Supplement to "Inference in dynamic discrete choice problems under local misspecification"

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May 25, 2018

Abstract

This document provides additional Monte Carlo simulation results for Bugni and Ura (2018).

S1 Additional simulation results for the first design

Table S1 is the last set of results for the Monte Carlo design described in Section 5. This table presents results under asymptotically overwhelming local misspecification (i.e. $\delta = 1/3$), with estimators scaled by the regular $\sqrt{n}$-rate. According to our theoretical results, the presence of overwhelming local misspecification implies that these estimators no longer converge at the regular $\sqrt{n}$-rate, but rather at $n^{1/3}$-rate. In accordance with this prediction, Table S1 reveals that the asymptotic bias of the estimators does not appear to converge when scaled by the regular $\sqrt{n}$-rate.

		K-MD($\mathbf{I}_{\lvert\tilde{A}\times X\rvert\times\lvert\tilde{A}\times X\rvert}$)			K-MD(W^*_{AV})			K-ML		
K	Statistic	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$\sqrt{n}$ Bias	0.98	1.14	1.30	1.04	1.23	1.42	1.10	1.29	1.47
	$\sqrt{n}$ SD	0.34	0.33	0.31	0.30	0.29	0.28	0.29	0.28	0.28
	n MSE	1.08	1.40	1.80	1.17	1.60	2.08	1.28	1.74	2.22
2	$\sqrt{n}$ Bias	0.90	1.12	1.30	0.97	1.22	1.41	1.05	1.27	1.46
	$\sqrt{n}$ SD	0.34	0.33	0.31	0.29	0.29	0.28	0.28	0.28	0.27
	n MSE	0.92	1.37	1.79	1.03	1.56	2.07	1.17	1.70	2.20
3	$\sqrt{n}$ Bias	0.90	1.12	1.30	0.97	1.22	1.41	1.05	1.27	1.46
	$\sqrt{n}$ SD	0.34	0.33	0.31	0.29	0.29	0.28	0.28	0.28	0.27
	n MSE	0.92	1.37	1.79	1.03	1.57	2.07	1.17	1.70	2.20
10	$\sqrt{n}$ Bias	0.90	1.12	1.30	0.97	1.22	1.41	1.05	1.27	1.46
	$\sqrt{n}$ SD	0.34	0.33	0.31	0.29	0.29	0.28	0.28	0.28	0.27
	n MSE	0.92	1.37	1.79	1.03	1.56	2.07	1.17	1.70	2.20

Table S1: Simulation results under local misspecification with $\tau_n \propto n^{-1/3}$ and using the regular scaling (i.e. $\sqrt{n}$).

S2 Simulation results for the second misspecification design

This section describes Monte Carlo simulation results for a second misspecification design. The econometric model is exactly as the one described in Section 5. The true DGP is analogous to the second illustration in Section 2.2, and it is inspired by the presence of unobserved heterogeneity along the lines of Arcidiacono and Miller (2011).

We simulate data composed of two types of agents, A and B. Both types of agents behave exactly according to the model and only differ in the parameter value of their utility functions. Recall from Section 5 that the utility function is specified as follows:

$$u_{\theta_u}(x,a) \;=\; -\theta_{u,1}\times 1[a=2]\;-\theta_{u,2}\times 1[a=1]x,$$

Agents of type A have $(\theta_{u,1},\theta_{u,2})=(1,0.05)$, while agents of type B have $(\theta_{u,1},\theta_{u,2})=(0,95,-0.05)$.

The econometric model is then misspecified in the sense that it presumes a homogenous sample. We use $\tau_n\in(0,1)$ to denote the proportion of agents of type B in the population. We impose local misspecification by setting $\tau_n\equiv n^{-\delta}$ with $\delta\in\{1/3,1/2,1\}$. The rest of the parameters used to implement the Monte Carlo simulation are exactly as in Section 5.

For the sake of comparison with the first simulation design, we present results for the estimator of $\theta_{u,2}$. The simulation results are qualitatively similar to the ones obtained in the previous section and support all of our theoretical conclusions. In particular, the results in Tables S2, S3, S4, and S5 are analogous to Tables 2, 3, 4, and S1, respectively. We refer to Section 5 for a description of these results.

		K-MD($\mathbf{I}_{\lvert\tilde{A}\times X\rvert\times\lvert\tilde{A}\times X\rvert}$)			K-MD(W^*_{AV})			K-ML		
K	Statistic	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$\sqrt{n}$ Bias	0.07	0.02	0.01	0.06	0.02	0.01	0.05	0.02	0.01
	$\sqrt{n}$ SD	0.24	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
2	$\sqrt{n}$ Bias	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\sqrt{n}$ SD	0.24	0.24	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
3	$\sqrt{n}$ Bias	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\sqrt{n}$ SD	0.24	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
10	$\sqrt{n}$ Bias	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	$\sqrt{n}$ SD	0.24	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05

Table S2: Simulation results in the second misspecification design under local misspecification with $\tau_n\propto n^{-1}$.

K	Statistic	K-MD($\mathbf{I}_{\lvert\tilde{A}\times X\rvert\times\lvert\tilde{A}\times X\rvert}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
	$\sqrt{n}$ Bias	-0.04	-0.08	-0.09	-0.05	-0.08	-0.09	-0.05	-0.08	-0.09
1	$\sqrt{n}$ SD	0.23	0.24	0.23	0.22	0.22	0.22	0.21	0.22	0.21
	n MSE	0.05	0.06	0.06	0.05	0.06	0.06	0.05	0.05	0.05
	$\sqrt{n}$ Bias	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
2	$\sqrt{n}$ SD	0.22	0.23	0.23	0.21	0.22	0.21	0.20	0.21	0.21
	n MSE	0.06	0.06	0.06	0.05	0.06	0.06	0.05	0.05	0.06
	$\sqrt{n}$ Bias	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
3	$\sqrt{n}$ SD	0.22	0.23	0.23	0.21	0.22	0.21	0.20	0.21	0.21
	n MSE	0.06	0.06	0.06	0.05	0.06	0.06	0.05	0.05	0.06
	$\sqrt{n}$ Bias	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10	-0.10
10	$\sqrt{n}$ SD	0.23	0.23	0.23	0.21	0.22	0.21	0.20	0.21	0.21
	n MSE	0.06	0.06	0.06	0.05	0.06	0.06	0.05	0.05	0.06

Table S3: Simulation results in the second misspecification design under local misspecification with $\tau_n \propto n^{-1/2}$.

K	Statistic	K-MD($\mathbf{I}_{\lvert\tilde{A}\times X\rvert\times\lvert\tilde{A}\times X\rvert}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
	$n^{1/3}$ Bias	-0.07	-0.09	-0.09	-0.07	-0.09	-0.09	-0.07	-0.09	-0.09
1	$n^{1/3}$ SD	0.08	0.08	0.07	0.08	0.07	0.06	0.08	0.07	0.06
	$n^{2/3}$ MSE	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	$n^{1/3}$ Bias	-0.09	-0.09	-0.10	-0.09	-0.09	-0.10	-0.09	-0.09	-0.10
2	$n^{1/3}$ SD	0.08	0.08	0.07	0.08	0.07	0.06	0.07	0.07	0.06
	$n^{2/3}$ MSE	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	$n^{1/3}$ Bias	-0.09	-0.09	-0.10	-0.09	-0.09	-0.10	-0.09	-0.09	-0.10
3	$n^{1/3}$ SD	0.08	0.08	0.07	0.08	0.07	0.06	0.07	0.07	0.06
	$n^{2/3}$ MSE	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
	$n^{1/3}$ Bias	-0.09	-0.09	-0.10	-0.09	-0.09	-0.10	-0.09	-0.09	-0.10
10	$n^{1/3}$ SD	0.08	0.08	0.07	0.08	0.07	0.06	0.07	0.07	0.06
	$n^{2/3}$ MSE	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table S4: Simulation results in the second misspecification design under local misspecification with $\tau_n \propto n^{-1/3}$ and using the correct scaling.

K	Statistic	K-MD($\mathbf{I}_{\|\tilde{A}\times X\|\times\|\tilde{A}\times X\|}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$\sqrt{n}$ Bias	-0.17	-0.25	-0.30	-0.17	-0.25	-0.30	-0.17	-0.25	-0.30
	$\sqrt{n}$ SD	0.20	0.22	0.22	0.20	0.20	0.20	0.19	0.20	0.20
	n MSE	0.07	0.11	0.13	0.07	0.10	0.13	0.07	0.10	0.13
2	$\sqrt{n}$ Bias	-0.23	-0.26	-0.30	-0.22	-0.26	-0.30	-0.22	-0.26	-0.30
	$\sqrt{n}$ SD	0.20	0.21	0.21	0.19	0.20	0.20	0.18	0.19	0.20
	n MSE	0.09	0.12	0.14	0.09	0.11	0.13	0.08	0.11	0.13
3	$\sqrt{n}$ Bias	-0.23	-0.27	-0.30	-0.22	-0.26	-0.30	-0.22	-0.26	-0.30
	$\sqrt{n}$ SD	0.20	0.21	0.21	0.19	0.20	0.20	0.18	0.19	0.20
	n MSE	0.09	0.12	0.14	0.09	0.11	0.13	0.08	0.11	0.13
10	$\sqrt{n}$ Bias	-0.23	-0.27	-0.30	-0.22	-0.26	-0.30	-0.22	-0.26	-0.30
	$\sqrt{n}$ SD	0.20	0.21	0.21	0.19	0.20	0.20	0.18	0.19	0.20
	n MSE	0.09	0.12	0.14	0.09	0.11	0.13	0.08	0.11	0.13

Table S5: Simulation results in the second misspecification design under local misspecification with $\tau_n \propto n^{-1/3}$ and using the regular scaling (i.e. $\sqrt{n}$).

S3 Simulation results for the third misspecification design

This section describes Monte Carlo simulation results for a third misspecification design. Once again, the econometric model is exactly as the one described in Section 5. The true DGP is as in the third illustration in Section 2.2, and considers agents that depart from rational behavior.

Given state variables (x, ϵ), our model predicts that agents choose the action that maximizes the expected discounted utility. Instead, we simulate agents who make choices according to a multinomial distribution with choice probabilities that are increasing in the action-specific expected discounted utility. Specifically, we use the choice probabilities in Eq. (2.10) for some $\tau_n > 0$.

The econometric model model is then misspecified in the sense that it presumes rationality, i.e., $\tau_n \to 0$. We impose local misspecification by setting $\tau_n \equiv 10n^{-\delta}$ with $\delta \in \{1/3, 1/2, 1\}$. The rest of the parameters used to implement the Monte Carlo simulation are exactly as in Section 5.

For the sake of comparison with previous simulation designs, we present results for the estimator of $\theta_{u,2}$. Once again, the simulation results are qualitatively similar to the ones obtained in the previous section and support all of our theoretical conclusions. The results in Tables S6, S7, S8, and S9 are analogous to Tables 2, 3, 4, and S1, respectively. We refer to Section 5 for a description of these results.

K	Statistic	K-MD($\mathbf{I}_{\|\tilde{A}\times X\|\times\|\tilde{A}\times X\|}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$\sqrt{n}$ Bias	0.07	0.03	0.01	0.06	0.02	0.01	0.06	0.02	0.01
	$\sqrt{n}$ SD	0.25	0.25	0.24	0.24	0.23	0.22	0.22	0.23	0.22
	n MSE	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
2	$\sqrt{n}$ Bias	0.00	0.01	0.00	0.00	0.01	0.00	0.01	0.01	0.00
	$\sqrt{n}$ SD	0.24	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
3	$\sqrt{n}$ Bias	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00
	$\sqrt{n}$ SD	0.25	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
10	$\sqrt{n}$ Bias	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01	0.00
	$\sqrt{n}$ SD	0.25	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.22
	n MSE	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05

Table S6: Simulation results in the third misspecification design under local misspecification with $\tau_n \propto n^{-1}$.

K	Statistic	K-MD($\mathbf{I}_{\|\tilde{A}\times X\|\times\|\tilde{A}\times X\|}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$\sqrt{n}$ Bias	-0.07	-0.12	-0.10	-0.09	-0.12	-0.11	-0.09	-0.12	-0.11
	$\sqrt{n}$ SD	0.25	0.24	0.24	0.23	0.22	0.22	0.22	0.22	0.22
	n MSE	0.07	0.07	0.07	0.06	0.07	0.06	0.06	0.06	0.06
2	$\sqrt{n}$ Bias	-0.15	-0.13	-0.11	-0.16	-0.14	-0.11	-0.16	-0.14	-0.11
	$\sqrt{n}$ SD	0.23	0.24	0.24	0.22	0.22	0.22	0.21	0.21	0.22
	n MSE	0.08	0.07	0.07	0.07	0.07	0.06	0.07	0.07	0.06
3	$\sqrt{n}$ Bias	-0.16	-0.13	-0.11	-0.16	-0.14	-0.11	-0.16	-0.14	-0.11
	$\sqrt{n}$ SD	0.24	0.24	0.24	0.22	0.22	0.22	0.21	0.22	0.22
	n MSE	0.08	0.07	0.07	0.07	0.07	0.06	0.07	0.07	0.06
10	$\sqrt{n}$ Bias	-0.16	-0.13	-0.11	-0.16	-0.14	-0.11	-0.16	-0.14	-0.11
	$\sqrt{n}$ SD	0.24	0.24	0.24	0.22	0.22	0.22	0.21	0.22	0.22
	n MSE	0.08	0.07	0.07	0.08	0.07	0.06	0.07	0.07	0.06

Table S7: Simulation results in the third misspecification design under local misspecification with $\tau_n \propto n^{-1/2}$.

K	Statistic	K-MD($\mathbf{I}_{\lvert\tilde{A}\times X\rvert\times\lvert\tilde{A}\times X\rvert}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$n^{1/3}$ Bias	-0.10	-0.16	-0.17	-0.11	-0.16	-0.17	-0.11	-0.16	-0.17
	$n^{1/3}$ SD	0.10	0.08	0.07	0.10	0.08	0.07	0.09	0.08	0.07
	$n^{2/3}$ MSE	0.02	0.03	0.03	0.02	0.03	0.03	0.02	0.03	0.03
2	$n^{1/3}$ Bias	-0.14	-0.17	-0.17	-0.15	-0.17	-0.17	-0.15	-0.17	-0.17
	$n^{1/3}$ SD	0.09	0.08	0.07	0.08	0.08	0.07	0.08	0.07	0.07
	$n^{2/3}$ MSE	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03
3	$n^{1/3}$ Bias	-0.14	-0.17	-0.17	-0.15	-0.17	-0.17	-0.15	-0.17	-0.17
	$n^{1/3}$ SD	0.09	0.08	0.07	0.08	0.08	0.07	0.08	0.07	0.07
	$n^{2/3}$ MSE	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03
10	$n^{1/3}$ Bias	-0.14	-0.17	-0.17	-0.15	-0.17	-0.17	-0.15	-0.17	-0.17
	$n^{1/3}$ SD	0.09	0.08	0.07	0.08	0.08	0.07	0.08	0.07	0.07
	$n^{2/3}$ MSE	0.03	0.03	0.03	0.03	0.03	0.04	0.03	0.03	0.03

Table S8: Simulation results in the third misspecification design under local misspecification with $\tau_n \propto n^{-1/3}$ and using the correct scaling.

K	Statistic	K-MD($\mathbf{I}_{\lvert\tilde{A}\times X\rvert\times\lvert\tilde{A}\times X\rvert}$)			K-MD(W^*_{AV})			K-ML		
		$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$	$n=200$	$n=500$	$n=1,000$
1	$\sqrt{n}$ Bias	-0.25	-0.45	-0.53	-0.26	-0.46	-0.54	-0.27	-0.46	-0.54
	$\sqrt{n}$ SD	0.25	0.24	0.23	0.23	0.22	0.22	0.22	0.21	0.21
	n MSE	0.12	0.25	0.34	0.12	0.26	0.34	0.12	0.26	0.34
2	$\sqrt{n}$ Bias	-0.34	-0.47	-0.54	-0.35	-0.48	-0.55	-0.36	-0.48	-0.55
	$\sqrt{n}$ SD	0.21	0.23	0.23	0.19	0.21	0.21	0.19	0.21	0.21
	n MSE	0.16	0.27	0.35	0.16	0.27	0.35	0.16	0.27	0.35
3	$\sqrt{n}$ Bias	-0.35	-0.47	-0.54	-0.36	-0.48	-0.55	-0.36	-0.48	-0.55
	$\sqrt{n}$ SD	0.21	0.23	0.23	0.20	0.21	0.21	0.19	0.21	0.21
	n MSE	0.17	0.27	0.35	0.17	0.27	0.35	0.17	0.27	0.35
10	$\sqrt{n}$ Bias	-0.35	-0.47	-0.54	-0.36	-0.48	-0.55	-0.37	-0.48	-0.55
	$\sqrt{n}$ SD	0.21	0.23	0.23	0.20	0.21	0.21	0.19	0.21	0.21
	n MSE	0.17	0.27	0.35	0.17	0.27	0.35	0.17	0.27	0.35

Table S9: Simulation results in the third misspecification design under local misspecification with $\tau_n \propto n^{-1/3}$ and using the regular scaling (i.e. $\sqrt{n}$).

References

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