Recent Advances in Mathematical Fluid Dynamics
Duke University

Speakers, Titles, and Abstracts

- ### *Dallas Albritton (Princeton University)*

Title: Kinetic shock profiles for the Landau equation

Abstract:

Abstract: Compressible Euler solutions develop jump discontinuities known as shocks. However, physical shocks are not, strictly speaking, discontinuous. Rather, they exhibit an internal structure which, in certain regimes, can be represented by a smooth function, the shock profile. We demonstrate the existence of weak shock profiles to the kinetic Landau equation. Joint work with Jacob Bedrossian (UCLA) and Matthew Novack (Purdue University).

- ### *Suncica Canic (University of California, Berkeley)*

Title: Fluid-structure interaction with multi-layered poroelastic structures

Abstract:

In this talk we will address fluid-structure interaction (FSI) between the flow of an incompressible, viscous fluid and a multi-layered poroelastic medium. The poroelastic medium consists of a thick poroelastic solid modeled by the Biot equations, and a thin poroelatic shell, located at the fluid-structure interface. These types of problems arise in many applications, including bioartificial organs design and drug-eluting stents. We will discuss two well-posedness results for this class of fluid-poroelastic structure interaction problems: one with linear and one with nonlinear coupling. The nonlinearly coupled problem is particularly difficult. In addition to the geometric nonlinearity due to the nonlinear coupling and nonlinearity in the model equations, the Biot equations pose additional difficulties due to their hyperbolic-parabolic nature, which contrasts classical FSI problems in which the structure model involves only elastodynamics equations and no filtrating fluid through the poroelastic matrix. We will summarize our recent well-posedness results and present our numerical simulations showing solutions to these problems.

- ***Jiajie Chen (New York University)***

Title: Stable nearly self-similar blowup of the 2D Boussinesq and 3D Euler equations with smooth data

Abstract:

Whether the 3D incompressible Euler equations can develop a finite-time singularity from smooth initial data is an outstanding open problem. In this talk, we will first review recent progress in singularity formation in incompressible fluids. Then, we will present a result inspired by the Hou-Luo scenario for a potential 3D Euler singularity, in which we prove finite time blowup of the 2D Boussinesq and 3D Euler equations with smooth initial data and boundary. To establish the blowup results, we construct an approximate self-similar blowup profile and prove its nonlinear stability with computer assistance. In the stability analysis, we decompose the linearized operator into the leading order operator and the remainder. We develop sharp functional inequalities using optimal transport and the symmetry properties of the velocity kernels to estimate the nonlocal terms from the velocity and use weighted energy estimates to establish the stability analysis of the leading order operator. The key role of computer assistance is to construct an approximate blowup profile and approximate space-time solutions with rigorous error control, which provides critical small parameters in the energy estimates for the stability analysis and allows us to control the remainder perturbatively. This is joint work with Tom Hou.

- ***Alexei Cheskidov (University of Illinois at Chicago)***

Title: Dissipation anomaly and anomalous dissipation in fluid flows

Abstract:

TBA

- ***Diego Cordoba (Instituto de Ciencias Matematicas)***

Title: TBA

Abstract:

TBA

- ***Theodore Drivas (Stony Brook University)***

Title: Irreversible features of the 2D Euler equations

Abstract:

We will discuss aspects of the long term dynamics of 2d perfect fluids. As an application of a certain stability of twisting for general hamiltonian flows, we will show generic loss of smoothness near stable steady states, the existence of many wandering neighborhoods, aging of the Lagrangian flow, along with other examples of complex behavior such as indefinite perimeter growth for special vortex patches.

- ***Francisco Gancedo (University of Seville)***

Title: Global-in-time dynamics for one-phase Muskat and two-phase Stokes gravity waves

Abstract:

In this talk we consider the evolution of interfaces evolving by incompressible flows. On the one hand, we study the one-phase Muskat problem, where the fluid is filtered in a porous medium. In the gravity-stable case, we show that initial Lipschitz graphs of arbitrary size provide global-in-time well-posedness. On the other hand, we study the interface dynamics given by two fluids of different densities evolving by the linear Stokes law. We show stability to the flat stable case and exponential growth in the unstable regime.

- ***Daniel Ginsberg (Princeton University)***

Title: On the Distribution of Heat in Integrable and Non-Integrable Magnetic Fields

Abstract:

We study the equilibrium temperature distribution in a model for strongly magnetized plasmas in dimension two and higher. Provided the magnetic field is sufficiently structured (integrable in the sense that it is fibered by co-dimension one invariant tori, on most of which the field lines ergodically wander) and the effective thermal diffusivity transverse to the tori is small, it is proved that the temperature distribution is well approximated by a function that only varies across the invariant surfaces. The same result holds for "nearly integrable" magnetic fields up to a "critical" size. In this case, a volume of non-integrability is defined in terms of the temperature defect distribution and related to the non-integrable structure of the magnetic field, confirming a physical conjecture of Paul-Hudson-Helander. Our proof crucially uses a certain quantitative ergodicity condition for the magnetic field lines on full measure set of invariant tori, which is automatic in two dimensions for magnetic fields without null points and, in higher dimensions, is guaranteed by a Diophantine condition on the rotational transform of the magnetic field. This is joint with Theo D. Drivas and Hezekiah Grayer II.

• *Nathan Glatt-Holtz (Tulane University)*

Title: Consistency Results for some Bayesian PDE inverse problems

Abstract:

Frequently one would like to estimate functional parameters u in a physical model defined by a partial differential equation from a collection of sparse and uncertain observations. Here a Bayesian methodology provides an attractive statistical approach for many such estimation problems, one which provides a comprehensive picture of uncertainties in the unknown. An important step in the validation of this Bayesian methodology is to establish conditions for posterior consistency. Specifically we would like to determine when $\mu_N \rightharpoonup \delta_{u_*}$ where μ_N is the Bayesian posterior conditioned on N observations of the solution and u_* is the true value of the unknown. In this talk we describe some rigorous approaches that we have recently developed tailored to address consistency for PDE inverse problems involving the recovery of an infinite dimensional unknown. We describe how our approach applies to a gallery of model problems including the recovery of a divergence free velocity field from the measurement of a solute which is advecting and diffusing in the fluid medium. This is joint work with Jeff Borggaard, Christian Frederiksen and Justin Krometis.

• *Zoran Grujic (University of Virginia)*

Title: On criticality of the Navier-Stokes diffusion

Abstract:

The main purpose of this talk is to present a mathematical evidence of criticality of the Navier-Stokes diffusion. In particular, considering a plausible candidate for a finite time blow-up, a two-parameter family of the dynamically rescaled profiles, we show that as soon as the hyper-diffusion exponent is greater than one, a new region in the parameter space (completely in the super-critical regime) is ruled out. As a matter of fact, the region is a neighborhood (in the parameter space) of the self-similar profile, i.e., the `approximately self-similar' blow-up is ruled out for all hyper-diffusive models.

• *Zineb Hassainia (New York University Abu Dhabi)*

Title: Invariant KAM tori around elliptical vortex patches for the planar Euler equations

- ***Tom Hou (Caltech)***

Title: Potentially singular behavior of 3D incompressible Navier-Stokes equations

Abstract:

Whether the 3D incompressible Navier-Stokes equations can develop a finite time singularity from smooth initial data is one of the most challenging problems in nonlinear PDEs. In this talk, I will present some new numerical evidence that the 3D Navier-Stokes equations develop nearly self-similar singular scaling properties with maximum vorticity increased by a factor of 10^7. This potentially singular behavior is induced by a potential finite time singularity of the 3D Euler equations. Unlike the Hou-Luo blowup scenario, the potential singularity of the 3D Euler and Navier-Stokes equations occurs at the origin. We have applied several blowup criteria to study the potentially singular behavior of the Navier-Stokes equations. The Beale-Kato-Majda blow-up criterion, the blowup criteria based on the growth of enstrophy and negative pressure, the Ladyzhenskaya-Prodi-Serrin regularity criteria all seem to imply that the Navier-Stokes equations develop nearly singular behavior. Finally, we present some new numerical evidence that the axisymmetric Navier-Stokes equations with fractional dimension slightly greater than 3 and decaying viscosity seem to develop asymptotically self-similar blowup.

- ***Marta Lewicka (University of Pittsburgh)***

Title: The Monge-Ampere system: flexibility in arbitrary dimension and codimension.

Abstract:

In this talk, we will be concerned with solutions to the Monge-Ampere system, obtained by means of convex integration. This system is a natural extension of the Monge-Ampere equation $\det\nabla^2 v=f$ in dimension d=2, given by the linearization of the Riemann curvature tensor in arbitrary dimension d, and arises in relation to:

(i) the problem of existence of isometric immersions and

(ii) the context of nonlinear elasticity.

Our main result consists of the ``stage'' construction, in which we achieve the Holder regularity $\mathcal{C}^{1,\alpha}$ of solutions approximating any subsolution, for any exponent $\alpha<\frac{1}{1+d(d+1)/k}$ where d is an arbitrary dimension and $k\geq 1$ is an arbitrary codimension in the problem.

When d=2 and k=1, we recover the previous result by Lewicka-Pakzad for the Monge-Ampere equation. Our construction can be translated to the isometric immersion problem, where for k=1 we recover the result by Conti-Delellis-Szekelyhidi, and for large k we quantify the result by Kallen.

- ***Nader Masmoudi (New York University)***

Title: Reversal in the Stationary Prandtl Equations

Abstract:

We investigate reversal and recirculation for the stationary Prandtl equations. Reversal describes the solution after the Goldstein singularity, and is characterized by regions in which $u > 0$ and $u < 0$ respectively. The classical point of view of regarding the stationary Prandtl system as an evolution equation in x completely breaks down since u changes sign. Instead, we view the problem as a quasilinear, mixed-type, free-boundary problem. This is a joint work with Sameer Iyer.

- ***Jonathan Mattingly (Duke University)***

Title: An example of Optimal enhanced dissipation and mixing for a time-periodic, Lipschitz velocity field on the 2-Torus

Abstract:

We consider the advection-diffusion equation on the 2-torus with a Lipschitz and time-periodic velocity field that alternates between two piecewise linear shear flows. We prove enhanced dissipation on the timescale $|\log\nu|$, where ν is the diffusivity parameter. This is the optimal decay rate as $\nu \to 0$ for uniformly-in-time Lipschitz velocity fields. We also establish exponential mixing for the $\nu = 0$ problem. This is joint work with Kyle Liss and Tarek Elgindi.

- ***Adam Oberman (McGill University)***

Title: Machine Learning and PDEs

Abstract:

In this introductory talk I'll talk about some connections between PDE regularization and Machine Learning, as well as Machine Learning applied to PDEs

- ***Laurel Ohm (Princeton University)***

Title: Problems inspired by the hydrodynamics of flexible filaments

Abstract:

Many fundamental biophysical processes, from cell division to cellular motility, involve dynamics of thin structures immersed in a very viscous fluid. Various popular models have been developed to describe this interaction mathematically, but much of our understanding of these models is only at the level of numerics and formal asymptotics. Here we seek to develop the PDE theory of filament hydrodynamics. We first propose a PDE framework for analyzing the error introduced by slender body theory (SBT), a common approximation used to facilitate computational simulations of immersed filaments in 3D. Given data prescribed only along a 1D curve, we develop a novel type of boundary value problem and obtain an error estimate for SBT in terms of the fiber radius. This places slender body theory on firm theoretical footing. We then consider other physically relevant scenarios in which the slender body PDE framework applies and shed light on the analysis of such problems.

- ***Jia Shi (MIT)***

Title: On the analyticity of the Muskat equation

Abstract:

The Muskat equation describes the interface of two liquids in a porous medium. We will show that if a solution to the Muskat problem in the case of same viscosity and different densities is sufficiently smooth, then it must be analytic except at the points where a turnover of the fluids happens. We will also show analyticity in a region that degenerates at the turnover points provided some additional conditions are satisfied.

- ***Roman Shvydkoy (University of Illinois at Chicago)***

Title: Generic alignment conjecture and global relaxation of kinetic models of collective behavior.

- ***Weiran Sun (Simon Fraser University)***

Title: Uniqueness of Weak Solutions to the Landau-Coulomb Equation

Abstract:

In this talk we will show some recent progress on the uniqueness of weak solutions to the Landau-Coulomb equation.

- ## *Andrei Tarfulea (Louisiana State University)*

Title: Regularity and continuation for the Boltzmann equation

Abstract:

The Boltzmann equation models a high-energy gas with elastic collisions. From the mathematical point of view, it presents a nonlocal degenerate-parabolic PDE with very few coercive quantities. The existence of global smooth solutions remains an open problem, and the state of the art is summarized by the conditional regularity program: as long as the hydrodynamic quantities (mass, energy, and entropy densities) remain "under control" (satisfying four time-independent inequalities), the solution is in fact smooth.

We eliminate two of the four inequalities from the conditional regularity result by showing that solutions to the Boltzmann equation dynamically (and instantly) fill any vacuum regions, even when the initial data contains such regions; the estimates only depend on an initial (possibly small) core of mass. We then examine how this mass spreading effect enhances known results on the construction, regularity estimates, uniqueness, and continuation for solutions starting from very rough initial data.

- ## *Edriss Titi (University of Cambridge)*

Title: Inviscid Voigt Regularization of Fluid Models: Blow-up Criterion, Computations and Statistical Properties

- ## *Jiahong Wu (Oklahoma State University)*

Title: Stabilizing phenomenon for electrically conducting fluids

Abstract:

Physical experiments have observed a remarkable stabilizing phenomenon: magnetic field can stabilize electrically conducting fluids. This talk presents two results that establish this observation as mathematically rigorous facts on the magnetohydrodynamic (MHD) equations. The first result is for a 3D incompressible MHD system with anisotropic dissipation. Without the magnetic field, the fluid is not known to be stable. But any perturbations near a suitable background magnetic field governed by this MHD system are shown to be asymptotically stable and decay algebraically in time. The second result concerns the 3D inviscid heat conductive compressible MHD system. Without the magnetic field, the fluid is governed by the 3D compressible Euler equations. Solutions of the compressible Euler equations can blow up in a finite time even when the initial data are smooth and small. However, this compressible MHD system is shown to be stable and decay near any background magnetic field satisfying a Diophantine condition.

● *Yao Yao (National University of Singapore)*

Title: Small scale formation for the 2D Boussinesq equation

Abstract:

In this talk, we consider the 2D incompressible Boussinesq equation without thermal diffusion, and aim to construct rigorous examples of small scale formations as time goes to infinity. In the viscous case, we construct examples of global-in-time smooth solutions where the H^1 norm of density grows to infinity algebraically in time. For the inviscid equation in the strip, we construct examples whose vorticity grows at least like t^3 and gradient of density grows at least like t^2 during the existence of a smooth solution. These growth results work for a broad class of initial data, where we only require certain symmetry and sign conditions. As an application, we also construct solutions to the 3D axisymmetric Euler equation whose velocity has infinite-in-time growth. This is a joint work with Alexander Kiselev and Jaemin Park.

● *Andrej Zlatos (University of California, San Diego)*

Title: Local regularity and finite time blow-up for the generalized SQG equation on the half-plane

Abstract:

The generalized SQG equation with parameter $\alpha\in[0,\frac 12]$ interpolates between the 2D Euler and SQG equations, for which $\alpha=0$ and $\alpha=\frac 12$, respectively. We show that this PDE with $\alpha\in(0,\frac 14]$ is locally well-posed on the half-plane in spaces of bounded integrable locally Lipschitz functions that are natural for its dynamic on domains with boundaries, and allow for some power growth of the derivative in the normal direction at the boundary. We also show existence of solutions exhibiting finite time blow-up in this whole local well-posedness parameter regime, which is the first finite time singularity result for equations (as opposed to patch models) of this type. Moreover, we prove sharpness of both these results by showing ill-posedness of the PDE in the above spaces when $\alpha>\frac 14$.