Recent Advances in Mathematical Fluid Dynamics
Duke University

Summer School Program

Speakers

Jacob Bedrossian	(UCLA)
Javier Gomez-Serrano	(Brown)
Helena Nussenzveig-Lopes	(FURJ)
Vladimir Sverak	(Minnesota)

Titles & Abstracts

Title: **Lagrangian chaos and almost-sure mixing in stochastic fluid mechanics**
(*Jacob Bedrossian****)***

Abstract: This course will introduce students to the basics of the random dynamical systems methods in order to study "chaos" of the Lagrangian flow map and almost-sure exponential mixing of passive scalars by stochastically forced fluid equations and similar random flows. First we discuss Lyapunov exponents, formulas for them, and how to estimate them. Time permitting, we will then discuss a general scheme for how to upgrade positive Lyapunov exponents to almost-sure exponential mixing estimates of passive scalars. While we will focus on finite-dimensional examples (i.e. Galerkin truncations of the Navier-Stokes equations rather than the full NSE themselves) we will discuss what ingredients need to be added and enhanced in order to extend the results to the infinite dimensional Navier-Stokes equations.

Title: **Global existence and finite time singularities in fluid mechanics** (*Javier Gomez-Serrano*)

Abstract: In this minicourse I will approach some of the recent advances in mathematical fluid mechanics, on one hand from the standpoint of constructing generic initial data for which there is global existence of solutions and on the other finding special initial data that leads to a singularity in finite time. Special emphasis will be put in the interaction with computers, which will be an integral part of the setup and the proofs. I will talk about different models such as the SQG equation, Euler, Muskat, Boussinesq, and many others.

Title: **Vanishing viscosity may prevent inviscid dissipation** (*Helena Nussenzeig-Lopes*)

Abstract: Inviscid dissipation of energy is a cornerstone of turbulence theory. In recent years considerable effort has been directed at producing examples of inviscid dissipation in fluid flows, notably by convex integration. The Onsager conjecture provides a threshold of regularity for flows which may exhibit inviscid dissipation. In these talks we discuss how vanishing viscosity may prevent inviscid dissipation at weaker regularity than that given by the Onsager scaling. We examine three scenarios for inviscid dissipation in 2D flows: (1) energy, which is the Onsager context, (2) enstrophy and (3) long time average of enstrophy.

Title: **Topics related to vortex rings** (*Vladimir Sverak*)

Abstract: The study of vortex rings has several aspects including for example the Hamilton-Poisson structure of Euler equations, various PDE estimates, classical asymptotic expansions, and evaluation of interesting integrals. We will discuss some of these topics, mentioning also their connections to broader themes in the PDE analysis of fluid flows. Joint work with Thierry Gallay will play an important part in the presentation.