No news is good news: voluntary disclosure in the face of litigation

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We study disclosure dynamics when the firm value evolves stochastically over time. The presence of litigation risk, arising from the failure to disclose unfavorable information, crowds out positive disclosures. Litigation risk mitigates firms’ tendency to use inefficient disclosure policies. From a policy perspective, we show that a stricter legal environment may be an efficient way to stimulate information transmission in capital markets, particularly when the nature of information is proprietary. We model the endogeneity of litigation risk in a dynamic setting and shed light on the empirical controversy regarding whether disclosure preempts or triggers litigation.

1. Introduction

Firms receive information on an ongoing basis. Productivity shocks originate from many sources, including innovation breakthroughs, the arrival of business opportunities, frictions in negotiations with labor unions, and breakdowns of supply-chain relationships. Capital market perceptions pressure managers to disclose information frequently because the stock price performance is affected by disclosures and lack thereof (see, e.g., Graham, Harvey, and Rajgopal, 2005).

Firm managers may thus feel inclined to disclose good news, but disclosing positive news can be costly. Concealing bad news likewise can be risky because that information might be revealed by external sources, perhaps triggering costly litigation. For example, in 2012, the US Justice Department announced that GlaxoSmithKline (GSK) had agreed to plead guilty and pay a $3 billion fine for withholding information about the cardiovascular risk of Avandia, its antidiabetes drug. Avandia’s problems began in 2007, when a study published in the New England Journal of Medicine (Nissen and Wolski, 2007) found that the drug carried a higher risk of heart attacks than alternative drugs. The 2012 settlement stems from claims made by four GSK employees,

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who tipped off the government about the firm’s concealment of two internal studies that preceded Nissen and Wolski (2007). Avandia prescriptions and GSK’s stock price dropped sharply after the publication of Nissen and Wolski (2007). In another example, Toyota was forced to pay a $1.2 billion penalty to settle the criminal probe into its handling of unintended acceleration problems that led to the recall of 8.1 million vehicles beginning in 2009.¹

We study managers’ incentives to disclose information when withholding negative information is costly due to the presence of litigation risk. How do managers delay the release of adverse information? We provide a model that highlights a fundamental asymmetry between positive and negative information. When the market expects positive information to be disclosed, no news is bad news. The market punishes a firm that withholds information. In contrast, when the market expects negative information to be disclosed, no news is good news. This asymmetry is important because silence, unlike a positive disclosure which cannot be imitated, is something that firms can imitate. The presence of litigation risk therefore allows firms to signal their good standing to the market by delaying the release of information.

We show that this trade-off determines both the timing and the content of disclosures. More important, we show that litigation risk may reduce market skepticism, thus mitigating the tendency of managers to rely on inefficient disclosure policies.

We present a continuous-time disclosure model. Specifically, we analyze a disclosure game between the manager of a firm and a mass of buyers (the market) when the firm’s asset value evolves stochastically over time. The evolution of the asset value is described by a continuous-time Markov chain that fluctuates between two possible states: low asset value and high asset value. The manager can disclose private information at any point in time and as often as desired. Unraveling is not possible in equilibrium because disclosing good news is costly (so the firm cannot do it continuously). Concealing bad news is risky because there the public news process has a positive arrival rate when the asset value is low and will trigger costly litigation if the manager fails to disclose the information before the news arrives. The market is competitive and continuously adjusts the firm’s stock price, based on all public information. The manager maximizes the present value of the firm’s future stock prices, perhaps because his compensation, at each point in time, is proportional to the firm’s stock price.

The presence of litigation risk suggests that managers will preempt the announcement of bad news by voluntarily disclosing it to avoid litigation costs. This preemption strategy is well documented empirically (see, e.g., Skinner, 1994; Lev, 1995; Johnson, Kasznik, and Nelson, 2001) but not well understood in theory. In fact, the strategy poses a conceptual difficulty: if the market expects the manager to reveal bad news at a given time, then the manager’s silence will be interpreted as a clear sign that the manager’s information is favorable, which will lead to a rise in the stock price. However, rewarding silence in this way cannot be part of an equilibrium because rewarding silence, unlike providing verifiable good news, is something that all firms can do, including those experiencing financial problems. Our analysis reveals that in equilibrium, the firm can release bad news but only probabilistically. Indeed, the equilibrium predicts that when falling stock prices reach a certain threshold, the firm will reveal bad news with a probability that depends on the arrival intensity of the public news, the cost of litigation, and the proprietary cost of disclosing good news. At that point, the stock price will remain constant until bad news is finally disclosed.

Litigation risk not only leads to the preemption of bad news but, more important, crowds out the disclosure of good news because silence is interpreted, per se, as a favorable signal of the firm’s prospects. The manager can reveal good news in two ways: by explicitly disclosing the good news and bearing the disclosure cost or by remaining quiet, thus signalling the good news.

¹ Former Attorney General Eric Holder called the settlement the largest US criminal penalty ever imposed on a car company and asserted, “We can say for certain that Toyota intentionally concealed information and misled the public about the safety issues behind these recalls.”
There is a pecking order: the manager prefers to use silence when the firm’s undervaluation is moderate and to use disclosure when the firm’s undervaluation is severe.

The presence of litigation risk may be desirable even from the firm’s perspective. By creating a new communication mechanism that allows managers to convey good news without disclosing it, the existence of litigation risk allows the firm to save on proprietary disclosure costs. From a policy perspective, this means that a harsh legal environment may be a cost-effective way of improving information transmission, especially in settings where information is highly proprietary.

Our model is stylized but tractable and allows us to study interesting applications.

In Section 5, we make litigation risk endogenous by considering the incentives to monitor the firm. For example, the US False Claims Act encourages people with knowledge of suspected false claims to sue on the government’s behalf. If the Justice Department joins such a lawsuit, the plaintiff can receive 15% to 25% of recoveries. This law creates significant incentives for whistle-blowers to monitor firms’ disclosure behavior, thereby creating endogenous litigation risk. To capture the endogeneity of litigation risk, we assume the presence of a whistle-blower who may investigate the firm at a cost. The whistle-blower receives a reward, paid by the firm, if he can establish that the firm concealed negative information. We demonstrate that the whistle-blower tends to investigate firms with relative low prior performance for whom the market uncertainty is relatively large (firms that have remained silent for a lengthy time). The presence of the whistle-blower means that, in equilibrium, there must always be a positive probability of concealment, no matter how large the litigation cost. This extension helps clarify the puzzling relation between litigation risk and disclosure documented empirically by Francis, Philbrick, and Schipper (1994) suggest that disclosure triggers litigation. In our setting, litigation and disclosure are simultaneously determined but are not causally linked.

In Section 6, we assume that withholding information for a longer period of time is more costly to the firm, perhaps because more people traded the stock at an inflated price. We characterize the amount of disclosure delay and its determinants and show that after a certain point investors will cease to update the stock price even though the manager may receive and withhold negative information—until the manager eventually discloses some information. In Section 7, we consider the case when the firm’s cash flows have a finite, predetermined life, perhaps because the firm’s asset is protected by a patent. In this setting, we show that firms tend to delay disclosures of adverse events until the expiration approaches, and to cluster the release of negative information just before its expiration. We also show that the longer the expiration, the more slowly the firm reveals the adverse information. To the best of our knowledge, these predictions are new and have not yet been tested.

Our results also apply to the problem of product quality certification (see, e.g., Dranove and Jin, 2010). Indeed, the model can be interpreted as one in which a monopolist sells a product of unknown quality to a mass of buyers. At each point in time, buyers purchase the good of unobservable quality at a price that equals the good’s expected quality. The monopolist has the option to certify the product’s quality at a cost to influence the trajectory of future prices. This certification-like interpretation, originally adopted by Jovanovic (1982), highlights the parallel between corporate disclosures and quality certification.

The rest of the article is organized as follows. Section 2 presents the setting. Section 3 analyzes the baseline model without litigation risk. Section 4 introduces litigation risk. Section 5 studies endogenous public news in settings where fact checkers monitor the firm’s disclosure behavior. Sections 6 and 7 consider two extensions. Section 8 concludes.

\(^2\) When adverse shocks are permanent, the result is stronger: the manager may prefer a high litigation risk to zero litigation risk. Numerical computations suggest that this result still holds if negative shocks are sufficiently persistent.

\(^3\) An article in the Wall Street Journal on July 24, 2014 states that Dr. William LaCorte, a “serial whistle-blower” received a $38 million cut under a federal law that encourages fraud reporting. Much of this sum was from a $250 million US settlement with Merck in 2008 over allegations that it overcharged Medicaid for the heartburn drug Pepcid.
Related literature. We study a continuous-time disclosure model, building on Jovanovic (1982) and Verrecchia (1983, 1990). Unlike existing literature, our model features a continuous flow of private information. Moreover, our model includes a continuous flow of public information and the presence of stochastic litigation costs.

The article most closely related to ours is Acharya, DeMarzo, and Kremer (2011). They consider a dynamic version of Dye (1985), where the manager may be privately informed about the asset value. When informed, the manager may disclose private information at one of two points in time: at the start of the game or at a known date after a public news signal is released. If the manager’s private information is not favorable, waiting for news has a positive option value, because the public signal might induce a higher price in the absence of disclosure than in its presence. By contrast, if the public signal turned out to be unfavorable, the manager could mitigate its negative price effect by disclosing private information. Their model explains clustering of disclosure in bad times: the less favorable the public signal, the higher the probability of disclosure.

Kremer, Guttman, and Skrypacz (2014) consider disclosure timing and the resulting price consequences. They study a two-period extension of the Dye (1985) model, where in each period, the manager may observe with some probability any of two pieces of information (if previously unobserved). They show that later disclosures are interpreted more favorably by the market because, in equilibrium, when partial disclosures are made earlier, the probability that the manager is hiding information is perceived to be higher.

In the presence of litigation risk, price dynamics resemble those in the recent literature on dynamic signalling. As in previous dynamic signalling models (Bar-Isaac, 2003; Daley and Green, 2012; Gul and Pesendorfer, 2012; Lee and Liu, 2013), the use of mixed strategies prevents the price from falling below a threshold and generates delay.

This article relates to the industrial organization literature studying imperfect competition with uncertain quality (see, e.g., Stokey, 1981; Conlisk, Gerstner, and Sobel, 1984; Bagwell and Riordan, 1991; Daughety and Reinganum, 2008b). Although in this strand of the literature firms choose prices as signals of product quality, in our article, the firm controls the price only indirectly by influencing buyers’ beliefs through certification. For example, whereas in Bagwell and Riordan (1991), high and declining prices signal high quality, in our article, this price pattern reflects a deterioration of market beliefs about quality.

Closely related is Daughety and Reinganum (2008a), who study information transmission in settings in which firms can either disclose or certify quality or signal it through the product’s price. In our model, when litigation risk is introduced, the signalling of quality via disclosure delay emerges, in equilibrium, as an alternative communication channel. In contrast to Daughety and Reinganum (2008a), signalling is preferred by high-quality firms, whereas disclosure is chosen by low-quality ones.

The literature studying the effect of quality certification on consumer choices and seller behavior is closely related. Dranove and Jin (2010) argue that “while most existing studies have examined the short-run consequences of quality disclosure, little is known about long-run effects.” Our article characterizes the distribution of steady-state prices and the market’s long-run uncertainty about quality.

We build on the literature on the relation of litigation risk and disclosure. Ongoing work by Dye (2014) extends Dye (1985) by incorporating litigation risk. Daughety and Reinganum (2008b) study a static model where a firm facing litigation risk may signal safety or quality by either pricing or quality disclosures. They show that, depending on the level of litigation risk, the equilibrium may result in too little or too much disclosure.

Our endogenous litigation extension is closely related to the literature on monitoring and auditing. In particular, early articles study the problem of monitoring when the monitor cannot commit to a monitoring strategy. Graetz, Reinganum, and Wilde (1986), for example, study the incentives of the Internal Revenue Service to audit taxpayers who underreport their taxable income when a fraction of the taxpayers are honest. Our endogenous monitoring extension, in
Section 5, can be considered as a dynamic extension of their work. Our endogenous litigation extension is also closely related to Halac and Prat (2014). In their model, a principal can monitor a myopic agent’s “good behavior” to mitigate the agent’s tendency to shirk in the absence of “recognition.” In contrast, in our model, the whistle-blower monitors the manager to extract rents from the manager’s bad behavior. Also, whereas in our model the whistle-blower chooses the intensity of monitoring, in Halac and Prac (2014), monitoring is a lumpy investment that entails a fixed cost and can depreciate. Such an investment gives rise to a detection technology that produces a verifiable (Poisson) signal that has a positive arrival intensity when the agent exerts effort. Hence, if the technology is in place, the intensity of monitoring is fixed.

2. Setting

The value of assets $V_t$ follows a continuous-time Markov chain with state space \{0, 1\}. It jumps from 0 to 1 with intensity $\lambda_0$ and from 1 to 0 with intensity $\lambda_1$. We can think of $\lambda_0$ as the frequency of an asset impairment. The impairment is permanent when $\lambda_1 = 0$, and transitory otherwise.

At the outset, the asset value is known to be 1, namely, $V_0 = 1$. From that point on, the manager privately observes shocks to the asset value. At any point, however, the manager can disclose private information at a cost. The cost of disclosure varies with the value disclosed. In particular, the cost of disclosure is $c > 0$ when the asset value is high and 0 when the asset value is low. Hence, disclosing bad news is costless.

The market has access to an alternative information source. Public information is represented by a Poisson process $N = \{N_t\}_{t \geq 0}$. If the value of assets is low, $N$ has an arrival rate $\mu$, whereas it has an arrival rate of 0 if the value of assets is high. Hence, observing an arrival is perfect evidence of low asset value, which we term bad news.

The manager is subject to legal liability. Whenever bad news arrives, the manager bears a fine with positive probability if he did not disclose that the asset value is low. We denote by $\theta$ the expected cost of withholding negative information conditional on the arrival of news. Thus, hiding information entails a flow expected cost of $\mu \theta$.

Let $F_t$ be the market information available at time $t$, $\sigma_t$ the manager’s disclosure strategy, and $\sigma$, the market conjecture. A mixed disclosure strategy can be represented by a cumulative distribution function (CDF). A strictly positive probability of disclosure at time $t$ represents a jump in the CDF. Formally, a strategy is a marked point process $(\tau_n, D_n)$ adapted to the filtration generated by $(N_t, V_t)$, where $\tau_n$ is the n-th disclosure and $D_n \in \{0, 1\}$ is the content of the disclosure. The type-$v$ disclosure time can be represented by a right-continuous, nondecreasing process $F^v$ such that $0 \leq F^v \leq 1$. The process $F^v_t$ is the CDF of the disclosure time; a jump in $F^v_t$ indicates a positive probability of having a disclosure $D_v$ at time $t$. We denote the price by $p_t = E^\mathbb{Q} (V_t | F_t)$. Whenever the CDF of the manager disclosure at time $t$ is absolutely continuous, we denote its hazard rate by $h^v_{rst}$; in other words, the disclosure probability in the interval of time $(t, t + dt)$ is $h^v_{rst} dt$.

By Bayes’ rule, we have

$$p_{t + dt} = p_t + (\lambda_1 (1 - p_t) - \lambda_0 p_t ) dt + \frac{(\mu + h^v_{r0} - h^v_{rst}) p_t (1 - p_t)}{p_t (1 - h^v_{rst} dt) + (1 - p_t)(1 - (\mu + h^v_{r1}) dt)} dt + o(dt). \tag{1}$$

In the limit as $dt \to 0$, in the absence of news arrivals and disclosure, prices evolve according to the following differential equation:

$$\dot{p}_t = \kappa (\bar{p} - p_t) + (\mu + h^v_{r0} - h^v_{rst}) p_t (1 - p_t). \tag{2}$$

\footnote{Nothing changes if $V_0$ is private information at the start.}

\footnote{We could also consider the case with positive Poisson shocks, in which an arrival represents a positive cash flow generated by a breakthrough or innovation. The characterization of the equilibrium would be similar, and the main economic forces would remain the same. We focus on the bad news case to lay the groundwork for the case with legal liability in Section 4.}
where $\tilde{p}$ is the stationary probability that the value of the asset is 1, which is given by

$$\tilde{p} := \frac{\lambda_1}{\lambda_0 + \lambda_1},$$

and $\kappa := \lambda_0 + \lambda_1$ represents the asset’s mean reversion, namely, the speed at which the distribution of the value of assets reverts to its stationary distribution $p$. This measure is the reciprocal of the persistence of shocks. In contrast, if at time $t$ the market expects the high-type manager to disclose information, then prices will drop to zero if the manager does not disclose.

We assume that manager preferences are increasing in the price and decreasing in the cost of disclosing information. In particular, the manager payoff given the price and the disclosure strategy is

$$U_t(\sigma, \tilde{\sigma}) := E_t^\sigma \left[ \int_t^\infty e^{-r(s-t)} p_s ds - \sum_{\tau_n \geq t} e^{-r(\tau_n-t)} c \cdot D_n - \int_t^\infty e^{-r(s-t)} \theta \cdot 1_{\{\text{hide}\}} \cdot dN_s \right], \quad (3)$$

where $1_{\{\text{hide}\}}$ is an indicator function that takes on value 1 if the manager is hiding negative information at time $t$ and zero otherwise. In Section 2, we discuss several possible interpretations for manager preferences.

In equilibrium, the manager’s disclosure strategy $\sigma$ maximizes (3) given the asset value $V_t$ and the evolution of prices given by (2). In this article, we focus on Markov Perfect Equilibria defined as

**Definition 1.** A Markov Perfect Equilibrium $(\sigma, \tilde{\sigma})$ is a Markov disclosure strategy $\sigma$ and market conjecture $\tilde{\sigma}$ such that $^6$

(i) Beliefs are $p_t = E_{\tilde{\sigma}}^\sigma(V_t)$, and, in absence of disclosure, evolve according to equation (2).

(ii) The manager’s disclosure strategy $\sigma$ maximizes the manager’s payoff (3) given the market conjecture $\tilde{\sigma}$.

(iii) The market beliefs about disclosure coincide with the manager’s disclosure strategy, that is, $\tilde{\sigma} = \sigma$.

The previous conditions are standard. First, at each point in time, the price is given by Bayes’ rule and consistent with the manager’s strategy. Second, the manager’s disclosure strategy maximizes the manager’s expected utility at each point in time and for all possible histories. Third, the market-conjectured disclosure strategy coincides with the disclosure strategy actually followed by the manager.

□ **Discussion and interpretation.**

**Manager preferences.** The manager preferences, represented by equation (3), have various interpretations. First, the manager preferences may capture incentives arising from stock-based compensation. Under this interpretation, as in Acharya, DeMarzo, and Kremer (2011) and Benmelech, Kandel, and Veronesi (2010), the manager chooses a disclosure strategy $\sigma$ that maximizes the present value of future prices net of disclosure expenses and litigation costs. The manager cares not only about the current price implications of his disclosures but also the long-term ones. This concern for future prices is natural and supported by a vast empirical literature documenting that the evolution of firms’ stock price affects manager wealth. These incentives may arise as a means of inducing the manager to exert effort, in the spirit of Benmelech, Kandel, and Veronesi (2010), but as our focus is on disclosure behavior, we take the manager’s incentives as given. The manager also cares about the present value of disclosure expenses. As

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$^6$ A strategy is Markov if for any two histories $H_t$ and $\tilde{H}_t$ ending in the same state $(p_t, V_t)$, we have that $\sigma(H_t) = \sigma(\tilde{H}_t)$. 

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mentioned above, the literature has often interpreted \( c \) as arising from the proprietary nature of the information. For this interpretation to be literally valid in our dynamic model, \( c \) should be priced; namely, it should be incorporated into the price as part of the firm’s future cash flows. Under this alternative formulation, the current price would be a nonlinear function of price. This alternative formulation is mathematically more complicated without adding much economic insight.\(^7\)

We can also interpret our model as representing quality certification by a monopoly. Equation (3) captures the profits of a monopoly selling a product of unobservable quality. At each point in time, the buyers purchase the good of unobservable quality \( v \in \{0, 1\} \) at a price that equals the good’s expected quality, \( p_t \). The monopoly can certify the product’s quality at a cost \( c \), to affect the trajectory of future prices. If we normalize the good’s production cost to zero, then the present value of the firm’s expected profits, given a certification strategy \( \sigma \), is given by (3). This certification-like interpretation, originally adopted by Jovanovic (1982), highlights the parallel between corporate disclosures and quality certification. The litigation risk in this case corresponds to the product liability associated with a low-quality product, as in the Avandia example mentioned in the Introduction.\(^8\) Last, the manager’s concern for the evolution of the stock price may be driven by career concerns insofar as prices can be interpreted as signals of the manager’s talent. Unlike standard career concerns models (e.g., Holmström, 1999; Stein, 1989), in our model, the manager controls the availability of information.

**Disclosure costs.** The presence of disclosure costs may arise from the proprietary nature of information (as in Verrecchia, 1983),\(^9\) the need to certify the information to make it credible (e.g., hiring an auditor, as in Jovanovic, 1982), or simply from the opportunity cost of the time required to prepare and disseminate the information.\(^10\) Verrecchia (1983), for instance, argues that “the release of a variety of accounting statistics about a firm may be useful to competitors, shareholders, or employees in a way which is harmful to a firm’s prospects. One example of this is the response of the United Auto Workers for fewer labor concessions in the face of an announcement by Chrysler Corporation’s chairman that that firm’s fortunes had improved. Other examples might include the reluctance of managers in certain highly competitive industries, such as personal computers or airlines, or certain politically sensitive industries, such as the oil industry or foreign automobile importers, to disclose favorable accounting data.”

For simplicity, we assume that negative disclosures are not costly. This asymmetry is not strictly required for the results to hold. Similar predictions hold if we assume that the cost of negative disclosures is positive, \( c_0 \in (0, c) \). The lower cost of negative disclosures is natural and can be justified by noting that, in an augmented version of our model, where managers can issue cheap talk messages, bad news disclosures are credible in equilibrium, without the need to certify them. In such an augmented model, even if the cost of verifying good and bad news is the same, the manager would never incur it. He would choose to communicate the bad news by releasing a cheap talk message.\(^11\)

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\(^7\) This alternative formulation is available from the authors upon request.

\(^8\) Milgrom (2008) and Dranove and Jin (2010) provide two excellent surveys of the literature of quality certification. Daughety and Reinganum (2013) provides a survey of the literature on product liability.

\(^9\) Ellis, Fee, and Thomas (2012) assert, “We find robust evidence in support of the hypothesis that proprietary costs are an important factor in firms’ disclosure choices regarding information about large customers.”

\(^10\) A number of large investors, such as Warren Buffett (1996) and analysts such as Candace Browning (2006), head of global research at Merrill Lynch, have called for managers to give up quarterly earnings guidance and hence avoid the myopic managerial behavior caused by attempts to meet market expectations.

\(^11\) Another situation in which bad news disclosure is relatively inexpensive occurs when the firm cash flows are generated via a Leontief production function with several inputs, any of which may be defective. Kremer (1993) O-ring technology is an example of this. To verify that the shuttle is in functioning order, one has to examine all its components, whereas to verify that it is dysfunctional, it suffices to find one defective component.
Last, it is reasonable to assume that bad news disclosures are less costly if the cost of disclosure comes from competitive advantages, as in the United Auto Workers example above. However, the assumption that disclosure costs are asymmetric is probably not very accurate if the cost represents the opportunity cost of the time required to prepare and disseminate information.

**Litigation cost.** We assume that disclosure of bad news preempts litigation risk. Though there is no consensus regarding the validity of this assumption in practice, Skinner (1994) argues that disclosures weakens the claim that managers act improperly by failing to disclose information promptly, thus lowering the probability of a lawsuit. Second, voluntary disclosure might reduce the contingent loss in the case of a lawsuit. By informing the market of bad news early on, the firm decreases the time that the stock trades at misleading prices, thus decreasing recoverable damages. Third, with lower potential damages, plaintiffs’ incentives to bring a lawsuit are reduced. Last, a stylized fact is that class-actions lawsuits tend to be precipitated by large stock price drops (Francis, Philbrick, and Schipper, 1994; Grundfest and Perino, 1997). Partially revealing bad news in a voluntary disclosure may reduce the probability of a lawsuit by preventing a single, large stock price drop later when the earnings are announced.

**Public information.** Public information affects disclosure patterns. Managers’ incentives to disclose private information at any point in time depend on the velocity with which the information leaks into the market via external sources (e.g., media coverage, analysts’ recommendations, peer firms’ disclosures) and the way the market interprets the absence of public information. When public information is noisy, managerial disclosures may be triggered by a news arrival and be used by the manager to counteract the sometimes adverse price effect of noisy information. Such reactive-like disclosures generate a clustering of disclosures in bad times (see Acharya, DeMarzo, and Kremer, 2011). For simplicity, we abstract away from this effect and instead focus on the case where a news arrival reveal the underlying state perfectly.¹²

**Real (cash-flow) effects.** Our model can be applied to settings in which the act of concealing information entails cash-flow effects for the firm. Such real costs may ensue, for example, when the manager continues unprofitable projects to avoid releasing negative information because the market would interpret the act of shutting down a project as a signal that the manager is hiding bad news. Grenadier and Malenko (2011) and Grenadier, Malenko, and Strebulaev (2014) study dynamic signalling in real option settings along these lines. Our analysis extends this literature along several dimensions. First, we introduce the possibility of certification, and analyze the interaction between certification and signalling by delay. Second, we develop a tractable model with time-varying private information about project profitability. Third, we analyze the case in which monitoring is endogenous.

### 3. Disclosure without litigation risk

As a benchmark, we consider the case without litigation risk (θ = 0). With some abuse of notation, let $U_v(p)$ be the manager’s payoff given that the price is $p_t = p$ and the asset value is $V_t = v \in \{0, 1\}$. Throughout, we restrict attention to Markov Perfect Equilibria in which $U_v(p)$ is nondecreasing in price. Monotone payoffs capture the idea that the manager’s disclosure decision is driven by the desire to increase the firm’s reputation. Similar monotonicity conditions are common in the reputation literature (e.g., in Benabou and Laroque, 1992; Lee and Liu, 2013).

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¹² The analysis with noisy public information is part of a previous draft and is available from the authors upon request.
When payoffs are monotone, Markov Perfect Equilibria are characterized by a disclosure threshold $p_*$ such that $F^*_0 = 0$ and

$$F^*_t = \begin{cases} 1 & \text{if there is } s \in [\tau_{t-1}, t] \text{ such that } V_s = 1 \text{ and } p_\tau - p_* \\ 0 & \text{otherwise.} \end{cases}$$

That is, the manager discloses at time $t$ if and only if both the price is lower than or equal to $p_*$ and the value of the asset is high. Anticipating this strategy, the market expects no disclosure when the price is above $p_*$. As a consequence, for any $p_t > p_*$ the price evolves according to (2). By contrast, for $p_t \leq p_*$, we have $p_t = V_t$. That is, if the manager does not disclose information when the price reaches the threshold $p_*$, the market infers that the asset value is low. As a result, the price drops from $p_*$ to zero and remains there until the manager again discloses good news (i.e., $V_t = 1$). Of course, this happens as soon as the asset value returns to the high state.

The discrete support of $V_t$ results in the existence of multiple equilibria. In particular, when the cost of disclosure is moderate, there exists a continuum of thresholds $p_*$ satisfying the equilibrium conditions. However, in our model, equilibria can be Pareto ordered.

**Definition 2.** The equilibrium threshold $p_\tau^*$ is Pareto dominant if and only if $U_i(p|p_\tau^*) \geq U_i(p|p_\tau)$ for all $p \in [0, 1]$, $p_* \in [p_*^-, p_*^+]$, and $\nu \in \{0, 1\}$.

This selection criterion is very weak because it requires the equilibrium to be Pareto optimal for all prices and all states. We can think of the Pareto-dominating equilibrium as the natural outcome of an extended game in which, at the outset, the manager informally announces the firm’s disclosure policy to the market. Though the manager cannot fully commit to disclosing regularly, he can issue a cheap talk message along the lines of “we will try to provide guidance on a quarterly basis.”

To solve for the equilibrium threshold, let’s consider the manager decision problem. The value function solves the Hamilton-Jacobi-Bellman (HJB) equation

$$0 = \max\{p + f(p)U_0(p) + \lambda_0[U_0(p) - U_1(p)] - rU_1(p), U_1(1) - c - U_1(p)\}$$

$$0 = \max\{p + f(p)U_0(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)] - rU_0(p), U_0(0) - U_0(p)\},$$
where \( f(p) := \kappa(\bar{p} - p) + \mu p(1 - p) \). In a monotone equilibrium, the HJB equation can be written as:

\[
\begin{align*}
    rU_1(p) &= p + f(p)U'_1(p) + \lambda_0[U_0(p) - U_1(p)] \\
    rU_0(p) &= p + f(p)U'_0(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)],
\end{align*}
\]

for \( p \in (p^*, 1) \), and

\[
\begin{align*}
    U_1(p) &= U_1(1) - c, \quad p \leq p^* \quad (6a) \\
    U_0(p) &= \frac{\lambda_1}{r + \lambda_1}[U_1(1) - c], \quad p \leq p^* \quad (6b).
\end{align*}
\]

The value function must satisfy the following optimality conditions:

\[
\begin{align*}
    U_1(1) - c &\geq 0 \quad (7a) \\
    U_1(p) &\geq U_1(1) - c \quad \text{for } p > p^* \quad (7b) \\
    U_1(p) &\leq U_1(1) - c \quad \text{for } p \leq p^*. \quad (7c)
\end{align*}
\]

Condition (7a) states that the manager finds it profitable to disclose information. Condition (7b) implies that the manager does not want to disclose early and release the information when prices are above \( p^* \). Similarly, Condition (7c) states that the manager does not want to delay disclosure when prices are below \( p^* \). In essence, the manager must solve an optimal stopping problem where the stopping time is endogenous, as it must be consistent with the market’s rational expectations. The following proposition describes the equilibrium set.

**Proposition 1.** Let \( \hat{p} \) be the unique positive root of the equation \( f(p) = 0 \). Then, for any \( p^* \in (\hat{p}, 1) \) satisfying

\[
\begin{align*}
    U_1(1) - c &\geq 0 \quad (8a) \\
    U'_1(p^*) &\geq 0 \quad (8b)
\end{align*}
\]

there is an equilibrium with threshold \( p^* \). If there are equilibrium disclosure thresholds \( \hat{p} \leq p^- < p^* \) such that

\[
\begin{align*}
    U_1(1|p^-) - c &\geq 0 \quad (9a) \\
    U'_1(p^-|p^*) &\geq 0 \quad (9b)
\end{align*}
\]

then, \( p^* \) is an equilibrium disclosure threshold if and only if \( p^* \in [p^-, p^+] \). Moreover, the least transparent equilibrium, \( p^- \), is the Pareto-dominant one.

This result is intuitive: given that disclosure is a deadweight cost, the most efficient equilibrium is the one that minimizes the frequency of disclosure, for this equilibrium minimizes also the present value of future disclosure expenses. The most transparent equilibrium, in terms of the probability of disclosure, arises when Condition (8a) is binding. By contrast, the most opaque equilibrium arises when Condition (8b) is binding. Notice again that the most opaque equilibrium is the manager’s preferred equilibrium for any initial price \( p \) and any asset value. Hence, the manager’s incentives to coordinate in the most opaque equilibrium will remain the same for all histories of the game.

Consider the manager’s payoff at the start of the game, namely, when the price is \( p = 1 \):

\[
U_1(1) = U^{ND}(1) - \frac{\delta}{1 - \delta} c,
\]
where $\delta = E_0(e^{-r_1})$. The first component, $U^{ND}(1)$, is the payoff the manager would obtain had he committed never to disclose. The second component is the present value of the disclosure cost the manager expects to bear over a lifetime, given this lack of commitment. The manager’s payoff is thus bounded above by the nondisclosure payoff $U^{ND}(1)$. This is natural: in our setting, information has no (social) value, so the disclosure expense is a loss borne by the manager only because he cannot commit not to disclose information when the price is severely depressed. However, ex ante, the average trajectory of future prices is unaffected by the manager’s disclosure policy; in equilibrium, the possibility of disclosure increases the volatility of the price but does not affect its expected level.

Consider the price dynamics: at the beginning of the game, the price drifts down until the asset undervaluation is so severe that disclosing good news becomes profitable to the manager. At that point, the price jumps up if the manager discloses good news or down if the manager withholds information. Notice that the failure to disclose at $p_t = p^*$ is followed by a period where (i) the price remains flat for some (random) time, and (ii) the information becomes symmetric. By contrast, the period following a disclosure is characterized by the price (mean) reverting toward its long-run value $\bar{p}$ and by the manager having private information about the true asset value.

4. Disclosure with litigation risk

In practice, concealing bad news from the public can be costly. In this section, we study the effect of litigation risk on disclosure dynamics. We first consider the special case of permanent shocks because most of the insights regarding the effect of litigation risk can be captured in this simple setting. Furthermore, this case is a good description of many real-world situations. The general case of transitory shocks is discussed in Section 4.

**Permanent shocks: myopic disclosure.** Assume that negative shocks are permanent (i.e., $\lambda_1 = 0$). As in previous sections, we begin by characterizing the evolution of prices. In the absence

---

17 Kothari, Shu, and Wysocki (2009) empirically document a similar pattern. They find evidence consistent with the view that managers tend to withhold bad news from investors and that prices tend to drift down, absent disclosure, and jump upward on the release of good news. This characterization has empirical support. Indeed, the idea that the inclination toward disclosure is negatively correlated with the level of stock prices is natural, and has been documented empirically. For example, Sletten (2012) argues that “stock price declines prompt managers to voluntarily disclose firm-value-related information (management forecasts) that was withheld prior to the decline because it was unfavorable but became favorable at a lower stock price.”

18 For example, permanent asset impairments arise when a regulator bans a pharmaceutical company from commercializing a drug due to safety concerns, a borrower defaults on its debt, or technological improvements drive a product out of the market.
of news and disclosures, the price drifts downward, due to the possibility of an undisclosed
impairment. Even though the asset’s long-term value is \( p = 0 \), the price is bounded from below by
\[
\hat{p} = \max \left( 1 - \frac{\lambda_0}{\mu}, 0 \right).
\]

The reason for the existence of this lower bound is that the absence of news is perceived as
favorable when news arrivals convey bad information.

We restrict attention to parameter values such that \( \hat{p} < \mu \theta \), because disclosure of bad news
would otherwise never occur in equilibrium. Similarly, we assume \( \mu \theta < 1 \), or else bad news
would be fully disclosed: information would be always symmetric. Depending on the cost of
disclosure \( c \), different equilibrium structures emerge. However, all equilibria are characterized by
a threshold for the price, \( p_\star \), such that whenever the price reaches the threshold, the manager may
disclose some information with positive probability. However, whether the manager discloses
good or bad news at \( p = p_\star \) depends on the magnitude of the disclosure cost relative to the
litigation cost \( \mu \theta \). We refer to a good news equilibrium as that arising when the manager discloses
good news at \( p = p_\star \) and a bad news equilibrium as that arising when the manager (may) only
disclose bad news at \( p_\star \).

The good news equilibrium is similar to the one analyzed in Section 3. Hence, we relegate
the analysis to Appendix B. Depending on the importance of litigation risk vis-à-vis disclosure
costs, there may not be a bad news equilibrium, or the bad news equilibrium may coexist with
good news equilibria. In these cases, we resort to Pareto optimality (see Definition 2) as the
refinement criterion. The next proposition provides the taxonomy of equilibria.

**Proposition 2.** There exist \( c < \bar{c} \) such that

(i) Good News Equilibrium: For any \( c \leq \bar{c} \), there is \( p^-_\star < p^+_\star \) such that for any \( p_\star \in [p^-_\star, p^+_\star] \)
there is an equilibrium with
\[
F_{0t} = 0
\]
\[
F_{1t} = \begin{cases} 1 & \text{if there is } s \in [0, t] \text{ such that } V_s = 1 \text{ and } p_s^- \leq p_* \\ 0 & \text{otherwise.} \end{cases}
\]

(ii) Bad News Equilibrium: For any \( c \geq \underline{c} \) there is an equilibrium with
\[
F_{0t} = F_{0t} = 1 - e^{-\int_{0}^{t} h_{0s} ds},
\]
\[
F_{1t} = 0,
\]
where
\[
h_{0s} = \left( \frac{\lambda_0}{1 - \mu \theta} - \mu \right) 1_{\{p^-_\star = \mu \theta\}}.
\]

(iii) If \( c < \underline{c} \), then any good news equilibria has a threshold strictly greater than \( \mu \theta \). That is, \( p^-_\star > \mu \theta \).

(iv) If \( c < \underline{c} \), the Pareto-dominating equilibrium is the good news equilibrium with threshold \( p^-_\star \)
and \( p^+_\star > \mu \theta \). If \( c \geq \underline{c} \), the Pareto-dominating equilibrium is the equilibrium with disclosure
of bad news with threshold \( \mu \theta \).

This taxonomy is intuitive. For low disclosure costs, only good news can be released in
equilibrium because otherwise, the manager would have an incentive to disclose good news
before the price reaches the threshold \( \mu \theta \). Conversely, for high disclosure costs, only the bad
news equilibrium can exist. Otherwise, in a good news equilibrium, the manager would have an
incentive to preempt bad news before the price reaches the equilibrium threshold. For intermediate
values of $c$, both types of equilibria may coexist, but the bad news equilibrium Pareto dominates even the most efficient good news equilibrium. Remarkably, on imposing Pareto dominance, we end up with a unique equilibrium for the whole range of parameters.

Intuitively, the presence of litigation risk should induce the manager to disclose bad news, especially when prices are relatively low. However, this presents a conceptual difficulty: if the market expects the manager to disclose bad news today, then not disclosing any information will be interpreted as evidence that the asset value is high. This will lead to an upward jump in the stock price, which in turn will destroy the manager’s incentives to disclose bad news in the first place. The temptation to withhold bad news, to benefit from the jump, would offset the benefit of preempting litigation. This suggests that the manager’s disclosure strategy must entail randomization.

In the bad news equilibrium, there is no disclosure when $V_t = 1$, and when $V_t = 0$, there is disclosure with a mean arrival rate

$$h_0 = \left(\lambda_0 \frac{p_0}{p_0(1 - p_0)} - \mu\right) 1_{[p_t - p_0]}.$$  

For $p_t > p_0$, there is no disclosure, and when $p_t = p_0$ the intensity of bad news disclosure is $h_0$ and the price process, $p_t$, has a reflecting barrier at $p_0$. At the outset, the price experiences a downward drift until it reaches $p_0$. If the asset value is low when the price reaches $p_0$, then the manager randomizes between disclosing and not disclosing the asset value. In response, the price remains flat until the manager reports bad news, at random time $\tau_1$. Naturally, such a disclosure causes the price to drop and stay at zero, given that the impairment is permanent.\(^{19}\)

Unlike the case without litigation risk examined in Section 2, here the manager strictly prefers to withhold good news. So litigation risk crowds out good news disclosures: that is, the presence of litigation risk not only prompts the manager to disclose bad news but also removes the incentive to disclose good news.\(^{20}\) The reason is that the absence of good news disclosures is now perceived by the market as a favorable signal of asset value. Because the market thinks that the manager will sometimes disclose bad news, it also interprets more favorably the act of withholding information, which weakens the manager’s incentives to incur the disclosure cost $c$. This effect is sometimes strong enough to offset the drift in the price, in particular once the price reaches the threshold.

The manager’s randomization keeps the price from jumping upward when it reaches the threshold but forces it either to remain constant in the absence of disclosure or to drop to zero in the presence of a disclosure. Of course, the manager must be indifferent between the choice of disclosing low values, to avoid the risk of litigation and concealing the bad news, to enjoy inflated prices. The threshold $p_0$ characterizes an optimal disclosure strategy if the manager’s value function satisfies the following HJB equations. For $p_t > p_0$,

$$rU_1(p) = p + f(p)U'_1(p) + \lambda \left[U_0(p) - U_1(p)\right]$$  \hspace{1cm} (10a)

$$rU_0(p) = p - \mu \theta + f(p)U'_0(p) + \mu \left[U_0(0) - U_0(p)\right].$$  \hspace{1cm} (10b)

These equations are analogous to those of the setting without litigation risk, except that in the low state, the manager’s instant payoff includes the expected litigation costs. These equations also show that the manager is exposed to shocks of two types. First, the asset value may experience a “real shock” that, even when not observed by the market, affects both the trajectory of prices and

\(^{19}\) If there were also public good news, then beliefs would jump up upon the arrival of good news and both types would restart to pool, as in Bar-Isaac (2003).

\(^{20}\) The crowding out of positive disclosures caused by the presence of litigation risk has some empirical support (Lev, 1995; Johnson, Kasznik, and Nelson, 2001), and has been explained as arising from the fact that litigation is triggered by “optimistic disclosures” (Lev, 1995). We provide an alternative (perhaps complementary) explanation: litigation risk crowds out good news because silence is not perceived negatively when withholding bad news is risky.

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expected litigation costs. Second, the manager may experience a “news shock”; a news arrival may reveal that the manager withheld information, triggering a drop in the stock price and potential litigation costs.

In addition, we have the following boundary conditions: first, because shocks are permanent, it must be the case that

\[ U_0(0) = 0; \]

second, the manager is willing to use a mixed strategy at \( p^* \) if and only if the following indifference condition is satisfied:

\[ U_0(p^*) = U_0(0). \]

Combining these two boundary conditions, we find that the disclosure threshold \( p^* \) is optimal for the manager if and only if

\[ p^* = \mu \theta. \]

The manager’s disclosure policy is thus myopic. However, this is an artifact of \( \lambda_1 = 0 \). As we show in the next section, when shocks are transitory, the equilibrium threshold is lower than \( \mu \theta \) because delaying disclosures has option value. Using Bayes’ rule, we find that the value of \( h_{0t} \) that ensures the price remains flat at \( p_t = \mu \theta \), absent disclosure, is given by

\[ h_{0t} = \left( \frac{\lambda_{00}}{1 - \mu \theta} - \mu \right) 1_{p_t = p^*}. \]

The propensity of disclosure, \( h_{0t} \), increases monotonically in the cost of litigation \( \theta \) and the likelihood of a shock \( \lambda_{00} \), but is u-shaped in the intensity of public news \( \mu \).

We next address the welfare effects of litigation risk. Perhaps surprisingly, we find that higher litigation risk increases the manager’s welfare under plausible circumstances. As mentioned, a higher litigation risk makes the market more positive about firms that do not disclose, effectively crowding out good news disclosures and reducing the firm’s overall disclosure expense. The manager may thus benefit from a higher litigation risk simply because he lacks commitment power to avoid incurring proprietary disclosure costs, and the presence of litigation risk may remove this incentive altogether.

Notice that the manager’s welfare is greater when \( \theta \) is very large versus when \( \theta \) approaches zero. In other words, the absence of litigation risk is dominated by a situation where the litigation risk is very large. Whereas in the former case the manager fully preempts bad news at no cost, in the latter case, the manager incurs excessive proprietary disclosure costs. We can indeed show that, for high litigation risks, the manager is better off in the presence of litigation risk than in its absence.

**Corollary 1.** There is \( \theta > 1/\mu \) such that for all \( \theta \geq \theta^* \), the manager’s expected payoff at time 0, \( U_1(1) \), is higher in the presence of litigation.

The literature typically views litigation risk as a mechanism that disciplines managers’ behavior. Corollary 1 shows that litigation risk also has a more subtle effect that benefits the firm: it allows the firm to signal favorable information by withholding information, thus reducing the need for the firm to engage in wasteful disclosures. Litigation risk thus creates an alternative information transmission mechanism.

**Temporary shocks: gambling for resurrection.** The case of permanent shocks analyzed in the previous section is tractable but somewhat restrictive, for the following two reasons. First, good and bad news don’t coexist in equilibrium. Second, in the bad news equilibrium, the manager adopts a myopic policy that ignores the option value associated with delaying bad news. By contrast, the case of temporary shocks features both aspects: first, the manager may disclose
good and bad news; second, the manager’s disclosure policy is not myopic; the manager is willing to delay disclosures of bad news for some time, and even to bear temporary losses, hoping that a positive shock would render the disclosure of bad news unnecessary.

We focus on the bad news equilibrium. It remains the case that the low-type manager discloses at a constant rate when \( p_t = p^* \); however, if the cost of good news disclosure is not too high, the high-type manager discloses with probability one when \( p_t = 0 \). In particular, we conjecture and verify that a bad news disclosure equilibrium is characterized as follows.

**Proposition 3.** Suppose that there exists some threshold \( p^* \) such that

\[
\max \left( \hat{p}, \frac{\mu \theta (r + \lambda_0)}{r + \kappa} \right) < p^* < \mu \theta,
\]

where \( \hat{p} \) is the unique positive root of \( f(p) = 0 \), and

\[
U_0(p^*) = \frac{\lambda_1}{r + \lambda_1} [U_i(1) - c],
\]

where \( U_i(p) \) is the solution to the HJB equation (11a)–(11b) with initial value

\[
U_0(p^*) = \frac{p^*}{r} - \frac{\mu \theta r + \lambda_0}{r + \kappa},
\]

\[
U_1(p^*) = \frac{p^*}{r} - \frac{\mu \theta}{r + \kappa}.
\]

Then, there exists an equilibrium such that:

(i) The high type discloses only when beliefs are zero, that is,

\[
F^{n}_{\tau_1} = \begin{cases} 1 & \text{if there is } s \in [\tau_{n-1}, \tau_1] \text{ such that } V_s = 1 \text{ and } p_{s} = 0 \\ 0 & \text{otherwise} \end{cases}
\]

(ii) The low type discloses only with positive probability when \( p_t \leq p^* \). In particular, the manager discloses with a mean arrival rate

\[
h^*_n = \left( \kappa \frac{p^* - \hat{p}}{p^*(1 - p^*)} - \mu \right) I_{\{p_t = p^*\}},
\]

so, on the equilibrium path,

\[
F^{n}_{\tau_0} = 1 - e^{\int_{\tau_{n-1}}^{\tau_n} h^*_n ds}.
\]

Figure 2 shows a sample path of the stock price in equilibrium. At the start, the price experiences a downward drift until it hits the threshold \( p^* \). This downward drift is caused purely by the increased likelihood of an undisclosed impairment. Conditional on low asset value, the manager begins randomizing between disclosing and not disclosing private information. The price remains flat until the manager reports bad news at time \( \tau_1 \). Naturally, this disclosure causes the price to drop to zero and stay there until the situation of the firm improves, at time \( \tau_2 \), and the manager discloses good news.

\[21\] If \( p_t < p^* \), then there is an atom in the disclosure strategy, in particular,

\[
F^{n}_{\tau_0} - F^{n}_{\tau_0} = \frac{p_{\tau_0}}{1 - p_{\tau_0}} \frac{1 - p^*}{p^*}.
\]

With perfect bad news, this is an off-equilibrium event because beliefs never enter the interval \((0, p^*)\) on the equilibrium path.

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The threshold $p_*$ characterizes an optimal disclosure strategy if the manager’s payoff satisfies the following HJB equation. For $p_t > p_*$,

\[ rU_1(p) = p + f(p)U'_1(p) + \lambda_0[U_0(p) - U_1(p)] \] (11a)

\[ rU_0(p) = p - \mu \theta + f(p)U'_0(p) + \lambda_1[U_1(p) - U_0(p)] + \mu[U_0(0) - U_0(p)]. \] (11b)

The manager’s decision to disclose bad news has the flavor of the real options problem analyzed by Dixit (1989), where a firm has the option, at any point in time, to shut down a project (i.e., disclose bad news) or restart it (i.e., disclose good news), based on the project’s observed profitability. The difference is that the payoffs are endogenous in our setting, because they are linked to the market’s rational belief about the asset value. When the stock price is low, disclosing bad news becomes profitable for the same reason that shutting down a loss-making project is optimal in Dixit’s model. Also, as in Dixit’s problem, the decision to disclose bad news today is linked to the option to disclose future good news: only when the manager discloses good news in the future, he has incentives to delay the disclosure of bad news today beyond the myopic cutoff. This speaks to a certain complementarity between the disclosure of bad news and good news.

To complete the characterization of the equilibrium, we need to characterize the boundary conditions. When $p_t = p_*$, the value function of the low-type manager satisfies

\[ U_0(p_*) = E \left[ \int_0^\tau e^{-rt}(p_* - \mu \theta)dt + e^{-\tau r}(U_0(0)1_{\{\tau = \tau_N\}} + U_1(p_*)1_{\{\tau = \tau_H\}}) \right], \]

where $\tau = \min\{\tau_N, \tau_D, \tau_H\}$, $\tau_N$ is the first arrival of public (bad) news, $\tau_D$ is the time at which the manager voluntarily discloses bad news, and $\tau_H$ is the time at which the value of assets jump from 0 to 1. Similarly, the boundary condition for a high-type manager is given by

\[ U_1(p_*) = \frac{p_* + \lambda_0U_0(p_*)}{r + \lambda_0}. \] (12)

In addition, the following condition applies when $p_t = 0$:

\[ U_0(0) = \frac{\lambda_1}{r + \lambda_1}U_1(0) \] (13)

\[ U_1(0) = U_1(1) - c. \] (14)
FIGURE 3

EFFECT OF CHANGES IN THE ARRIVAL RATE OF BAD NEWS, $\mu$, AND LITIGATION COST $\theta$ ON MANAGER’S PAYOFF. BASELINE PARAMETERS: $r = 0.1$, $c = 0.5$, $\beta = 0.1$, $\kappa = 1.2$, $\theta = 1$, $\mu = 0.2$, AND $\theta = 1$

Because the manager is using a mixed strategy when $p_t = p^*$, he must be indifferent between disclosing and not disclosing negative information. Hence, we can determine the threshold $p^*$ using the indifference condition for the manager’s mixed strategy:

$$U_0(p^*) = U_0(0).$$

(iii) $U_0(p) \geq U_0(0)$ for $p > p^*$.

The disclosure threshold is lower than the myopic threshold $\mu\theta$, which means that the manager delays bad news, relative to the case with permanent shocks. When the price reaches $\mu\theta$, the manager has the option to wait further and see whether the asset recovers its value. If it does, the manager avoids the negative price consequences of bad news disclosures. Of course, this gamble makes sense if there is a positive probability of “resurrection” (i.e., $\lambda_1 > 0$).22 This is, in essence, Verrecchia’s (1983) alternative explanation.23

5. Endogenous litigation risk

In practice, plaintiffs are more likely to bring a lawsuit against a firm when they believe that the firm is withholding (or has withheld) information (see Skinner, 1994). This speaks to

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22 The idea that managers may withhold bad news hoping that the firm’s financial standing will improve is borne out by the survey evidence in Graham, Harvey, and Rajgopal (2005). Some Chief Financial Officers (CFOs) claim that they delay bad news disclosures in the hope that they may never have to release the bad news if the firm’s status improves.

23 Verrecchia (1983) argues, “An alternative to my explanation for why a manager delays the reporting of ‘bad news’ is that he hopes that during the interim some ‘good news’ will occur to offset what he has to say. The disadvantage of this explanation is that it ignores the fact that rational traders will correctly infer ‘bad news’ as soon as it becomes apparent that the information is being withheld.”
the endogenous nature of litigation risk. In the United States, for example, whistle-blowers can investigate a firm’s violations and obtain a reward if they can prove the firm’s violation in court.\textsuperscript{24}

In this section, we endogenize litigation risk and evaluate the effects of whistle-blower programs. Specifically, we endogenize the monitoring intensity, \( \mu \). For simplicity, we restrict attention to the case with permanent shocks, that is, we assume \( \lambda_1 = 0 \).

Suppose that a whistle-blower scrutinizes the firm and can investigate whether the firm has concealed information. At each point, the whistle-blower chooses a monitoring intensity \( \mu_t \) such that he discovers a manager hiding negative information with a mean arrival rate \( \mu_t \in [0, \tilde{\mu}] \). For simplicity, we assume that the flow cost of monitoring borne by the whistle-blower is linear, \( k \cdot \mu_t \). If the whistle-blower proves that the firm hid bad news, he obtains a reward \( b \). For example, the whistle-blower may be a lawyer looking for litigation opportunities, where \( b = \omega \cdot \theta \) and \( \omega \in (0, 1) \) represents the fraction of the damages payment the lawyer receives as the (contingent) fee.

The whistle-blower payoff, conditional on finding the firm withholding information at time \( t \), is

\[
e^{-rt} b - \int_0^t e^{-rs} k \cdot \mu_s \, ds.\]

In the Appendix, we show that in equilibrium, there is a threshold \( p_m \) such that the whistle-blower begins investigating the firm when beliefs reach a threshold \( p_m \) and only at that point the firm starts disclosing negative information with positive probability. Hence, we find that the equilibrium is characterized as follows.

**Proposition 4.** Suppose that \( b > k \). In the model with endogenous monitoring, there is an equilibrium in which the whistle-blower finds that the firm is hiding information with mean arrival rate

\[
\mu_t = 1_{\{p_t \leq p_m\}} \mu_m \text{ with threshold } p_m = 1 - \frac{k}{b},
\]

where the monitoring intensity is

\[
\mu_m = \min\left\{ \frac{1}{\theta} \left( 1 - \frac{k}{b} \right), \tilde{\mu} \right\}.
\]

The manager’s disclosure threshold is \( p_s = \mu_m \theta \), and the disclosure mean arrival rate is

\[
h_{\theta} = \left( \frac{\lambda_0}{1 - p_s} - \mu_m \right) 1_{\{p_s = p_m\}}.
\]

The monitoring threshold is increasing in \( \omega \) and \( \theta \) and decreasing in \( k \). The whistle-blower uses a monitoring intensity \( \mu_m \), which may increase or decrease in \( \theta \). This result is explained by the effect that \( \theta \) has on the manager’s disclosure strategy. Increasing \( \theta \) makes monitoring more profitable, conditional on finding the violation, but it also increases the probability that the manager will preempt litigation by disclosing bad news, thus reducing the probability that the whistle-blower gets rewarded. Notice that with endogenous monitoring, the manager must always hide information with positive probability, otherwise, the whistle-blower would lack incentives to investigate, which in turn would make withholding profitable to the manager.\textsuperscript{25}

To conclude this section, consider the total cost associated with the information transmission taking place in this model. Figure 4 displays the effect of monitoring on the total cost of producing

\textsuperscript{24} For example, in 2013, the Securities and Exchange Commission (SEC) implemented a whistle-blower program awarding a whistle-blower between 10\% and 30\% of the money collected in a case. In September 2014, the SEC announced a $30 million award to a whistle-blower, the SEC’s largest ever whistle-blower award.

\textsuperscript{25} Rahman (2012) finds that optimal contracts that provide simultaneous incentives to agents and monitors necessary require some misbehavior (in equilibrium) by the agents.
the information. Without monitoring, the cost stems from the disclosure costs. By contrast, with monitoring, the cost stems from the present value of the monitoring costs \( k \cdot \mu \). In Figure 2, we compare the costs with and without litigation \((\theta = 0)\) as a welfare measure. We include neither the litigation cost borne by the firm nor the rewards obtained by the whistle-blower, which are transfers between risk-neutral parties. The blue line represents the equilibrium cost, and the red line is the cost without litigation. These lines coincide when the equilibrium entails no monitoring. For high monitoring costs, the equilibrium without litigation generates lower information costs, and vice versa. Figure 4b shows that monitoring is part of the equilibrium only when the frequency of shocks is high, but in that case the equilibrium also entails excessive monitoring. On the contrary, when the frequency is low, there is no monitoring in equilibrium, so only good news is disclosed.

The previous comparison ignores the fact that the amount of information produced by the two regimes differs. To quantify this effect, we measure the expected delay between the moment the manager receives negative information and the moment this information reaches the market,
through either the firm’s disclosure or the whistle-blower’s investigation. As expected, the regime with litigation reduces the amount of delay. Delay increases (decreases) in the cost of monitoring (mean arrival of negative shocks).

6. When litigation costs depend on withholding period length

In this section, we consider the case in which the cost of withholding negative information depends on the length of the withholding period, consistent with current regulations in US financial markets (The Private Securities Reform Act of 1995, Rule 10b-5). This is consistent with the fact that, in reality, damage payments are proportional to the length of the period during which the firm concealed negative information.

We restrict attention to the case of permanent shocks and consider the case in which \( c \) is high enough so that it is never optimal to disclose good news (the case with small \( c \) is qualitatively similar to the case with constant litigation risk discussed in the previous sections). Let us denote by \( \tau_L \) the time at which the firm suffers the negative shock—namely, when \( V_t \) switches from one to zero. At time \( t \geq \tau_L \), if the firm is caught hiding information, then the expected cost from hiding information is \( \theta(t - \tau_L) \), where \( \theta(\cdot) \) is a continuously differentiable increasing function.

Moreover, if the firm voluntarily discloses negative information at time \( t \geq \tau_L \), it suffers a cost \( \psi(t - \tau_L) \), where \( \psi(\cdot) \) is continuously differentiable and increasing. We assume that for any \( t \geq \tau_L \), the expected cost is lower if the firm voluntarily discloses its negative information, that is, \( \theta(\cdot) \leq \psi(\cdot) \).

The continuation value at time \( t \) of a low-type firm with time \( \tau_L \) negative shock is

\[
U_0(t, \tau_0) = E_t \left[ \int_{\tau_L}^t e^{-r(t-s)} p_s ds - e^{-r(t-\tau_L)} (1_{\{\tau_D > t\}} \theta(\tau - \tau_L) + 1_{\{\tau_N > t\}} \psi(\tau - \tau_L)) \right],
\]

where \( \tau = \min\{\tau_D, \tau_N\} \). It is clear from equation (16) that the equilibrium cannot be a function of \( p_t \) only because the date of the negative shock \( \tau_L \) has a direct effect on the manager’s payoff. For this reason, in this section, we construct an equilibrium using \((t, \tau_L)\) as the main state variables. In particular, we construct an equilibrium in which the firm discloses at time \( \tau_D = \tau_L + \tau^* \); that is, the firm discloses with a fixed delay \( \tau^* \). Given this strategy, and in the absence of negative public information, the market beliefs are

\[
p_t = \Pr(\tau_L > t | \tau_D > t, \tau_N > t).
\]

Using the memoryless property of the exponential distribution, we have that for any \( t \geq \tau^* \),

\[
p_t = \Pr(\tau_L > t | \tau_N > t^*) = \frac{\lambda_0 - \mu}{\lambda_0 e^{\lambda_0 - \mu} - \mu}.
\]

Using equation (17), we can characterize the equilibrium as follows:

**Proposition 5.** Suppose that

\[
\psi''(t) + \mu \theta'(t) \geq (r + \mu) \psi'(t)
\]

and

\[
\lim_{t \to \infty} \mu \theta(t) - (r + \mu) \psi(t) + \psi'(t) \leq \max \left( 1 - \frac{\lambda_0}{\mu}, 0 \right).
\]

Then, there is an equilibrium in which the low type discloses at time \( \tau_D = \tau_L + \tau^* \), where \( \tau^* \) is the unique solution to

\[
\frac{\lambda_0 - \mu}{\lambda_0 e^{\lambda_0 - \mu} - \mu} = \mu \theta(\tau^*) - (r + \mu) \psi(\tau^*) + \psi'(\tau^*).
\]
Proof. Using the observation that
\[ e^{-r(t-\tau)}\psi(t\tau - \tau_L) = \psi(t - \tau_L) + \int_{t}^{\tau_D} e^{-r(s-\tau)}(\psi(s - \tau_L) - r\psi(s - \tau_L))ds, \]
we can write the continuation payoff in equation (16), given any price path \( p_t \), and disclosure date \( \tau_D \), as:
\[ U_0(t, \tau_L) = \int_{t}^{\tau_D} e^{-r(t+s)}(p_s - C(s - \tau_L))ds - \psi(t - \tau_L), \]
where
\[ C(t) \equiv \mu_\theta(t) - (r + \mu_\psi(t) + \psi(t). \]
Given the market conjecture that the manager discloses at time \( \tilde{\tau}_D = \tau_L + \tilde{\tau} \) and assumption (18), we find that \( p_t - C(t - \tau_L) \) is a decreasing function of \( t \). Accordingly, it is optimal for the manager to disclose at time \( \tau_D = \inf\{t > \tau_L, p_t = C(t - \tau_L)\} \). Equations (18) and (19) guarantee that there is a unique solution to \( p_t = C(t - \tau_L) \) such that \( \tilde{\tau}_D = \tau_D \). This solution is given by \( \tau_D = \tau_L + \tau^* \), where \( \tau^* \) is the unique solution to equation (20). \qed

The comparative statics are intuitive. The amount of delay depends on the difference between the cost of hiding and disclosing it, \( \mu_\theta(\tau^*) - \psi(\tau^*) \). The manager has lower incentives to delay information when this difference is high. In addition, at any point in time, the manager must decide between disclosing today at a cost \( \psi(t - \tau_L) \) and disclosing tomorrow at a cost \( e^{-d\tau^t}\psi(t + d\tau - \tau_L) \); this trade-off is captured by the term \( \psi(\tau^*) - r\psi(\tau^*) \) in equation (20). Accordingly, there is less delay when the cost of disclosing negative news is increasing in the amount of delay.

It is worth noting that unlike in the case of Markov Perfect Equilibrium, we have a pure strategy equilibrium in this case. This happens because we are using both the current calendar time as well as the time of the shock as state variables. However, the economic intuition for having delay is the same as the mixed strategies in the previous case. When \( \tau^* = 0 \), so that there is no delay, we find that \( p_t = 1 \) in absence of disclosure and public news, which makes it optimal for the manager to hide information. Moreover, the dynamics of prices resemble the one in the case of a mixed disclosure strategy; in fact, as in the Markov case, the presence of delay generates a reflecting barrier at \( p_{\tau^*} \), so that \( p_t = p_{\tau^*} \) for all \( t \geq \tau^* \). The construction of the equilibrium can be extended to the case with transitory shocks by resetting the clock to zero every time that the firm discloses news.

7. Finite maturity asset

Next, we consider a setting in which the value of the firm’s assets is expected to be altered by a future event. For example, consider an invention whose patent expires at time \( T \), thus reducing the invention’s market value.

Assume that the asset expires in period \( T \). Thereafter, profits will diminish. Before time \( T \), the company receives information, continuously, about the quality of its product: for example, the firm may learn that its drug increases the user’s probability of having a heart attack. The company has an incentive to delay and even conceal this information because, if made public, the information would reduce the demand, lowering both the manufacturer’s expected revenues and the stock price. Concealing the information, however, is risky from a legal standpoint and, as before, may trigger litigation costs. In the following, we study how the patent length affects the speed at which the firm will disclose adverse information. To model this situation, suppose that the value of the asset in the high state is
\[ V_1(t) = \begin{cases} V - e^{-\rho(T-t)}(\bar{V} - V) & \text{if } t \leq T \\ V & \text{if } t > T. \end{cases} \]

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FIGURE 5

EVOLUTION OF BELIEFS IN ABSENCE OF DISCLOSURE AND INTENSITY OF DISCLOSURE OF NEGATIVE INFORMATION FOR DIFFERENT ASSET MATURITY. HERE, $T_1 = 1$, $T_2 = 2$, AND $T_3 = 3$ ARE THREE POSSIBLE MATURITIES $T$. $T_1$ AND $T_*$ CORRESPOND TO THE THRESHOLDS FOR THE CASE WITH MATURITY $T_1$. THE VALUE OF $T_*$ AND $T_*$ FOR $T_1$, $T_2$, AND $T_3$ ARE $T_* = (0.23, 0.74, 1.29)$ AND $T_{**} = (0.45, 1.45, 2.45)$, RESPECTIVELY.

Before the patent expires, the firm earns a flow of profits $\overline{v}$. After the patent expires, competition reduces the profits of the firm to $v$. Under this specification, the value of the patent is given by (21), where $\overline{V} = \overline{v}/\rho$, $V = v/\rho$, and $\rho$ is the market’s discount rate. As before, the value of the firm in the low state is $V_0(t) = 0$ for all $t \geq 0$. The optimal disclosure strategy (of bad news) is given by a threshold $p_*(t)$, such that the low-type manager discloses with positive probability only when $p_{\text{L}} \leq p_(t)$. As before, the equilibrium must entail the use of mixed strategies. It can be shown that if the negative shock is permanent and the value of the patent is decreasing over time, then the optimal disclosure rule is the myopic rule. The manager does not disclose any information when $p_V V(t) > \mu \theta$, discloses with positive probability when $p_V V(t) = \mu \theta$, and discloses immediately when $V(t) < \mu \theta$. We show in the Appendix that there is $T_* < T_{**}$ such that the manager does not disclose any information before time $T_*$, discloses negative information with intensity $h_{0\epsilon}$ when $T_* < t < T_{**}$, and discloses immediately when $t > T_{**}$. This means that, conditionally on not disclosing negative information, $p_\epsilon = 1$ for $t \geq T_{**}$.

Figure 5 provides a numerical example showing the dynamics of disclosure in this case. Two elements are worth noting. First, disclosure tends to cluster around expiration. Furthermore, the longer the maturity, the lower the disclosure rate (i.e., the longer the disclosure delay).

8. Concluding remarks

- Litigation risk is an important driver of firms’ voluntary disclosures. Skinner (1994) finds that managers use voluntary disclosures to preempt large negative earnings surprises more often than other types of earnings news. Lev (1995) and Skinner (1994) document that managers can reduce stockholder litigation costs by voluntarily disclosing adverse earnings news “early,” namely, before the mandated release date. The extant disclosure theory has nonetheless largely
ignored the role of litigation risk as a determinant of corporate disclosure. Disclosure theory is also overwhelmingly static, ignoring the fact that firms’ information environment exhibits a continuous flow of private and public information, which means that a manager cannot conceal information forever because external sources will end up revealing it. We argue that these two aspects of the disclosure environment—information flows and litigation risk—are indispensable for understanding both the amount and the nature of the information that firms disclose in capital markets.

The contribution of this article is threefold. First, we prove that litigation not only stimulates transparency in capital markets, but also mitigates firms’ wasteful disclosure behavior and the proprietary costs that positive disclosures typically induce. Second, because litigation risk crowds out good news (toward negative disclosures), it makes the market less skeptical, in the absence of disclosure. Litigation thus introduces an additional information transmission mechanism, by allowing firms to credibly signal favorable information through disclosure delay. Third, we show that litigation risk may explain some counterintuitive evidence regarding the time series of corporate disclosures. We prove, for example, that declining stock prices heightens both the litigation risk and the rate of negative disclosures. Thus, contrary to common interpretations of these facts (see, e.g., Francis, Philbrick, and Schipper, 1994), the disclosure of bad news does not cause litigation risk; in our model, both outcomes are simultaneously determined.

Our analysis is relevant to industries in which negative disclosures are socially important and the risk of litigation is significant. We think that pharmaceutical firms, for example, might face incentives to delay disclosures of negative information, about drug side effects, clustering the disclosures around patent expiration dates, and that this effect could be exacerbated by longer patents. Though the significance of this effect has not been tested empirically, it seems appropriate—from a policy perspective—that the design of a patent system incorporates the impact of patent length on the transmission of (socially relevant) information. Longer patents may foster innovation but may also destroy information.

We conclude with some general thoughts regarding the scope of our results. The academic interest in corporate disclosures was built on the intuition that, by disciplining the behavior of managers, transparency plays a crucial role in capital markets. As a February 2015 article in The Economist, puts it, “The public company is one of capitalism’s greatest invention .... Being listed makes a firm open to scrutiny.” The disciplining role of transparency is present in our model. Indeed, the model developed in Section 4 can be interpreted as one where managers’ negative disclosures have positive cash-flow effects arising not necessarily from the preemption of litigation risk but from a more efficient investment policy. For example, a negative disclosure may lead to the termination of negative net present value projects, thereby avoiding the expected loses that such projects would generate if continued. In practice, as in our model, managers face incentives to inefficiently delay such decisions, because of the reputation and stock price implications that often follow the decision to discontinue business segments. Timely disclosures facilitate the implementation of such measures.

A robust finding in the disclosure/certification literature is the notion that firms tend to incur excessive certification costs, as they suffer from an intertemporal inconsistency. In our setting, litigation risk may impose new costs to the firms (especially in the presence of frivolous litigation) but it also mitigates firms’ certification frenzy by lowering the skepticism with which markets penalize the firms that remain quiet for a long period of time. An analogy can be traced from our model to the signalling literature, which shows that signalling motives induce wasteful investments, driven by firms’ desire to differentiate themselves from low-value firms.

27 Although there is consensus regarding the notion that litigation risk is a determinant of corporate disclosure, the sign of the effect is not clear. For example, Francis, Philbrick, and Schipper (1994) argue that disclosures trigger litigation. This disagreement may be due to the fact that litigation risk is endogenous and cannot be measured by the incidence of litigation.
The driving force is similar to that arising in our certification setting: markets penalize the absence of signalling, as if this reflected adverse information. We think that the presence of litigation risk would alter this result in the same way as in our setting. By forcing low-value firms to reveal adverse information, litigation reduces the benefit from engaging in costly signalling, thus mitigating the underlying inefficiency. Hence, by crowding out wasteful signalling, litigation risk may be welfare improving. We focus on disclosure instead of signalling to avoid dealing with complications of signalling models, for example, multiplicity. However, we believe the general lessons from our analysis regarding the role of litigation risk on information transmission should apply to these cases as well.

**Appendix A**

Appendix A contains the proofs of Section 3 and Appendix B contains the proofs of Section 4.

□ **Proofs of Section 3.**

*Proof of Proposition 1.* We start showing that for any \( p_\ast \in (\hat{p}, 1) \) satisfying

\[
U_1(1) - c \geq 0 \quad (A1)
\]

\[
U_\ast(p_\ast) \geq 0, \quad (A2)
\]

there is an equilibrium with threshold \( p_\ast \). We need to show that (A1) and (A2) imply that \( U_\ast(p) \leq U_1(1) - c \) for \( p \geq p_\ast \). To show this, it is enough to show that \( U_1 \) is nondecreasing. From the boundary condition, we already have \( U_\ast(p) = U_1(1) - c \); if \( U_\ast(p) \) is nondecreasing, then it follows that \( U_\ast(p) \geq U_1(1) - c \) for all \( p > p_\ast \).

Differentiating the HJB equation, we get

\[
rU_\ast''(p) = 1 + f'(p)U_\ast'(p) + f(p)U_\ast'(p) + \lambda c(p) - \mu U_\ast(p) \quad (A3)
\]

\[
rU_\ast'(p) = 1 + f'(p)U_\ast'(p) + f(p)U_\ast'(p) - \lambda_0 c(p). \quad (A4)
\]

Suppose that \( U_\ast(p) \) is decreasing in some interval, then there is \( p \) such that \( U_\ast(p) = 0 \). Let’s define \( p^1 = \inf\{p > p_\ast : U_\ast(p) < 0\} \). Then, by (A4), we have that

\[-f(p^1)U_\ast'(p^1) = 1 + \lambda_0 U_\ast''(p^1). \]

By Lemma A1 below, it must be the case that \( U_\ast''(p^1) > 0 \). This means that \( U_\ast'(p^1) > 0 \), which is a contradiction with \( p^1 = \inf\{p > p_\ast : U_\ast(p) < 0\} \). Hence, \( U_\ast(p) \) must be nondecreasing.

Next, we show that if there are equilibrium disclosure thresholds \( \hat{p} \leq p_\ast < p_\ast^* \) such that

\[
U_1(1)p_\ast^\ast - c = 0 \quad (A5)
\]

\[
U_\ast(p_\ast^\ast) = 0, \quad (A6)
\]

then, \( p_\ast \) is an equilibrium disclosure threshold if and only if \( p_\ast \in [p_\ast^\ast, p_\ast^*] \) and the equilibrium threshold \( p_\ast^* \) determines the Pareto-dominant equilibrium.

First, we show in Lemma A4 that \( U_\ast(p_\ast^\ast) \) crosses 0 only once. This means that \( p_\ast^\ast \) is unique and \( U_\ast(p_\ast^\ast) \geq 0 \) for \( p_\ast \geq p_\ast^\ast \), and \( U_\ast(p_\ast^\ast) < 0 \) for \( p_\ast < p_\ast^\ast \). Lemma A3 shows that for any equilibrium satisfying (A1) and (A2) we have that \( U_\ast(p) \) is nonincreasing in the threshold \( p_\ast \); hence, \( U_\ast(p_\ast) - c \geq 0 \) for all \( p_\ast \leq p_\ast^* \). These two observations imply that \( p_\ast \) satisfies conditions (8a) and (8b) if and only if \( p_\ast \in [p_\ast^\ast, p_\ast^*] \). Moreover, Lemma A3 immediately implies that \( p_\ast^* \) is the Pareto-dominant equilibrium.

**Lemma A1.** Suppose there is \( p^1 \geq p_\ast \), such that \( U_\ast(p^1) = 0 \), then \( U_\ast(p^1) > 0 \).

*Proof.* Let \( \Delta(p) := U_1(p) - U_\ast(p) \), which satisfies

\[
(r + \kappa)\Delta(p) = f(p)\Delta'(p) + \mu[U_\ast(p) - U_\ast(0)]. \quad (A7)
\]

Evaluating (A7) at \( p_\ast \), we get that \( U_\ast(p_\ast) > 0 \). If \( U_\ast(p) \) is nondecreasing for \( p > p_\ast \), we are done. Suppose that \( U_\ast(p) \) is decreasing for some \( p > p_\ast \), then there must be some \( p > p_\ast \) such that \( U_\ast(p) = 0 \). Let \( p^0 = \inf\{p \geq p_\ast : U_\ast(p) < 0\} \). We have two possibilities, \( p^0 \geq p^1 \) or \( p^0 < p^1 \). Suppose that \( p^0 < p^1 \), if this is the case, using equation (A3), we get

\[-f(p^0)U_\ast''(p^0) = 1 + \lambda U_\ast'(p^0) > 0. \]

This means that \( U_\ast'(p^0) > 0 \), which contradicts the fact that \( p^0 = \inf\{p > p_\ast : U_\ast(p) < 0\} \), so it must be the case that \( p^0 \geq p^1 \). However then, by definition of \( p^0 \), we have \( U_\ast(p^0) > 0 \). □
Lemma A2. Let \( p_1^* < p_2^* \) be two equilibrium thresholds, then \( U_1(1|p_1^*) > U_1(1|p_2^*) \).

Proof. The solution to the HJB equation satisfies (Davis, 1993)

\[
U_i(1) = E \left[ \int_0^\infty e^{-rt} p_i dt - c \sum_{s_i \geq 0} e^{-rt_i} D_s \right]
\]

\[
= E \left[ \int_0^\infty e^{-rt} E(V_i|\mathcal{F}_t) dt - c \sum_{s_i \geq 0} e^{-rt_i} D_s \right]
\]

\[
= \int_0^\infty e^{-rt} E(V_i) dt - cE \left[ \sum_{s_i \geq 0} e^{-rt_i} D_s \right]
\]

\[
= \int_0^\infty e^{-rt} E(V_i) dt - \frac{\delta}{1 - \delta}. 
\]

where \( \delta := E(e^{-rt}) \) and \( t_d \) is the (random) time of disclosure. Let \( t_j^* \) and \( t_j^* \) be the first disclosure times for \( p_1^* \) and \( p_2^* \), respectively. To show that \( U_1(1|p_1^*) > U_1(1|p_2^*) \), it is sufficient to show that \( t_j^* \geq t_j^* \) and that \( t_j^*(\omega) > t_j^*(\omega) \) for a positive measure set of states \( \omega \).

Let \( t_N := \inf\{t \geq 0 : dN_i = 1\} \) and let \( T_i^* \), \( i = 1, 2 \) be given by \( \phi_i = p_i^* \), where \( \phi_i \) is the solution to the differential equation

\[
\frac{dp_i}{dt} = \kappa(\bar{p} - p_i) + \mu p_i(1 - p_i). \quad p_0 = 1.
\]

By construction, we have \( T_2^* > T_1^* \). We consider several cases:

(i) If \( t_N < T_2^* \), then \( t_1^* = t_2^* \).

(ii) If \( t_N > T_2^* \) and \( V_{T_2^*} = 1 \), then \( t_2^* = T_2^* > t_1^* \).

(iii) If \( t_N > T_2^* \) and \( V_{T_2^*} = 0 \), we have several subcases. Let \( \bar{t} = \inf\{t > T_2^* : V_i = 1\} \).

(a) If \( t_N < T_2^* \), then \( t_1^* = t_2^* = \inf\{t \geq t_N : V_i = 1\} \).

(b) If \( t_N > T_2^* \) and \( \bar{t} < T_1^* \), then \( t_2^* = \sigma < T_1^* \).

(c) If \( t_N > T_2^* \) and \( \bar{t} > T_1^* \), then \( t_2^* = t_2^* = \bar{t} \).

Accordingly, \( t_j^* > t_2^* \) a.s. and \( \text{Pr}(t_j^* > t_2^*) > 0 \), which means that \( E(e^{-rt_j^*}) < E(e^{-rt_2^*}) \) and \( U_1(1|p_j^*) > U_1(1|p_2^*) \). \( \square \)

Lemma A3. Suppose that \( U_i(p_i|p_* \geq 0 \) and \( U_1(1|p_* \) - \( c \geq 0 \), then \( U_0(p|p_*) \) and \( U_1(p|p_*) \) are nonincreasing in \( p_* \).

Proof. Following the same computation as in (Davis 1993) we can integrate the HJB equation to get

\[
U_0(p_i) = \int_{t_N}^{T_i} e^{-(\rho + \lambda + \mu)T_i - t} (p_* + \lambda_i U_i(p_* + \mu U_0(0)) ds + e^{-(\rho + \lambda + \mu)T_i - t} U_0(0)
\]

\[
U_1(p_i) = \int_{t_N}^{T_i} e^{-(\rho + \lambda + \mu)T_i - t} (p_* + \lambda_i U_i(p_* + \mu U_0(0)) ds + e^{-(\rho + \lambda + \mu)T_i - t} U_0(1 - c)
\]

where \( T_i \) is the time it gets for beliefs to reach \( p_* \) in absence of any shock. Differentiating with respect to \( p_* \), we get

\[
\frac{\partial}{\partial p_*} U_0(p_i) = \int_{t_N}^{T_i} e^{-(\rho + \lambda + \mu)T_i - t} \left( \lambda_i \frac{\partial}{\partial p_*} U_i(p_* + \mu U_0(0)) ds + e^{-(\rho + \lambda + \mu)T_i - t} \frac{\partial}{\partial p_*} U_0(0)
\]

\[
\frac{\partial}{\partial p_*} (p_* + \lambda_i U_i(p_* + \mu U_0(0)) - (r + \lambda_i + \mu) e^{-(\rho + \lambda + \mu)T_i - t} U_0(p_*)) \frac{\partial T_i}{\partial p_*}
\]

\[
\frac{\partial}{\partial p_*} U_1(p_i) = \int_{t_N}^{T_i} e^{-(\rho + \lambda + \mu)T_i - t} \lambda_0 \frac{\partial}{\partial p_*} U_0(p_* ds + e^{-(\rho + \lambda + \mu)T_i - t} \frac{\partial}{\partial p_*} U_1(1)
\]

\[
+ [e^{-(\rho + \lambda + \mu)T_i - t} (p_* + \lambda_i U_i(p_* + \mu U_0(0)) - (r + \lambda_i + \mu) e^{-(\rho + \lambda + \mu)T_i - t} U_0(p_*)) \frac{\partial T_i}{\partial p_*}
\]

Noting that \( U_i(p_* = U_1(1) - c \) and \( U_0(p_* = U_0(0) = \lambda_i [U_1(1) - c]/(r + \lambda_i) \),

\[
\frac{\partial}{\partial p_*} U_0(p_i) = \int_{t_N}^{T_i} e^{-(\rho + \lambda + \mu)T_i - t} \left( \frac{\partial}{\partial p_*} U_i(p_* + \mu U_0(0)) ds + e^{-(\rho + \lambda + \mu)T_i - t} \frac{\partial}{\partial p_*} U_0(0)
\]

\[
+ e^{-(\rho + \lambda + \mu)T_i - t} \frac{\partial T_i}{\partial p_*}
\]

\[
(A8)
\]

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Proofs of Section 4.

\[
\frac{\partial}{\partial p} U_i(p_t) = \int_0^{\tau} e^{-(r+\lambda_0)(t-s)} \frac{\partial}{\partial p} U_s(p_s) ds + e^{-(r+\lambda_0)(\tau-t)} \frac{\partial}{\partial p} U_\tau(p_\tau) + \frac{r(r+\lambda_0)}{r + \lambda_1} U_i(p_i) \frac{\partial T_i}{\partial p}.
\]

(A9)

Evaluating the HJB equation at \( p_* \), we get

\[
p_* = \frac{r(r+\lambda_1+\lambda_0)}{r+\lambda_1} U_i(p_i) = -f(p_*) U'_i(p_*),
\]

which is greater than or equal to zero if \( U'_i(p_*) \geq 0 \). Evaluating (A8) and (A9) at \( T_* \), we get

\[
\frac{\partial}{\partial p} U(p_0) = \frac{\lambda_1}{r+\lambda_1} U(1) + p_* \frac{\partial T_*}{\partial p}.
\]

Hence, using that \( U(1) \) is decreasing in \( p_* \) (Lemma A2) and \( \partial T_*/\partial p_* < 0 \), we get that \( U(p_0) = U_0(0) \) and \( U(p_*) \) are also decreasing in \( p_* \). Then, by working backward from \( t = T_* \), it is straightforward that (A8) and (A9) must be negative for all \( t \leq T_* \) and hence for all \( p \geq p_* \).

Lemma A4. Suppose that \( U_i(1|p_*) - c \geq 0 \), then \( U'_i(p_*|p_*) = 0 \implies \frac{\partial}{\partial p} U'_i(p_*|p_*) > 0 \).

Proof. Rearranging the HJB equation, we can write

\[
U'_i(p_*|p_*) = \frac{r U_i(p_*|p_*) - p - \lambda_0 U_0(p_*|p_*) - U_i(p_*|p_*)}{f(p_*)}.
\]

Evaluating at \( p = p_* \) and using the boundary conditions, equations (6a) and (6b), yields

\[
U'_i(p_*|p_*) = \frac{r U_i(p_*|p_*) - p_* + U_i(p_*|p_*) \frac{\lambda_0}{r+\lambda_1}}{f(p_*)} = \frac{r(r+\lambda_1) U_i(p_*|p_*) - p_*}{f(p_*)}.
\]

(A10)

Now, we can show that

\[
U'_i(p_*|p_*) = 0 \implies \frac{\partial}{\partial p} U'_i(p_*|p_*) > 0.
\]

Differentiating equation (A10) with respect to \( p_* \) yields

\[
\frac{\partial}{\partial p} U'_i(p_*|p_*) = \frac{r(r+\lambda_1) U_i(p_*|p_*)}{f(p_*)} \left|_{p_0} - 1 \right. + \frac{f'(p_*)}{f(p_*)} U'_i(p_*|p_*)
\]

\[
= \frac{r(r+\lambda_1) U_i(p_*|p_*)}{f(p_*)} \left|_{p_0} - 1 \right. + \frac{f'(p_*)}{f(p_*)} U'_i(p_*|p_*) > 0.
\]

However, from Lemma A3, we know that \( \frac{U_i(p_*|p_*)}{p_0} < 0 \). This along with \( f(p_*) < 0 \) proves the lemma.

Appendix B

Appendix B contains the proofs of the results presented in Section 4.

Proofs of Section 4.

B.1 Permanent shocks: proof of Proposition 2. We start introducing some notation. The evolution of prices, absent public news, is given by

\[
p_t = -\lambda_0 p_t + \mu p_t (1 - p_t).
\]

(B1)

Solving (B1), we get that the beliefs at time \( t \), given initial beliefs \( p_0 \), are

\[
\phi_t(p_0) = \frac{p_0(\lambda_0 - \mu)}{e^{\lambda (1-p_0)} - \mu (1 - p_0) - p_0 \mu}.
\]

(B2)
Using this equation, we can define $T(p_0; p_*)$ as the unique solution of $\phi_T(p_0) = p_*$. The function $T(p_0; p_*)$ thus represents the time required for the price $p_i$ to reach $p_*$ when the initial belief is $p_0$. Sometimes, when there is no risk of confusion, we omit the argument $p_*$ and just write $T(p)$.

\[ T(p_0; p_*) = \frac{1}{\lambda_0 - \mu} \ln \left( \frac{p_0 \lambda_0 - \mu + p_* \mu}{p_* \lambda_0 - \mu + p_0 \mu} \right). \]  

(B3)

### Step 1. Equilibrium with disclosure of good news.

The value functions obey

\[ rU_i(p) = p + f(p)U_i(p) + \lambda_0 [U_i(p) - U_i(p)] \]  

(B4)

\[ rU_0(p) = p - \mu \theta + f(p)U_0(p) - \mu U_0(p). \]  

(B5)

With boundary conditions

\[ U_i(p) = U_i(1) - c, \quad \text{for all } p \leq p_* \]  

(B6)

\[ U_0(p) = 0, \quad \text{for } p \leq p_* \]  

(B7)

**Lemma A5.** The solution to the HJB equation is

\[ U_0(p) = \int_0^{T(p)} e^{-r + \mu \theta} \left( \phi(p) - \mu \theta \right) dt \]

\[ U_i(p) = \int_0^{T(p)} e^{-r + \mu \theta} \left( \frac{\mu - \lambda_0 e^{-(\mu + \lambda_0)T}}{\mu - \lambda_0} \phi(p) - \frac{1 - e^{-(\mu + \lambda_0)T}r}{r + \mu} \lambda_0 \mu \theta \right) dt + e^{-r + \mu \theta} U_i(1) - c. \]

The term accompanying $\mu \theta$ follows by simple integration. For the term accompanying $\phi(p)$, we change the order of integration to get

\[ \int_0^{T(p)} \int_0^{T(p)} e^{-r + \mu \theta} e^{-(\mu + \lambda_0)T} \phi_\ast(p) ds dt = \int_0^{T(p)} \int_0^{T(p)} e^{\beta - \gamma k} e^{-(\mu + \lambda_0)T} \phi_\ast(p) ds dt \]

\[ = \int_0^{T(p)} e^{\beta - \gamma k} \phi_\ast(p) \int_0^{T(p)} e^{\gamma k - \gamma k T} dt ds \]

\[ = \int_0^{T(p)} e^{\beta - \gamma k} \phi_\ast(p) \frac{1 - e^{-\gamma k T}}{\mu - \lambda_0} ds. \]

Hence,

\[ \int_0^{T(p)} e^{-r + \mu \theta} \left( \phi(p) + \lambda_0 \int_0^{T(p)} e^{-(\mu + \lambda_0)T} \phi_\ast(p) ds \right) dt = \int_0^{T(p)} e^{-r + \mu \theta} \frac{\mu - \lambda_0 e^{-(\mu + \lambda_0)T}}{\mu - \lambda_0} \phi(p) dt. \]

**Lemma A6.** Assume $U_i(p_\ast | p_\ast) \geq 0$ and $U_i(1 | p_\ast) - c \geq 0$, then \( \frac{\partial}{\partial p_0} U(1 | p_\ast) < 0 \), for all $p_\ast \in [\mu \theta, 1]$.  

**Proof of Lemma A6.** We first note that $\frac{\partial}{\partial p_0} e^{-(\mu + \lambda_0)T} \phi_\ast(1) = 1$. Hence, using Lemma A12, the two hypotheses become

\[ U_i(1 | p_\ast) - c = \int_0^T e^{-(\mu + \lambda_0)T} \left( 1 - \frac{1 - e^{-(\mu + \lambda_0)T}}{r + \mu} \lambda_0 \mu \theta \right) dt - \frac{c}{1 - e^{-(\mu + \lambda_0)T}} \geq 0, \]  

(B10)
Suppose that \( U - e \), or equivalently, \( -e > (\mu_\theta, 0) \text{ and } (\lambda = \lambda_0 \text{ yields} -e \left( -e \right) > 0 \).

This implies that \( -e - e \lambda_0 \mu \ge 0 \), or equivalently,

\[
\begin{aligned}
   p_* &= \frac{\lambda_0 - \mu}{e^{T(x, y)}}, \\
   &\text{or}
   \end{aligned}
\]

Now differentiating (B10) with respect to \( T \) yields

\[
\begin{aligned}
   \frac{\partial (U_1(p_*|p_*)) - c}{\partial T} &= -(r + \lambda_0) \frac{e^{T(x, y)}(U_1(1|p_*)) - c)}{1 - e^{T(x, y)}} \\
   &\quad + \int_0^T \frac{e^{T(x, y)}(U_1(1|p_*)) - c)}{1 - e^{T(x, y)}} \lambda_0 \mu dt \\
   &\ge -p_* + 1 - \int_0^T \frac{e^{T(x, y)}(U_1(1|p_*)) - c)}{1 - e^{T(x, y)}} \lambda_0 \mu dt,
\end{aligned}
\]

where the last inequality follows from (B11). Now, replacing \( p_* \) by (B12), yields

\[
\begin{aligned}
   \frac{\partial (U_1(1|p_*)) - c}{\partial T} &\ge 1 - \frac{\lambda_0 - \mu}{e^{T(x, y)}(\lambda_0 - \mu)} \\
   &\quad - \int_0^T \frac{e^{T(x, y)}(U_1(1|p_*)) - c)}{1 - e^{T(x, y)}} \lambda_0 \mu dt \\
   &= \lambda_0(1 - e^{T(x, y)}(\lambda_0 - \mu)) \frac{\mu \theta (e^{T(x, y)}(\lambda_0 - \mu))}{(\lambda_0 - \mu)}.
\end{aligned}
\]

Now notice that

\[
\lambda_0(1 - e^{T(x, y)}(\lambda_0 - \mu)) \frac{\mu \theta (e^{T(x, y)}(\lambda_0 - \mu))}{(\lambda_0 - \mu)} = 0 \iff T = 0 \text{ or } T = \frac{\ln \frac{\lambda_0 - \mu}{\lambda_0 - \mu}}{\lambda_0 - \mu}
\]

and

\[
\lim_{T \to 0} \left[ \lambda_0(1 - e^{T(x, y)}(\lambda_0 - \mu)) \frac{\mu \theta (e^{T(x, y)}(\lambda_0 - \mu))}{(\lambda_0 - \mu)} \right] = \lambda_0(1 - \mu \theta) > 0.
\]

This means that \( \frac{\partial (U_1(p_*|p_*)) - c}{\partial T} \) is positive for all \( T \in [0, \frac{\ln \lambda_0 - \mu}{\lambda_0 - \mu}] \). This implies that \( \frac{\partial (U_1(p_*|p_*)) - c}{\partial p_*} \) is negative for all \( p_* \in \left[ \frac{\ln \lambda_0 - \mu}{\lambda_0 - \mu}, 1 \right] \supseteq [\mu \theta, 1] \).

**Lemma A7.** Suppose that \( U_i(1|p_*|p_i) \ge 0 \text{ and } U_i(1|p_*|p_i) - c \ge 0 \), then \( U_i(p_*|p_i) \) is nonincreasing in \( p_* \).

**Proof of Lemma A7.** This follows directly upon adapting the proof of Lemma A3.

**Lemma A8.** Suppose that \( U_i(1|p_*|p_i) - c \ge 0 \), then \( U_i(p_*|p_i) = 0 \Rightarrow \frac{\partial}{\partial p_*} U_i(p_*|p_i) > 0 \).

**Proof of Lemma A8.** This follows directly from adapting the proof of Lemma A10.

**Lemma A9.** In any equilibrium with good news disclosure, the disclosure threshold must be greater or equal than \( \mu \theta \).

**Proof of Lemma A9.** Suppose there is an equilibrium with disclosure threshold \( p_* < \mu \theta \). The value function of the low type is

\[
U_0(p) = \int_0^{T_{(p)}} e^{-\mu \theta t} \phi(p) - \mu \theta dt.
\]
so for \( p < p_* \), we have \( U_i(p) < 0 \). This means that the low type would have incentives to disclose its type, which contradicts the fact that this is an equilibrium with disclosure of good news. \( \square \)

**Lemma A10.** Suppose there exist \( p^*_+ \leq p^*_- \), such that

\[
\begin{align*}
U_i(p^+_*) = 0 \\
U_i(1|p^+_*) - c = 0
\end{align*}
\]

then \( p_* \) is an equilibrium threshold if and only if \( p_* \in [\max(p^+_*, \mu, p^*_-)]. \)

**Proof of Lemma A10.** This follows directly after slightly adapting the proof of Proposition 1 and using Lemma A9. \( \square \)

**Lemma A11.** There exists \( \underline{c} \leq \overline{c} \) such that: (i) a necessary and sufficient condition for the existence of equilibria with good news is that \( c \leq \overline{c} \) where

\[
U_i(1|\mu, \theta) - \overline{c} = 0,
\]

(ii) there is \( \underline{c} \) such that

\[
\begin{align*}
p^*_- = \mu, \\
U_i'(p^+_*) = 0.
\end{align*}
\]

The constants \( \underline{c}, \overline{c}, T \) are

\[
\begin{align*}
\underline{c} &= 1 - e^{-r - \lambda_0 t} \left( \frac{r + \mu - (r + \mu + \lambda_0) \mu\theta}{(r + \mu)(r + \lambda_0)} + \frac{\lambda_0 \mu \theta}{(r + \mu)(\lambda_0 - \mu)} \right) \\
\overline{c} &= e^{\left(\ln \frac{\lambda_0 - \mu + \lambda_0 \theta}{\lambda_0 - \mu}\right) \frac{r + \lambda_0 \theta}{r + \lambda_0}(r + \mu)} \left( \frac{r - \lambda_0 \mu \theta - (\mu \theta r + \mu - \lambda_0)}{(r + \lambda_0)(r + \mu)} \right) \\
T &= \ln \frac{\lambda_0 - \mu + \lambda_0 \theta}{\lambda_0 - \mu}.
\end{align*}
\]

**Proof of Lemma A11.** Observe that

\[
U_i(1|p_*) - c = 0 \Rightarrow U_i'(p_*) \geq 0.
\]

Also,

\[
U_i'(p_*) \leq 0 \Rightarrow U_i(1|p_*) - c \geq 0.
\]

Now recall that \( p^*_+ \) is defined by \( U_i(1|p^*_+) - c = 0 \). Then, from Lemma A6 and the Implicit Function Theorem, we have that \( \frac{dx}{dt} < 0 \). Defining \( \overline{c} \) by

\[
U_i(1|\mu, \theta) - \overline{c} = 0,
\]

we have that \( p^*_+ \geq \mu \theta \) for all \( c \leq \overline{c} \). Moreover, the fact that \( U_i'(p^*_+|p^*_+) > 0 \) implies that either \( U_i'(p_*,|p_*+) > 0 \) in the interval \([\mu, \mu, p^*_+]) \) or it crosses zero at most once and from below at some \( p^*_- \leq p^*_+ \). Hence, for any \( c < \overline{c} \), there must be a unique interval of equilibrium thresholds: \([\min(\mu, \mu, p^*_+), p^*_+]). The value of \( \overline{c} \) can be computed as

\[
\overline{c} = e^{\left(\ln \frac{\lambda_0 - \mu + \lambda_0 \theta}{\lambda_0 - \mu}\right) \frac{r + \lambda_0 \theta}{r + \lambda_0}(r + \mu)} \left( \frac{r - \lambda_0 \mu \theta - (\mu \theta r + \mu - \lambda_0)}{(r + \lambda_0)(r + \mu)} \right).
\]

Finally, we denote by \( \underline{c} \) the value of \( c \) such that \( p^*_- = \mu \theta \), where

\[
U_i'(\mu, \theta|\mu, \theta) = 0.
\]

Using the HJB equation, we can verify that

\[
U_i'(\mu, \theta|\mu, \theta) = 0 \Rightarrow U_i(1|\mu, \theta) = U_i(1|\mu, \theta) - \underline{c} = \frac{\mu \theta}{r + \lambda_0},
\]

where

\[
U_i(1|\mu, \theta) = \int_0^{T(\mu, \theta)} e^{-\psi + \lambda_0 t} \left( \frac{\lambda_0 \theta e^{-r \psi + \lambda_0 t}}{\mu + \lambda_0} + \phi(1) - \frac{1 - e^{-r \psi + \lambda_0 t}}{r + \mu} \right) dt + e^{-r \psi + \lambda_0 T(\mu, \theta)} \frac{\mu \theta}{r + \lambda_0}.
\]

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The value of $\zeta$ can thus be computed as
\begin{equation}
\zeta = (1 - e^{-r + \lambda\theta T}) \left( \frac{r + \mu - (r + \mu + \lambda_0)\mu\theta}{(r + \mu)(r + \lambda_0)} + \frac{\lambda_0\mu\theta}{(r + \mu)(\lambda_0 - \mu)} \right) - \frac{1 - e^{r + \mu\theta T}}{1 - e^{r + \lambda_0 T}},
\end{equation}
where $T = \ln \frac{ln\frac{\mu - \lambda_0}{\lambda_0 - \mu}}{\lambda_0 - \mu}$.

\[\Box\] \hspace{1cm} \textbf{Step 2. Equilibrium with disclosure of bad news.} \hspace{1cm} The value functions obey
\begin{equation}
r U_i(p) = p + f(p) U_i'(p) + \lambda_0[U_i(p) - U_i(p)]. \tag{B13}
\end{equation}
\begin{equation}
r U_0(p) = p - \mu \theta + f(p) U_0'(p) - \mu U_0(p). \tag{B14}
\end{equation}
With boundary conditions
\begin{equation}
U_i(p_t) = \frac{\mu \theta}{r + \lambda_0}, \quad \text{for all } p \leq p_t. \tag{B15}
\end{equation}
\begin{equation}
U_0(p) = 0, \quad \text{for } p \leq p_t. \tag{B16}
\end{equation}
The time that it takes to reach the threshold is
\begin{equation}
T = \ln \frac{\mu - \lambda_0}{\lambda_0 - \mu}.
\end{equation}
The value functions can be written as
\begin{equation}
U_i(p) = \int_0^{T(p)} e^{-(r + \lambda_0)(\phi(p) + \lambda_0 U_0(\phi(p)))}ds + \frac{\mu \theta}{r + \lambda_0} e^{-T(p)(r + \lambda_0)} \tag{B17}
\end{equation}
\begin{equation}
U_0(p) = \int_0^{T(p)} e^{-(r + \theta)(\phi(p) - \mu)\theta}ds. \tag{B18}
\end{equation}
\textbf{Lemma A12.} The solution to the HJB equation is
\begin{equation}
U_i(p) = \int_0^{T(p)} e^{-(r + \lambda_0\mu)(\phi(p) - \mu\theta)}dt 
\end{equation}
\begin{equation}
U_i(p) = \int_0^{T(p)} e^{-(r + \lambda_0\mu)(\phi(p) - \mu\theta)} - \frac{1 - e^{-(r + \mu\lambda_0)(T(p)-t)}}{r + \mu} \lambda_0\mu\theta \lambda_0 \mu\theta dt + e^{-T(p)(r + \lambda_0)} \frac{\mu\theta}{r + \lambda_0}. \tag{B19}
\end{equation}
\begin{equation}
U_i(p) = U_i(1) - c, \quad \text{for all } p \leq p_t. \tag{B20}
\end{equation}
\begin{equation}
U_0(p) = 0, \quad \text{for } p \leq p_t. \tag{B21}
\end{equation}
\textbf{Proof.} Identical to Lemma A5, but replacing $U_i(p_t) = \mu \theta / (r + \lambda_0)$.
\[\Box\]

\textbf{Lemma A13.} There is an equilibrium with bad news if and only if $c \geq \check{c}$, where $\check{c}$ is defined by
\begin{equation}
U_i(1) - \check{c} = U_i(\mu \theta) = \frac{\mu \theta}{r + \lambda_0}, \tag{B22}
\end{equation}
and the value of $\check{c}$ has been computed at the end of the proof of Lemma A11 as $\zeta$.

\textbf{Proof of Lemma A13.} A necessary and sufficient condition for an equilibrium with disclosure of bad news is that the high type does not disclose its type when $p = \mu \theta$. This happens if and only if
\begin{equation}
U_i(1) - c \leq \frac{\mu \theta}{r + \lambda_0}. \tag{B23}
\end{equation}
Noting that $U_i(1) = A(1, \mu \theta) + \delta_{\mu \theta}$, where $A$ and $\delta$ are as in the proof of Lemma A14, does not depend on $c$, we can conclude from (B23) that an equilibrium with disclosure of good news exists if and only if
\begin{equation}
c \geq \check{c} := U_i(1) - \frac{\mu \theta}{r + \lambda_0}. \tag{B24}
\end{equation}
Thus, the definition of $\check{c}$ in equation (B24) coincides with $\zeta$ computed at the end of Lemma A11.
\[\Box\]
Step 3 Comparing equilibria with good and bad news.

Lemma A14. Let \( U^n_l(p) \) and \( U^n_r(p) \) be the value function in an equilibrium with disclosure of good news and bad news with disclosure threshold \( p_n = \mu \theta \), respectively. Then, for all \( p \) and for all \( v \in [0, 1] \), we have that \( U^n_l(p) \geq U^n_r(p) \).

Proof. There is nothing to prove for the low type as

\[
U^n_l(p) = U^n_r(p) = \int_0^{T(p)} e^{−r+μθ(μ−r(1−δ))}dt.
\]

Let’s define

\[
A(p, p_n) = \int_{T(p)}^{T(p)+\lambda_{0}T} e^{−r+λ_{0}Y} \left( \frac{μ−μθ}{μ−λ_{0}}(μ−r(1−δ))ϕ(1) + \frac{1−e^{−r+μθT(p)−1}}{r+μ}\lambda_{0}μθ \right) dt
\]

\[
δ(p, p_n) = e^{−r+λ_{0}T(p, p_n)}
\]

\[
δ = δ_1(1, μθ).
\]

We have the following equilibrium condition for an equilibrium with disclosure of bad news.

\[
U^n_l(1) − c = \frac{A(1, μθ) − c}{1−δ} \leq \frac{μθ}{r+λ_{0}}.
\]

Similarly, we have that

\[
U^n_r(p) = \frac{A(1, μθ) − c}{1−δ}.
\]

Hence,

\[
U^n_l(p) − U^n_r(p) = δ(p, μθ) \left[ \frac{μθ}{r+λ_{0}} − \left( U^n_r(1) − c \right) \right] = δ(p, μθ) \left[ \frac{μθ}{r+λ_{0}} − \frac{A(1, μθ) − c}{1−δ} \right] \geq 0,
\]

where in the last inequality, we use (B22)

Lemma A15. Let’s define

\[
n := (1−e^{−r+λ_{0}Y}) \left( \frac{r+μ−(r+μ+λ_{0})μθ}{(r+μ)(r+λ_{0})} + \frac{λ_{0}μθ}{(r+μ)(λ_{0}−μ)} \right) \frac{1−e^{−(λ_{0}−λθ)(1−1)}}{r+λ_{0}}.
\]

where \( T = T(μθ) = \frac{ln(μθ−r)}{r+λ_{0}} \). Then,

(i) If \( c < n \), then any equilibria with good news has a threshold strictly greater than \( μθ \). In particular, \( p_{n}^− > μθ \).

(ii) If \( c < n \), then the Pareto-dominating equilibrium is the equilibrium with disclosure of good news and threshold \( p_{n}^− > μθ \). Alternatively, if \( c > n \), then the Pareto-dominating equilibrium is the equilibrium with disclosure of bad news with threshold \( μθ \).

Proof. Let \( U^n_l(p) \) be the value function in an equilibrium with disclosure of good news with threshold \( p_n \). Let also \( p_n^−(c) \) be the threshold satisfying the smooth pasting condition \( U^n_l(p_n | p_n) = 0 \) for a cost of disclosure \( c \). From the proof of Lemma A11, we have that \( p_n^−(c) = μθ \) and \( U^n_l(1 | p_n^−(c)) = \frac{p_n^−(c)}{p_n^−(c)+1} \). Using the Implicit Function Theorem, we get

\[
\frac{d}{dc}p_n^−(c) = \frac{r+λ_{0}}{1−(r+λ_{0})p_n^−(c)U^n_l(1 | p_n^−(c))/p_n^−(c)} < 0,
\]

where we have used that \( p_n^−(c)U^n_l(1 | p_n^−(c))/p_n^−(c) < 0 \) (Lemma A6). Hence, the result in 1. follows from \( p_n^−(c) = μθ \).

For (ii), observe that a bad news equilibrium does not exist if \( c < n \) (Lemma A13). Hence, we only need to consider equilibrium with good news, and we know that in this case, \( p_n^− \) Pareto dominates any other equilibrium (Lemma A7). When \( c < n \), equilibriums with good news disclosure and bad news disclosure may coexist. However, from Lemma A14, when this is the case, the equilibrium with disclosure of bad news is Pareto dominant.

Proof of Corollary 1. Proof. In absence of litigation risk, the manager ex ante payoff is \( U^n_l(1) = U^{ND}(1) − cθ(1)/(1−δ(1)) \). If we take \( δ \geq 1/μ \), we get \( p_n = 1 \), so the manager’s payoff with litigation risk is \( U^n_l(1) = 1/(r+λ_{0}) = U^{ND}(1) \). Accordingly, in this case, the manager payoff is strictly higher in the presence of litigation risk. By continuity of \( U^n_l(1) \) with respect to \( θ \), there is \( θ < 1/μ \), such that the inequality continue to hold.
B.2 General case: proof of Proposition 3.

B.2.1 HJB equation. With transitory shocks, the HJB equation for \( p_t > p_* \),
\[
    rU_0(p) = p + f(p)U_1'(p) + \lambda_0[U_0(p) - U_1(p)]
\]
\[
    rU_0(p) = p - \mu \theta + f(p)U_1'(p) + \lambda_1[U_0(p) - U_1(p)] + \mu[U_0(0) - U_0(p)].
\]

The low-type manager value function satisfies the boundary condition
\[
    U_0(p_*) = E \left[ \int_0^\tau e^{-rt}(p_* - \mu \theta + (\mu + \lambda)U_0(0) + \lambda_1U_1(p_*))dt \right] + U_0(0)I_{\{\tau > \tau_0\}}.
\]

where \( \tau_r \) is the first arrival of public (bad) news, \( \tau_p \) is the time at which the manager voluntarily discloses bad news, and \( \tau_i \) is the time at which the value of assets jump from 0 to 1. We can solve for the expected payoff of a low-type manager, as given by
\[
    U_0(p_*) = \int_0^\tau e^{-rt}(p_* - \mu \theta + (\mu + \lambda)U_0(0) + \lambda_1U_1(p_*))dt
\]
\[
    U_0(p_*) = \frac{p_* - \mu \theta}{r + \mu + \lambda_1 + \lambda} + \frac{\mu + \lambda}{r + \mu + \lambda_1 + \lambda} U_0(0) + \frac{\lambda_1}{r + \mu + \lambda_1 + \lambda} U_1(p_*).
\]

(B23)

Similarly, the value function of the high-type manager satisfies the boundary condition
\[
    U_1(p_*) = \frac{p_* + \lambda_0 U_0(p_*)}{r + \lambda_0}.
\]

(B24)

In addition, we have the following conditions when \( p_* = 0 \):
\[
    U_0(0) = \frac{\lambda_1}{r + \lambda_1} U_1(0)
\]
\[
    U_1(0) = U_1(1) - c.
\]

(B25)

(B26)

As the manager is using a mixed strategy when \( p_* = p_* \), he must be indifferent between disclosing and not disclosing negative information, otherwise, he would not be willing to randomize. Hence, we can determine the threshold \( p_* \) using the indifference condition for a mixed strategy:
\[
    U_0(p_*) = U_0(0).
\]

(B27)

We can solve for \( U_0(p_*) \) by combining equations (B23) and (B27), which gives us
\[
    U_0(p_*) = \frac{p_* - \mu \theta + \lambda_1 U_1(p_*)}{r + \lambda_1}.
\]

(B28)

Then, combining (B24) with (B28), we get
\[
    U_0(p_*) = \frac{p_* r}{r + \lambda_0} - \frac{\mu \theta}{r + \lambda_0}.
\]

(B29)

Similarly, let \( U_0''(p_*) \) be the right-hand second derivative of \( U_0 \) at \( p_* \). Evaluating the HJB equation at \( p_* \) and using the initial conditions (B29) and (B30), we get that \( U_0''(p_*) = 0 \) and
\[
    U_0''(p) = 0 \Rightarrow U_0''(p) = \frac{1 + \lambda_1 U_1''(p)}{-f(p)}.
\]

(B32)

Using the HJB equation for \( U_1 \) and the initial conditions, we get (B29) and (B30), we get
\[
    f(p_*)U_1''(p_*) = 0.
\]
By assumption $p_0 > \hat{p}$ so $f(p_0) < 0$, which means that $U^0_i(p_0) = 0$. Differentiating the HJB equation for $U_i$ we get

$$U^0_{i+}(p) = 0 \Rightarrow U^0_{i+}(p) = \frac{1 + \lambda_0 U^0_{i0}(p)}{-f(p)}.$$  \hfill (B33)

In particular, we get that $U^0_{i+}(p_0) = 0$. Using (B32) and (B33) we find that $U^0_{i0}(p_0) > 0$ and $U^0_{i+}(p_0) > 0$. Which means that there is $e > 0$ such that $U^0_{i0}(p) > 0$ and $U^0_{i+}(p) > 0$ for $p \in (p_0, p_0 + e)$. Let $\tilde{p} = \inf\{p > p_0, U^0_{i0}(p) < 0 \text{ or } U^0_{i+}(p) < 0\}$. As $U_i$ and $U_i'$ are continuously differentiable in $(p_0, 1)$, we have that either $U^0_i(\tilde{p}) = 0$ and $U^0_i(\tilde{p}) \geq 0$ or $U^0_i(\tilde{p}) = 0$ and $U^0_i(\tilde{p}) \geq 0$. Without loss of generality, suppose that $U^0_i(\tilde{p}) = 0$ and $U^0_i(\tilde{p}) \geq 0$. Equation (B32) implies that $U^0_i(\tilde{p}) > 0$. Suppose that $U^0_i(\tilde{p}) > 0$, then there is $\tilde{\epsilon}$ such that $U^0_i(p) > 0$ and $U^0_i(p) > 0$ for all $p \in (\tilde{p}, \tilde{p} + \tilde{\epsilon})$, contradicting the definition of $\tilde{p}$ as the inf $\{p > p_0, U^0_i(p) < 0 \text{ or } U^0_i(p) < 0\}$. On the other hand, if $U^0_i(\tilde{p}) = 0$, then equation (B33) implies that $U^0_i(\tilde{p}) > 0$, which means that we can also find $\tilde{\epsilon}$ such that $U^0_i(\tilde{p}) > 0$ and $U^0_i(\tilde{p}) > 0$ for all $p \in (\tilde{p}, \tilde{p} + \tilde{\epsilon})$. Hence, $U_i$ must be nondecreasing in $[p_0, 1]$. A symmetric argument can be used to show that $U_i$ is nondecreasing.

**Step 2. Optimality of disclosure strategy.** The next step is to verify that the disclosure strategy is optimal. First, we verify that the disclosure strategy is optimal whenever $V_i = 0$. By construction, $U_0(p_0) = U_0(0)$ so the manager is indifferent between disclosing negative information or not when $p_1 = p_0$. Moreover, given that $U_0(p)$ is nondecreasing, the manager does not have incentives to deviate and disclose if $p > p_0$.

Next, we verify that the disclosure strategy is also optimal when $V_i = 1$. The manager disclosure strategy is optimal if the following two conditions are satisfied:

(i) $U_i(1) \geq c$.

(ii) $U_i(p) \geq U_i(1) - c$ for $p \geq p_0$.

For (i) note that, by construction (equation (B31)),

$$U_i(1) - c = \left(1 + \frac{r}{\lambda_i}\right) U_0(p_0) = \left(1 + \frac{r}{\lambda_i}\right) \left(p_0 - \frac{\mu \theta r + \lambda_0}{r + \kappa}\right),$$

which is always positive given the assumption that

$$p_0 \geq \frac{\mu \theta r + \lambda_0}{r + \kappa}.$$  

For (ii), note that as $U_i$ is increasing, (ii) is satisfied if and only if $U_i(p_0) \geq U_i(1) - c$. This happens if and only if

$$\left(1 + \frac{r}{\lambda_i}\right) \left(p_0 - \frac{\mu \theta r + \lambda_0}{r + \kappa}\right) \leq \frac{p_0}{r} - \frac{\mu \theta r + \lambda_0}{r + \kappa},$$

which is true for all $p_0 \leq \mu \theta$.

**References**


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28 $U_i$ is twice continuously differentiable for $p > p_0$. © The RAND Corporation 2016.


Supporting information

Additional supporting information may be found in the online version of this article at the publisher’s website:

Web Appendix