The background of the slide is a map of the United States, where several states and regions are shaded in a light red color. The shading is somewhat irregular, covering parts of the Northeast, the Midwest, and the South. The text is overlaid on this map.

# **Measures of partisan fairness**

**Mira Bernstein**

**Duke “Geometry of Redistricting” Conference**

**November 3, 2017**

# What is fairness?

---

*“The parties have not shown us, and I have not been able to discover . . . . statements of principled, well-accepted rules of fairness that should govern districting.”*

- Justice Anthony Kennedy, *Vieth v Jubelirer* (2004)

# Is fairness the same as proportionality?

---

Most people's intuitive notion of fairness:

*If a party gets  $X\%$  of the vote, it should get about  $X\%$  of the legislative seats*

# The Supreme Court says NO!

---

*“... the mere lack of proportional representation will not be sufficient to prove unconstitutional discrimination.”*

- Plurality Opinion, *Davis v. Bandemer*, 1986

*“Nor do I believe that ... proportional representation ... is consistent with our history, our traditions, or our political institutions.”*

- Justice Sandra Day O'Connor, *Davis v. Bandemer*, 1986

# **(Though they do say proportionality is fair...)**

---

*“... judicial interest should be at its lowest ebb when a State purports fairly to allocate political power to the parties in accordance with their voting strength and ... through districting, provide a rough sort of proportional representation in the legislative halls of the State.”*

- Majority in *Gaffney v. Cummings* (1973)

# ***Gill v Whitford* oral arguments**

---

JUSTICE BREYER: If party A wins a majority of votes, party A controls the legislature. That seems fair. And if party A loses a majority of votes, it still controls the legislature. That doesn't seem fair. And can we say that without going into what I agree is pretty good gobbledygook?

CHIEF JUSTICE ROBERTS: And if you need a convenient label for that approach, you can call it proportional representation, which has never been accepted as a political principle in the history of this country.

# ***Gill v Whitford* oral arguments**

---

MR. SMITH: Your Honor, we are not arguing for proportional representation. We are arguing for partisan symmetry, a map which within rough bounds at least treats the two parties relatively equal in terms of their ability to translate votes into seats.

CHIEF JUSTICE ROBERTS: That sounds exactly like proportional representation to me.

# ***Gill v Whitford* oral arguments**

---

MR. SMITH: Proportional representation is when you give the same percentage of seats as they have in percentage of votes. That's what proportional representation means. And our -- our claim simply doesn't remotely do that. It says if party A at 54 percent gets 58 percent of the seats, party B when it gets 54 percent ought to get 58 percent of the seats. That's symmetry. That's what the political scientists say is the right way to think about a map that does not distort the outcome and put a thumb on the scale.

# A toy example

---

- The state of Utopia has 100 seats in its state legislature.
- There are two parties, Purple and Orange.
- Purple won **55%** of the vote. How many of the seats should they win?

# Simulating Utopia (first with 10 districts)

---

Step 1: For each district, pick a random number from 0 to 1 to be the fraction of people who voted for Purple.

[0.75, 0.60, 0.37, 0.59, 0.073, 0.42, 0.60, 0.38, 0.75, 0.28]



**37% of voters in District 3  
voted for Purple**



**75% of voters in District 9  
voted for Purple**

# Simulating Utopia (first with 10 districts)

---

Step 2: Average these numbers together. That's the overall fraction of Utopians who voted for Purple. Call that **V**.

Step 3: Compute what percent of seats Purple won. Call that **S**. In our example:

[**0.75**, **0.60**, 0.37, **0.59**, 0.07, 0.42, **0.60**, 0.38, **0.75**, 0.28]

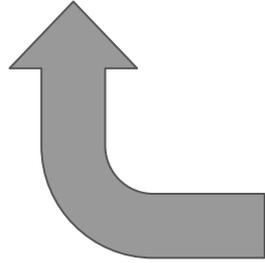
$$S = 0.5$$

$$V = 0.48$$

# Simulating Utopia (first with 10 districts)

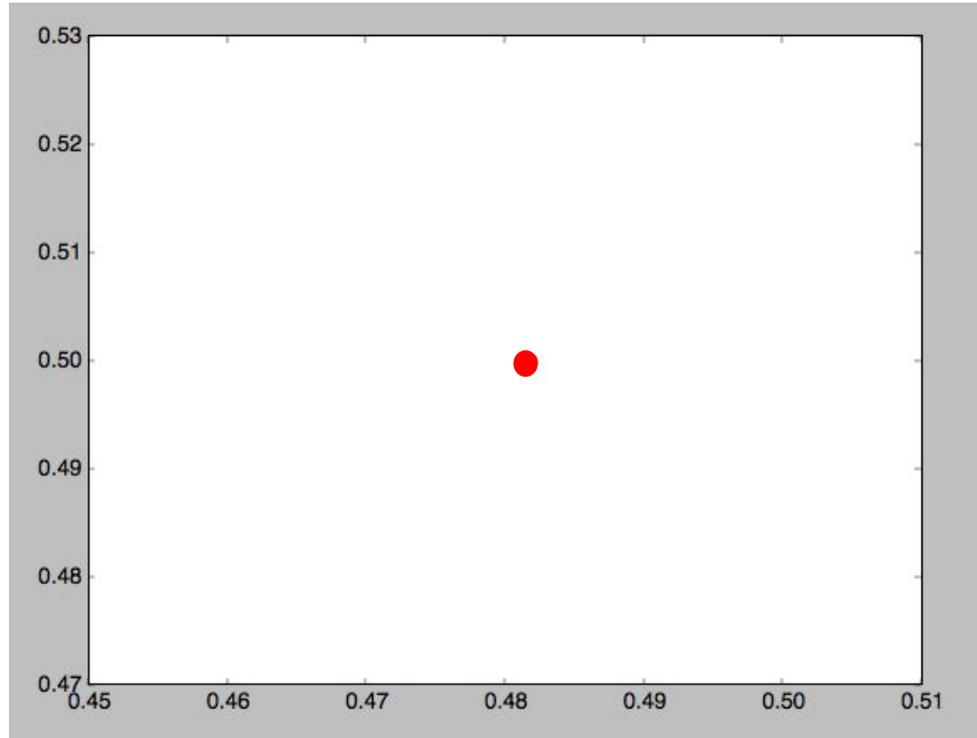
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[0.75, 0.60, 0.37, 0.59, 0.07, 0.42, 0.60, 0.38, 0.75, 0.28]



Step 4:

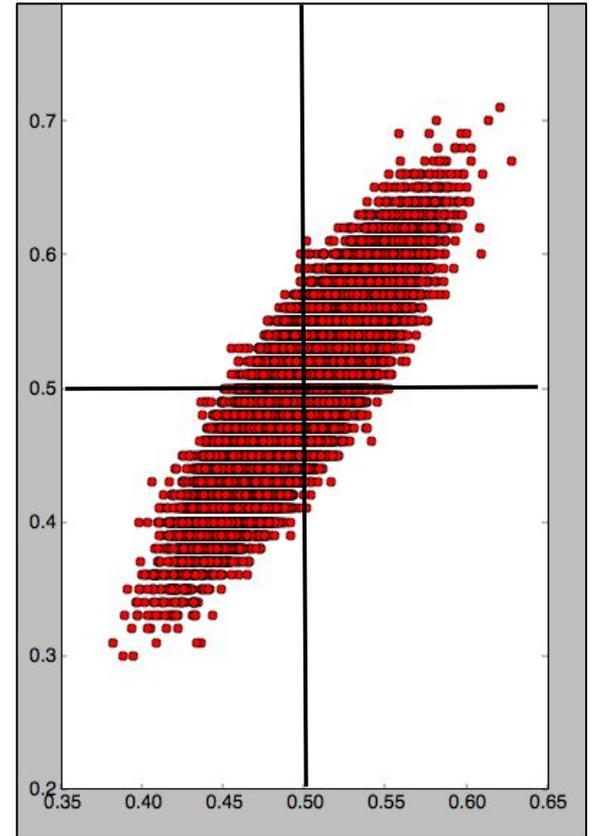
Plot the point **(v,s)**.



# Simulating Utopia (with 100 districts)

Now go back to 100 districts and do this 50,000 times.

This gives us 50,000 elections with different win margins for Purple.

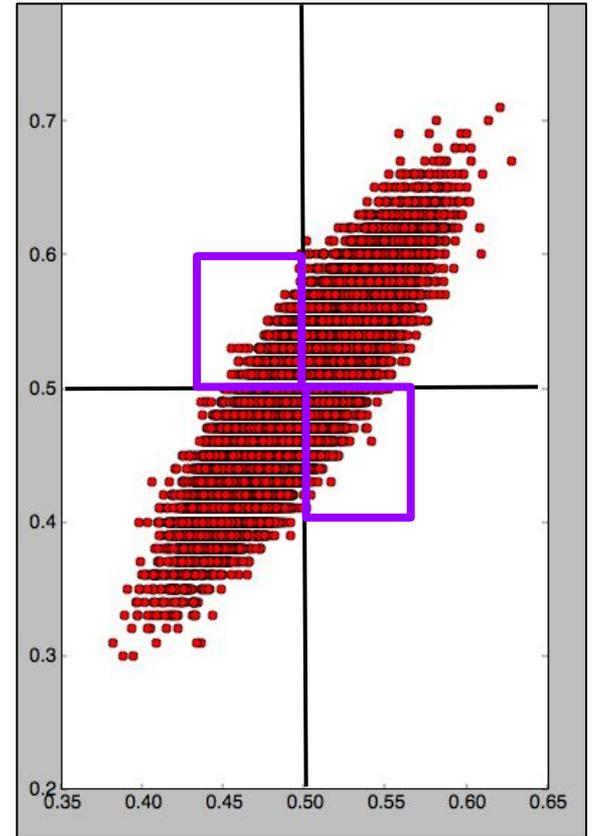


# Simulating Utopia (with 100 districts)

Now go back to 100 districts and do this 50,000 times.

This gives us 50,000 elections with different win margins for Purple.

*Note that in ~13% plans, a party that gets fewer than 1/2 the votes wins more than 1/2 the seats.*

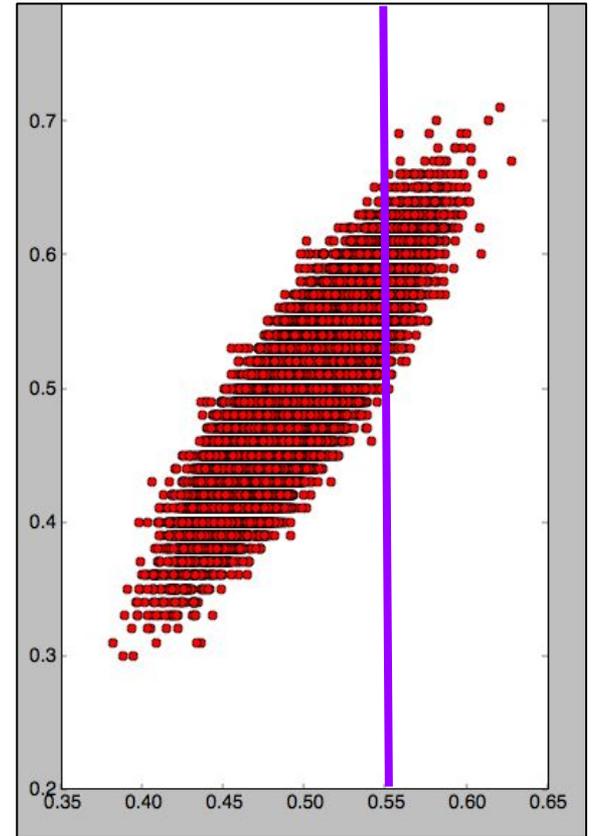


# Simulating Utopia (with 100 districts)

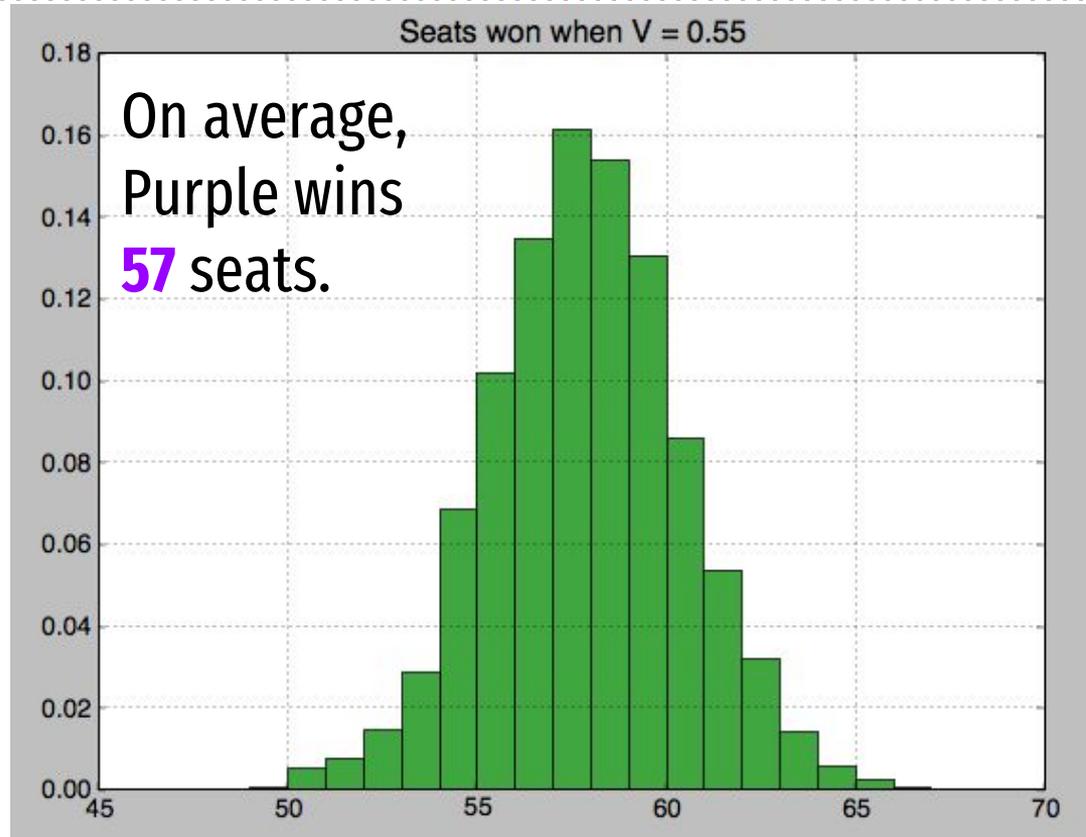
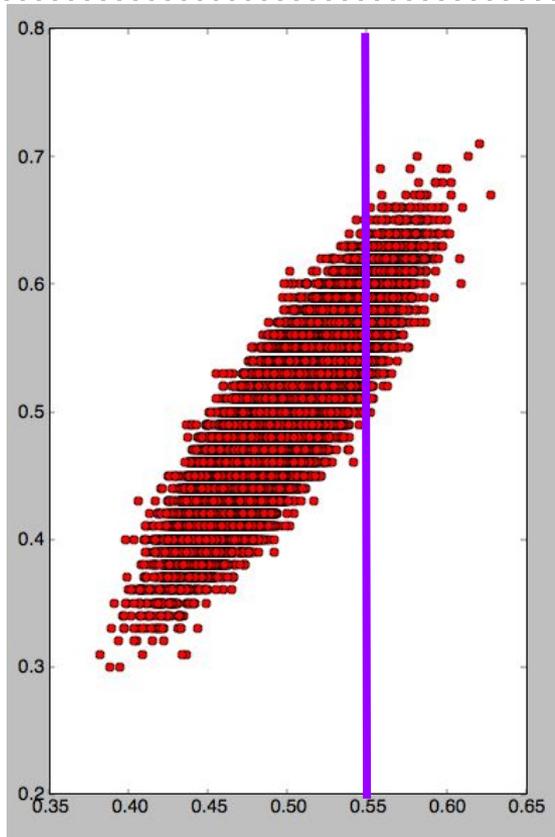
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Let's look just at the elections where Purple won 55% of the vote.

How many seats did they get?



# Simulating Utopia ( $V = .55$ )



# Simulating Utopia, version 2

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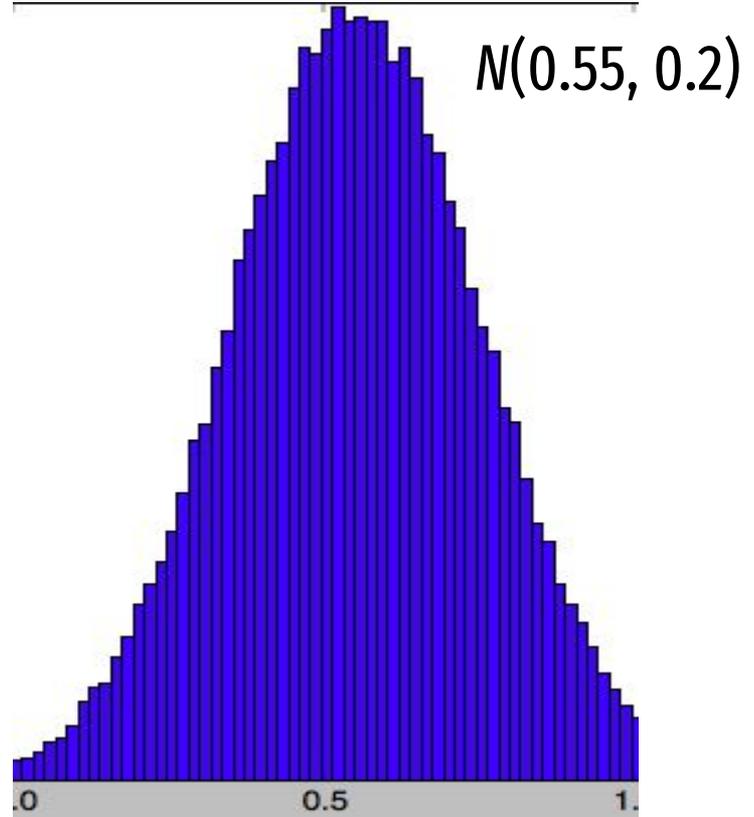
Our simulation was unrealistic:

- Not all win margins for districts are equally likely. Districts are (or should be?) more commonly won by 60% than by 99%.
- We assumed that Purple got 55% of the total vote purely by luck. A more likely scenario is that Purple is actually more popular than Orange.

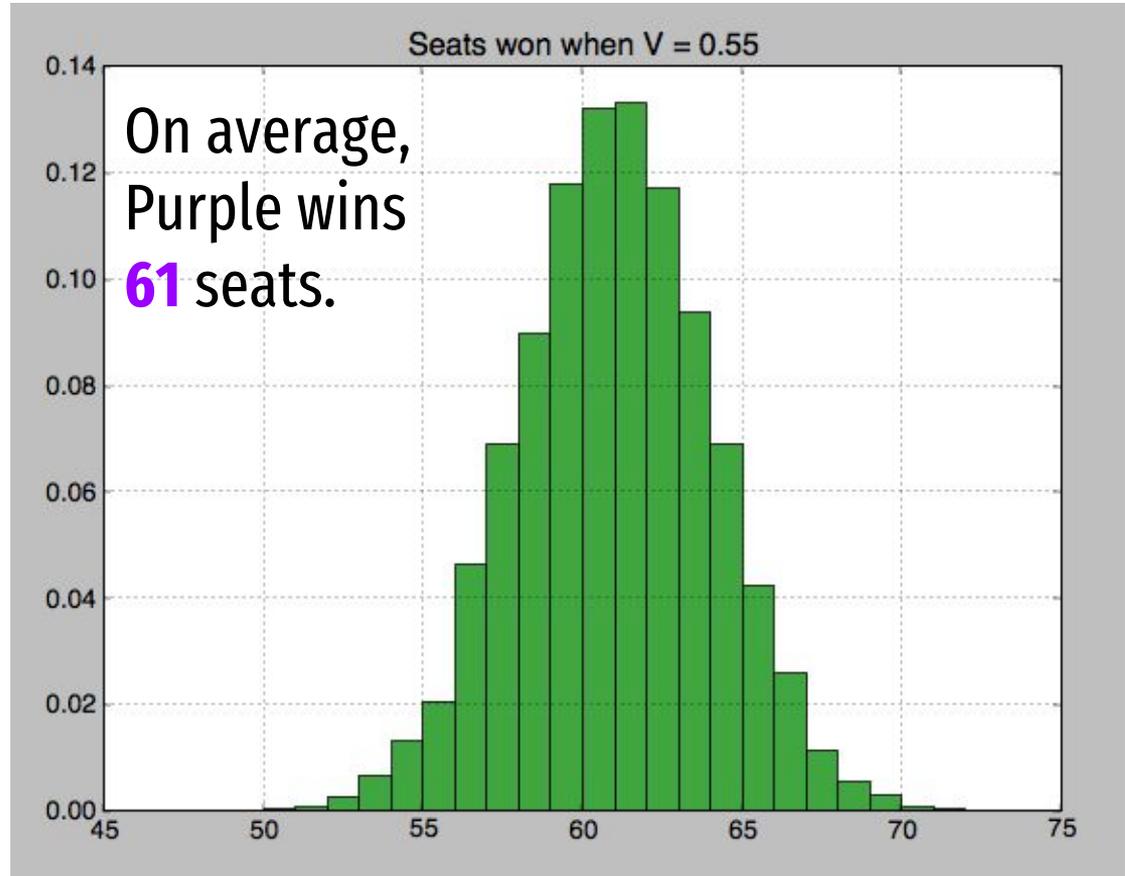
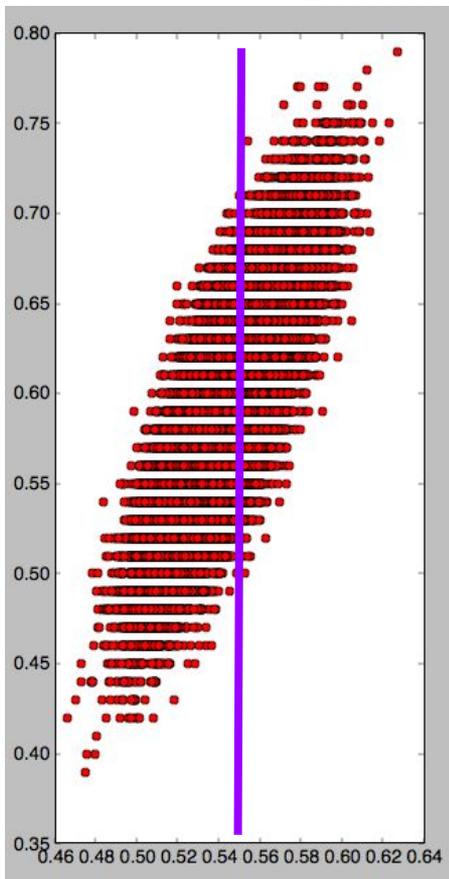
# Simulating Utopia, version 2

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Instead of picking Purple's popularity in individual districts uniformly from 0 to 1, let's use a truncated normal distribution centered at 0.55.



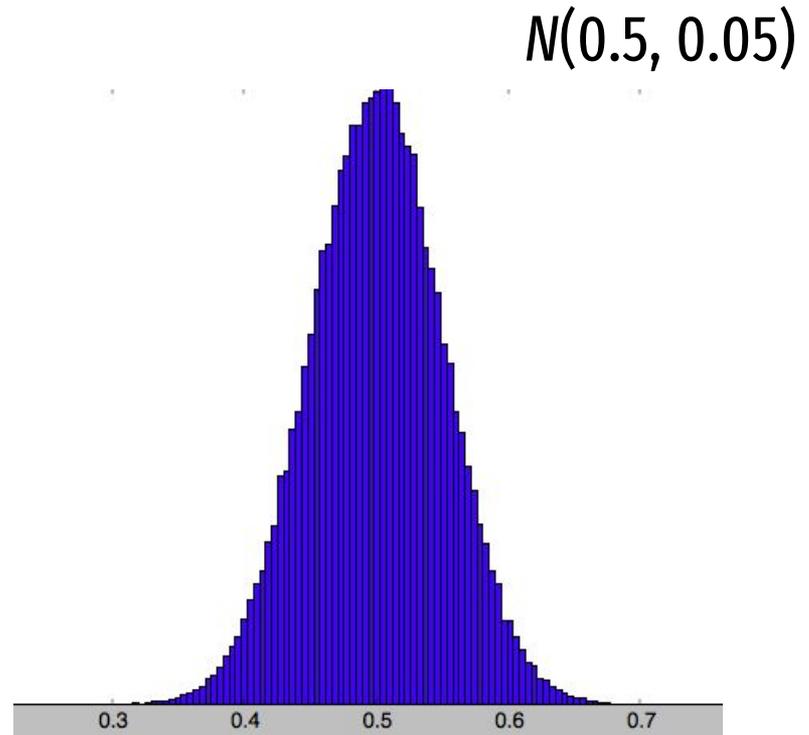
# Simulating Utopia, version 2



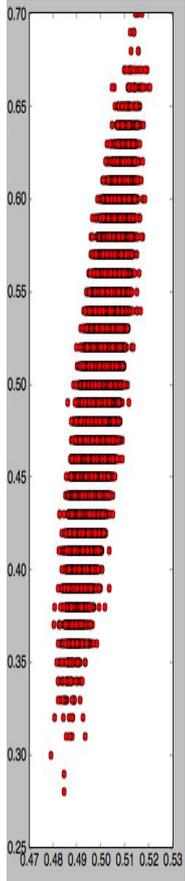
# Simulating Utopia with competitive districts

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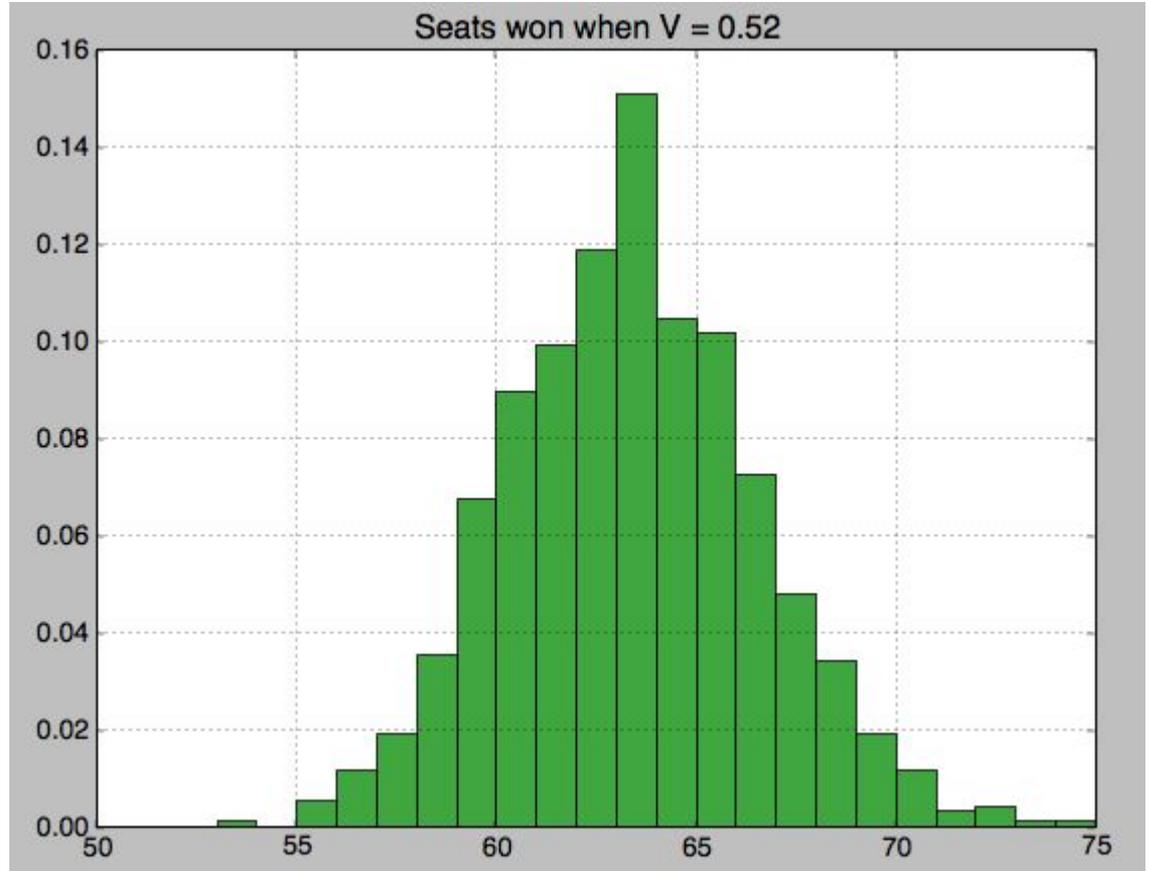
Say purple and orange are balanced overall but the vast majority of districts are 40% to 60% purple.



# Simulating Utopia with competitive districts



On average, if Purple wins **52%** of the votes, they win **63%** of the seats.

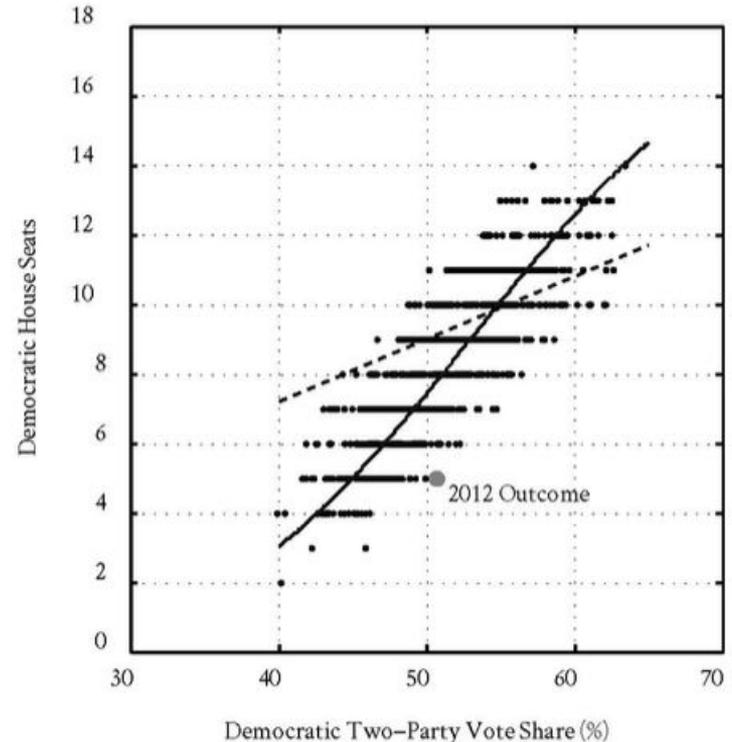


# Less utopian simulations

*Three Tests for Practical Evaluation of Partisan Gerrymandering*  
68 STAN. L. REV. 1263 (2016)

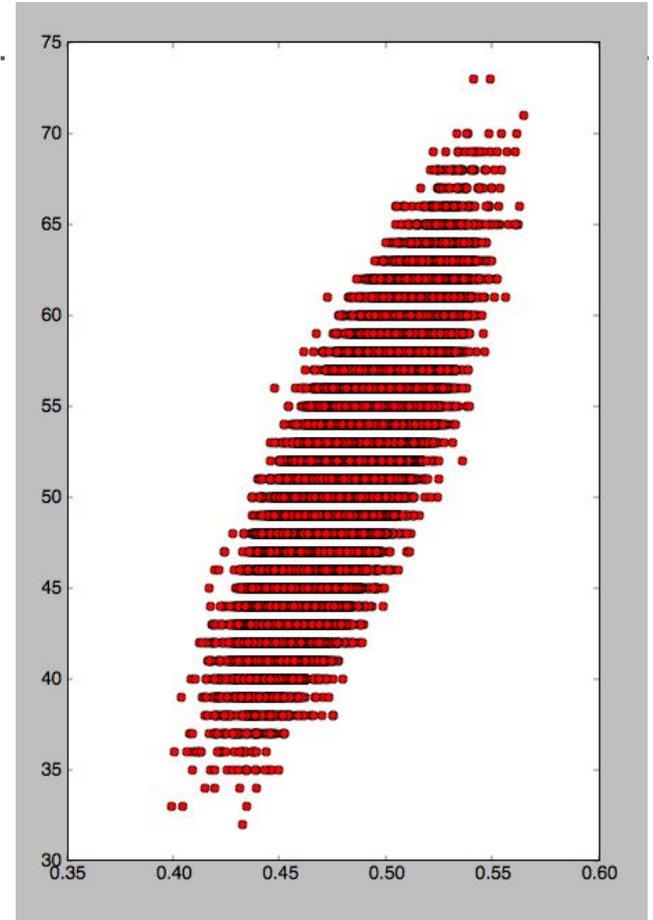
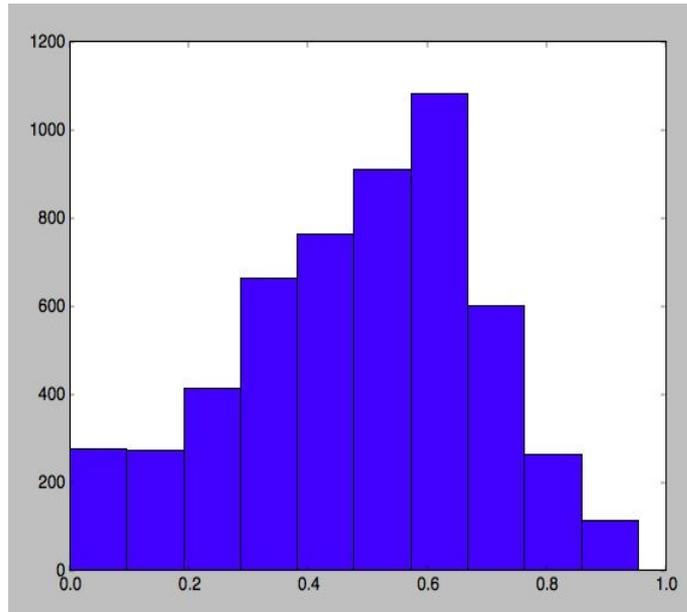
Sam Wang's idea:  
pick actual districts  
from around the  
country, at random  
  
(based on 2012 election)

**Figure 2**  
Simulated Pennsylvania House Delegations



# Less utopian simulations

Florida 2016: draw district probabilities at random from precinct probabilities



# Summary so far

---

Our electoral system (geographic single-member districts) has a built-in “winner’s bonus”: the party that wins the election gets **more** than its proportional share of votes.

- This has nothing to do with gerrymandering

# Summary so far

---

Our electoral system (geographic single-member districts) has a built-in “winner’s bonus”: the party that wins the election gets **more** than its proportional share of votes.

- This has nothing to do with gerrymandering
- ***In fact, to get proportional representation in this system, you have to gerrymander!***

# Summary so far

---

- ***How big is the winner's bonus built into our system?***

# Summary so far

---

- ***How big is the winner's bonus built into our system?***  
It depends on the partisan distribution of the voters.

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It depends on the partisan distribution of the voters.

- ***How big should the winner's bonus be?***

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It depends on the partisan distribution of the voters.
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That is a value judgment, not a mathematical question.

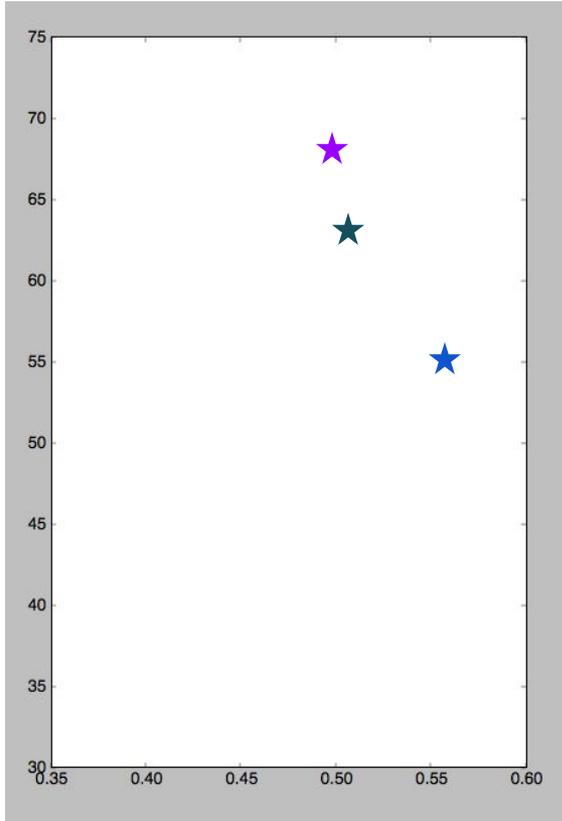
# Summary so far

---

- ***How big is the winner's bonus built into our system?***  
It depends on the partisan distribution of the voters.
- ***How big should the winner's bonus be?***  
That is a value judgment, not a mathematical question.
- ***Then how can you tell if a plan is “fair” without imposing your value judgment on others?***

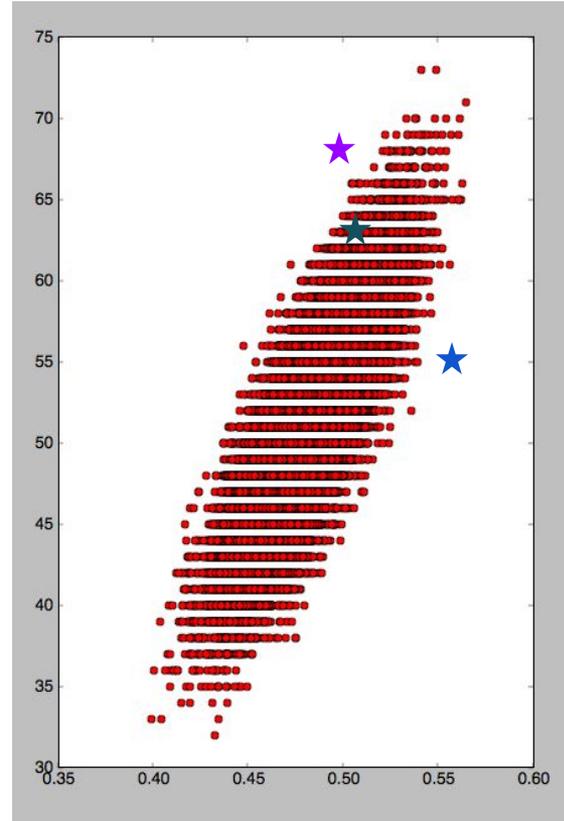
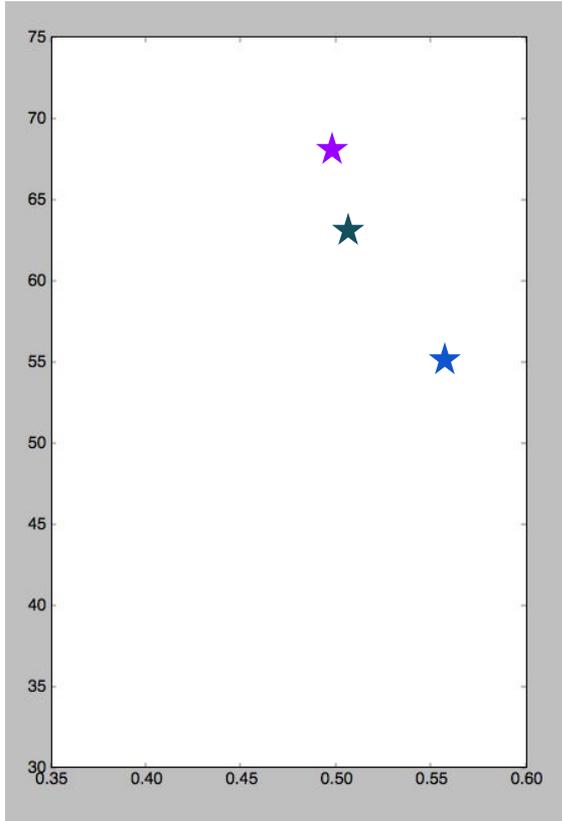
# Outlier analysis to the rescue?

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# Outlier analysis to the rescue?

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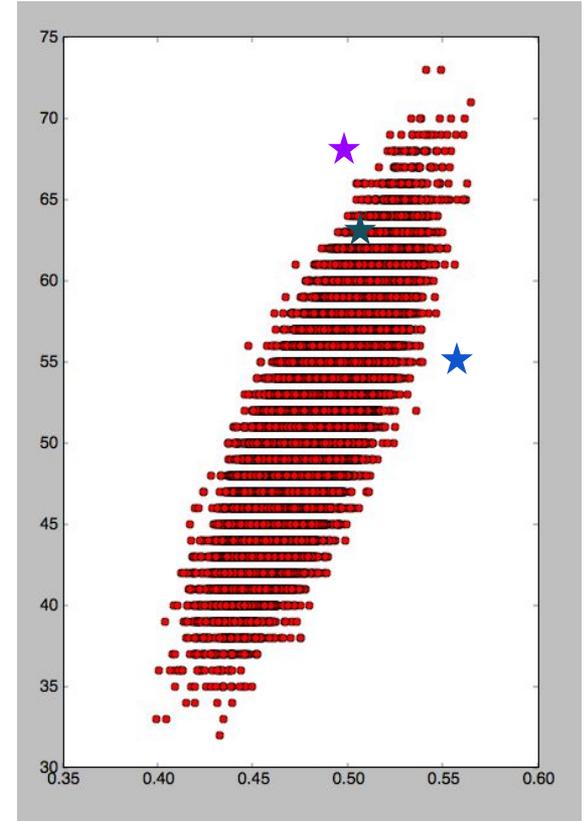


# Outlier analysis to the rescue?

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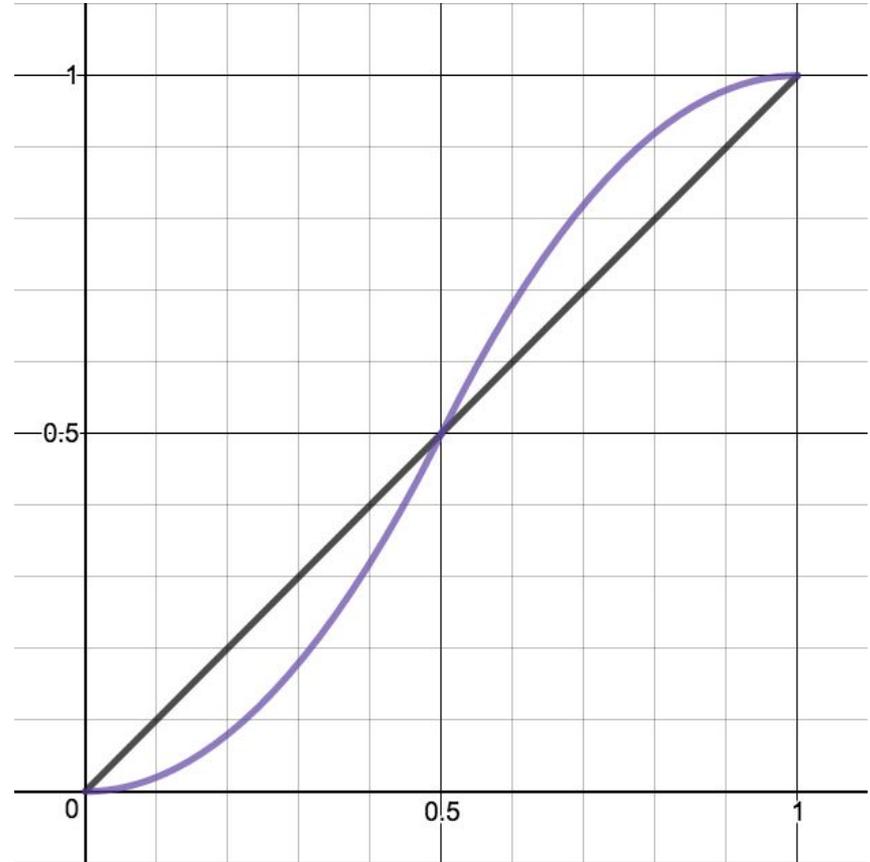
Yes, but...

- Extremely powerful and important
- A good indication of intentional gerrymandering
- But how can we say it's evidence of “discriminatory effect” unless we specify what “fair” means?



# Partisan Symmetry

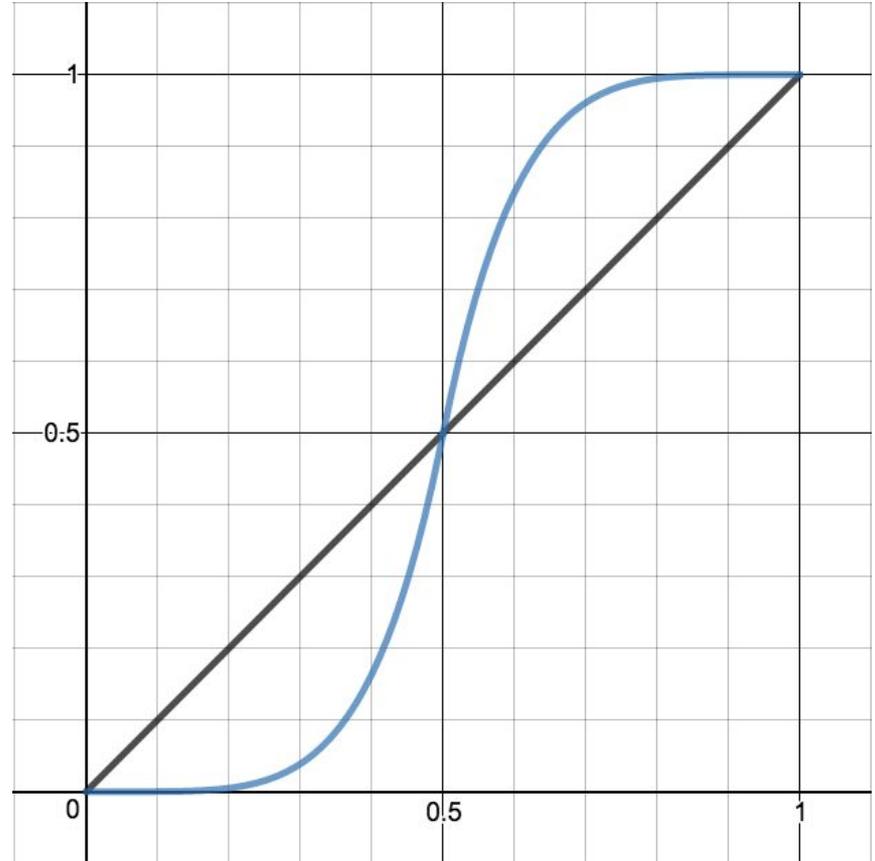
Rather than prescribing the “fair” value of  $S$  for a given  $V$ , we insist only that the plan must treat the two parties symmetrically.



# Partisan Symmetry

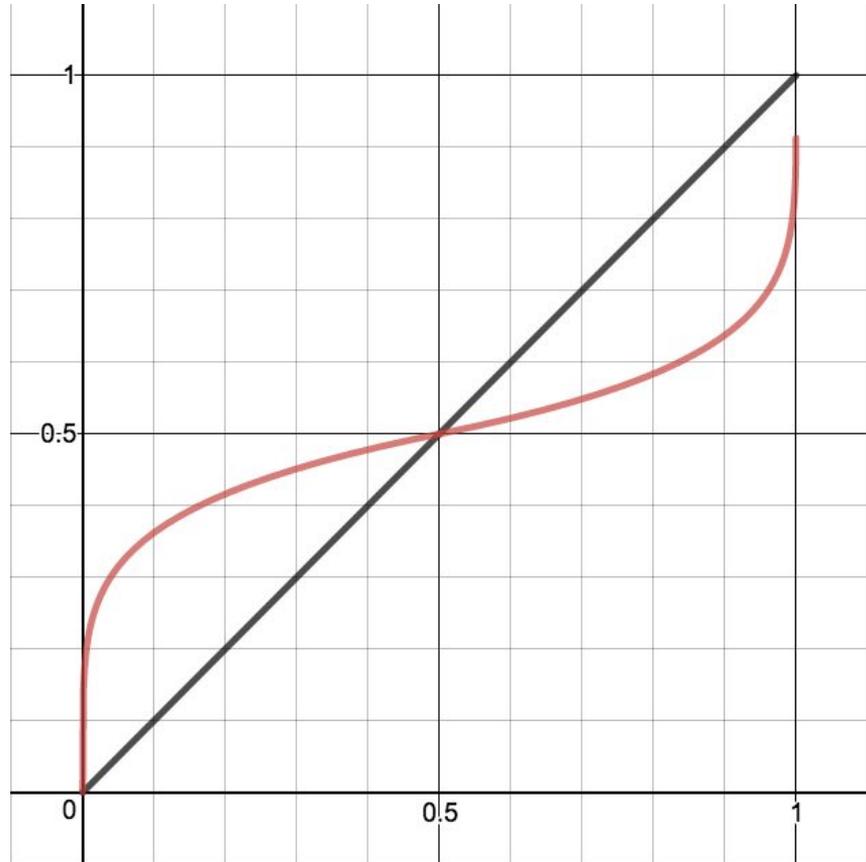
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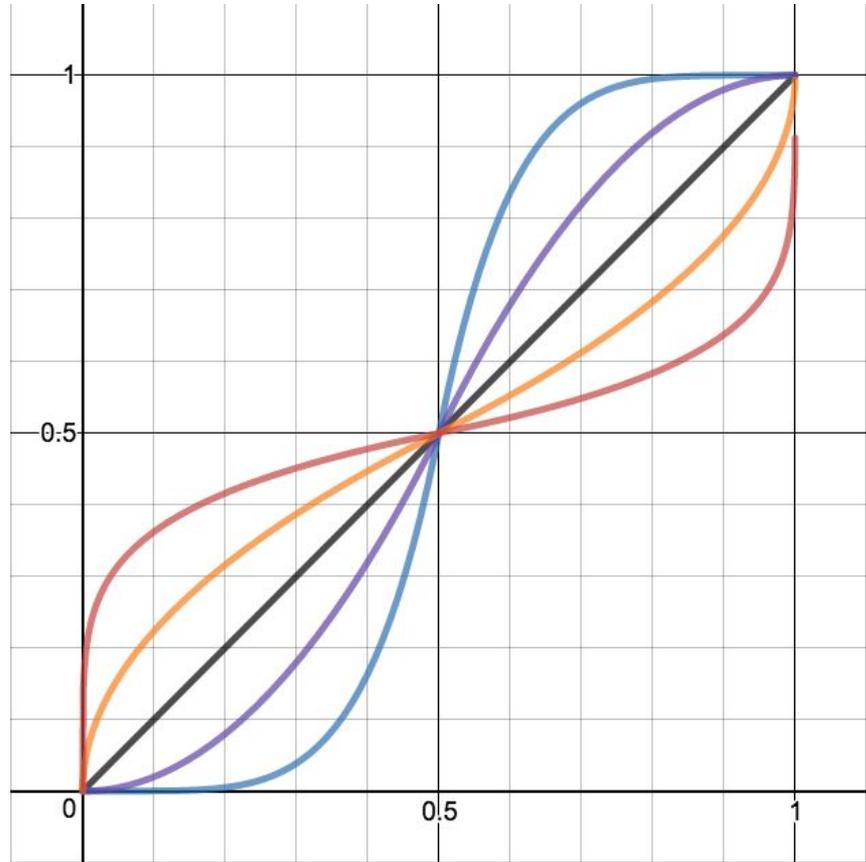
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# Evaluating the symmetry of a plan

---

Necessarily entails counterfactuals: how would this plan treat the parties under different (realistic) scenarios?

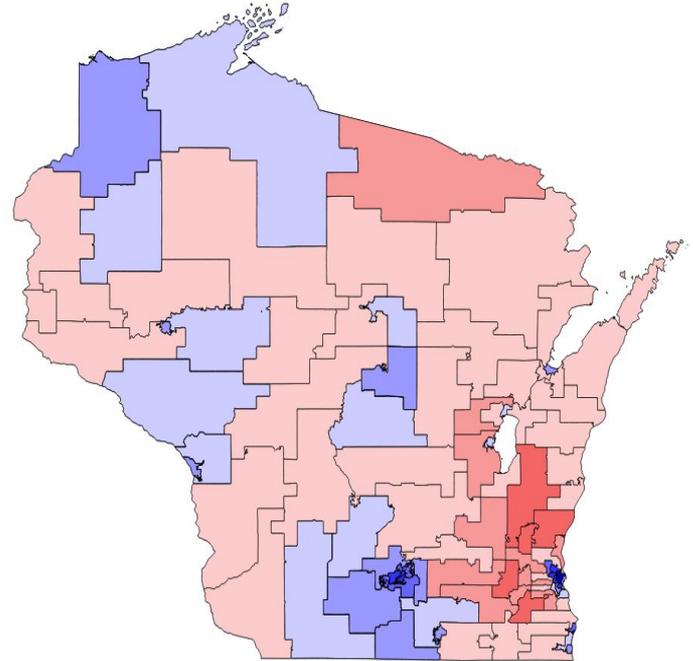
- *In the last election, the Democrats got a huge winner's bonus. Would the Republicans have gotten the same bonus if they had won a majority of the votes?*
- *Republicans got a majority of votes and a majority of seats. If they had gotten a minority of votes, do we believe they would have gotten a minority of seats?*

# Needed: a model of partisanship

---

Partisan preference depends on...

- **place:** some areas are always more Republican than others

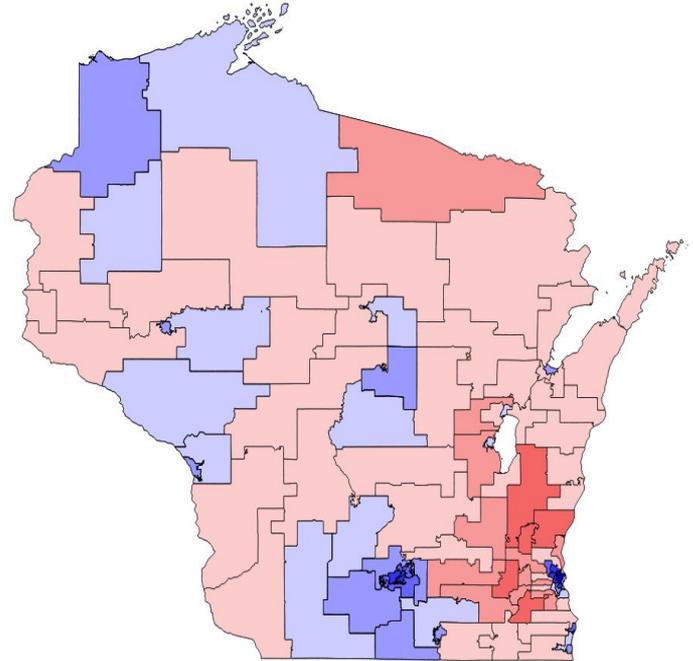


# Needed: a model of partisanship

---

Partisan preference depends on...

- **place:** some areas are always more Republican than others
- **time:** the whole country experiences swings left and right as the political climate changes

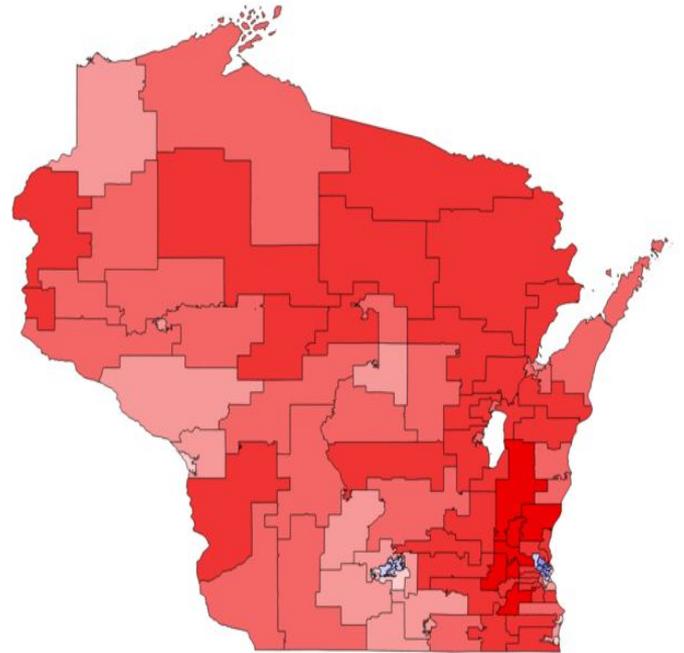


# Needed: a model of partisanship

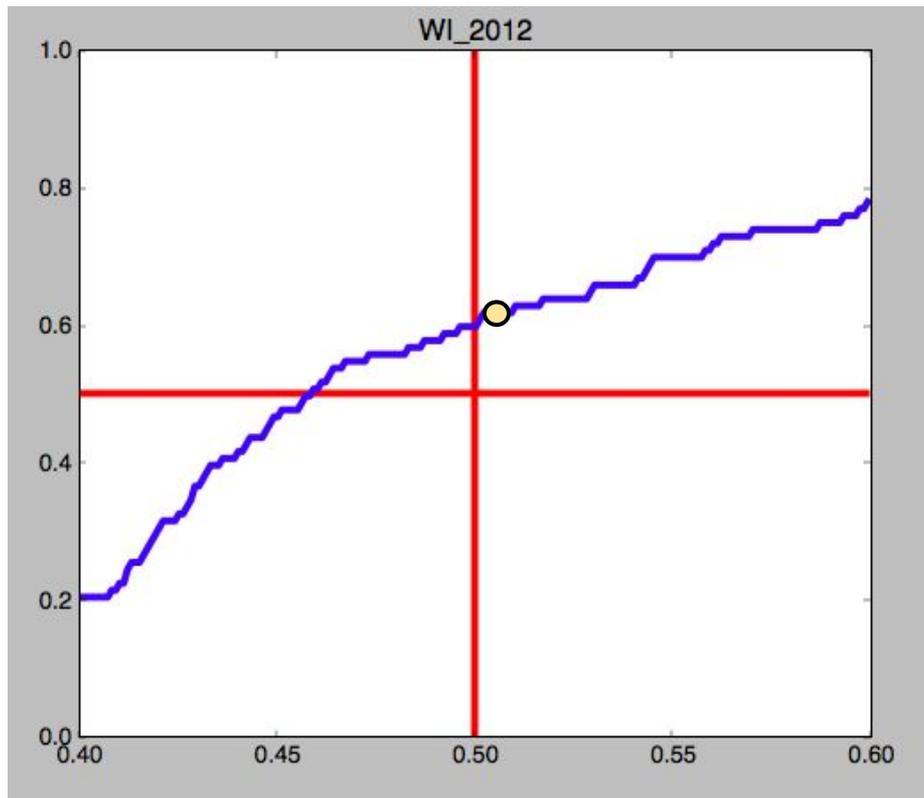
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## **Model assumption:**

*The effects of place and of time are independent.*



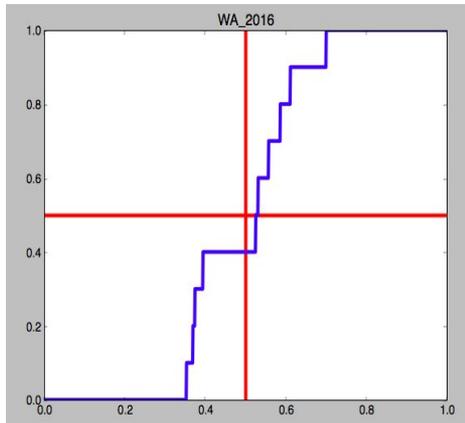
# A model of partisanship!



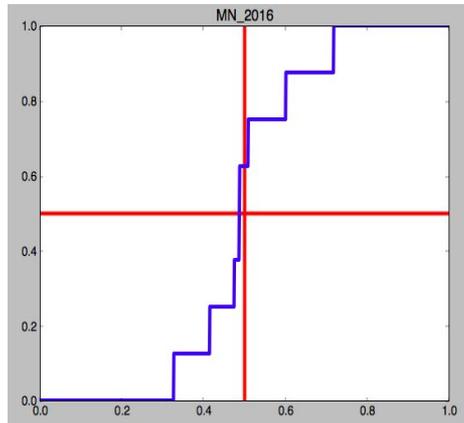
v	s
40%	21%
42%	29%
44%	41%
46%	51%
48%	56%
50%	60%
52%	64%
54%	66%
56%	71%
58%	74%

***“Uniform  
partisan  
swing”***

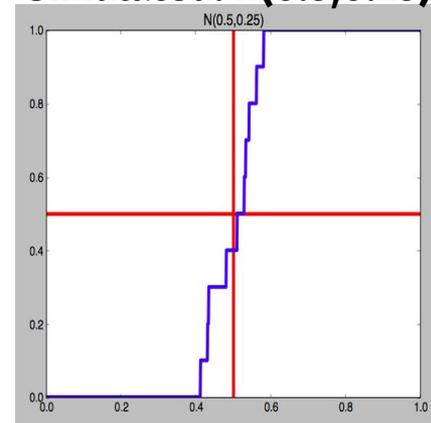
### WA 2016



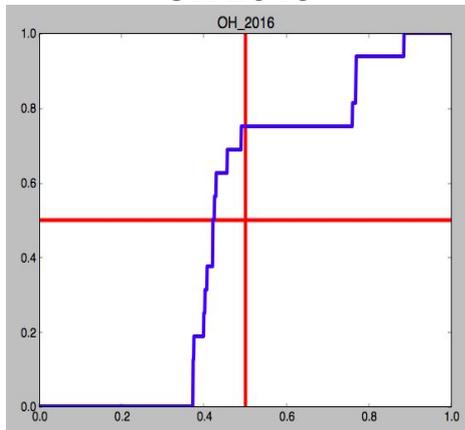
### MN 2016



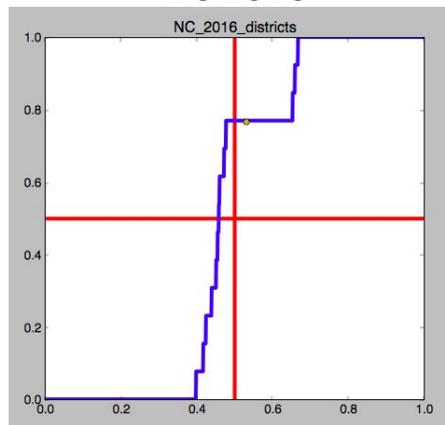
### Simulated: $N(0.5, 0.25)$



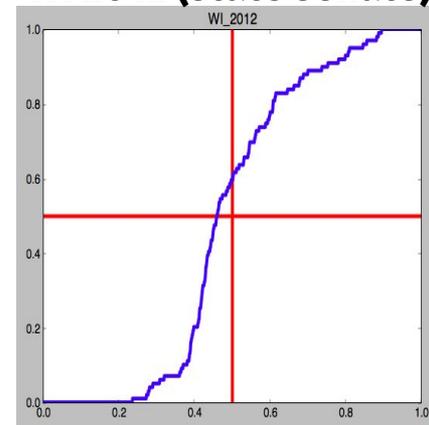
### OH 2016



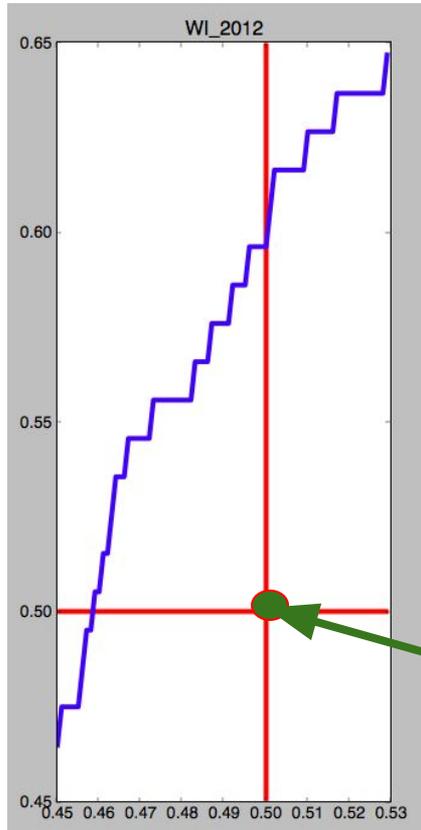
### NC 2016



### WI 2012 (state senate)

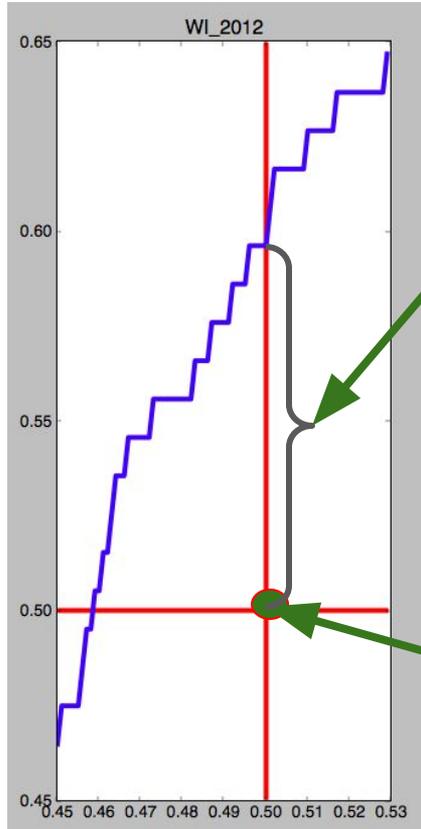


# Measures of asymmetry



**(0.5, 0.5)**: the one point required be on any symmetric curve

# Measures of asymmetry

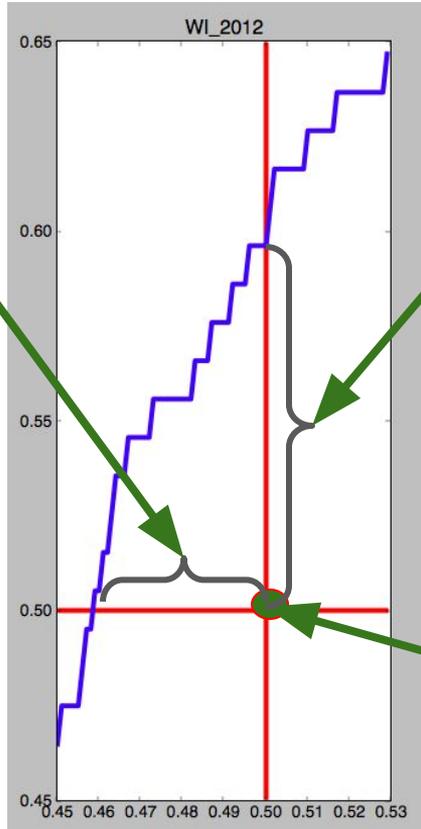


***Partisan bias:*** how much of an unfair advantage the party would have if the vote were evenly split

***(0.5, 0.5):*** the one point required be on any symmetric curve

# Measures of asymmetry

How far the party can fall from a majority of votes and still get a majority of seats



**Partisan bias:** how much of an unfair advantage the party would have if the vote were evenly split

**(0.5, 0.5):** the one point required be on any symmetric curve

# The issue of hypotheticals

---

Justice Kennedy, *LULAC v. Perry* (2006):

*The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside.*

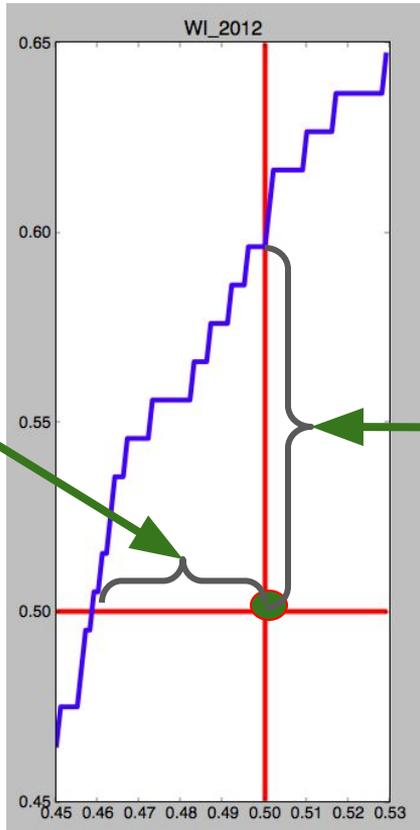
# The issue of hypotheticals

---

*“Even assuming a court could choose reliably among different models of shifting voter preferences, we are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.”*

# Same measures without hypotheticals

**Median - mean:**  
*% voting for the party in the median district*  
minus  
*% statewide vote for the party (mean)*



If the party won  $X\%$  of the statewide vote:  
**Partisan bias:**  $\frac{1}{2}^*$   
(*% districts where they got  $> X\%$*   
minus  
*% districts where they got  $< X\%$* )

# Efficiency gap

---

- New standard: first proposed in 2014 - 2015
- a symmetry measure that is easy to describe and avoids hypotheticals (sort of)
- **“... captures, in a single tidy number, all of the packing and cracking decisions that go into a district plan.”**

# Efficiency gap

---

$i$	$V_i^A$	$V_i^B$	Winner	$W_i^A$	$W_i^B$	$W_i^A - W_i^B$
1	95	5	A	45	5	40
2	40	60	B	40	10	30
3	75	25	A	25	25	0
4	45	55	B	45	5	40
5	45	55	B	45	5	40
All	300	200	2A : 3B	200	50	150

# Efficiency gap

$i$	$V_i^A$	$V_i^B$	Winner	$W_i^A$	$W_i^B$	$W_i^A - W_i^B$
1	95	5	A	45	5	40
2	40	60	B	40	10	30
3	75	25	A	25	25	0
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5	45	55	B	45	5	40
All	300	200	2A : 3B	200	50	150

*Efficiency gap:*

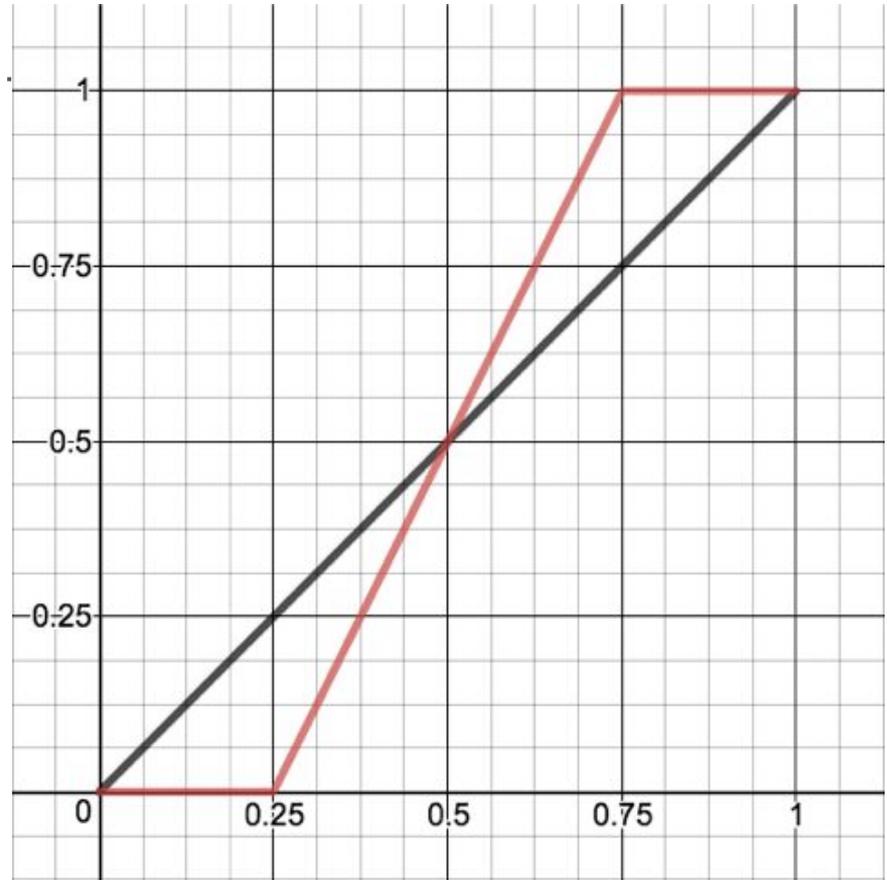
$$EG = (W^A - W^B) / V$$

In our example:  $EG = 150/500 = 0.3$

# Efficiency gap

With some mild assumptions, everything simplifies to

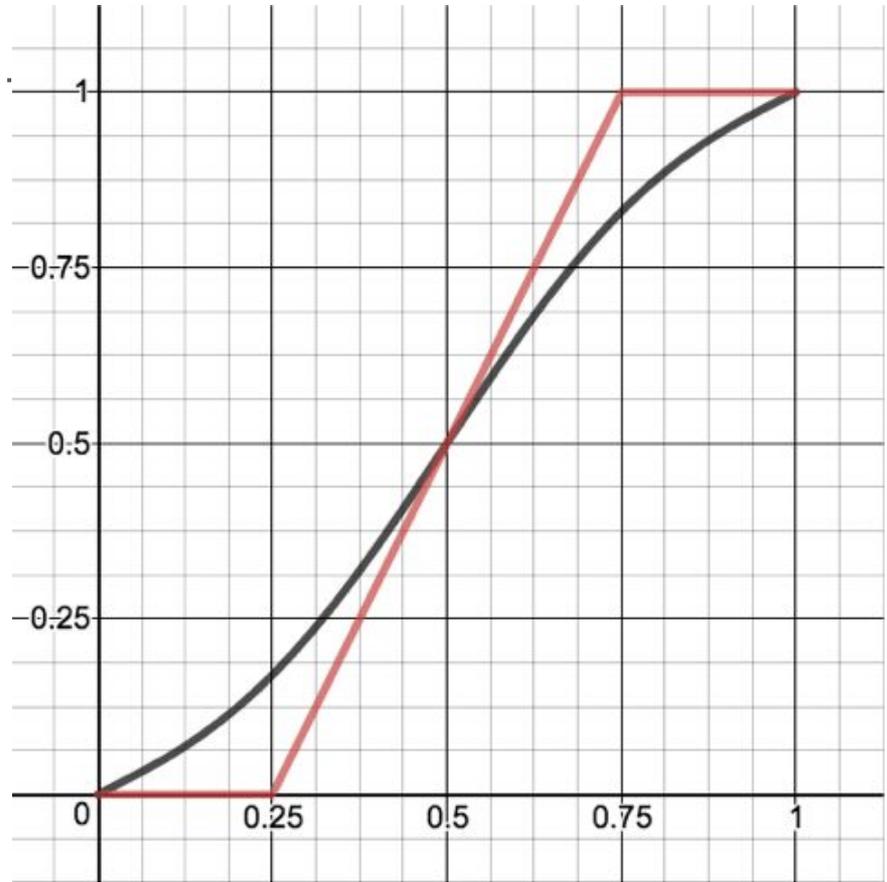
$$EG = 2V - S - \frac{1}{2}$$



# Efficiency gap

With some mild assumptions, everything simplifies to

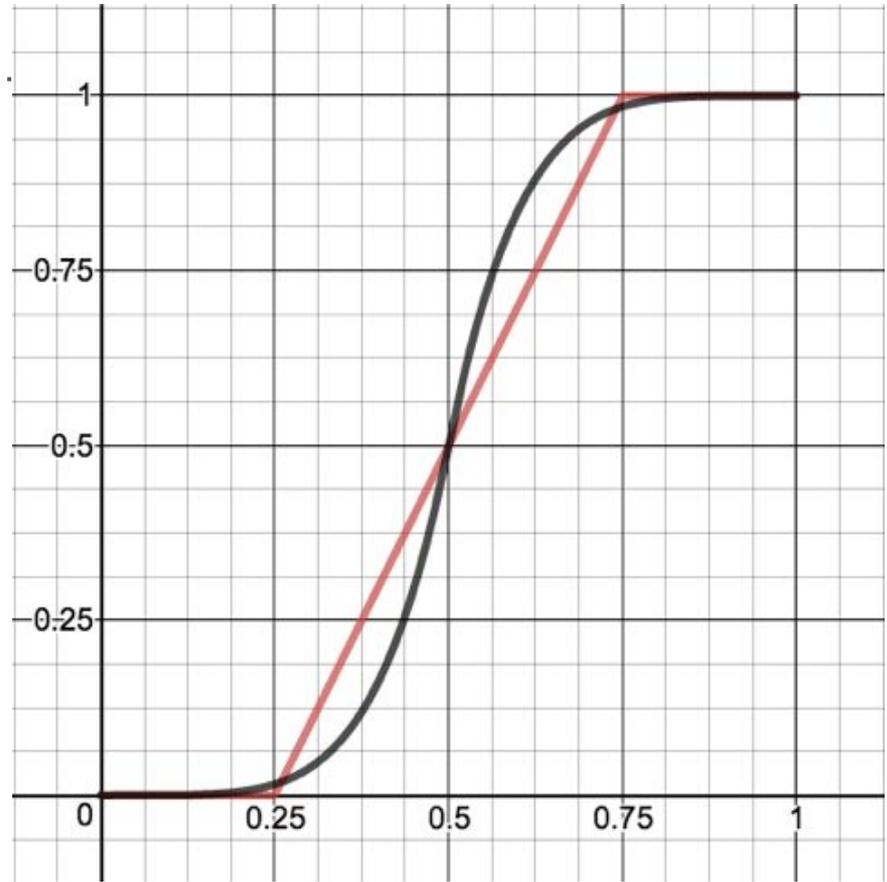
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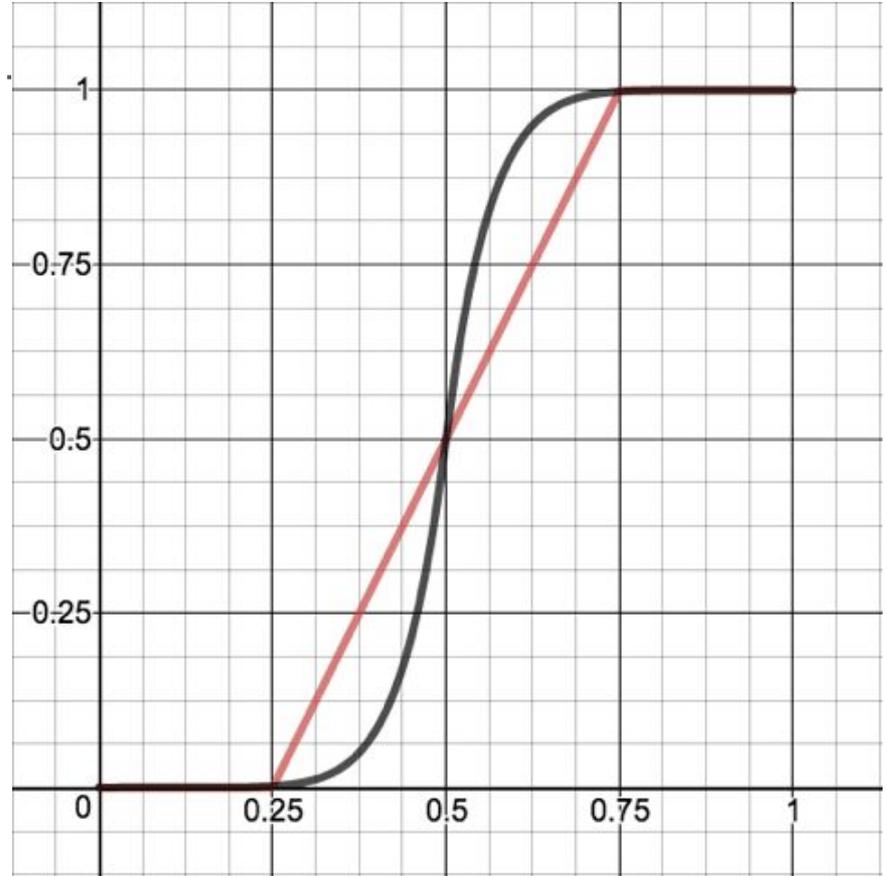
$$EG = 2V - S - \frac{1}{2}$$



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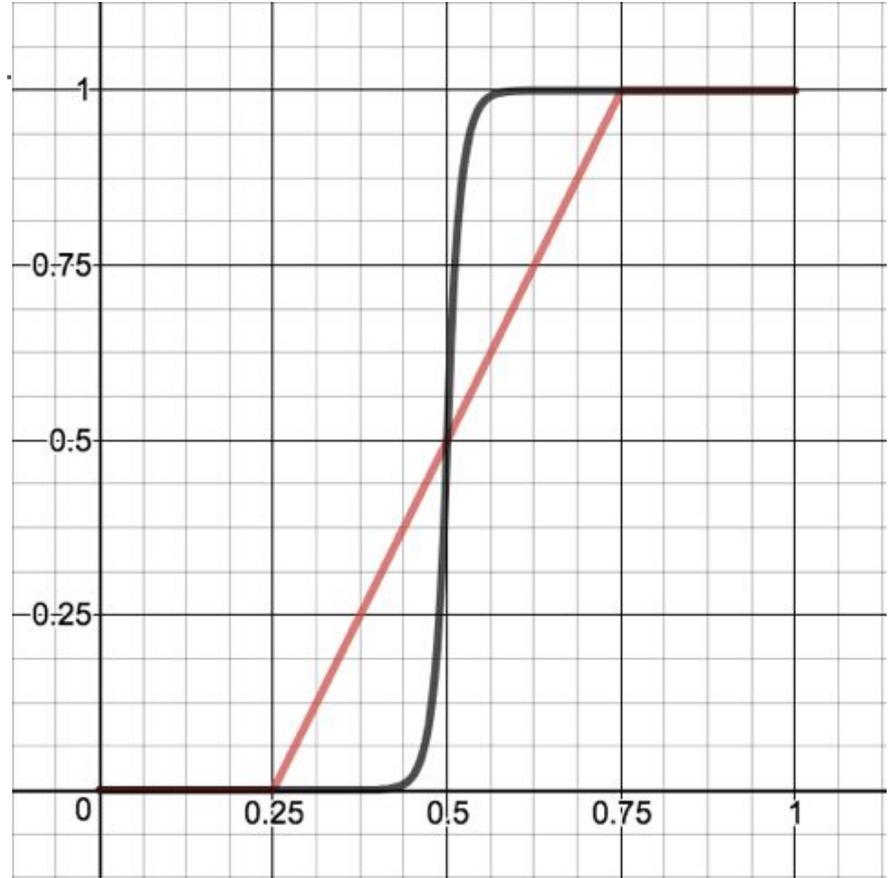
$$EG = 2V - S - \frac{1}{2}$$



# Efficiency gap

With some mild assumptions, everything simplifies to

$$EG = 2V - S - \frac{1}{2}$$



# Efficiency gap

---

*“Scholars have long recognized that [single-member district] systems such as the American one tend to provide a `winner's bonus" of surplus seats to the majority party, and the efficiency gap is consistent with this understanding.”*

— Stephanopoulos-McGhee, 2015

## Efficiency gap

---

*“But the gap offers what scholars to date have been unable to supply: a normative guide as to how large this bonus should be. To produce partisan fairness, in the sense of equal wasted votes for each party, the bonus should be a precisely twofold increase in seat share for a given increase in vote share.”*

— Stephanopoulos-McGhee, 2015

# Efficiency gap

---

What if we compared wasted votes slightly differently?

# Efficiency gap

---

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$$EG = (W^R - W^B)/V = W^R/V - W^B/V$$

# Efficiency gap

---

What if we compared wasted votes slightly differently?

$$EG = (W^R - W^B)/V = W^R/V - W^B/V$$

$$\text{“FH”} = W^R/V^R - W^B/V^B$$

# Efficiency gap

---

What if we compared wasted votes slightly differently?

$$EG = (W^R - W^B)/V = W^R/V - W^B/V$$

$$\text{“FH”} = W^R/V^R - W^B/V^B$$

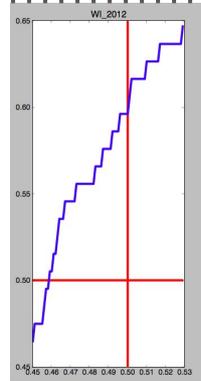
Using the same simple algebra as before, we obtain:

$$\text{FH} \cong 0 \text{ if and only if } S \cong V$$

# The measures cited in *Gill v. Whitford*

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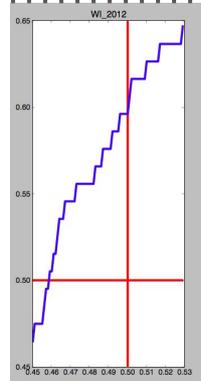
- *partisan bias*
  - explicitly dismissed by Kennedy in 2006



# The measures cited in *Gill v. Whitford*

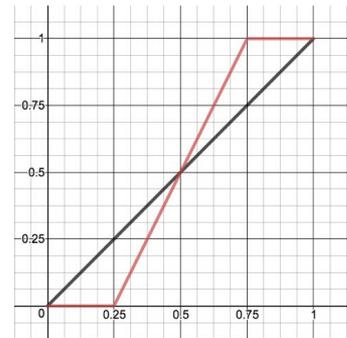
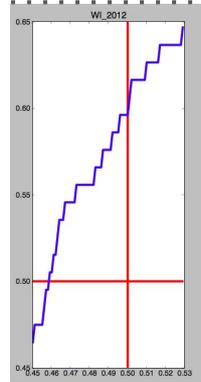
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- *partisan bias*
  - explicitly dismissed by Kennedy in 2006
- *median-mean*
  - similar to partisan bias, but sounds nicer when stated without hypotheticals

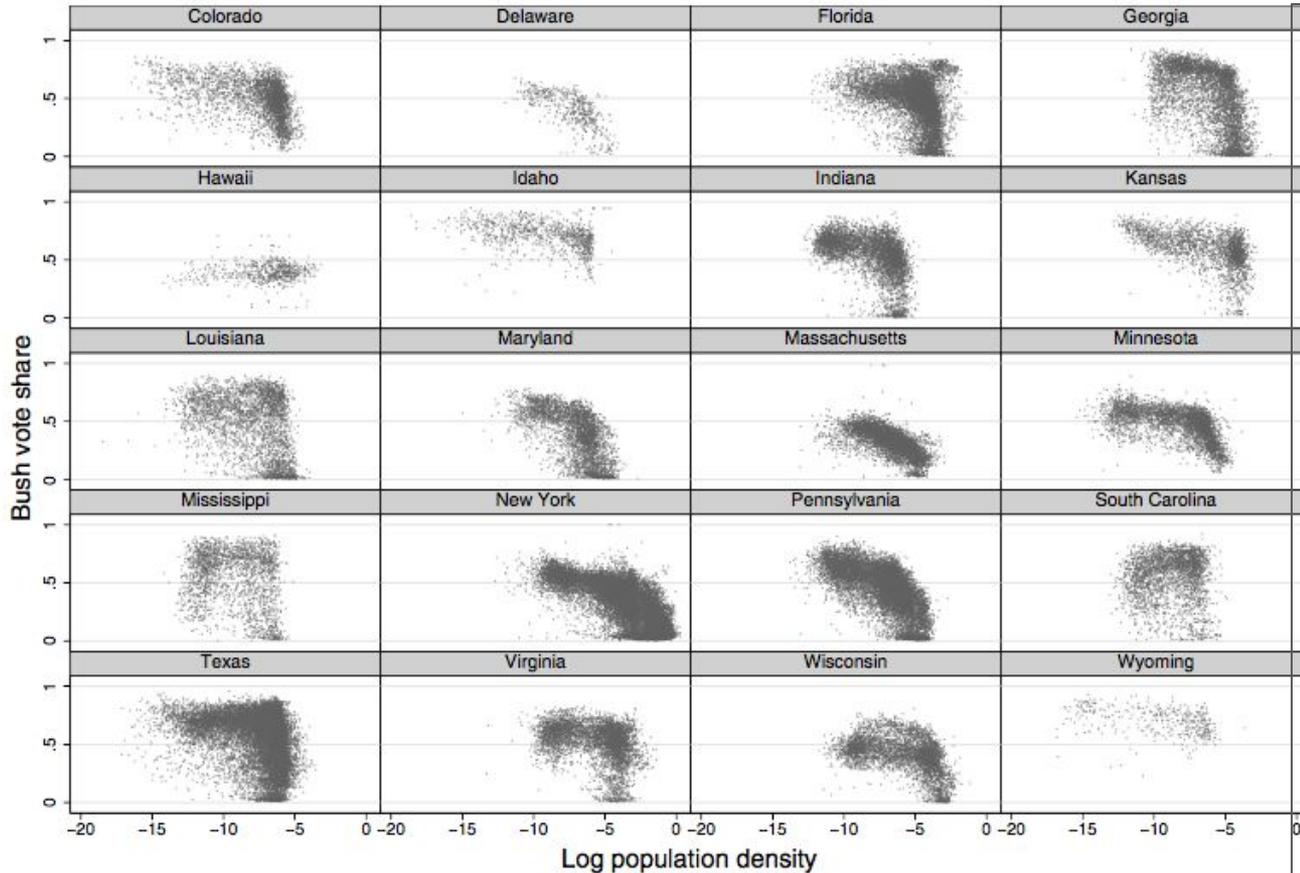


# The measures cited in *Gill v. Whitford*

- *partisan bias*
  - explicitly dismissed by Kennedy in 2006
- *median-mean*
  - similar to partisan bias, but sounds nicer when stated without hypotheticals
- *efficiency gap*
  - no hypotheticals, but other problems



# Problem for all the measures: voter geography



Chen & Rodden (2013): Unintentional gerrymandering

# **Problem for all the measures: voter geography**

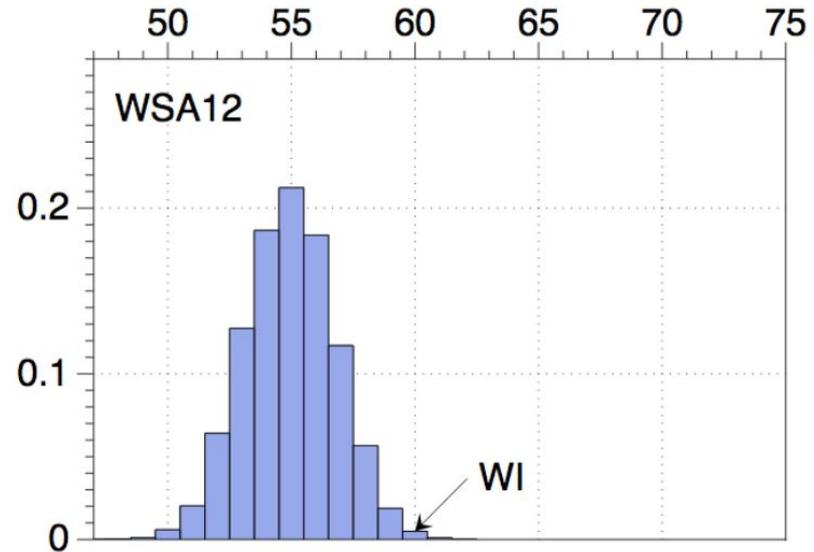
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**Sampling from the space of maps and outlier analysis!**



## **In summary...**

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- Most people want proportionality, but it does not arise naturally, both because of winner's bonus and because of voter geography
- We have some good measures of partisan symmetry, but Kennedy doesn't like them and they don't correct for voter geography
- EG is problematic, also doesn't consider geography

## In summary...

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- Sampling from the “space of reasonable maps” does correct for geography -- an enormous step forward!
  - Gives a baseline and effect size for whatever quantity you decide to measure. But you still have to decide what to measure...
  - What if a plan is an outlier in a direction we like (but maybe someone else doesn't)?

# The big non-mathematical questions remain...

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*Not just for the courts, but for redistricting reform!*

**What is “fairness”? What are we trying to achieve?**

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**What is “fairness”? What are we trying to achieve?**

- proportionality? (of what?)
- symmetry?
- neutrality?
- responsiveness?
- compactness for its own sake? (why?)