Measures of partisan fairness

Mira Bernstein
Duke “Geometry of Redistricting” Conference
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What is fairness?

“The parties have not shown us, and I have not been able to discover .... statements of principled, well-accepted rules of fairness that should govern districting.”

Is fairness the same as proportionality?

Most people’s intuitive notion of fairness:

*If a party gets X% of the vote, it should get about X% of the legislative seats*
The Supreme Court says NO!

“... the mere lack of proportional representation will not be sufficient to prove unconstitutional discrimination.”
- Plurality Opinion, *Davis v. Bandemer*, 1986

“Nor do I believe that ... proportional representation ... is consistent with our history, our traditions, or our political institutions.”
- Justice Sandra Day O’Connor, *Davis v. Bandemer*, 1986
“... judicial interest should be at its lowest ebb when a State purports fairly to allocate political power to the parties in accordance with their voting strength and ... through districting, provide a rough sort of proportional representation in the legislative halls of the State.”

- Majority in Gaffney v. Cummings (1973)
Gill v Whitford oral arguments

JUSTICE BREYER: If party A wins a majority of votes, party A controls the legislature. That seems fair. And if party A loses a majority of votes, it still controls the legislature. That doesn't seem fair. And can we say that without going into what I agree is pretty good gobbledygook?

CHIEF JUSTICE ROBERTS: And if you need a convenient label for that approach, you can call it proportional representation, which has never been accepted as a political principle in the history of this country.
MR. SMITH: Your Honor, we are not arguing for proportional representation. We are arguing for partisan symmetry, a map which within rough bounds at least treats the two parties relatively equal in terms of their ability to translate votes into seats.

CHIEF JUSTICE ROBERTS: That sounds exactly like proportional representation to me.
MR. SMITH: Proportional representation is when you give the same percentage of seats as they have in percentage of votes. That's what proportional representation means. And our -- our claim simply doesn't remotely do that. It says if party A at 54 percent gets 58 percent of the seats, party B when it gets 54 percent ought to get 58 percent of the seats. That's symmetry. That's what the political scientists say is the right way to think about a map that does not distort the outcome and put a thumb on the scale.
A toy example

- The state of Utopia has 100 seats in its state legislature.
- There are two parties, Purple and Orange.
- Purple won 55% of the vote. How many of the seats should they win?
Simulating Utopia (first with 10 districts)

Step 1: For each district, pick a random number from 0 to 1 to be the fraction of people who voted for Purple.

[0.75, 0.60, 0.37, 0.59, 0.073, 0.42, 0.60, 0.38, 0.75, 0.28]

37% of voters in District 3 voted for Purple
75% of voters in District 9 voted for Purple
Simulating Utopia (first with 10 districts)

Step 2: Average these numbers together. That’s the overall fraction of Utopians who voted for Purple. Call that $V$.

Step 3: Compute what percent of seats Purple won. Call that $S$. In our example:

$$[0.75, 0.60, 0.37, 0.59, 0.07, 0.42, 0.60, 0.38, 0.75, 0.28]$$

$S = 0.5$

$V = 0.48$
Simulating Utopia (first with 10 districts)

[0.75, 0.60, 0.37, 0.59, 0.07, 0.42, 0.60, 0.38, 0.75, 0.28]

Step 4:
Plot the point \((V, S)\).
Simulating Utopia (with 100 districts)

Now go back to 100 districts and do this 50,000 times.

This gives us 50,000 elections with different win margins for Purple.
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Note that in ~13% plans, a party that gets fewer than 1/2 the votes wins more than 1/2 the seats.
Simulating Utopia (with 100 districts)

Let’s look just at the elections where Purple won 55% of the vote.

How many seats did they get?
Simulating Utopia ($V = .55$)

On average, Purple wins **57** seats.
**Simulating Utopia, version 2**

Our simulation was unrealistic:

- Not all win margins for districts are equally likely. Districts are (or should be?) more commonly won by 60% than by 99%.
- We assumed that Purple got 55% of the total vote purely by luck. A more likely scenario is that Purple is actually more popular than Orange.
Simulating Utopia, version 2

Instead of picking Purple’s popularity in individual districts uniformly from 0 to 1, let’s use a truncated normal distribution centered at 0.55.
Simulating Utopia, version 2

On average, Purple wins 61 seats.
Simulating Utopia with competitive districts

Say purple and orange are balanced overall but the vast majority of districts are 40% to 60% purple.
Simulating Utopia with competitive districts

On average, if Purple wins 52% of the votes, they win 63% of the seats.
Less utopian simulations

Sam Wang’s idea: pick actual districts from around the country, at random (based on 2012 election)
Less utopian simulations

Florida 2016: draw district probabilities at random from precinct probabilities
Summary so far

Our electoral system (geographic single-member districts) has a built-in “winner’s bonus”: the party that wins the election gets more than its proportional share of votes.

- This has nothing to do with gerrymandering
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- This has nothing to do with gerrymandering
- In fact, to get proportional representation in this system, you have to gerrymander!
Summary so far

- How big is the winner’s bonus built into our system?
Summary so far

- *How big is the winner’s bonus built into our system?*
  
  It depends on the partisan distribution of the voters.
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  It depends on the partisan distribution of the voters.

- *How big should the winner’s bonus be?*
  That is a value judgment, not a mathematical question.

- *Then how can you tell if a plan is “fair” without imposing your value judgment on others?*
Outlier analysis to the rescue?
Outlier analysis to the rescue?
Outlier analysis to the rescue?

Yes, but...

- Extremely powerful and important
- A good indication of intentional gerrymandering
- But how can we say it’s evidence of “discriminatory effect” unless we specify what “fair” means?
Partisan Symmetry

Rather than prescribing the “fair” value of $S$ for a given $V$, we insist only that the plan must treat the two parties symmetrically.
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Evaluating the symmetry of a plan

Necessarily entails counterfactuals: how would this plan treat the parties under different (realistic) scenarios?

➢ In the last election, the Democrats got a huge winner’s bonus. Would the Republicans have gotten the same bonus if they had won a majority of the votes?

➢ Republicans got a majority of votes and a majority of seats. If they had gotten a minority of votes, do we believe they would have gotten a minority of seats?
Needed: a model of partisanship

Partisan preference depends on...

- **place**: some areas are always more Republican than others
Needed: a model of partisanship

Partisan preference depends on...

- **place**: some areas are always more Republican than others
- **time**: the whole country experiences swings left and right as the political climate changes
Needed: a model of partisanship

Model assumption:
The effects of place and of time are independent.
A model of partisanship!

“Uniform partisan swing”

<table>
<thead>
<tr>
<th>V</th>
<th>S</th>
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<tbody>
<tr>
<td>40%</td>
<td>21%</td>
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<tr>
<td>42%</td>
<td>29%</td>
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<td>44%</td>
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<td>56%</td>
<td>71%</td>
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<td>58%</td>
<td>74%</td>
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WA 2016

MN 2016

Simulated: N(0.5,0.25)

OH 2016

NC 2016

WI 2012 (state senate)
Measures of asymmetry

(0.5, 0.5): the one point required be on any symmetric curve
Measures of asymmetry

Partisan bias: how much of an unfair advantage the party would have if the vote were evenly split

(0.5, 0.5): the one point required be on any symmetric curve
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How far the party can fall from a majority of votes and still get a majority of seats
The issue of hypotheticals


The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside.
The issue of hypotheticals

“Even assuming a court could choose reliably among different models of shifting voter preferences, we are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.”
Same measures without hypotheticals

Median - mean:
% voting for the party in the median district minus % statewide vote for the party (mean)

If the party won X% of the statewide vote:
Partisan bias: $\frac{1}{2}$
(% districts where they got > X% minus % districts where they got < X%)
Efficiency gap

- New standard: first proposed in 2014 - 2015
- A symmetry measure that is easy to describe and avoids hypotheticals (sort of)
- "... captures, in a single tidy number, all of the packing and cracking decisions that go into a district plan."
### Efficiency gap

<table>
<thead>
<tr>
<th>$i$</th>
<th>$V_i^A$</th>
<th>$V_i^B$</th>
<th>Winner</th>
<th>$W_i^A$</th>
<th>$W_i^B$</th>
<th>$W_i^A - W_i^B$</th>
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<tbody>
<tr>
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<td>5</td>
<td><strong>A</strong></td>
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<tr>
<td>2</td>
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<td>60</td>
<td><strong>B</strong></td>
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<td>10</td>
<td>30</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>25</td>
<td><strong>A</strong></td>
<td>25</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>45</td>
<td>55</td>
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<td>55</td>
<td><strong>B</strong></td>
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<td>40</td>
</tr>
<tr>
<td>All</td>
<td>300</td>
<td>200</td>
<td><strong>2A : 3B</strong></td>
<td>200</td>
<td>50</td>
<td>150</td>
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</table>


## Efficiency gap

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**Efficiency gap:**

\[
EG = \frac{(W^A - W^B)}{V}
\]

In our example: \(EG = \frac{150}{500} = 0.3\)
Efficiency gap

With some mild assumptions, everything simplifies to

\[ \text{EG} = 2V - S - \frac{1}{2} \]
Efficiency gap

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Efficiency gap

“Scholars have long recognized that [single-member district] systems such as the American one tend to provide a "winner's bonus" of surplus seats to the majority party, and the efficiency gap is consistent with this understanding.”

— Stephanopoulos-McGhee, 2015
Efficiency gap

“But the gap offers what scholars to date have been unable to supply: a normative guide as to how large this bonus should be. To produce partisan fairness, in the sense of equal wasted votes for each party, the bonus should be a precisely twofold increase in seat share for a given increase in vote share.”

— Stephanopoulos-McGhee, 2015
Efficiency gap

What if we compared wasted votes slightly differently?
Efficiency gap

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\[ EG = (W^R - W^B)/V = W^R/V - W^B/V \]
Efficiency gap

What if we compared wasted votes slightly differently?

\[ EG = \frac{(W^R - W^B)}{V} = \frac{W^R}{V} - \frac{W^B}{V} \]

“FH” = \[\frac{W^R}{V^R} - \frac{W^B}{V^B}\]
Efficiency gap

What if we compared wasted votes slightly differently?

$$\text{EG} = \frac{W^R - W^B}{V} = \frac{W^R}{V} - \frac{W^B}{V}$$

\[\text{“FH”} = \frac{W^R}{V^R} - \frac{W^B}{V^B}\]

Using the same simple algebra as before, we obtain:

\[\text{FH} \approx 0 \text{ if and only if } S \approx V\]
The measures cited in *Gill v. Whitford*

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- **partisan bias**
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- **median-mean**
  - similar to partisan bias, but sounds nicer when stated without hypotheticals
- **efficiency gap**
  - no hypotheticals, but other problems
Problem for all the measures: voter geography

Chen & Rodden (2013): Unintentional gerrymandering
In each particular case, how do we tell how much of the observed asymmetry is due to “unintentional gerrymandering”?  

Problem for all the measures: voter geography
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Problem for all the measures: voter geography

Sampling from the space of maps and outlier analysis!
In summary...

- Most people want proportionality, but it does not arise naturally, both because of winner’s bonus and because of voter geography
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- Most people want proportionality, but it does not arise naturally, both because of winner’s bonus and because of voter geography.
- We have some good measures of partisan symmetry, but Kennedy doesn’t like them and they don’t correct for voter geography.
- EG is problematic, also doesn’t consider geography.
In summary...

- Sampling from the “space of reasonable maps” does correct for geography -- an enormous step forward!
  - Gives a baseline and effect size for whatever quantity you decide to measure. But you still have to decide what to measure...
  - What if a plan is an outlier in a direction we like (but maybe someone else doesn’t)?
The big non-mathematical questions remain...

Not just for the courts, but for redistricting reform!

What is “fairness”? What are we trying to achieve?
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- symmetry?
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What is “fairness”? What are we trying to achieve?

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- neutrality?
The big non-mathematical questions remain...

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What is “fairness”? What are we trying to achieve?

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- symmetry?
- neutrality?
- responsiveness?
The big non-mathematical questions remain...

Not just for the courts, but for redistricting reform!

What is “fairness”? What are we trying to achieve?

- proportionality? (of what?)
- symmetry?
- neutrality?
- responsiveness?
- compactness for its own sake? (why?)