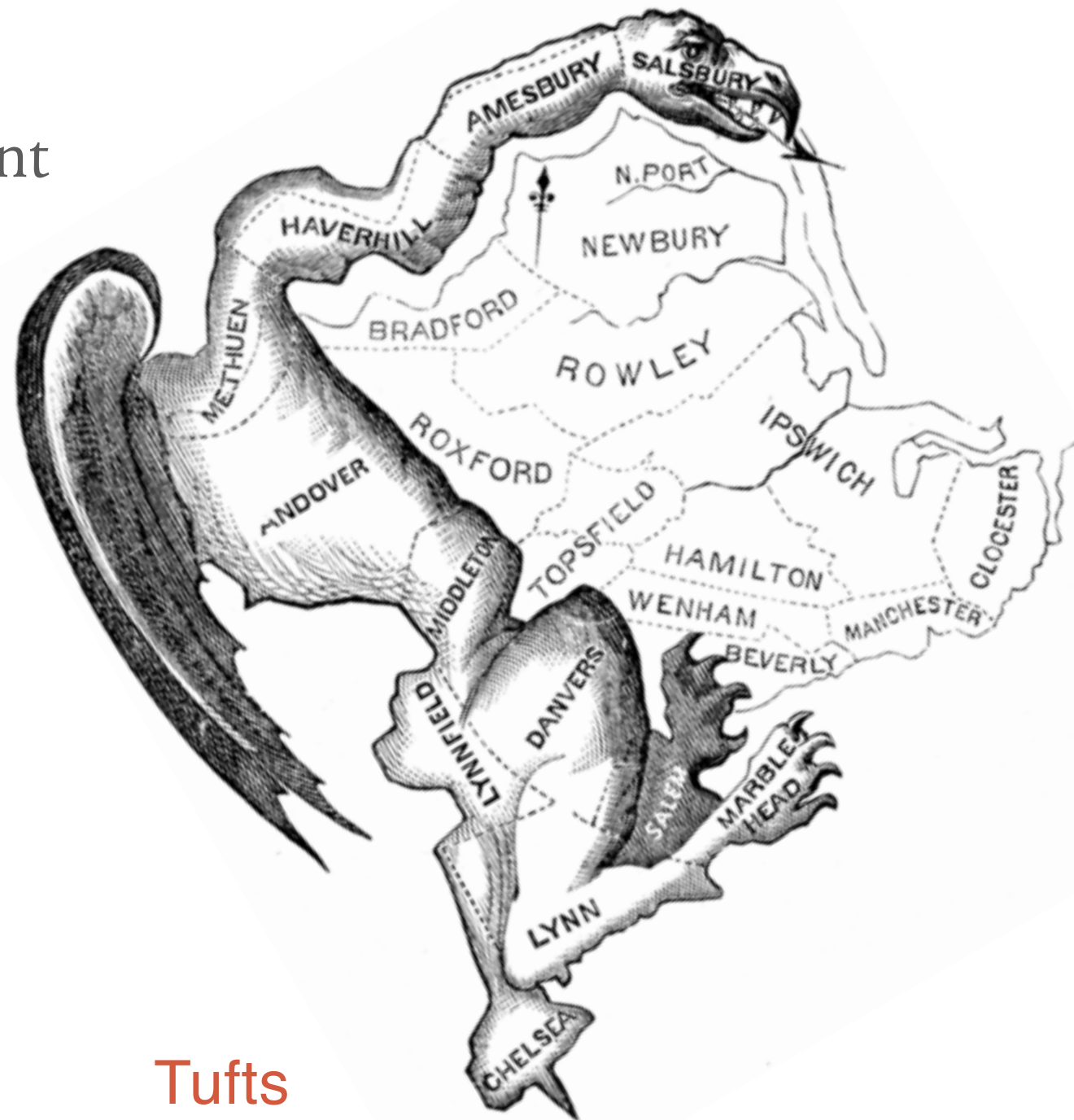
*120 state House districts*

*50 state Senate districts*

# HOW TO GERRYMANDER WHEN PLURALITIES RULE

---

- Very simplest principle: to win a single district, arrange the placement of the lines to ensure you get most votes within the district.
- Famous example: Elbridge Gerry's salamander gave us the term "gerrymander"
- Districting plan designed to favor Democratic-Republican party over Federalists



Tufts

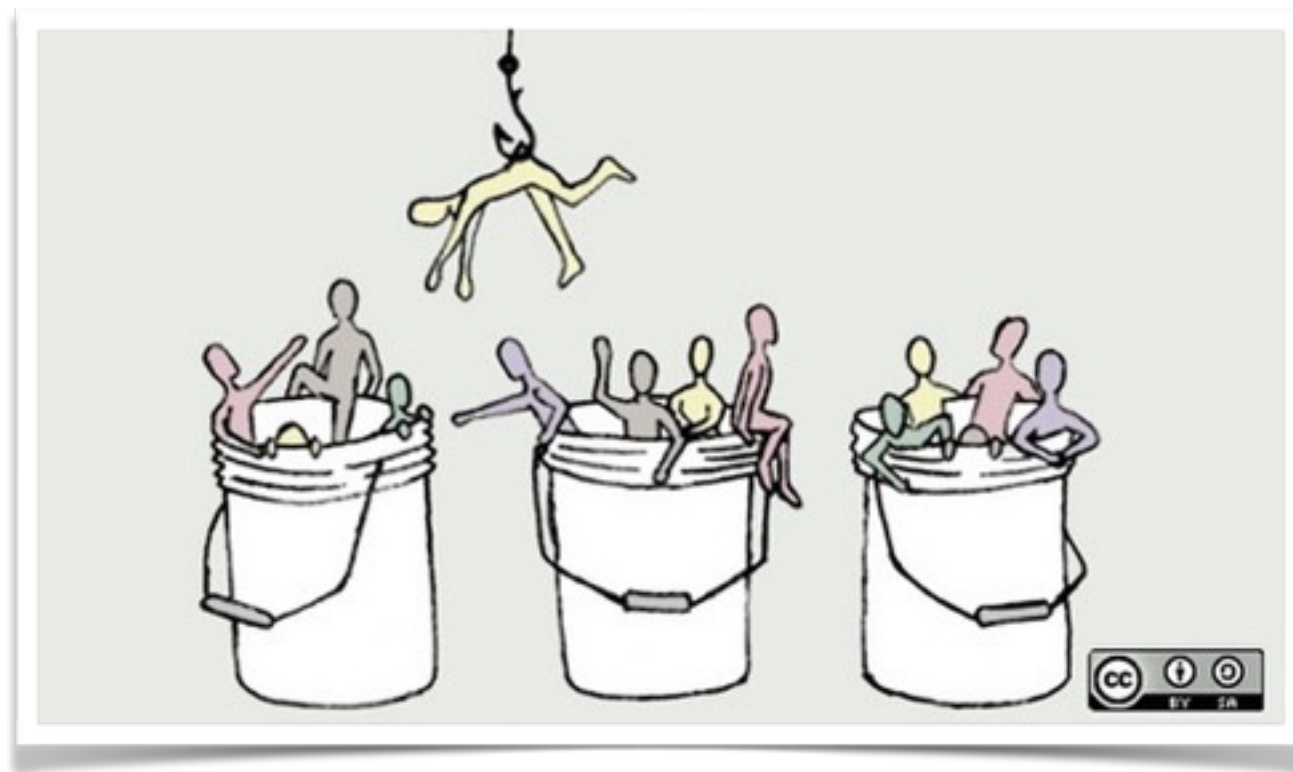


# HOW TO GERRYMANDER

.....

- Suppose (a) you know exactly which people vote your way, and (b) you have total freedom to separate people into buckets arbitrarily.

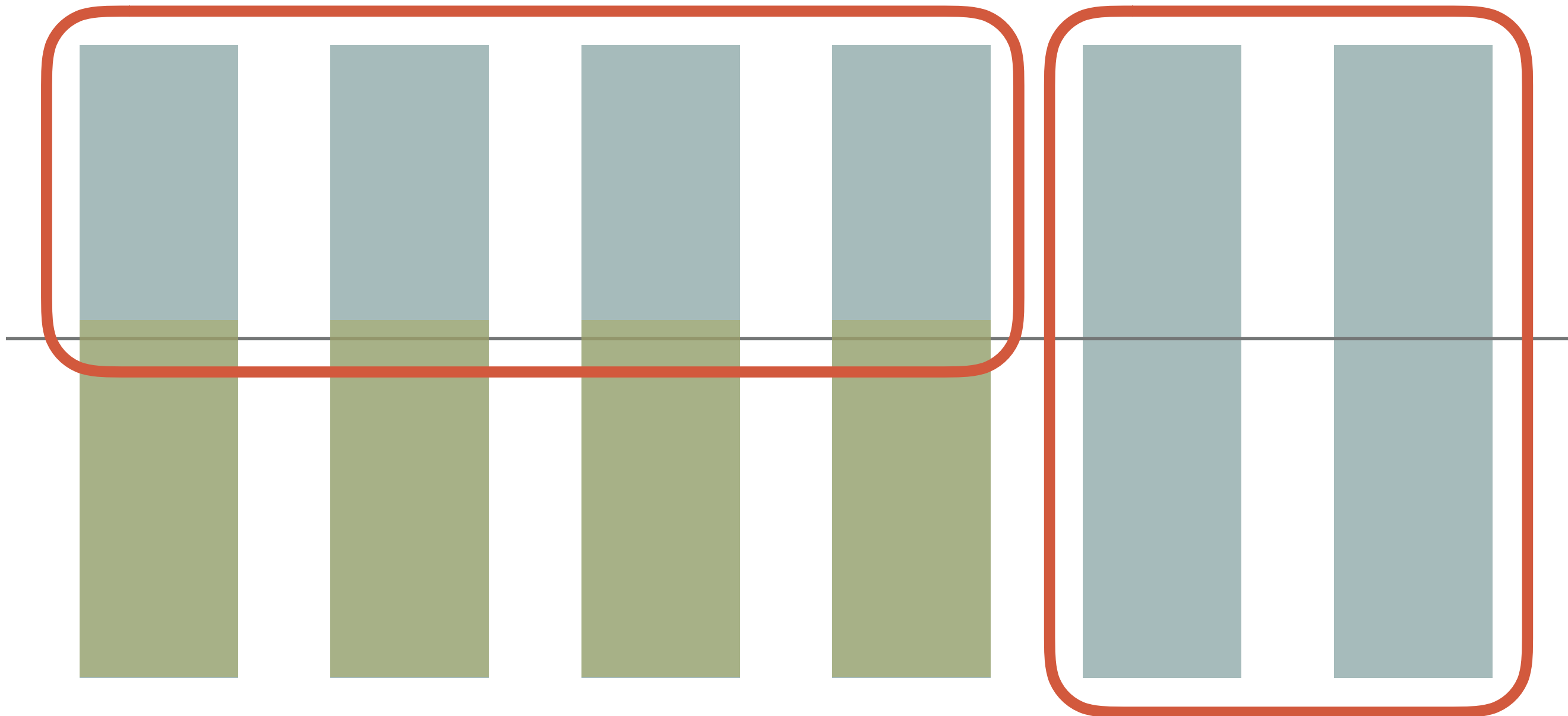
**Goal: win the most buckets.**



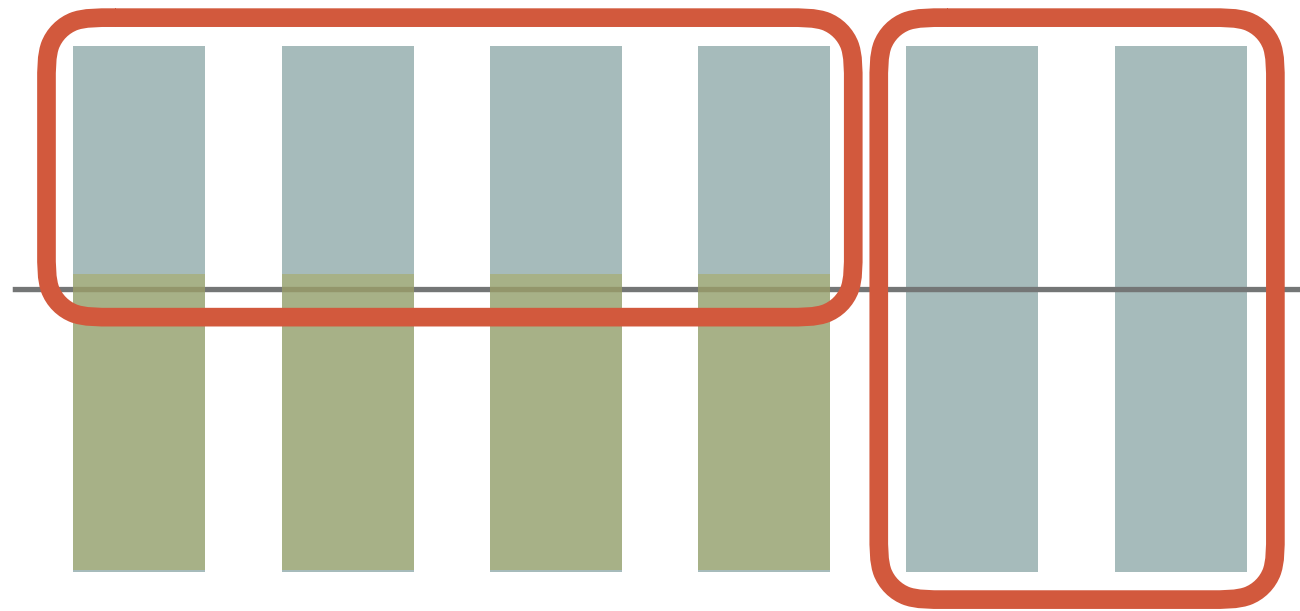
- You'd make a narrow majority in as many buckets as possible and you wouldn't waste voters in any others.

Cracking

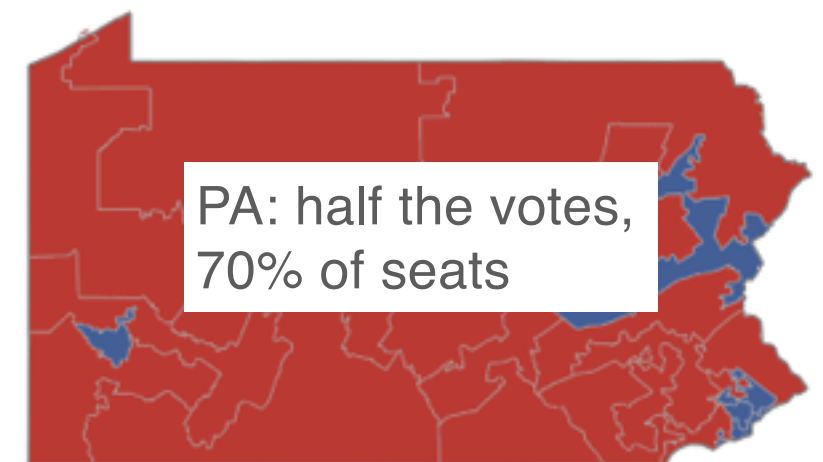
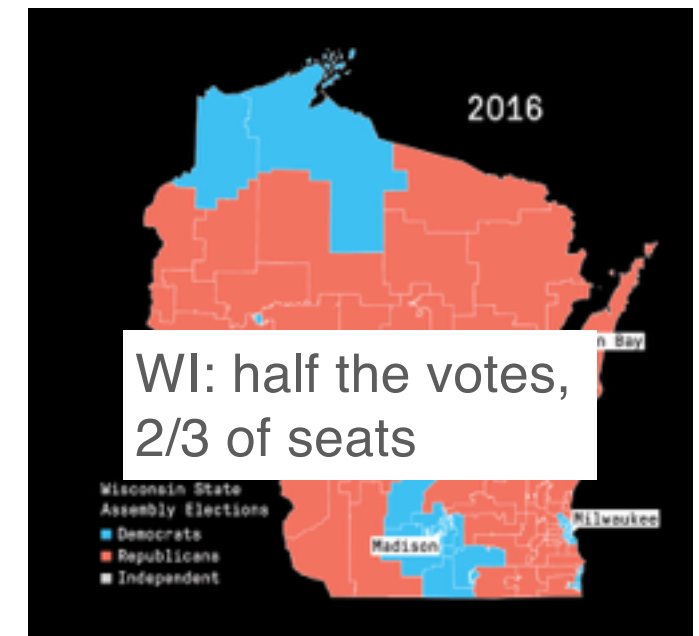
Packing



*Efficient majorities for you = Packing and Cracking for your opponents*



- So theoretically it's possible to get a seat share that is double your vote share, if you were unconstrained by geography. How does it actually play out?
- Key point: Rs now have 32/50 legislatures, 33/50 governors, and “trifectas” in 27/50 states.
- Was not always so, and both parties gerrymander rampantly when they can!



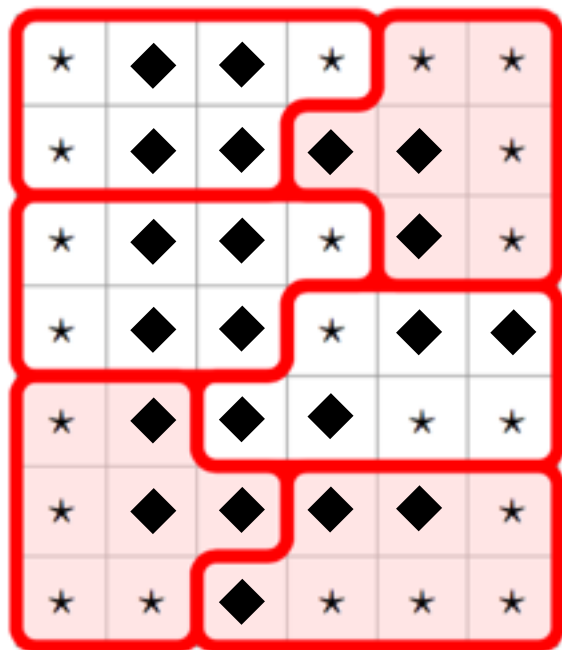


# WHY DOES SHAPE MATTER?

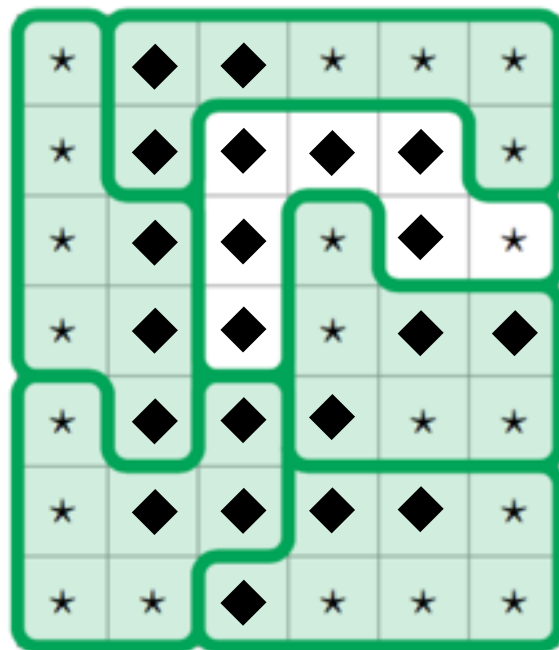
---

- Any careful composition of demographics (such as packing/cracking) requires your pen to follow the distribution
- Limiting degrees of freedom limits the power of the pen

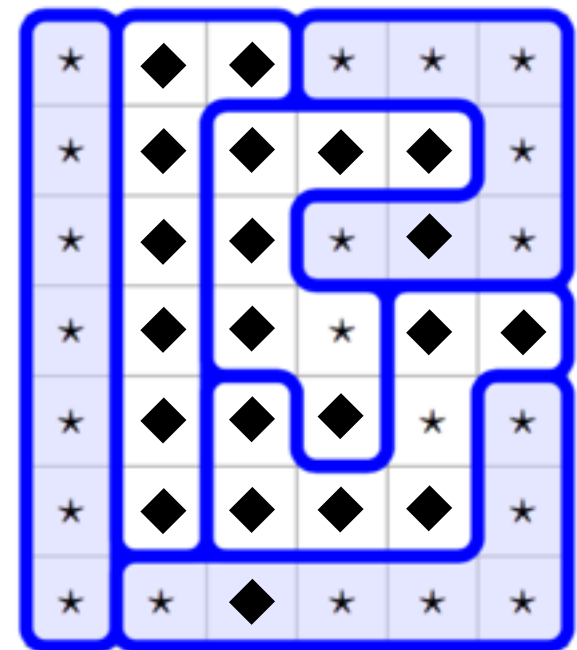
In these pictures, the two sides have an equal number of voters, but Team ★ can get all but one seat!



Neutral



◆ Packed & Cracked



Both sides Packed



# COURTS OFTEN CONNECT SHAPE (ONLY) TO RACE

---

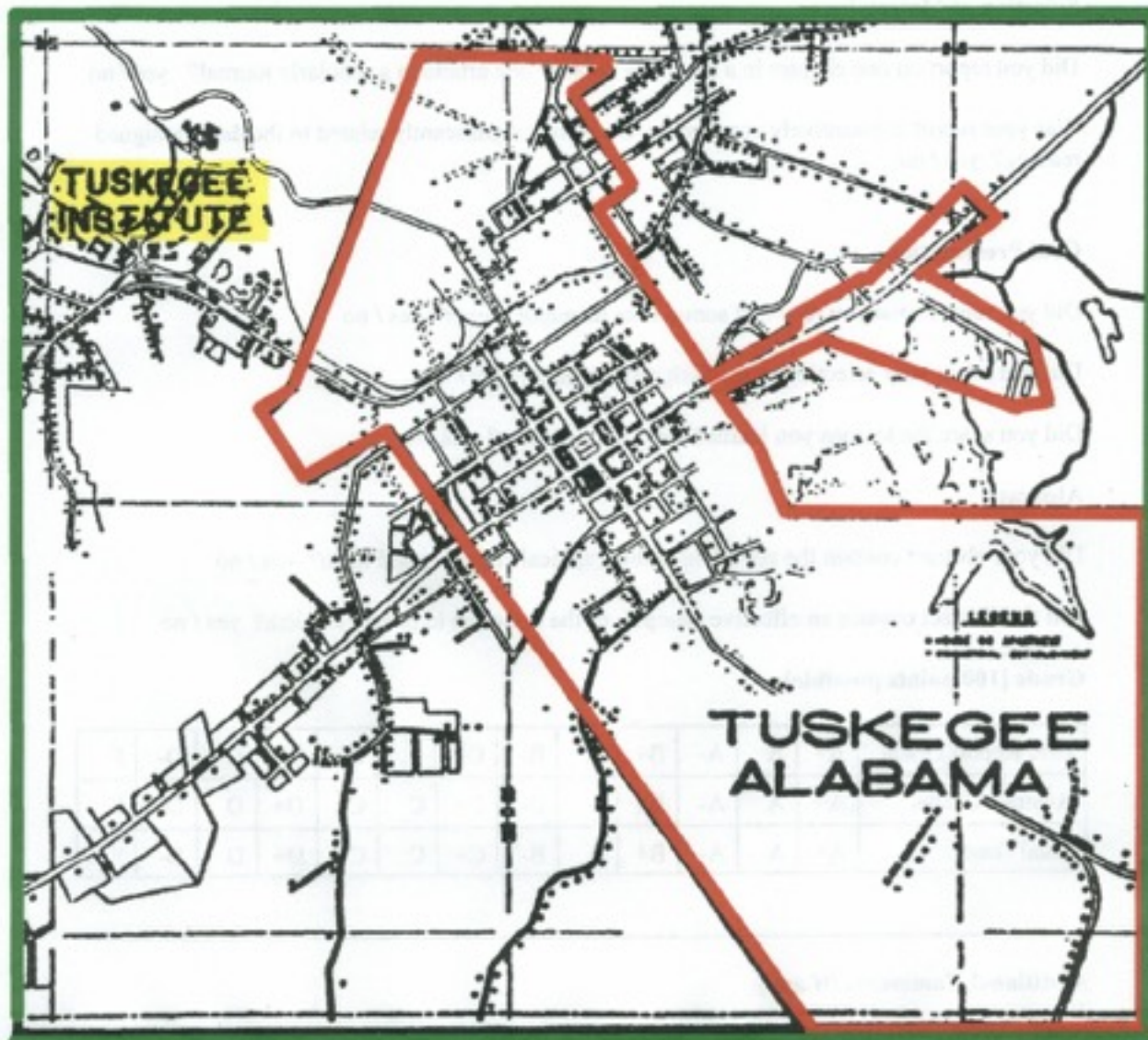
- Distended shapes indicate an agenda, but it could be anything: racial gerrymandering, partisan gerrymandering, incumbent gerrymandering, keeping grandma's house in the district, etc
- Justice Kennedy, writing in *Miller v. Johnson* (1995):  
“**Shape is relevant**... because it may be **persuasive circumstantial evidence** that **race** for its own sake, and not other districting principles, was the legislature's dominant and controlling rationale in drawing its district.”
- Why race?

# MID-TWENTIETH CENTURY GERRYMANDERS OF NOTE

---

## APPENDIX TO OPINION OF THE COURT.

CHART SHOWING TUSKEGEE, ALABAMA, BEFORE AND AFTER ACT 140



(The entire area of the square comprised the City prior to Act 140. The irregular red-bordered figure within the square represents the post-enactment city.)

- Gomillion v. Lightfoot (1960)
- Tuskegee redrew its lines in 1957

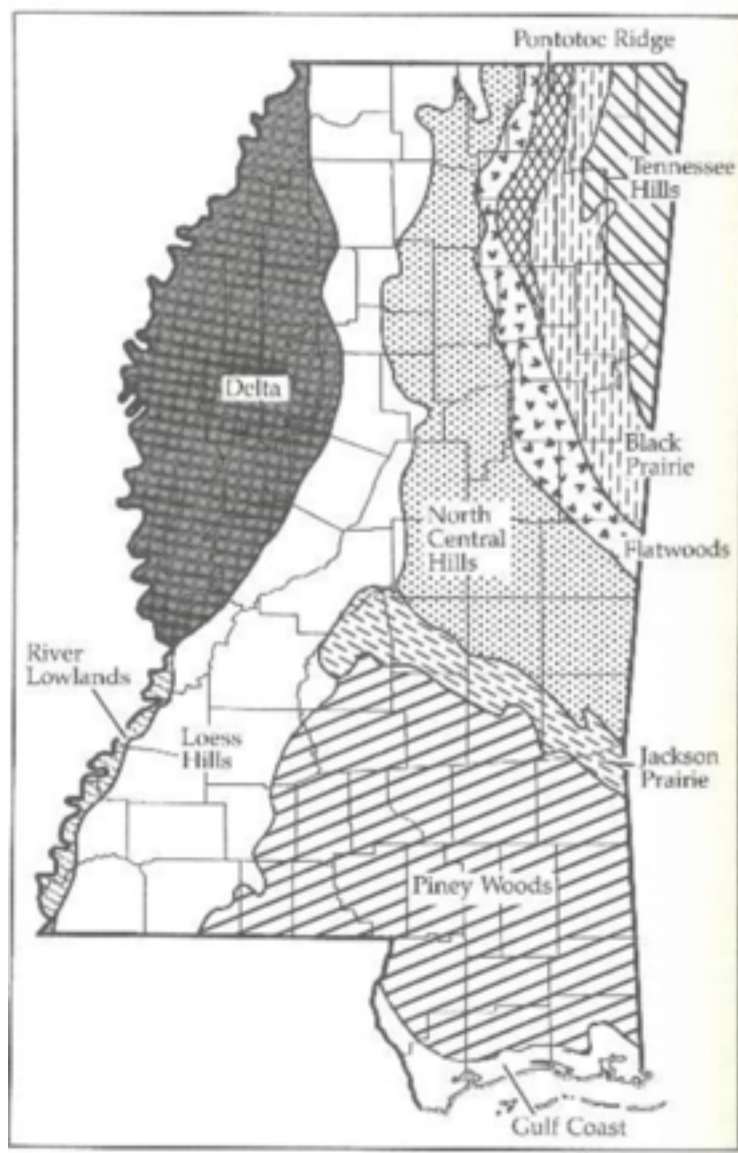
Before: **square**

After: **28-sided polygon**

Before: **79% Black**

After: **100% White**

# MISSISSIPPI REDRAWS THE LINES



Delta traditionally preserved



MS's Black population is concentrated in Delta region in state's Northwest



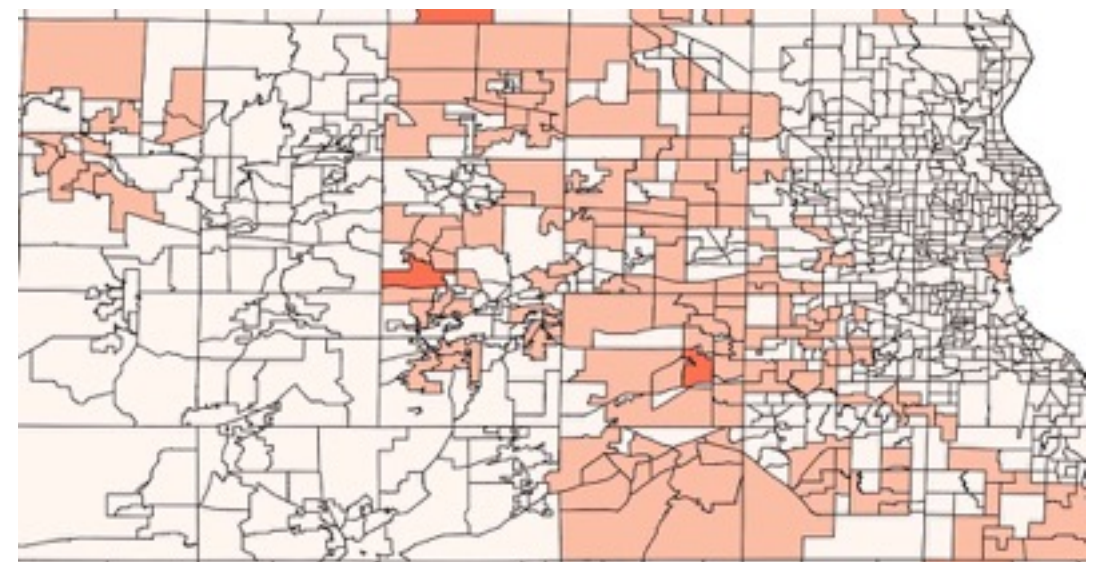
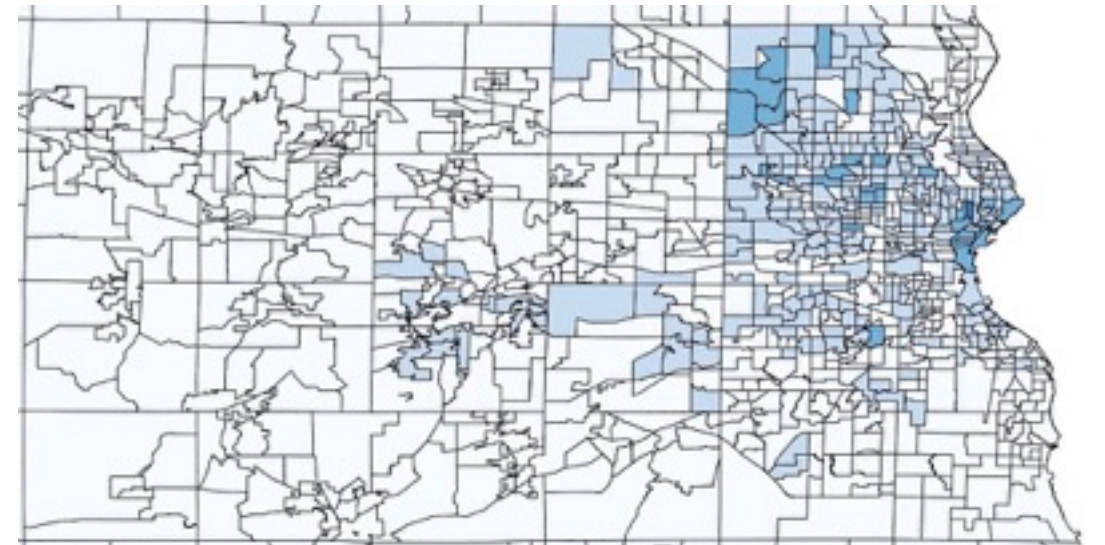
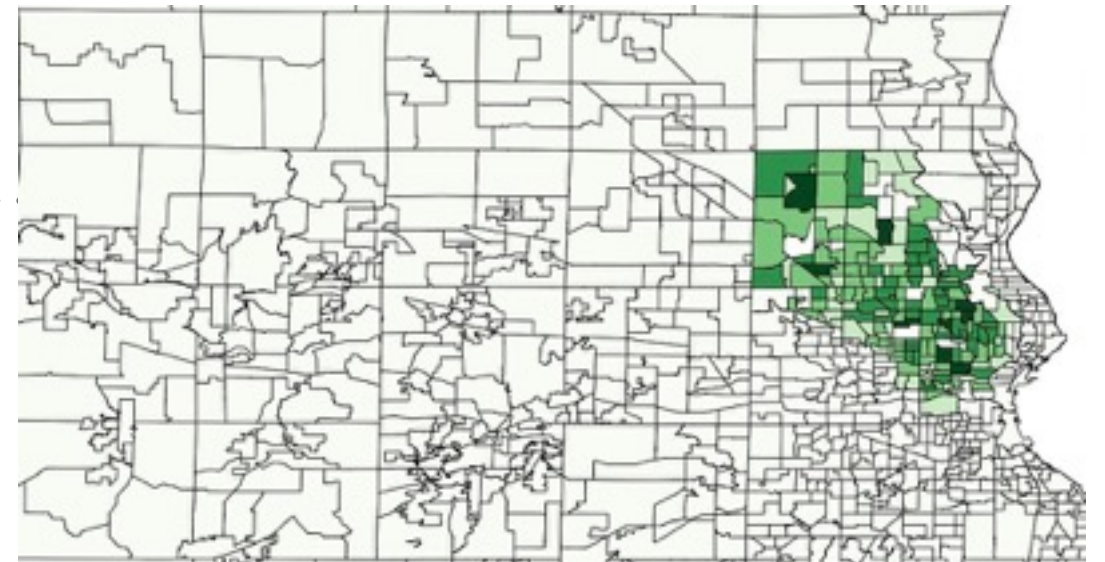
Broken up in 1960s—zero majority-Black districts



# RACE AND PARTY ENTWINED

.....

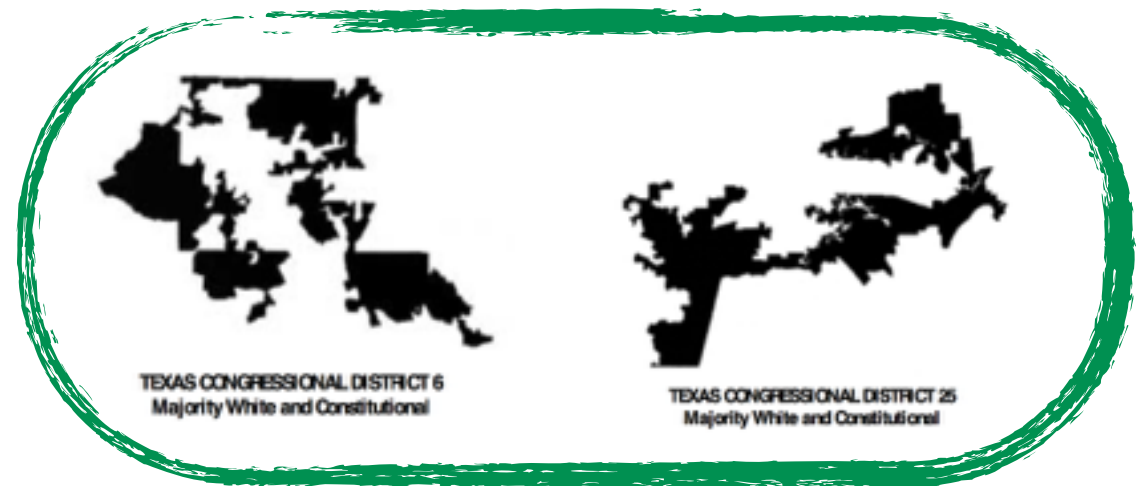
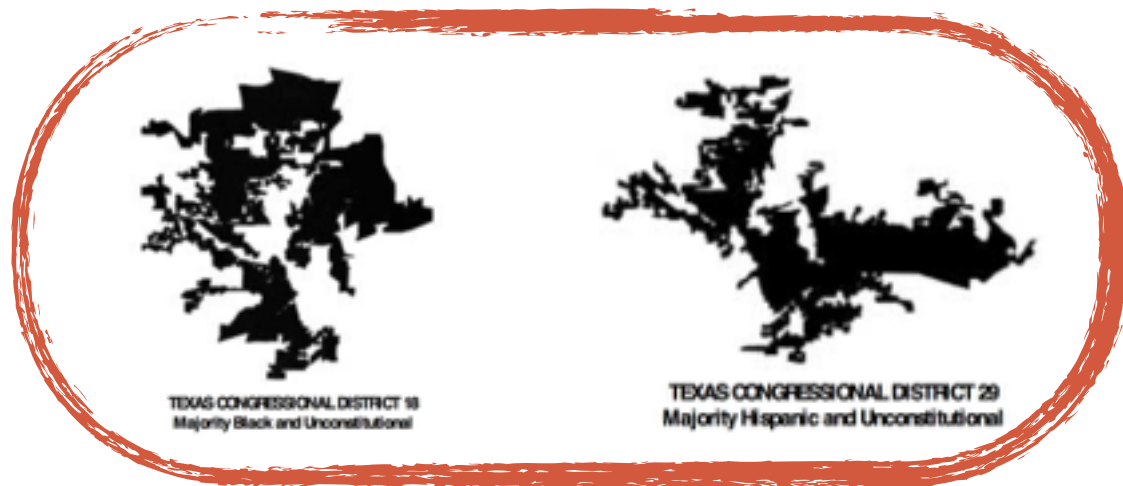
- These days, pronounced “conjoined polarization” effects
- Race can be a very effective proxy for party preference, and vice versa



# “SHAW LINE” AND BEYOND

---




- Court has derided “tortured,” “irregular,” “bizarre,” “irrational” shapes, and has thrown out maps on the basis of shape, when race is in the mix—but no standard
- Shaw v Reno, Bush v Vera, etc: on one hand, no standard; on the other, grumbling about “endless beauty contests” (!)



- We’re left with a muddle.




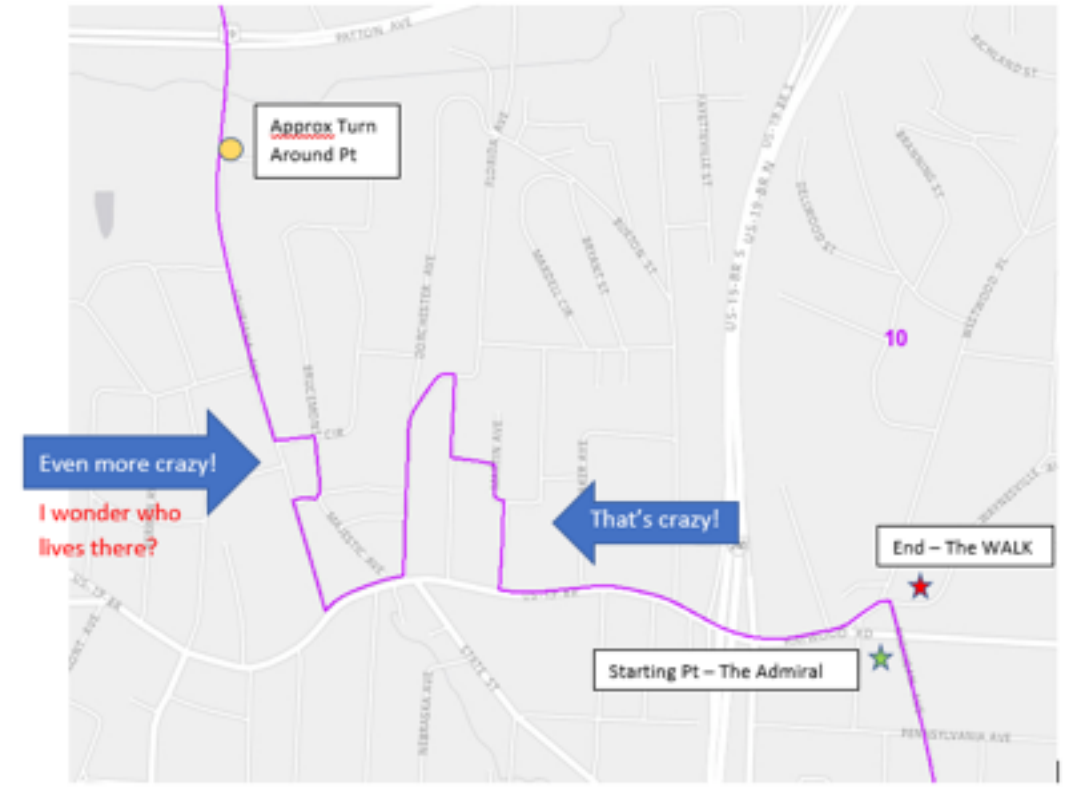
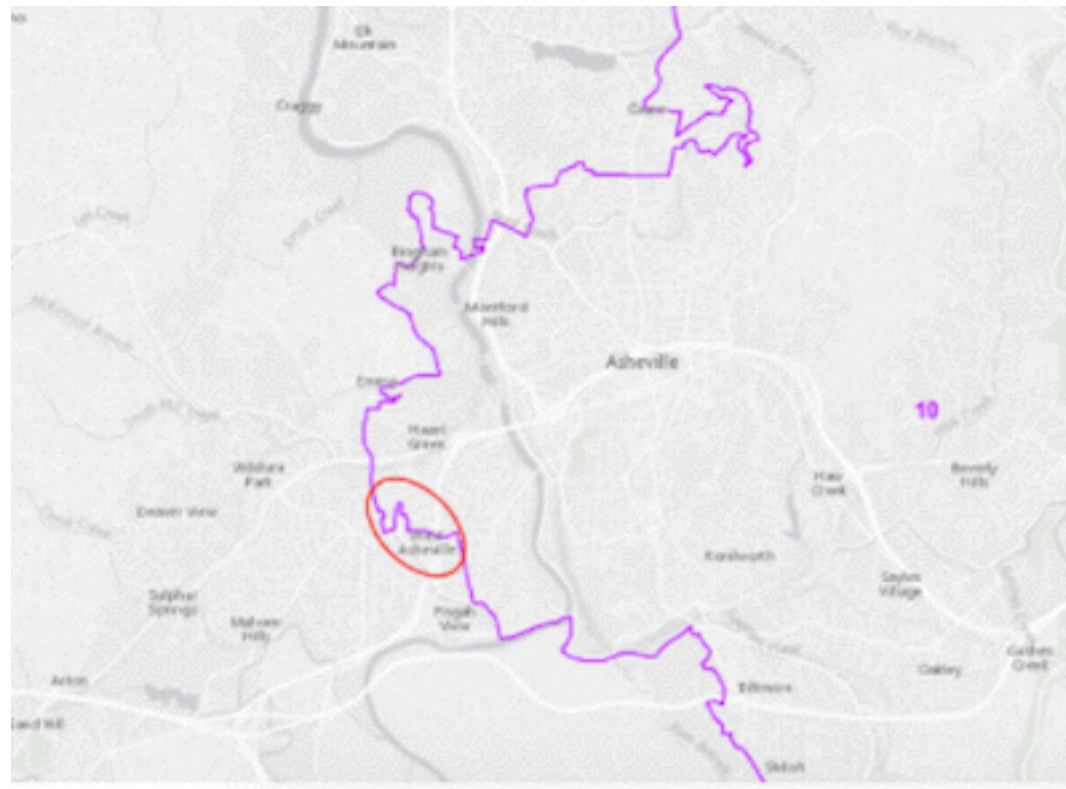
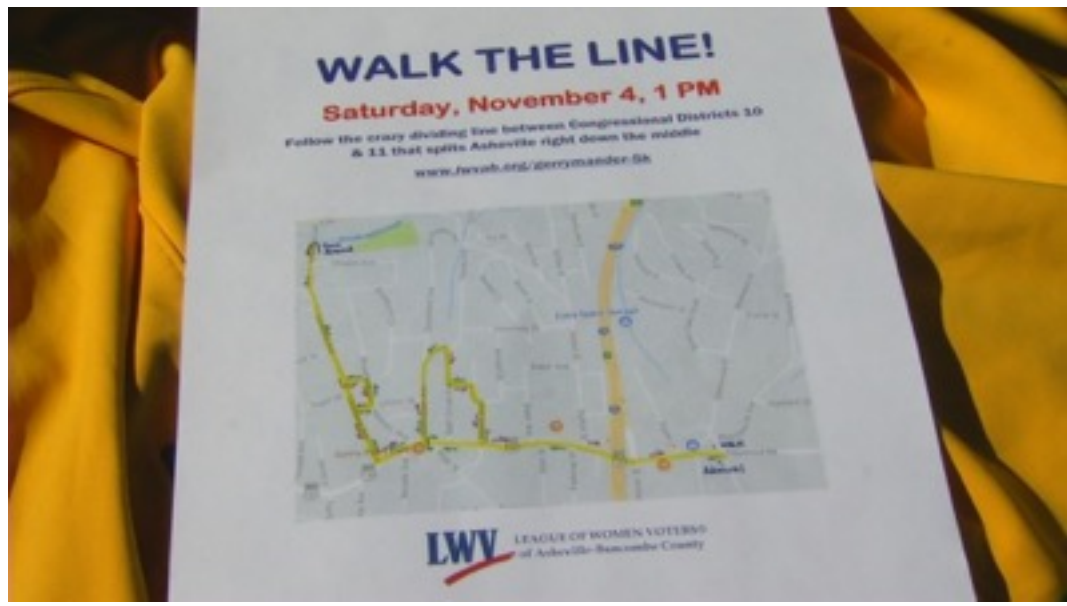
AMERICA

# Not-So-Fun Run: Joggers In 'Gerrymander 5K' Must Run Oddly Shaped Route

October 30, 2017 · 6:01 PM ET

LAUREL WAMSLEY 



**SO, WHAT ARE THE  
RULES?**



# DISTRICTING PRINCIPLES, TRADITIONAL AND OTHER

---

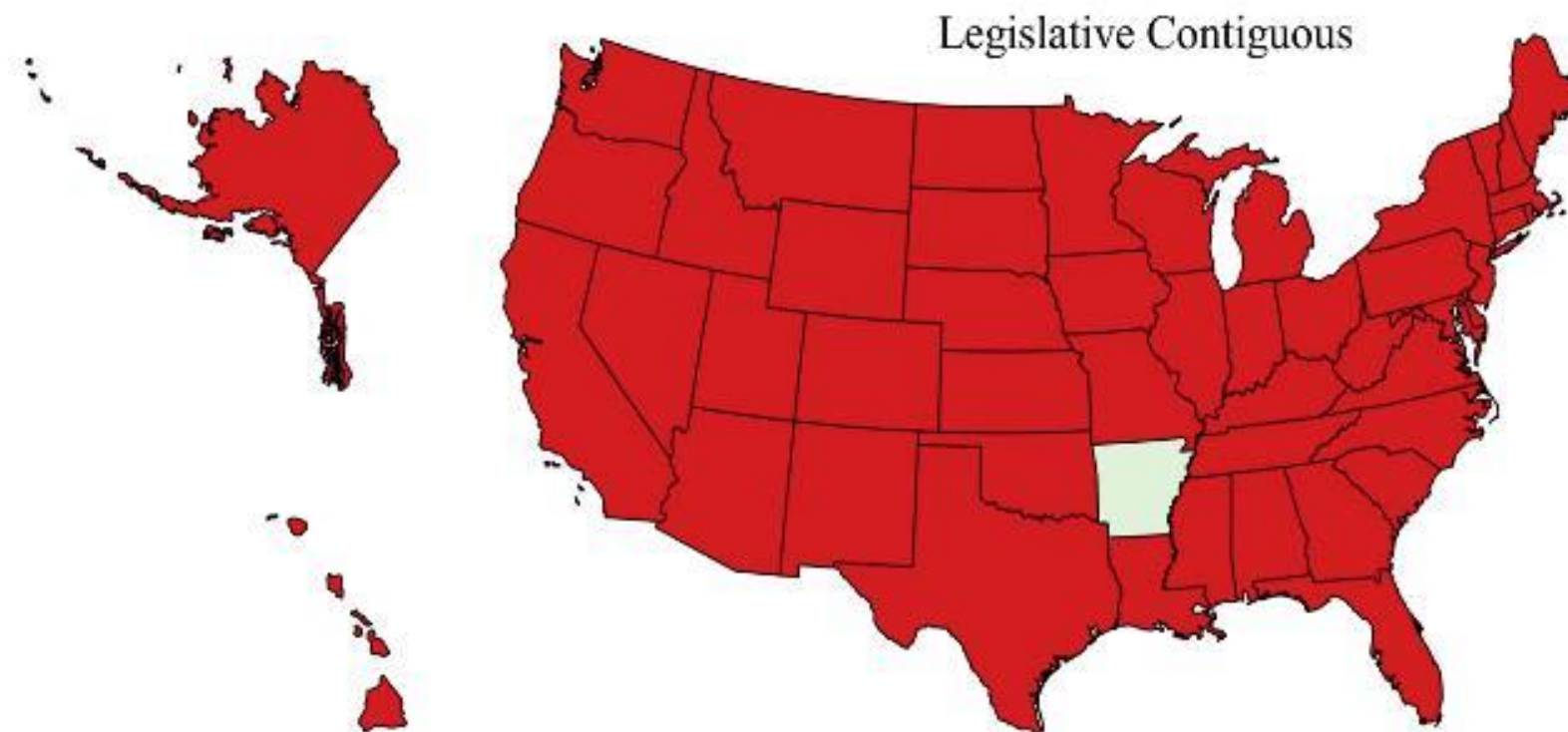
- Overview of what principles redistricting bodies **can** / **must** / **can't** take into account.
- First, **population equality** is taken quite seriously nationwide.

*Massachusetts districts from 113th Congress, by 2010 population*

1	2	3	4	5	6	7	8	9
727515	727514	732090	727514	727515	731681	727514	732884	727514

(largest deviation <0.7%)

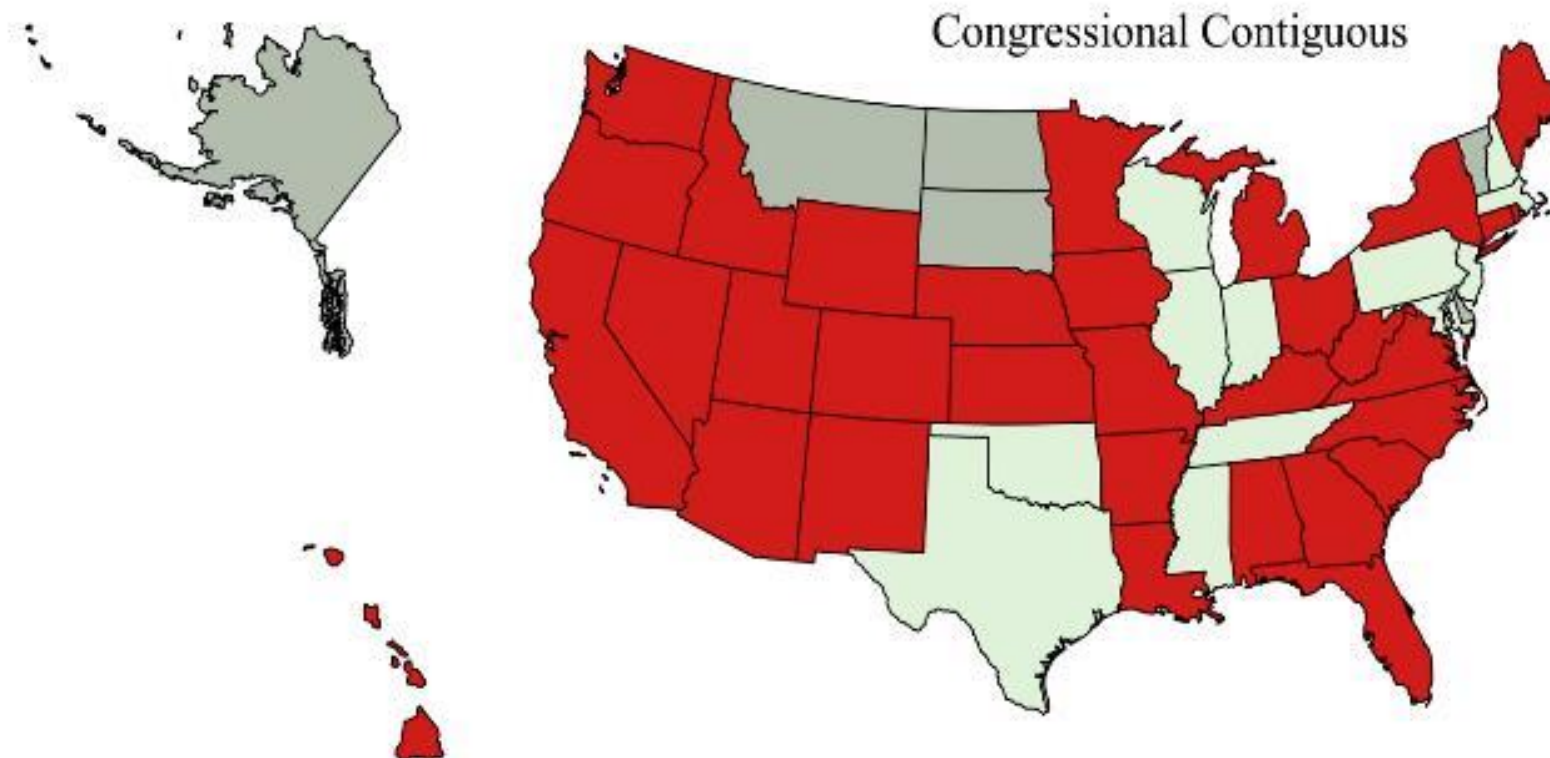
- There are two other principles that sound mathematical: **contiguity** and **compactness**.

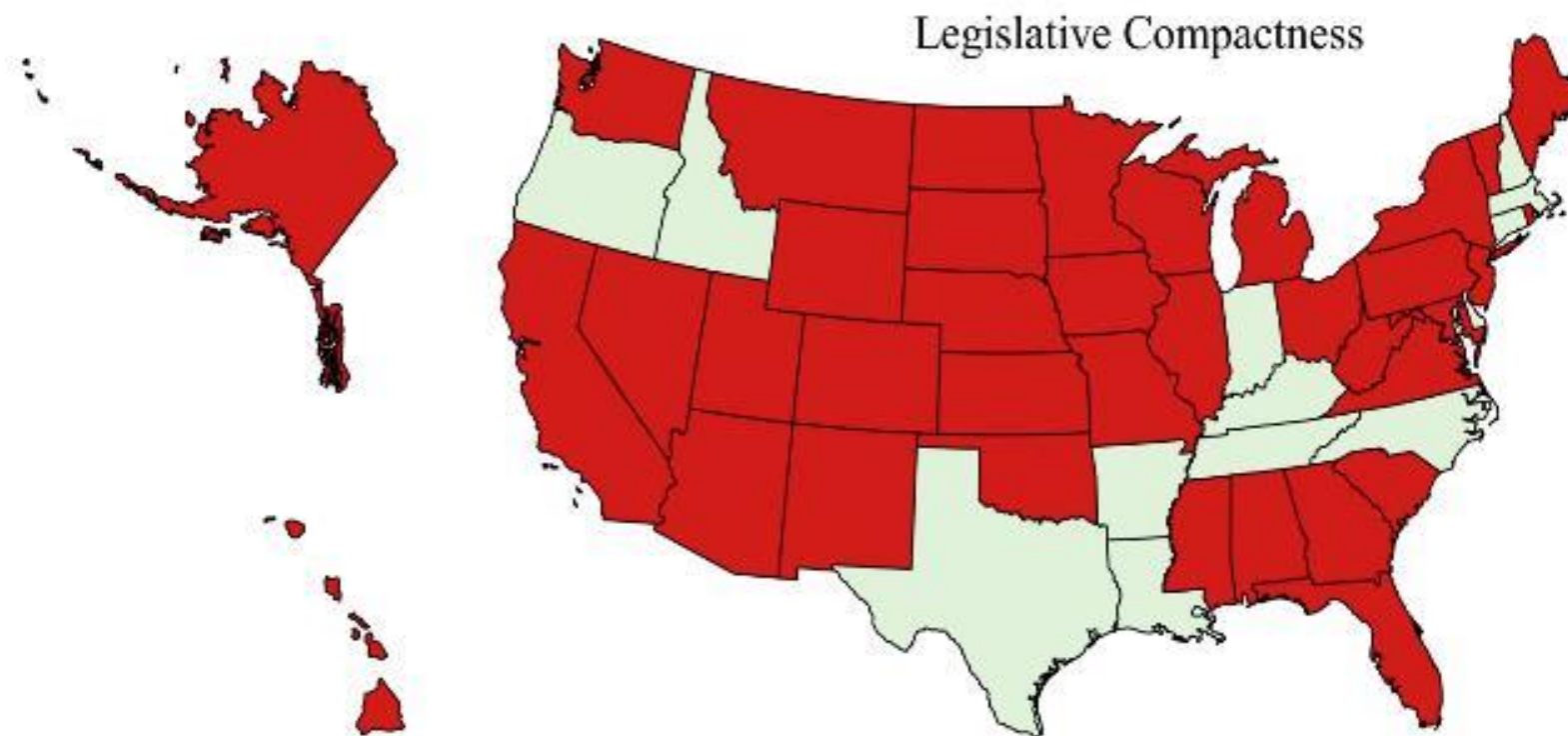


## Contiguous

- Consideration Required
- Not Specified
- N/A

- Judicially recognized in *Shaw v. Reno* (1993)
- Districts can't be in geographically separate pieces
- Relatively easy and non-controversial

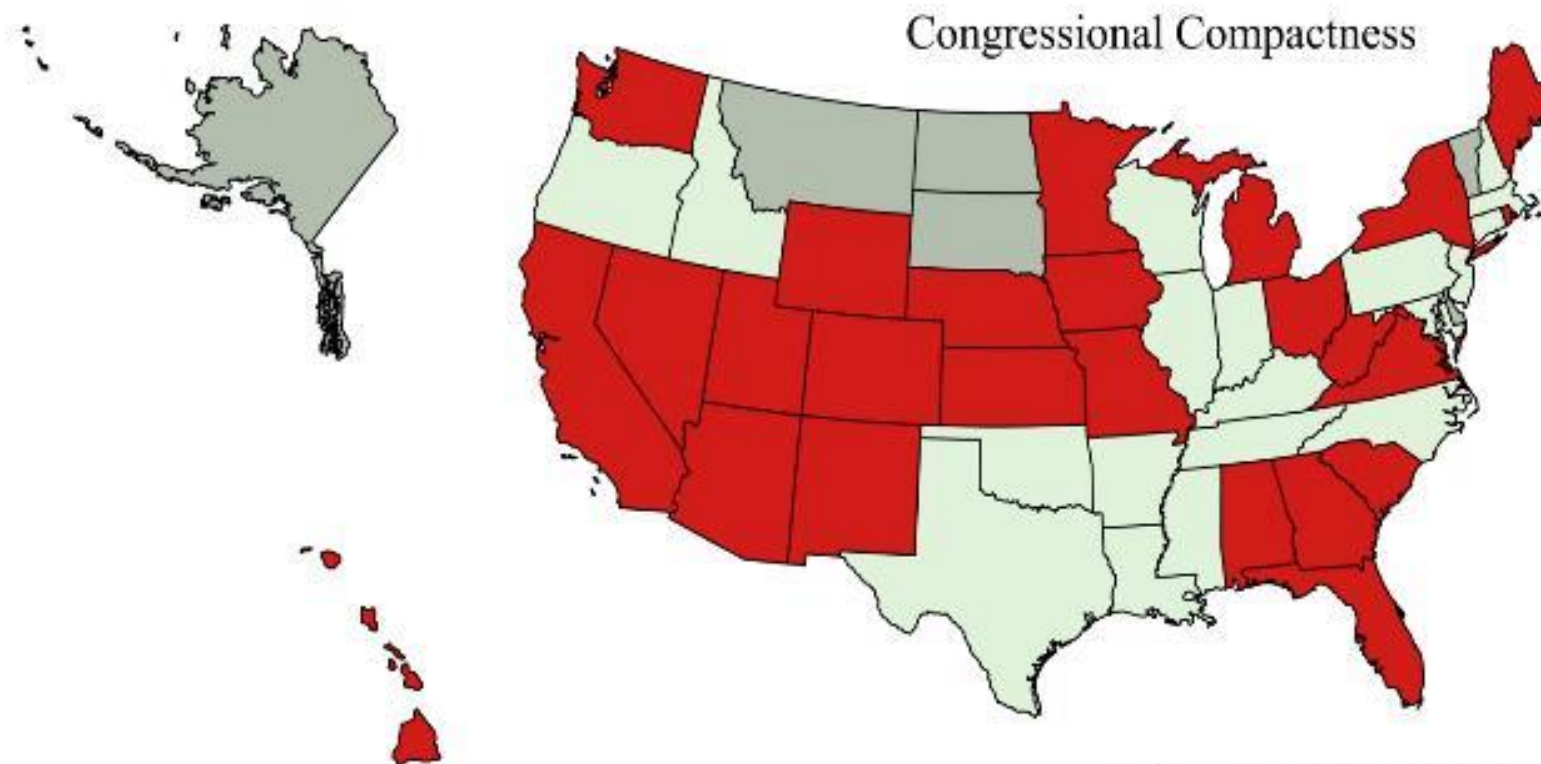




### Compactness

- Consideration Required
- Not Specified
- N/A

- Judicially recognized in *Shaw v. Reno* (1993)
- Geographic compactness
- Few jurisdictions define compactness



# LESS MATHY-SOUNDING REQUIREMENTS

---

- There are three more widespread principles and then some scattered others.
  - **VRA:** All states are bound by the federal Voting Rights Act of 1965, which takes minority representation into account.
  - **Political boundaries:** avoid splitting cities/counties/towns.
  - **Communities of interest:** keep them together when possible.



# VOTING RIGHTS ACT OF 1965

---

- Originally aimed at eliminating devices blocking the black vote
- Some historically problematic regions (see: poll tax) had to pass “preclearance” with all new plans (until 2013)
- VRA frequently renewed and expanded

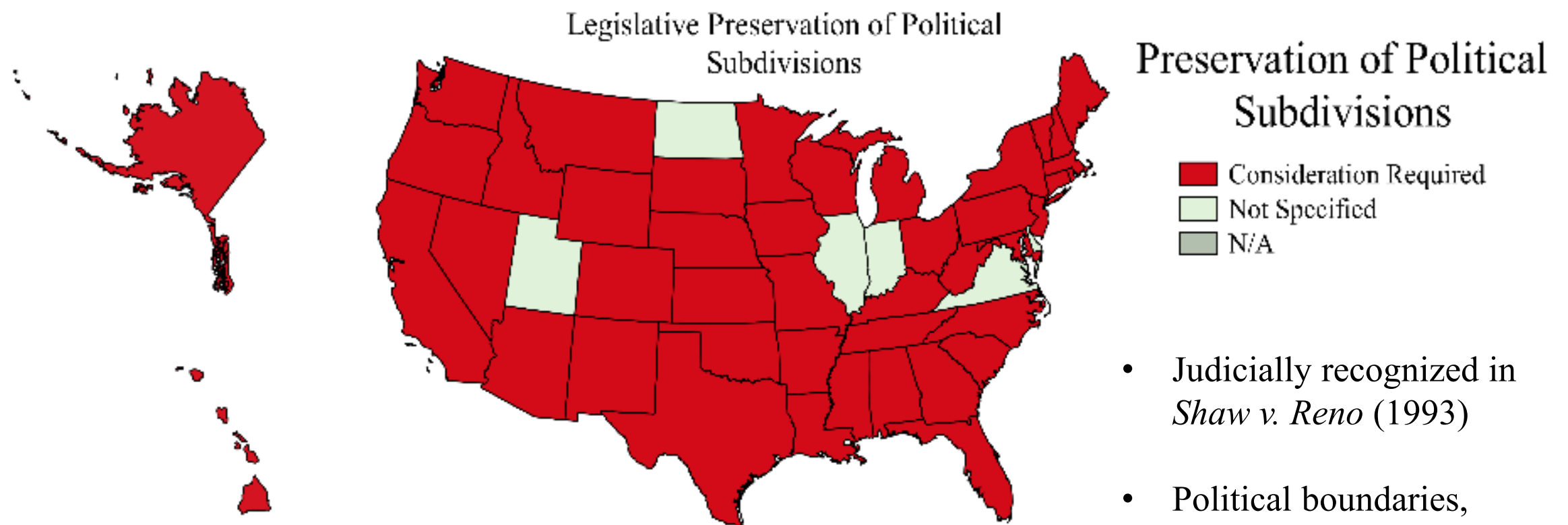
- ★ Language minorities added (1975)
- ★ **Results not intents** count (1982)
- ★ “Gingles factors” to detect vote denial/dilution (1986)

*Group sufficiently large and compact?*

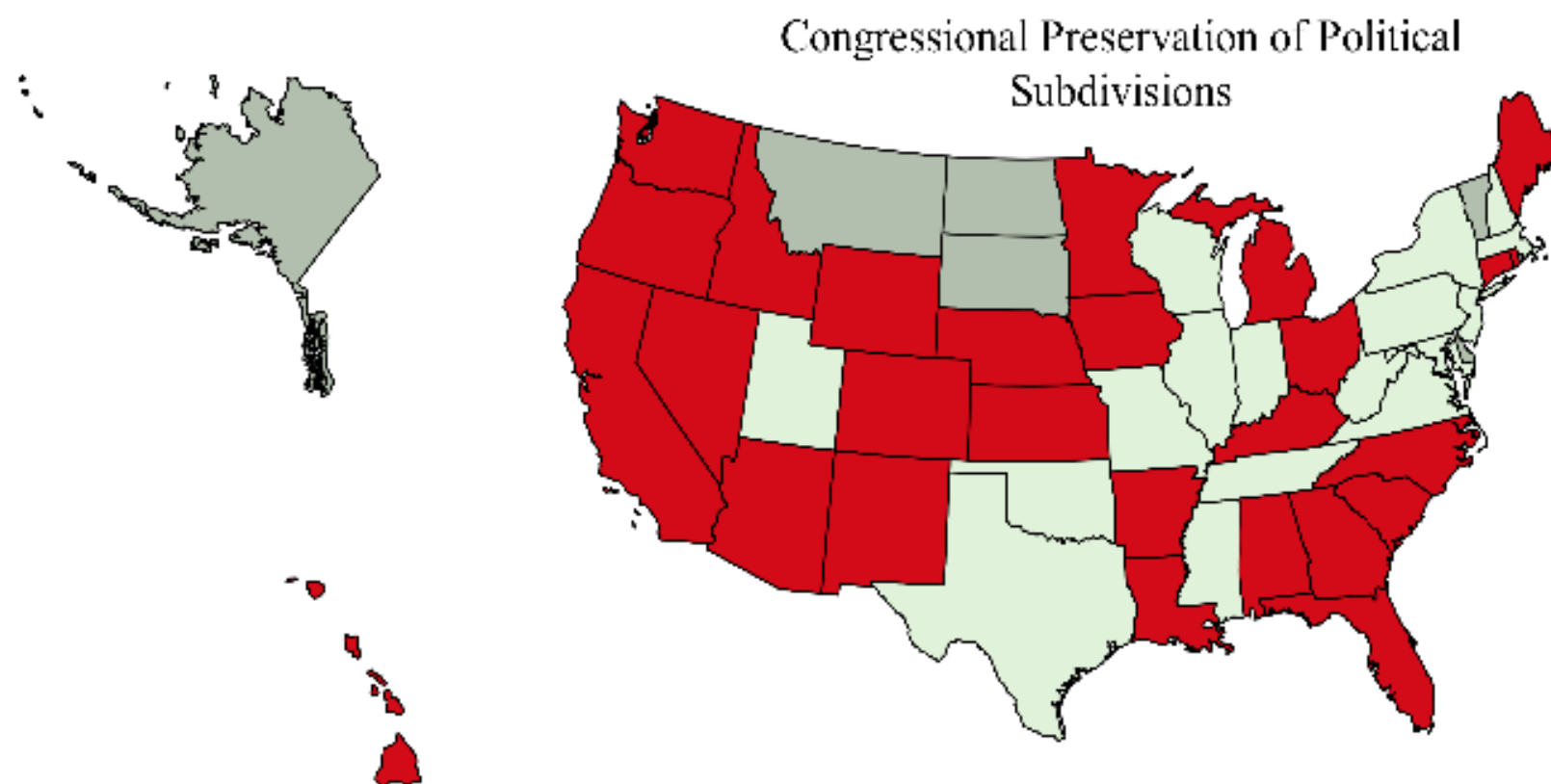
*Minority votes as a bloc?*

*Majority bloc votes against minority?*





- Judicially recognized in *Shaw v. Reno* (1993)
- Political boundaries, e.g. counties, cities, wards



- Not always clear cut
- Splitting jurisdictions

(maps made by NCSL and borrowed from Megan Gall, LCCR/NAACP LDF)

A map of Alaska and Hawaii. The state of Alaska is shown in red, with the Aleutian Islands chain extending from the southwestern tip. The Hawaiian Islands are shown in red in the lower right. A small black rectangle is located on the Aleutian Islands, indicating the study area.



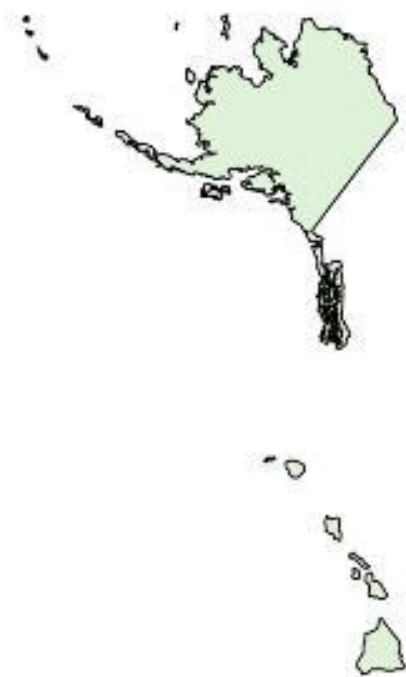
## A map of the United States showing the state of Alaska in light green and the Hawaiian Islands in red. The map is oriented with Alaska at the top and Hawaii at the bottom.



- Judicially recognized in *Abrams v. Johnson* (1997)
- Groups with similar geography, social interactions, trade, interests, or political ties
  - Non-racial communities of interest
- A subjective concept

see: James Gardner, *Representation without Party*, p937





Legislative Preservation of District  
Cores



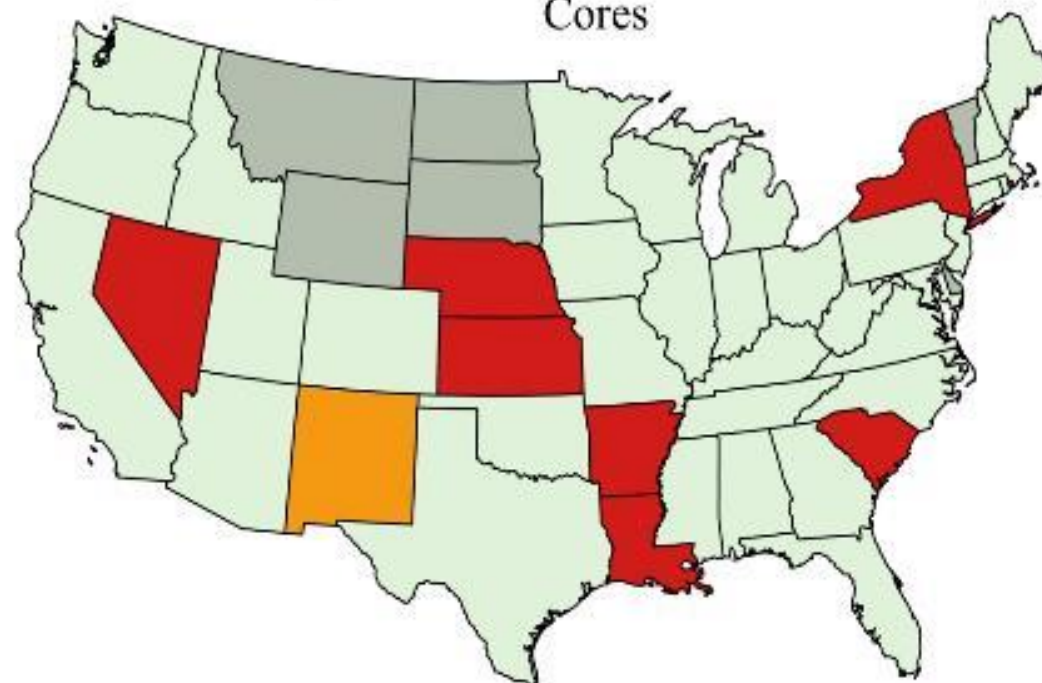
Preservation of District  
Cores

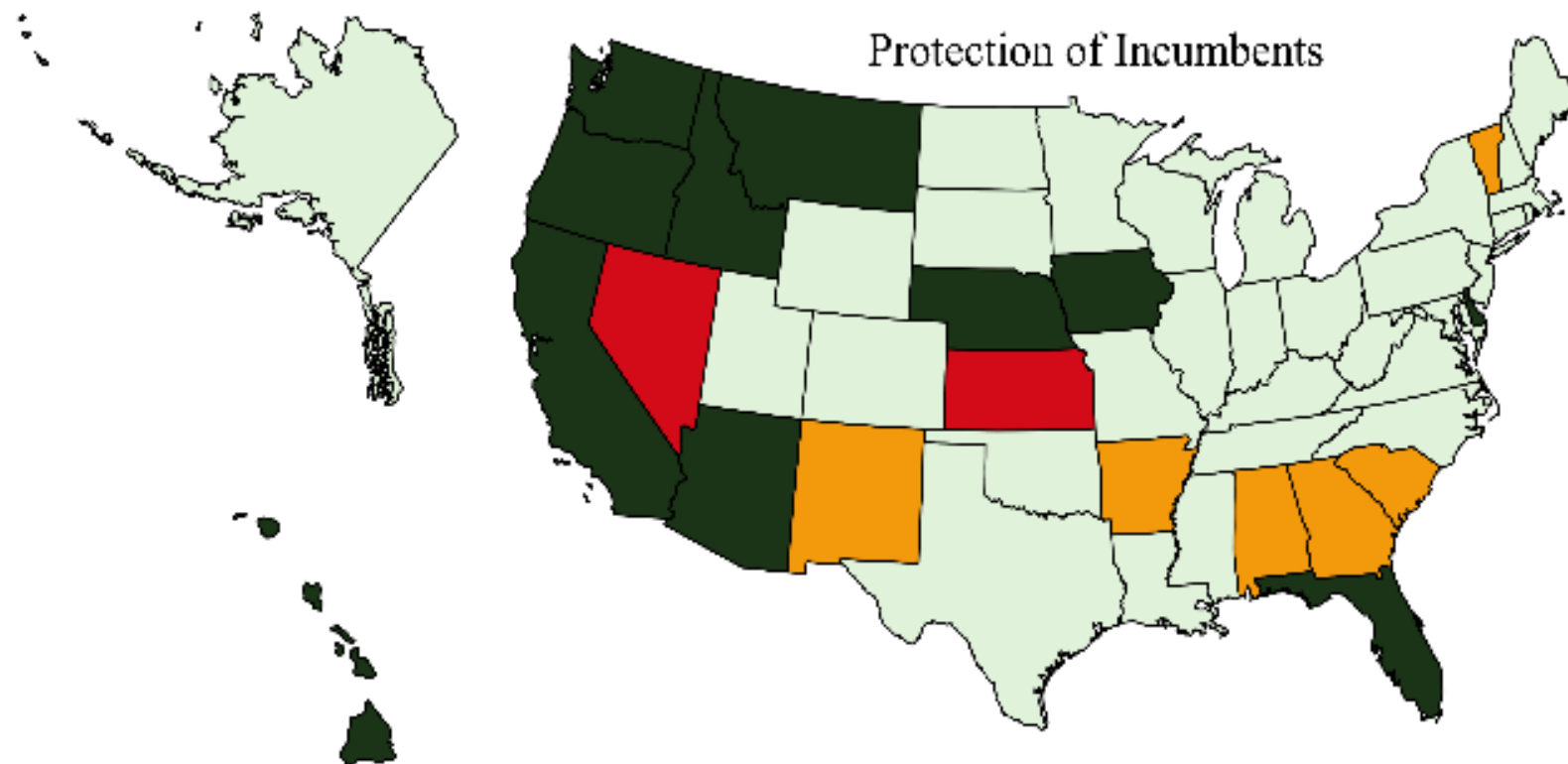
- Consideration Required
- Consideration Allowed
- Not Specified
- N/A

- Judicially recognized in *Abrams v. Johnson* (1997)
- Preserving prior district cores








Congressional Preservation of District  
Cores

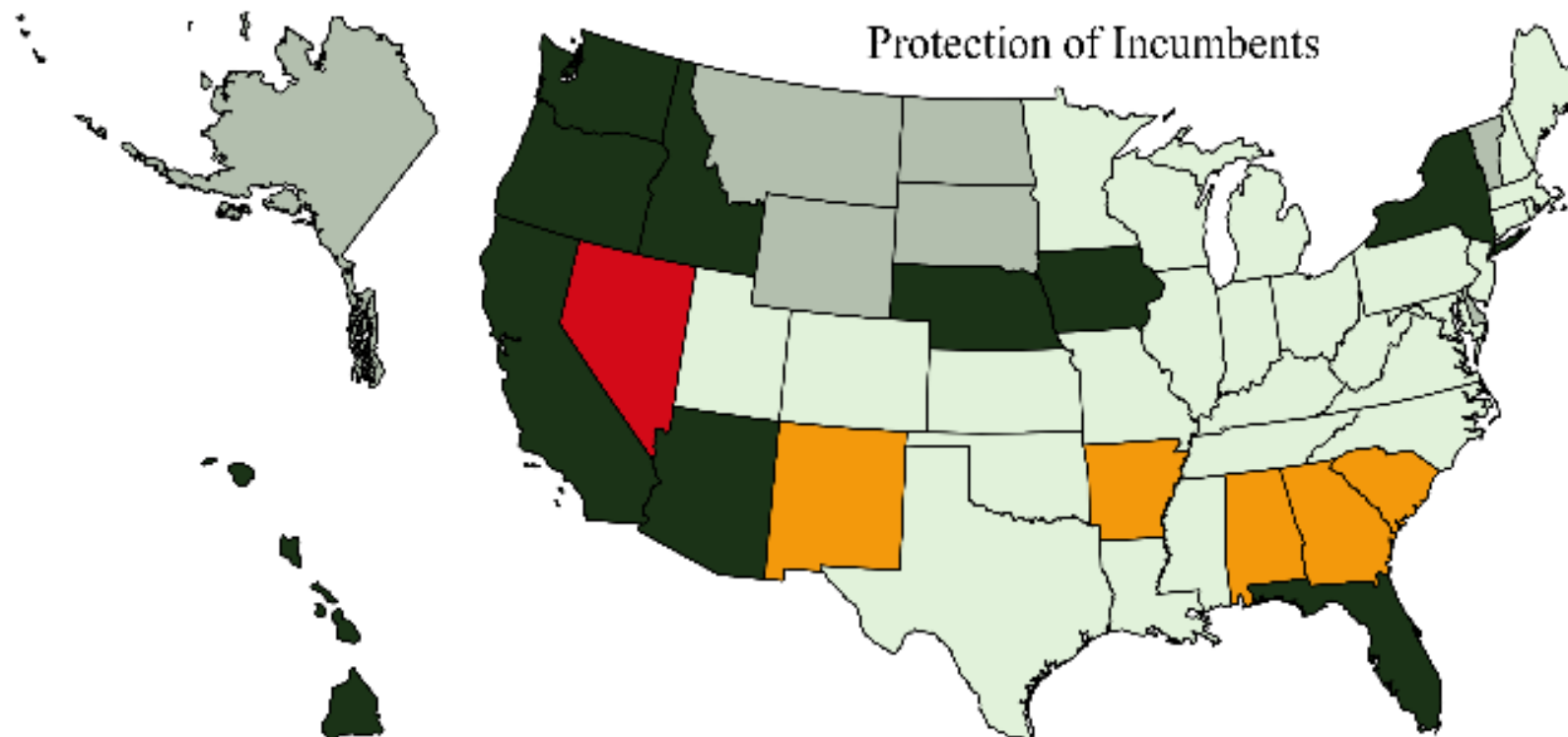




## Protection of Incumbents

-  Consideration Required  
 Consideration Allowed  
 Consideration Prohibited  
 Not Specified  
 N/A

- Judicially recognized in *Abrams v. Johnson* (1997)
- Exactly what it sounds like
- Only principle that is prohibited in some areas



Data Citation: National Conference of State Legislatures, accessed in June 2017, <http://www.ncsl.org/research/redistricting/redistricting->

**HOW CAN WE MEASURE COMPLIANCE  
WITH THESE RULES?**

**LET'S TRY TO MEASURE  
COMPACTNESS  
FOR STARTERS**

# HOW IS COMPACTNESS MEASURED?

---

► There are many metrics in the literature, mostly classifiable three ways:

★ **isoperimetry** - a measure of *efficiency*

- ◉ **Polsby-Popper**:  $A/P^2$  and variants
- ◉ total perimeter

★ **convexity** - a measure of *indentedness*

- ◉ **Reock**: compare to circumcircle
- ◉ compare to convex hull

★ **dispersion** - a measure of *sprawl*

- ◉ average distance between points
- ◉ moment of inertia

*Louisiana House of Reps v. Ashcroft*  
*Martinez v. Bush*

*Perez v. Perry*  
*Vesilind v. VA State Board of Elections*  
*Page v. Judd*

*Sanders v. Dooly County*  
*Sessions v. Texas*

*Session v. Perry*  
*U.S.v. County of Los Angeles*  
*Harris v. McCrory*

*Johnson v. Mille*  
*Cromartie v. Hunt*  
*Moon v. Meadows*

*The City of Greensboro et al v.*  
*Guilford County Board of Elections*  
*Romo v. Detzner*

*Missouri NAACP v. Ferguson-*  
*Florissant School District*  
*Whitford v. Nichol*

# ISOPERIMETRY / AREA VS. PERIMETER / POLSBY-POPPER

---

- Suppose you have a district of area  $A$  and perimeter  $P$ . You could create a score of the form  $A/P$  or  $A/P^2$ .
- Why  $A/P^2$ ? It seems to protect you from scale effects; when you dilate a shape by a factor  $k$ , the perimeter is scaled by  $k$  and the area by  $k^2$ , so this score is invariant.



# RIGOROUS BOUNDS

---

## Isoperimetric Theorem (Steiner 1838):

For any shape with area  $A$  and perimeter  $P$ ,

$$A/P^2 \leq 1/4\pi,$$

with equality only for circles.

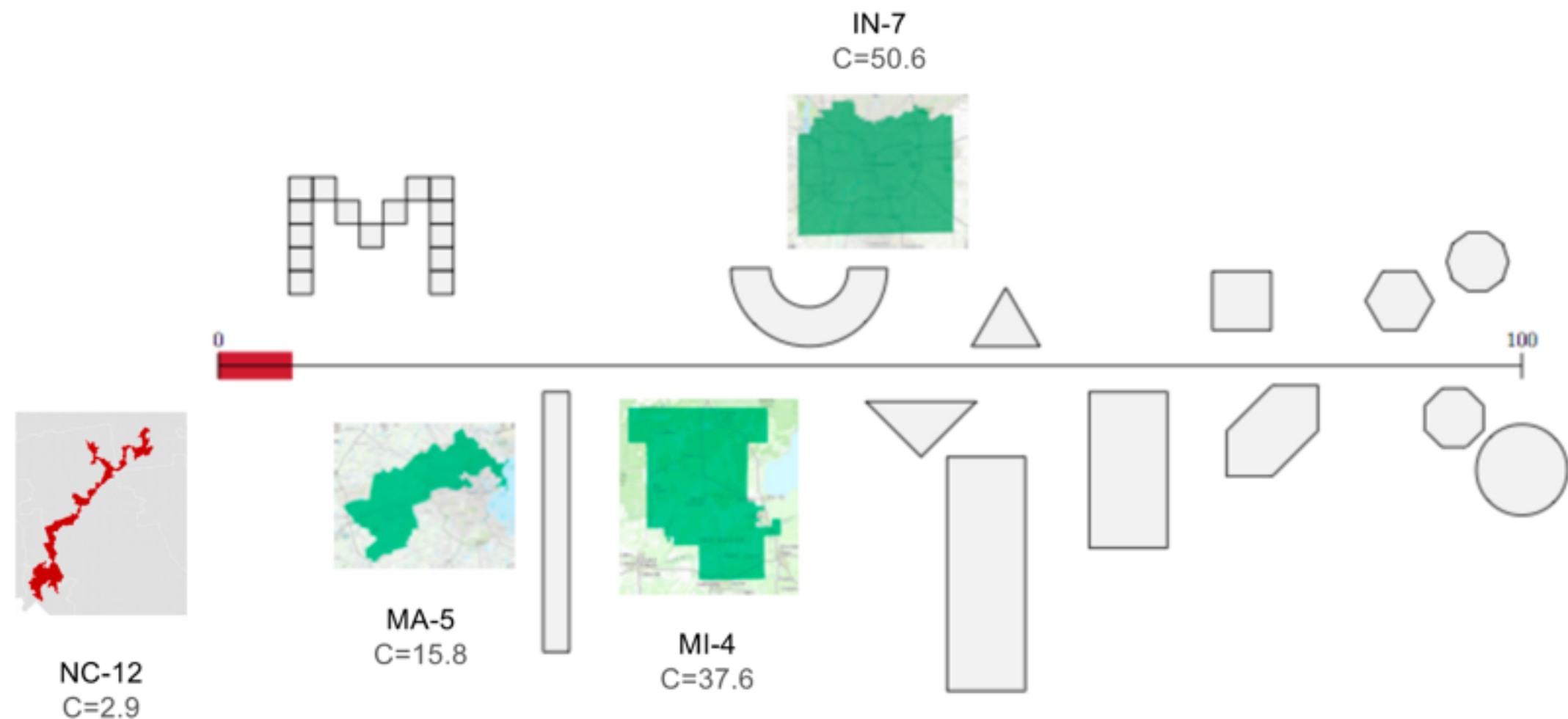
- So for any shape  $S$ , if we define  $C(S) = 400\pi A/P^2$ , we get a nice statistic of shape efficiency, because

$$0 \leq 400\pi A/P^2 \leq 100.$$



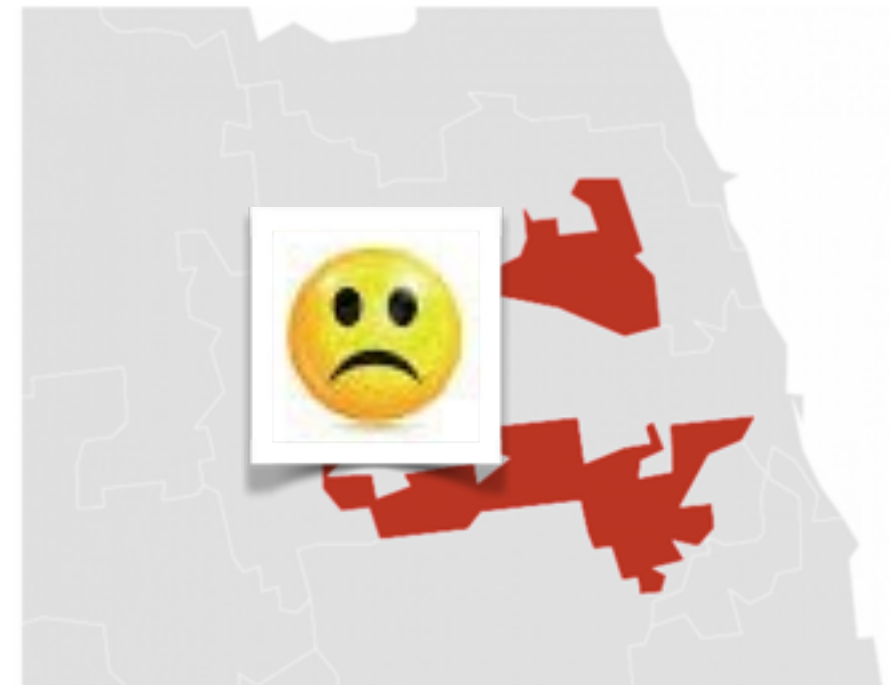
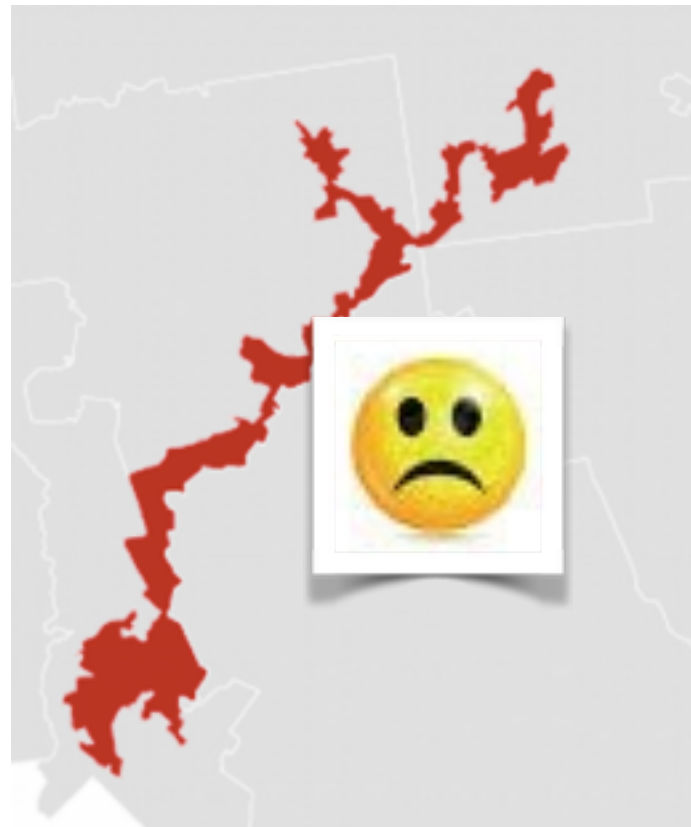
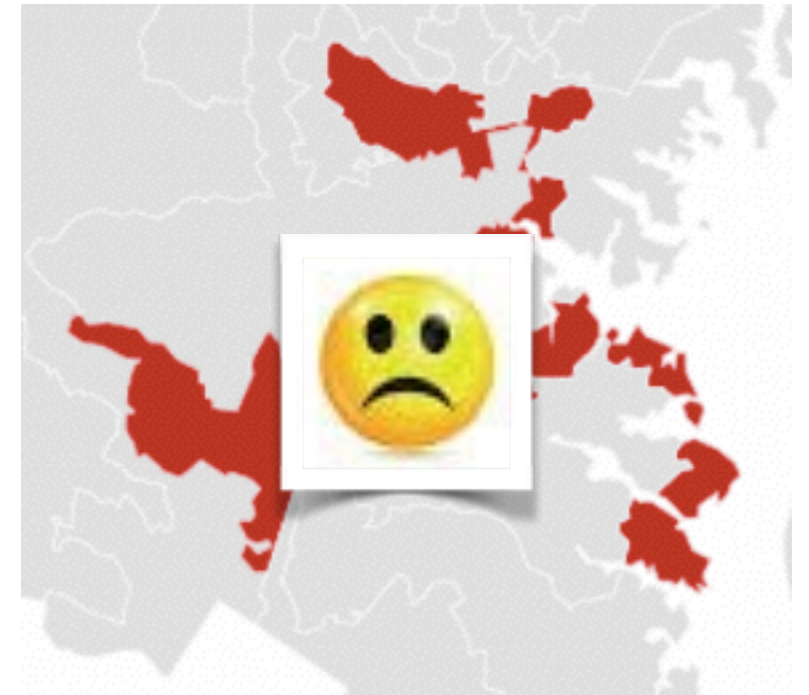
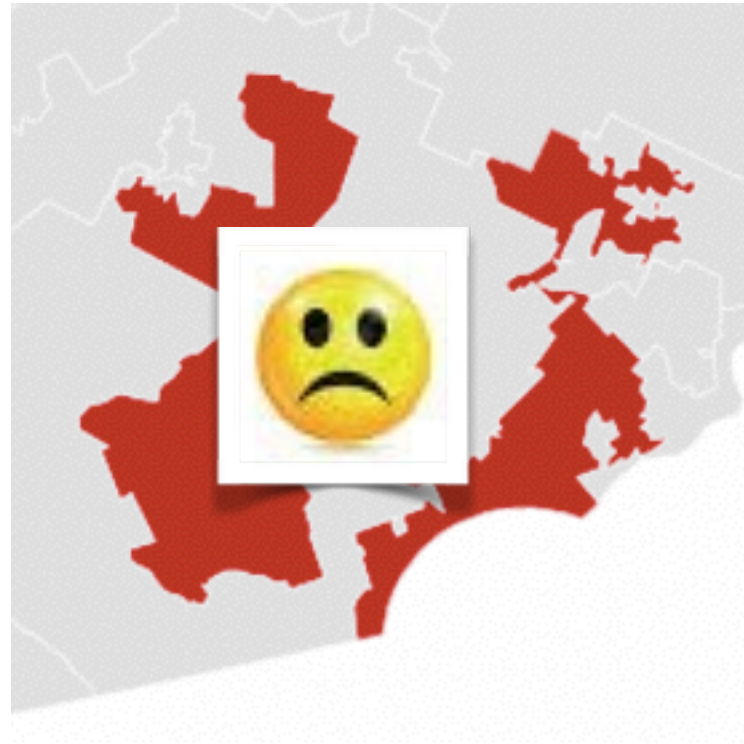
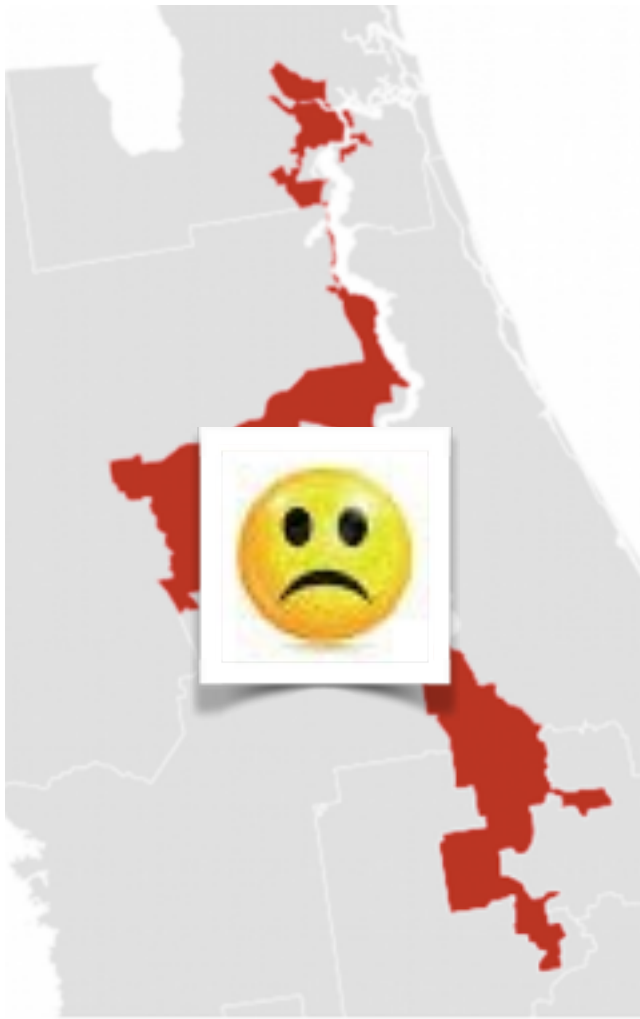
# A RANGE OF ISOPERIMETRIC RATIOS

- This compactness score  $C(S) = 400\pi A/P^2$  works by *comparing the area of a region by the area of a circle of the same perimeter*.



- Idea: circles are the most efficient, so you're dividing (actual area) by (max possible area). This gives you a *percentage efficiency* for any shape.





# INDENTATION / CONVEXITY / REOCK

---

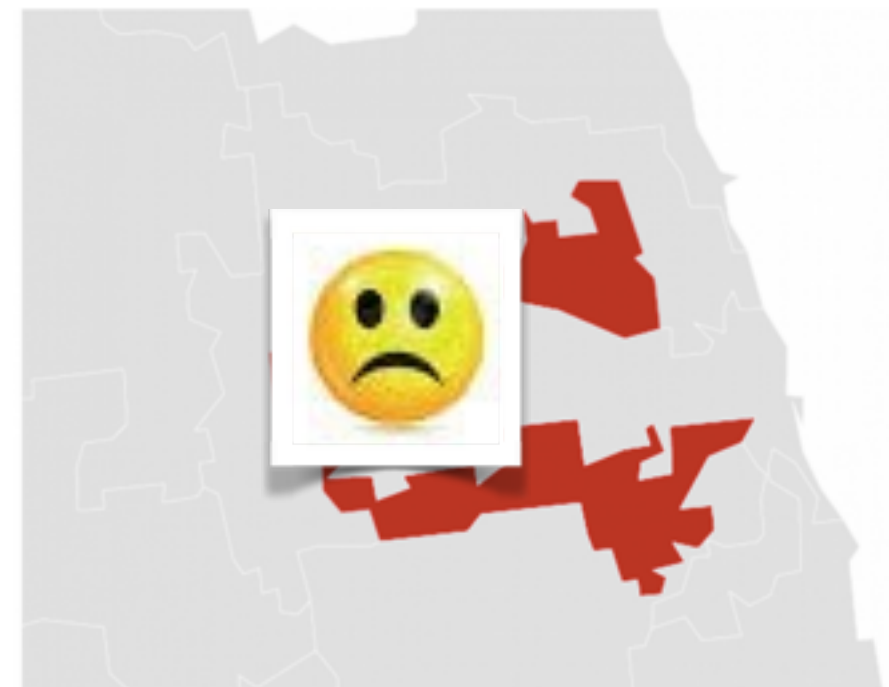
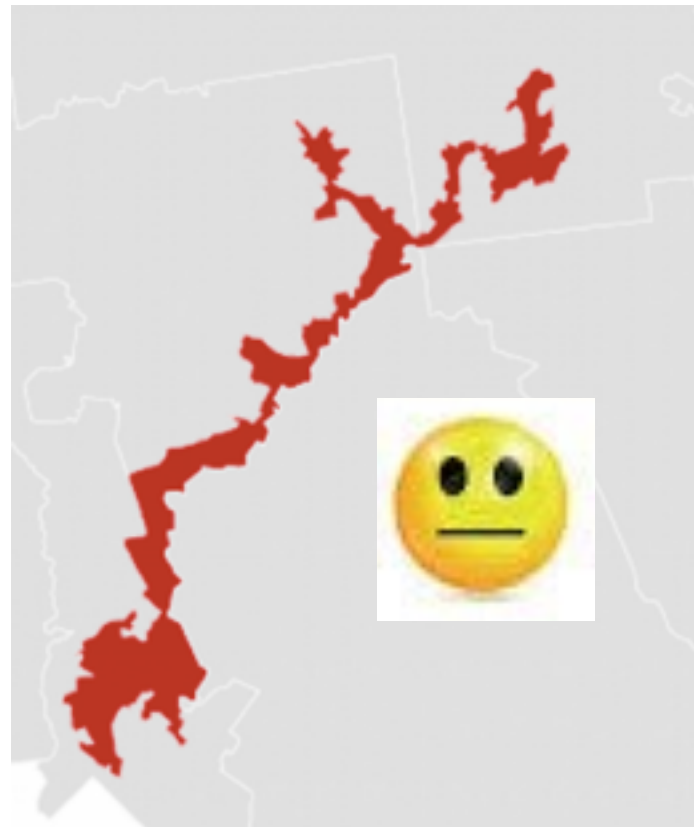
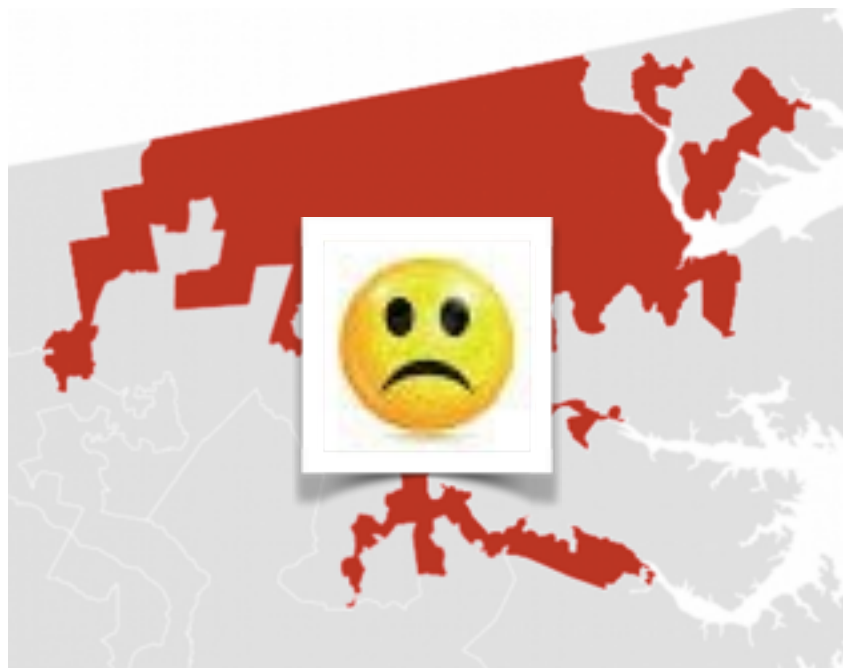
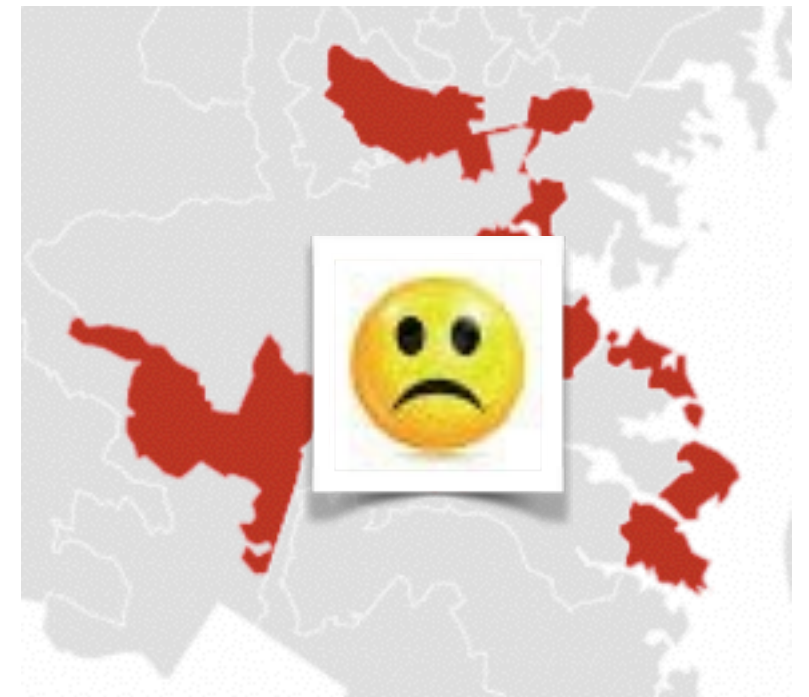
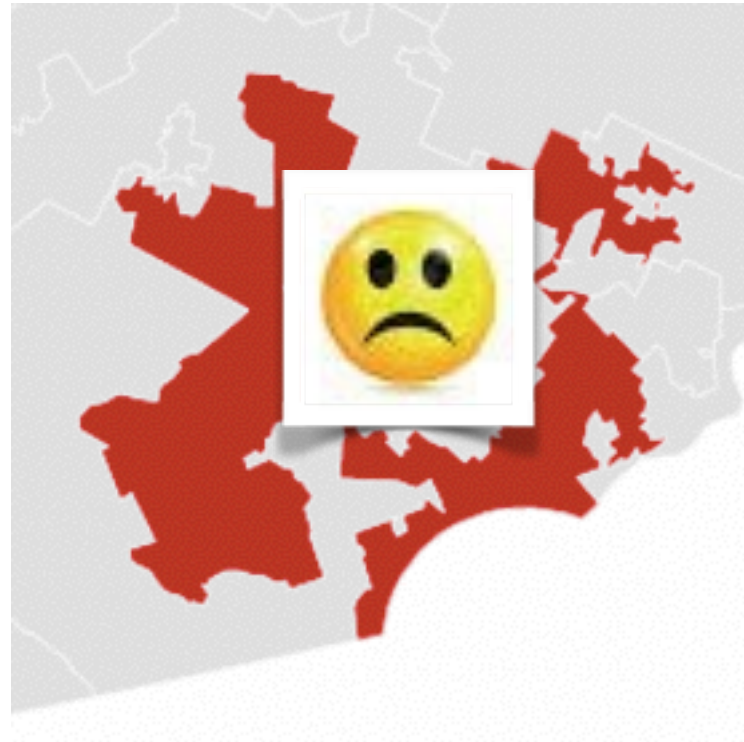
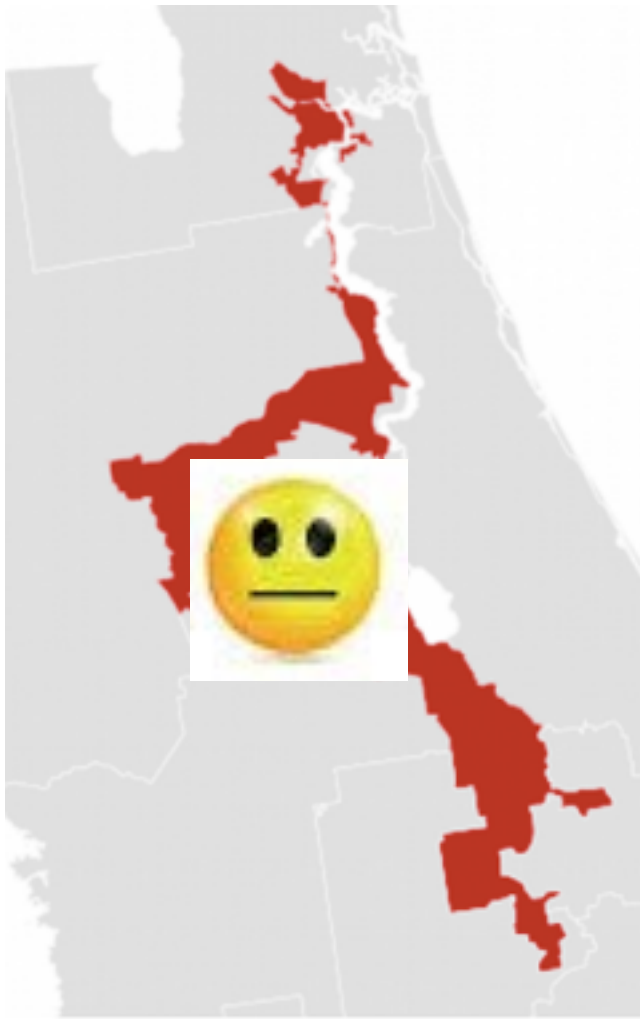
- Mathematically, a region is **convex** if it contains the line segment between any two of its points
- The **convex hull** is the “rubber-band enclosure”–smallest convex body containing the region

- **Convex hull score:**  
 $A(\text{region})/A(\text{conv hull})$



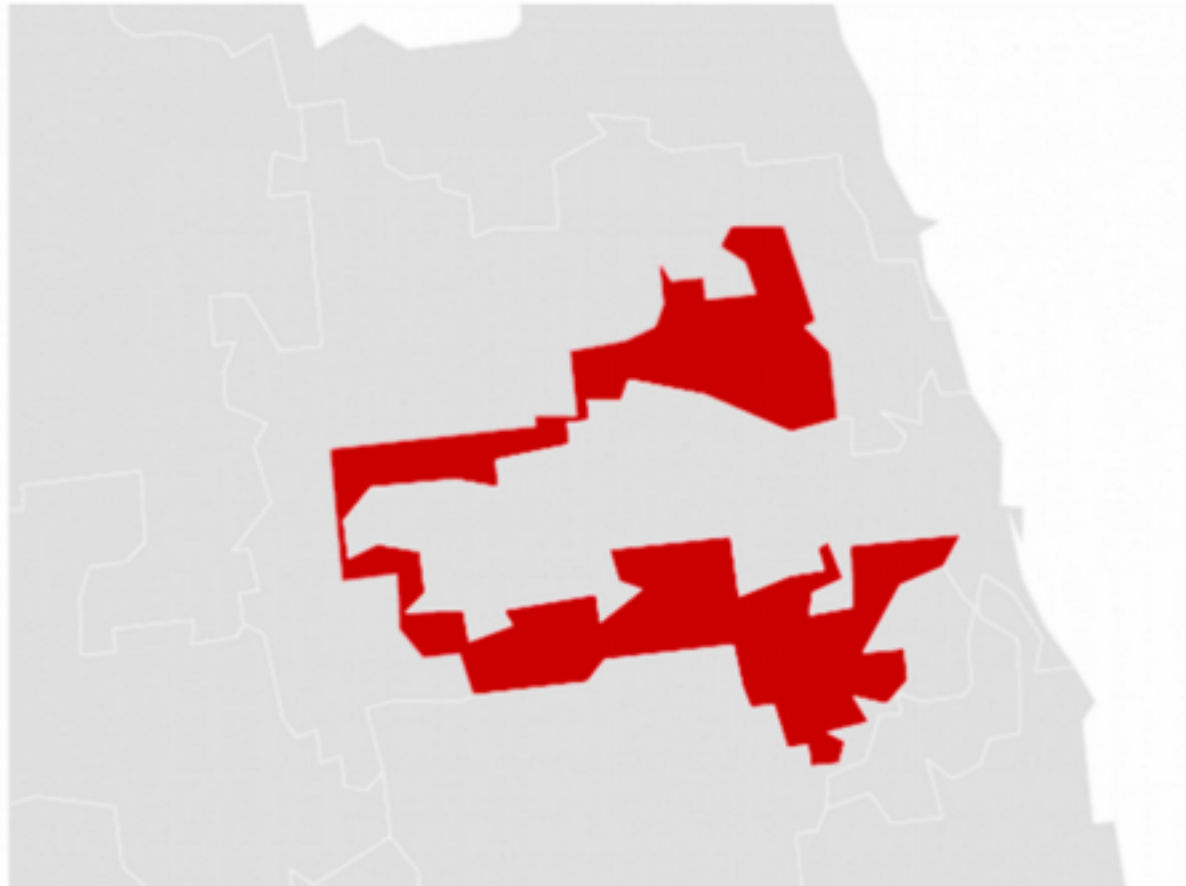
- **Reock score:**  
 $A(\text{region})/$   
 $A(\text{circumcircle})$







# SO, DOES IT WORK?



Year	Democrat	Votes	Pct
1992	Luis Gutiérrez	90,452	77.6%
1994	Luis Gutiérrez (inc.)	46,695	75.2%
1996	Luis Gutiérrez (inc.)	85,278	93.6%
1998	Luis Gutiérrez (inc.)	54,244	81.7%
2000	Luis Gutiérrez (inc.)	89,487	88.6%
2002	Luis Gutiérrez (inc.)	67,339	79.7%
2004	Luis Gutiérrez (inc.)	104,761	83.7%
2006	Luis Gutiérrez (inc.)	69,910	85.8%
2008	Luis Gutiérrez (inc.)	112,529	80.6%
2010	Luis Gutiérrez (inc.)	63,273	77.4%
2012	Luis Gutiérrez (inc.)	133,226	83%
2014	Luis Gutiérrez (inc.)	79,666	78.1%
2016	Luis Gutiérrez (inc.)	171,297	100%

*Packing!*

...but it turns out to be *friendly* packing!

highway

Humboldt Park (Puerto Rican)

Pilsen (Mexican)



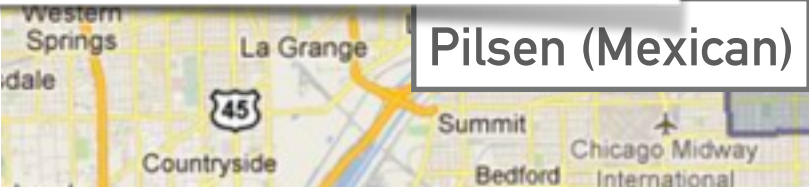


# SO, DOES IT WORK?

Year	Democrat	Votes	Pct
1992	Luis Gutiérrez	90,452	77.6%
1994	Luis Gutiérrez (inc.)	46,695	75.2%
1996	Luis Gutiérrez (inc.)	85,278	93.6%
1998	Luis Gutiérrez (inc.)	54,244	81.7%
2000	Luis Gutiérrez (inc.)	89,487	88.6%
2002	Luis Gutiérrez (inc.)	67,339	79.7%
2004	Luis Gutiérrez (inc.)	104,761	83.7%
2006	Luis Gutiérrez (inc.)	69,910	85.8%
2008	Luis Gutiérrez (inc.)	112,529	80.6%

*Packing!*


...but it turns out to be *friendly* packing!



(KINDA)

# STATES THAT DEFINE COMPACTNESS

.....

- **Arizona:** map should begin with a “grid-like pattern”; some talk of the score  $A/P^2$
- **California:** “To the extent practicable... districts must also encourage compactness, defined by lines that do not bypass nearby population in favor of more distant population.”
- **Colorado:** sum the perimeter of all districts 

the only one concrete enough to score a plan
- **Michigan:** “as compact as possible, measured by drawing a circle around the district, and assessing the area within the circle (and within the landmass of the state) but outside the district lines.”
- **Montana:** general appearance, and the degree to which it fosters "functional compactness" through "travel and transportation, communication, and geography."



# 2017 CODE OF IOWA, SECTION 42.4, REDISTRICTING STANDARDS

---

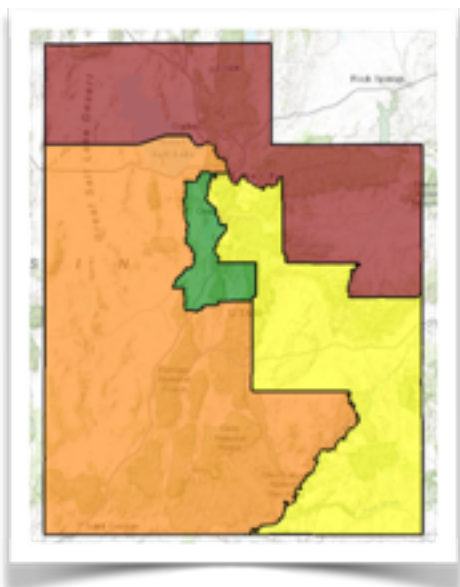
- Districts shall be **reasonably compact** in form, to the extent consistent with the standards established by subsections 1, 2, and 3. In general, reasonably compact districts are those which are square, rectangular, or hexagonal in shape, and not irregularly shaped, to the extent permitted by natural or political boundaries. If it is necessary to compare the relative compactness of two or more districts, or of two or more alternative districting plans, the tests prescribed by paragraphs “a” and “b” shall be used.
- a.**Length-width compactness.** The compactness of a district is greatest when the length of the district and the width of the district are equal. The measure of a district’s compactness is the absolute value of the difference between the length and the width of the district. In general, the length-width compactness of a district is calculated by measuring the distance from the northernmost point or portion of the boundary of a district to the southernmost point or portion of the boundary of the same district and the distance from the westernmost point or portion of the boundary of the district to the easternmost point or portion of the boundary of the same district. The absolute values computed for individual districts under this paragraph may be cumulated for all districts in a plan in order to compare the overall compactness of two or more alternative districting plans for the state, or for a portion of the state.
- b.**Perimeter compactness.** The compactness of a district is greatest when the distance needed to traverse the perimeter boundary of a district is as short as possible. The total perimeter distance computed for individual districts under this paragraph may be cumulated for all districts in a plan in order to compare the overall compactness of two or more alternative districting plans for the state, or for a portion of the state.



# SO WHAT IS COMPACTNESS?

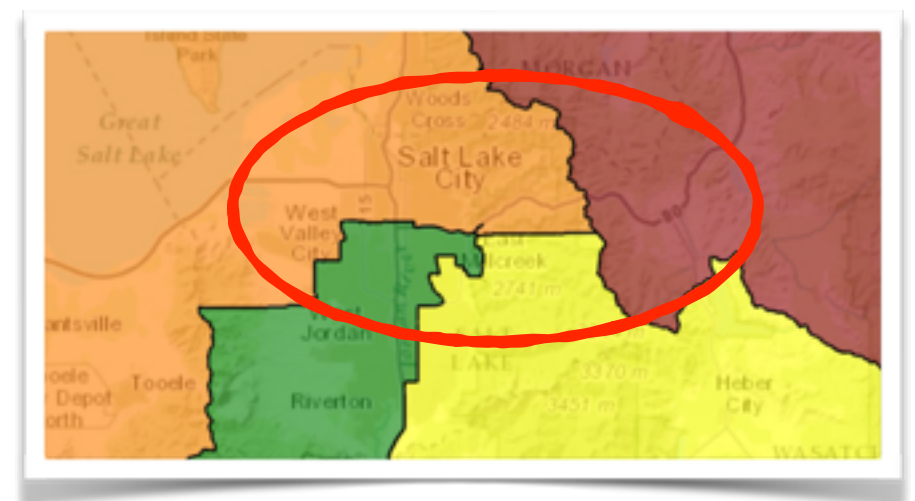
---

- In practice, usually: “you know it when you see it”
- i.e., eyeball test
- e.g., Utah debuted [redistrictutah.com](http://redistrictutah.com) to allow public creation of plans, listing compactness as a **requirement**...  
In practice, committee simply tossed maps that looked bad.



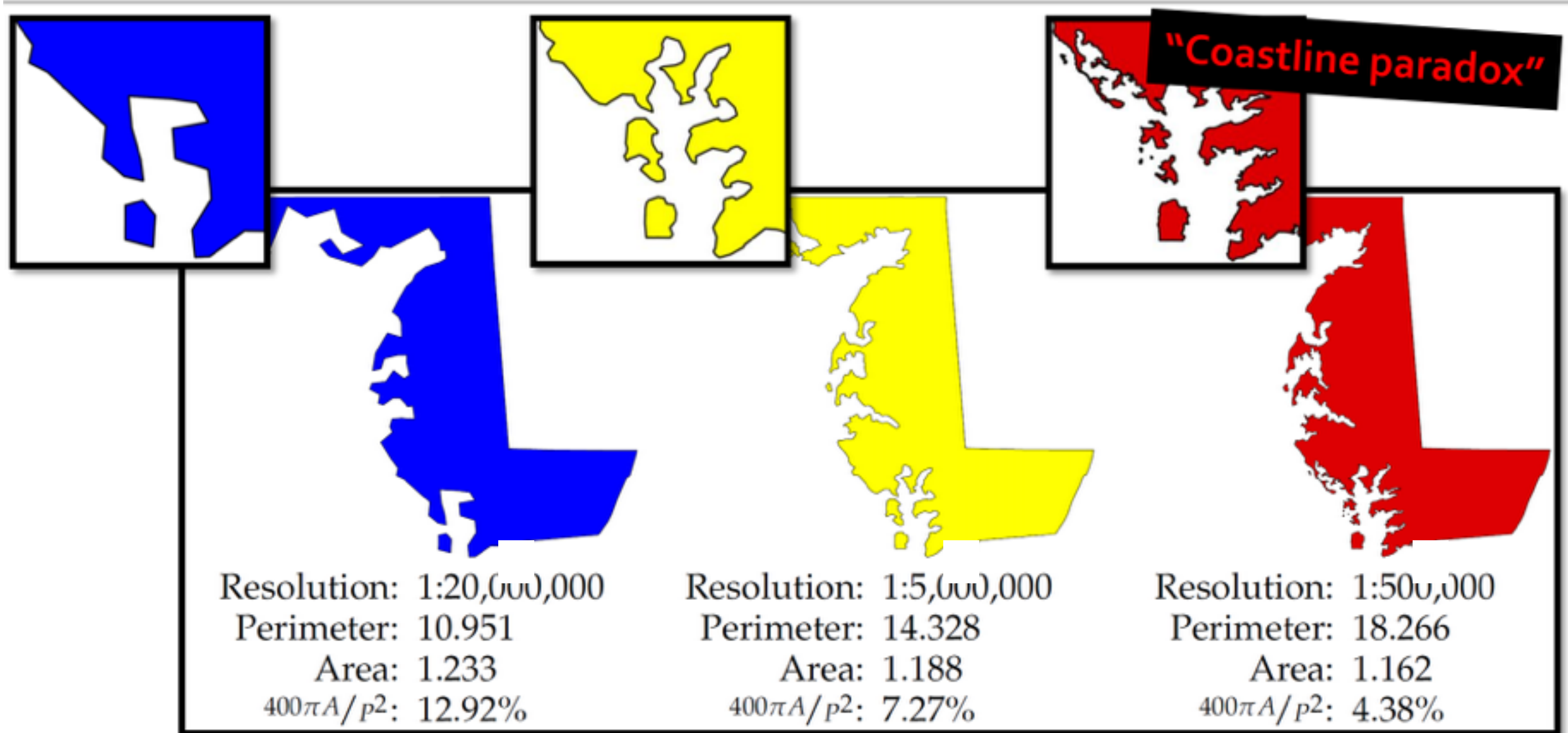
outcome?  
current map looks fine...  
but splits SLC four ways!

Dems got 33% of 2016  
Congressional vote and  
0/4 seats—*cracking*



# A CLOSER LOOK AT ISOPERIMETRY

---



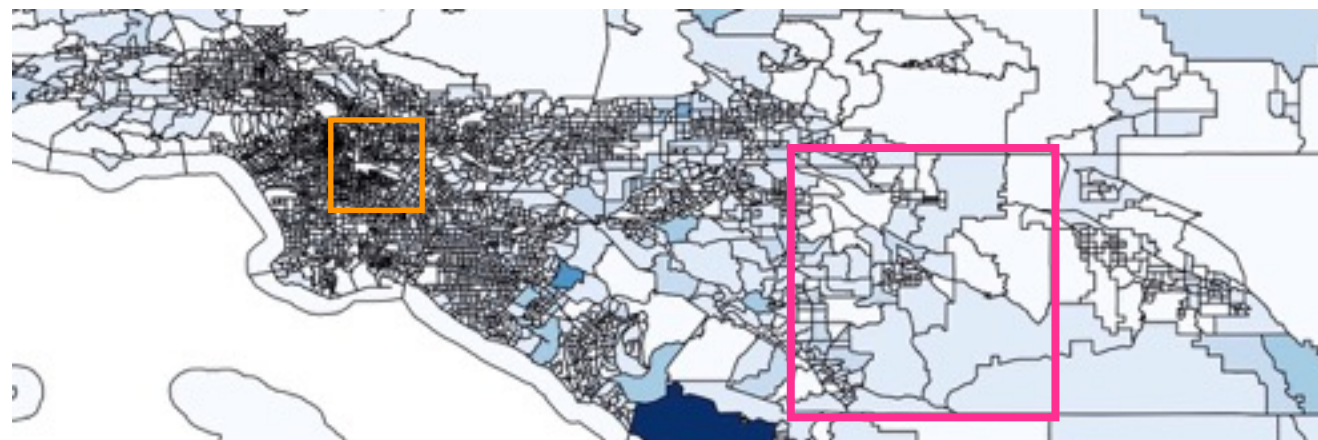
Example courtesy Mira Bernstein and Assaf Bar-Natan

## Maryland district 1

# THINKING ABOUT THE “GUTS” OF A DISTRICT

.....

- What is the right abstraction to capture the relevant information? (i.e., what *object* should we study?)



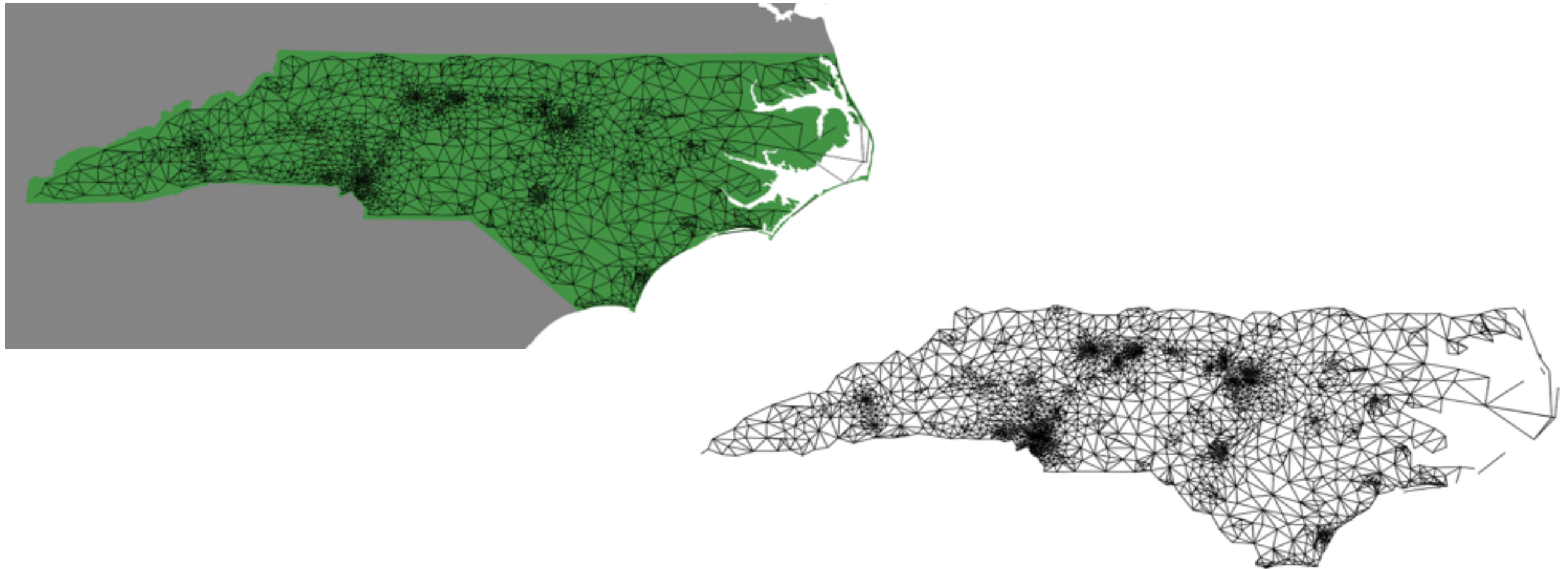
*(Perhaps all squares are not created equal)*

- The census data comes in discrete units: census blocks (0-100 people), block groups (600-3000), and tracts (1200-8000)
- Could break down a state into its census units, form graph to see the *guts* of a state and its district plan

Q: What are the edges... adjacency? distance/travel time? commonalities?

# BUILDING A CENSUS-DATA **GRAPH**

---

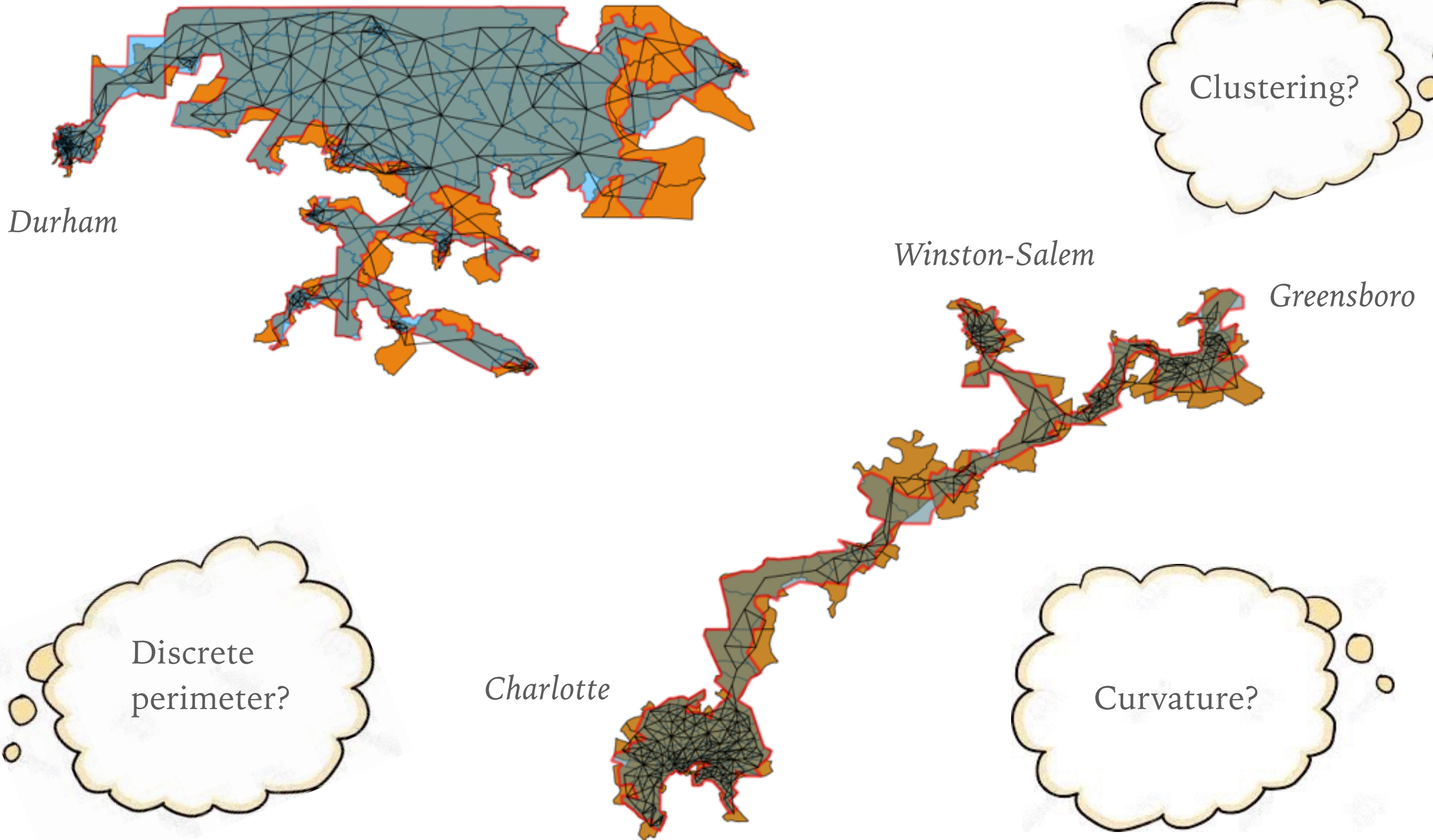


- Census data comes in blocks, block groups, and tracts
- Tracts typically have 4000 people; NC has 2195 tracts
- This graph shows one vertex for every tract in NC, with edges between tracts that share a border



# THE GRAPH “SEES” MORE POLITICALLY RELEVANT DATA

---

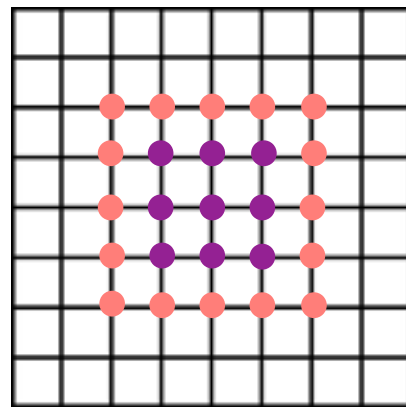




# WHAT CAN YOU DO WITH A GRAPH?

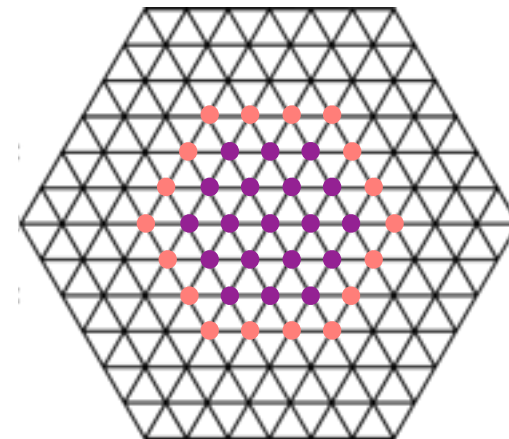
.....

- Use discrete/coarse definitions of area and perimeter, counting **area** as the total number of nodes and **perimeter** as the number of boundary nodes



$$A = n^2, \quad P = 4n - 4$$

$$A/P^2 \rightarrow 1/16$$



$$A = 3n^2 - 3n + 1, \quad P = 6n - 6$$

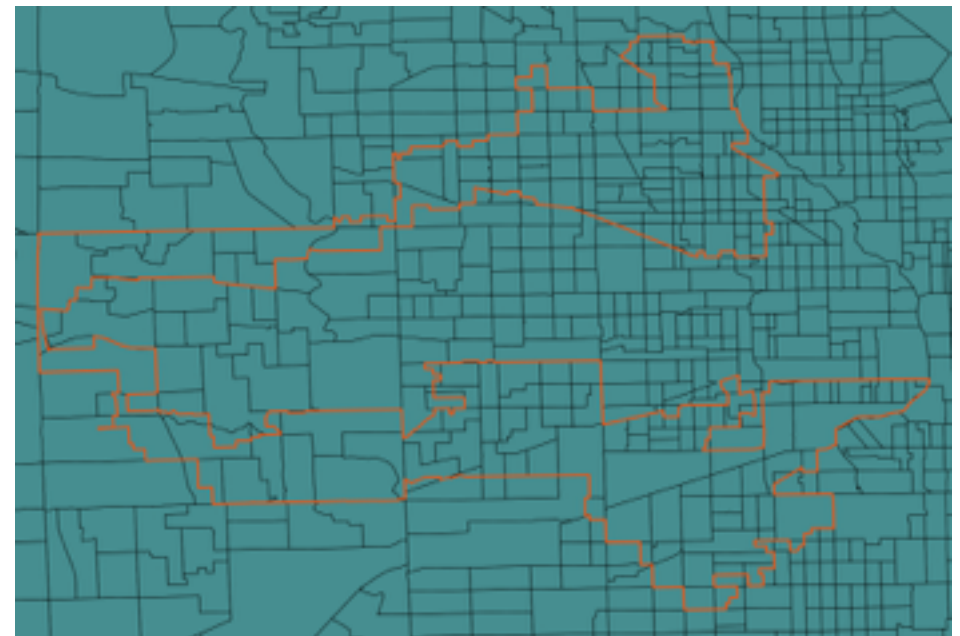
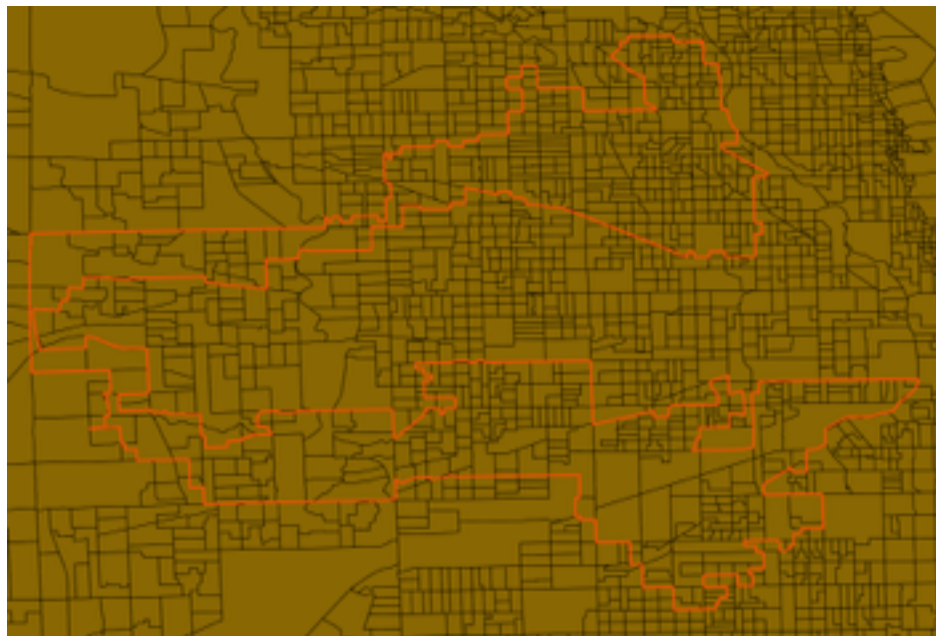
$$A/P^2 \rightarrow 1/12$$

- Behaves well under refinement if the pattern is stable

# DISCRETIZED POLSBY-POPPER

---

- Current project with Bridget Tenner: compare discrete  $A/P^2$  to classical
- Discrete score lightens coastline penalty
- Better protects from scale effects
- Weights perimeter heavily as it cuts through cities

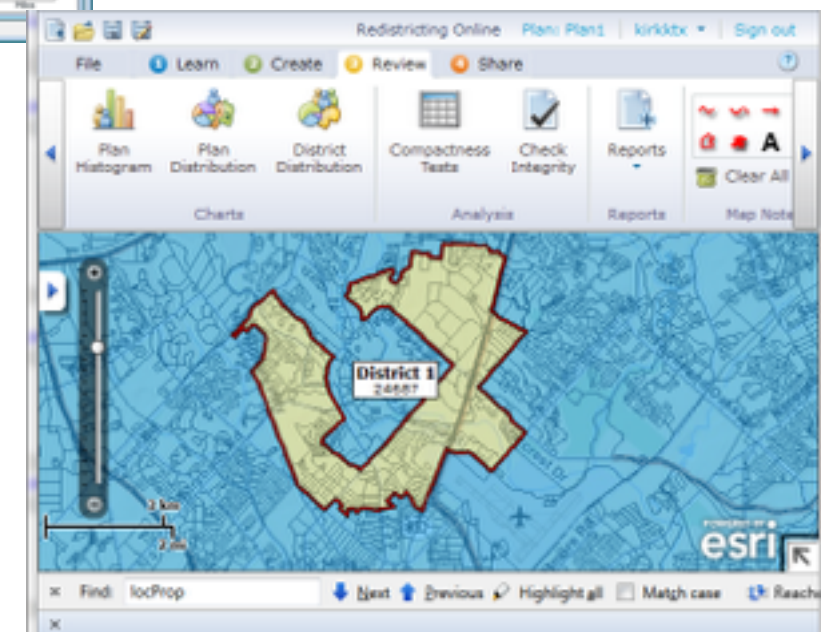
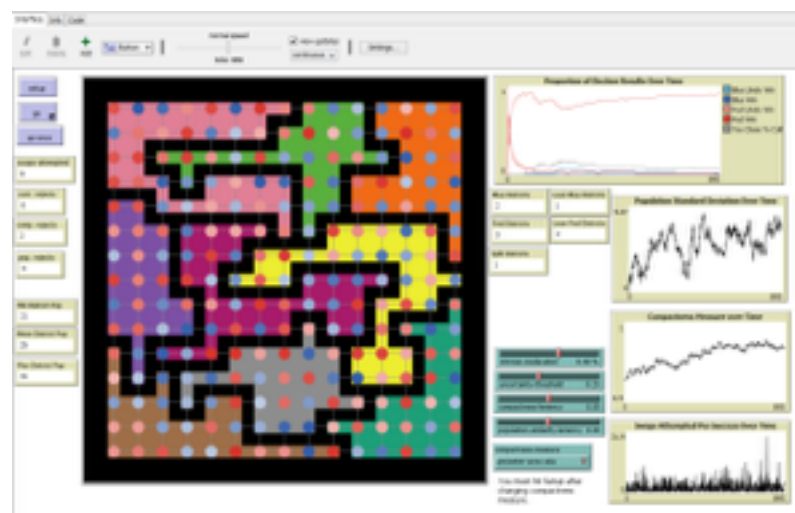
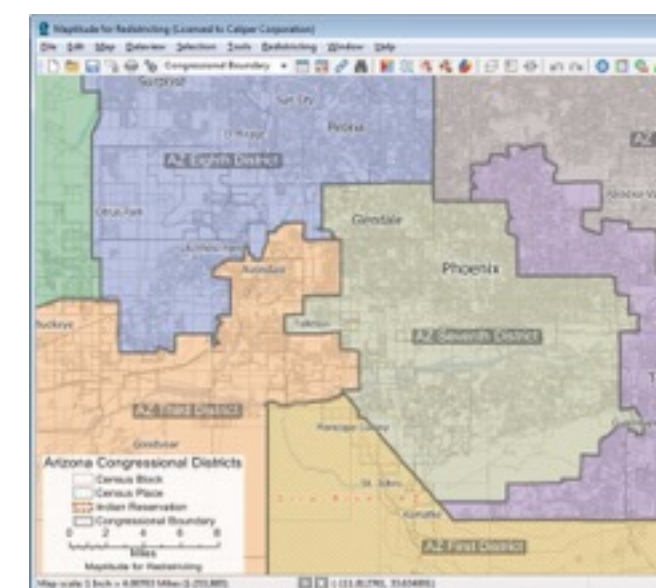
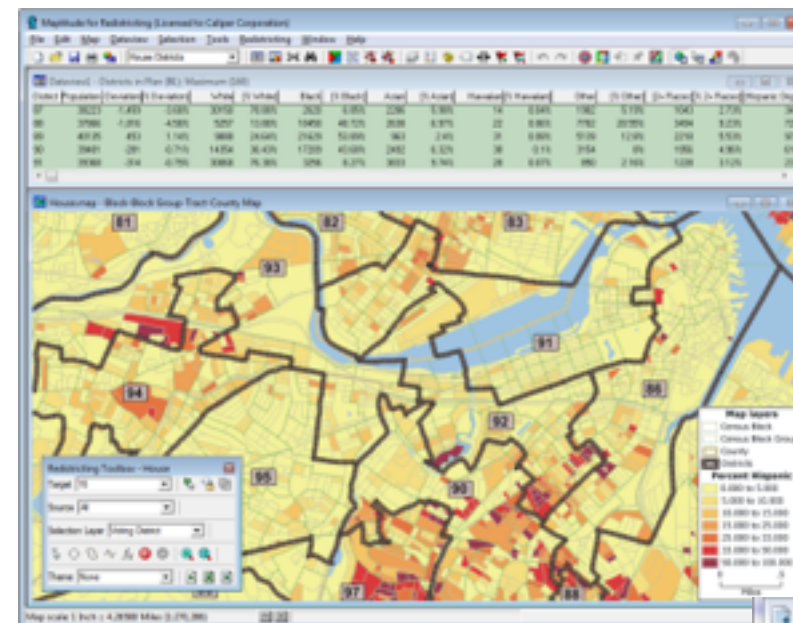
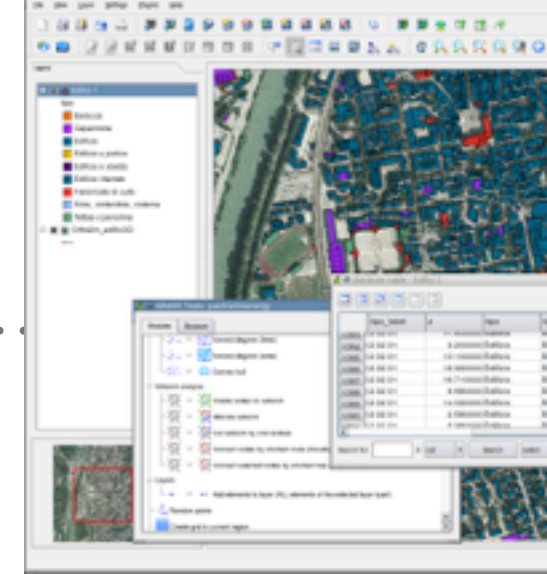
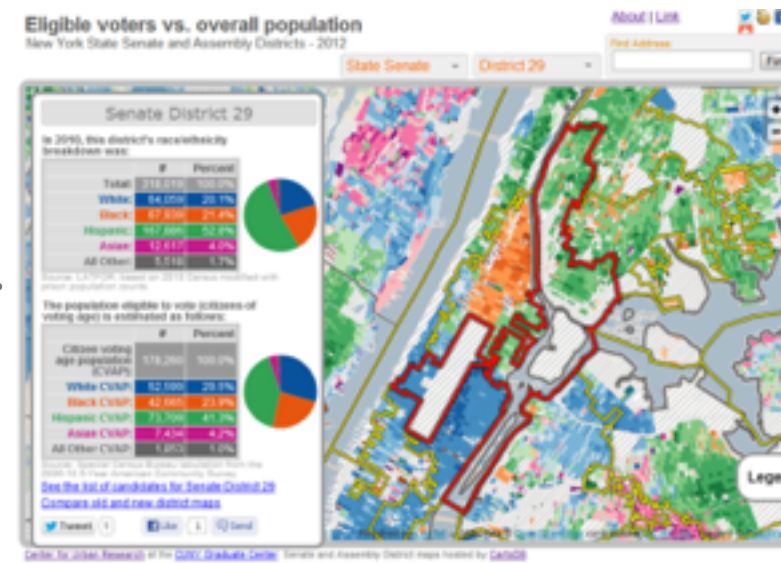


# THE ROLE OF TECHNOLOGY



# RICH DATA, AD HOC METHODS

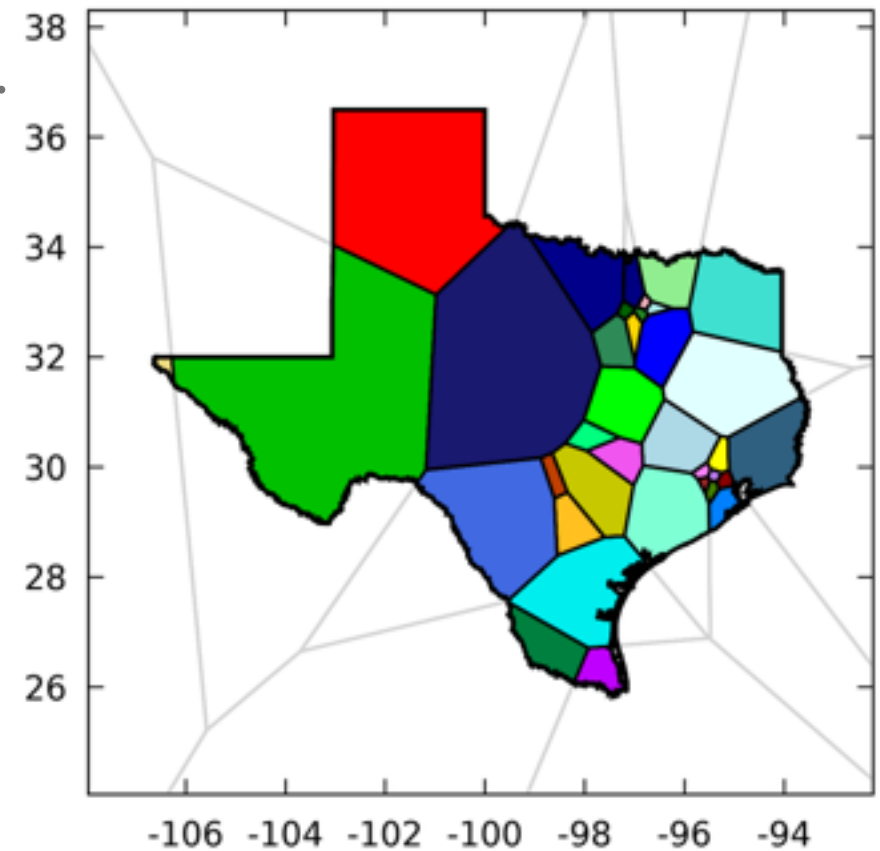
- We have incredible descriptive and predictive data, plus the ability to overlay it on spatial “shapefiles”
- But maps are still built by hand
- What is the baseline?





# CAN'T WE JUST AUTOMATE?

- Algorithms can take into account: population equality, contiguity, compactness. Can even try to optimize.
- Can handle county splits with a score, but communities of interest? Racial fairness? Tradeoffs in priorities among competing norms?



source: Cohen-Addad-Klein-Young

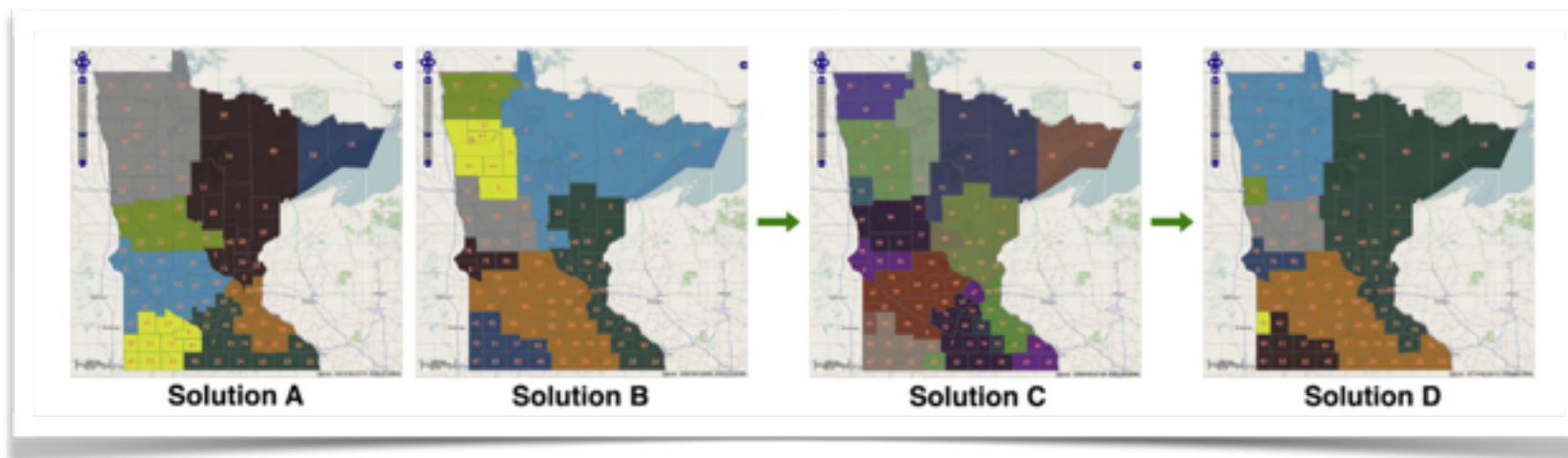


*“Blue Waters meets Maxine Waters”*

# HOW TO USE COMPUTERS BETTER

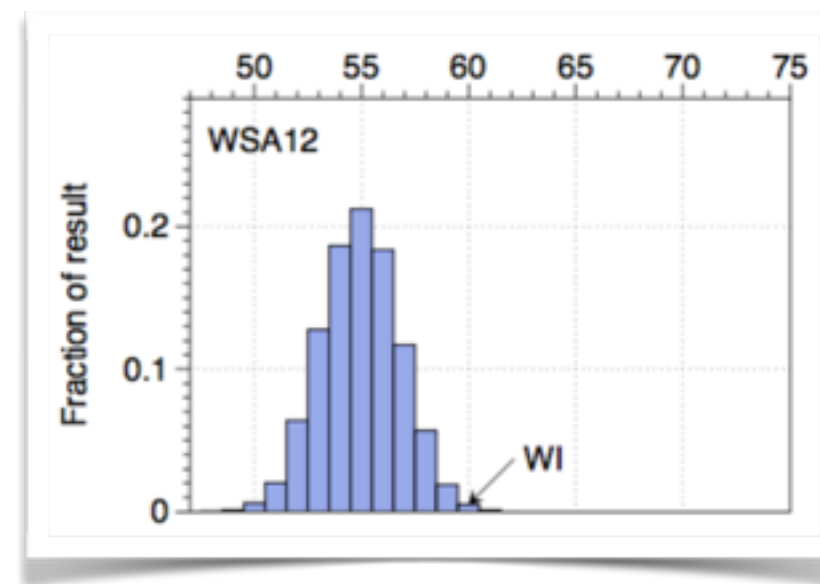
.....

- Multiple teams developing MCMC (random walk) algorithms to study the space of **all possible** maps by many local swaps
- Evolutionary algorithms/Genetic algorithms: can make local *mutations* to a map or *crossovers* between two maps



source: Cho-Liu

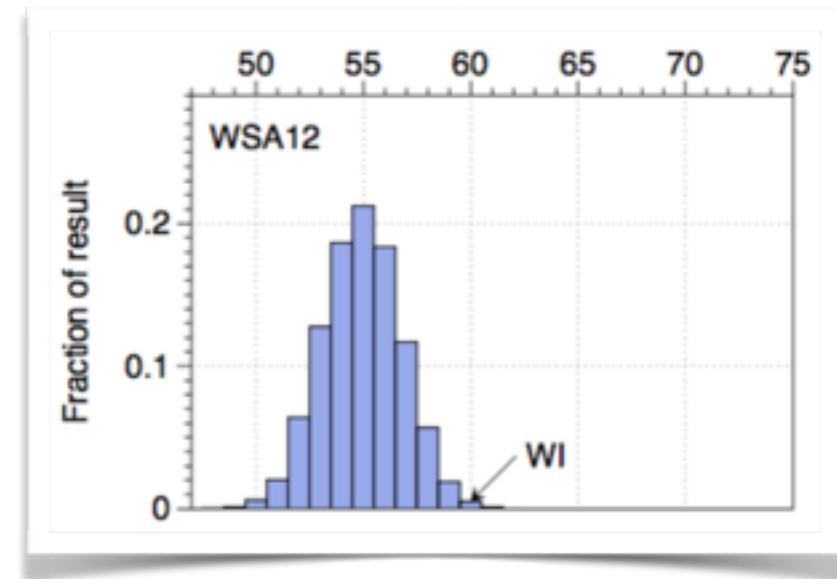
- **How to use a sampler:**  
Evaluate a plan against hundreds of thousands of alternate plans produced by the algorithm.



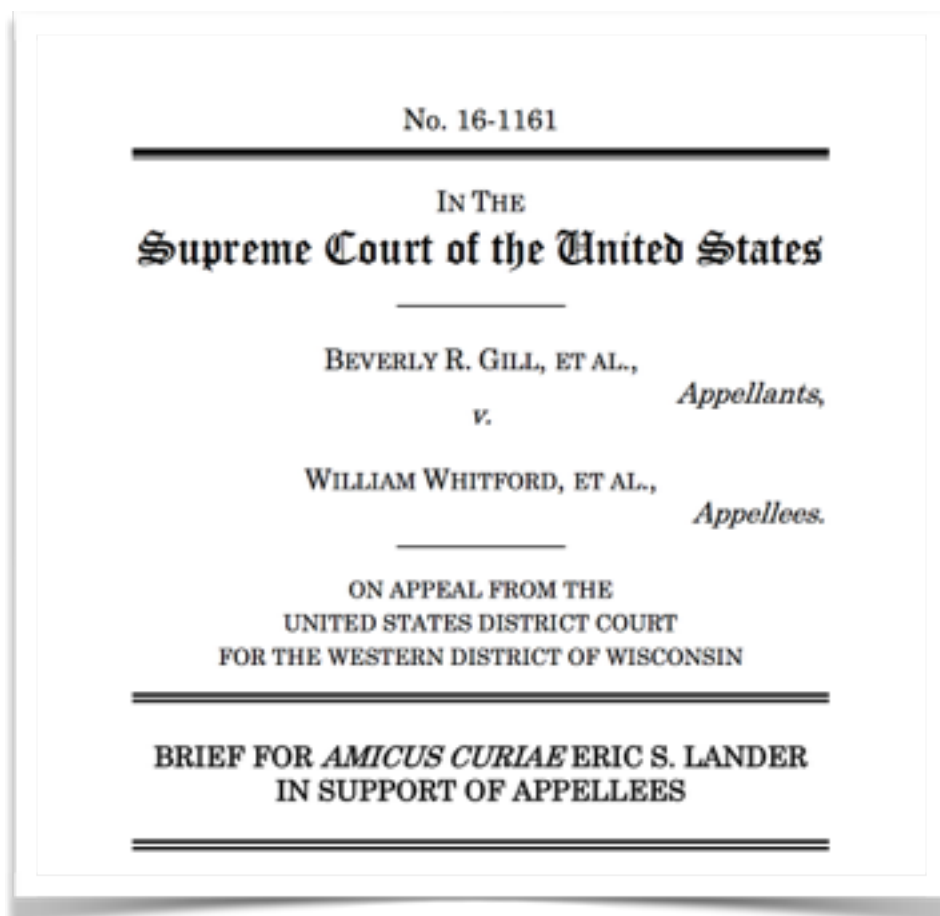
source:  
Herschlag-  
Ravier-  
Mattingly

# SAMPLING AND OUTLIER ANALYSIS

- How to use a sampler:  
Evaluate a plan against hundreds of thousands of alternate plans produced by the algorithm.



source:  
Herschlag-  
Ravier-  
Mattingly



20 And if there is, you say is this an  
21 extreme outlier in respect to asymmetry? And  
22 there we have Eric Lander's brief, okay? You  
23 know that one.  
24 And -- and we look through thousands  
25 and thousands of maps, and somebody did it with

Heritage Reporting Corporation

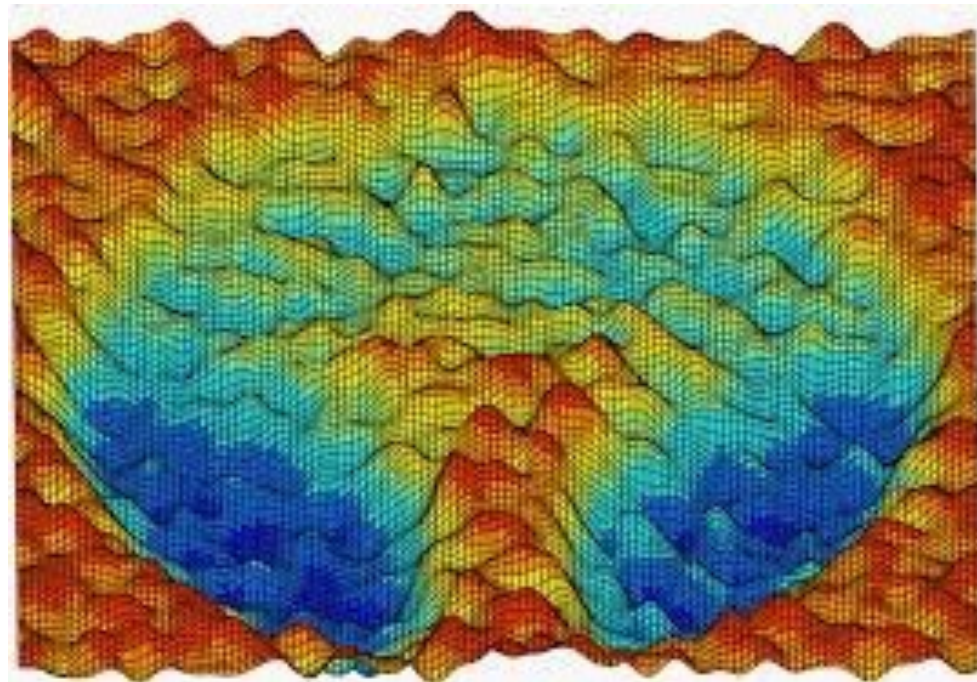
Official - Subject to Final Review

13

1 real maps and said how bad is this compared to,  
2 you know, the worst in the country.

# WHAT IS THE SHAPE OF THE SPACE OF MAPS?

.....



- Rugged? Smooth? Disconnected? Depends on constraint choices.
- Climb hills or explore blindly?
- Can we characterize the sampling distribution?
- Are the sampled plans neutral? fair? reasonable? ...

*Looking for a good practical math problem? Start here!*



# THANK YOU!

visit

**[sites.tufts.edu/gerrymandr](https://sites.tufts.edu/gerrymandr)**

to find out more about the  
Metric Geometry and Gerrymandering Group

and come talk math nitty gritties tonight

7-8:30pm

in the Math Department (Physics Building)