Uncertainty Shocks and Entrepreneurship

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Abstract

Aggregate startup rates are strongly and negatively correlated with economic policy uncertainty, while exit rates are only very weakly and positively correlated with uncertainty. This observation can be reconciled within a model of endogenous entrepreneurial choice, where entrepreneurial capital is illiquid and thus partially irreversible. Heightened uncertainty leads to a delay in entry due to a real options effect, but simultaneously raises exit propensities for poorly performing entrepreneurs and lowers exit propensities for others. At the aggregate level, this interaction of lowered entry propensities and slightly higher exit propensities depresses the size of the entrepreneurial sector while increasing aggregate labor supply for many years. This in turn leads to recessions that are deeper and more prolonged relative to firm-dynamics models where the extensive margin of adjustment did not exist. The results suggest that business-cycle models that abstract from the extensive margin might substantially under-estimate the impact of uncertainty shocks.

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1 Introduction

What is the effect of uncertainty shocks on entrepreneurship, and how does it affect the overall population of households, as well as the broader aggregate economy? In this paper, I bring in new evidence regarding the relationship between uncertainty shocks and entrepreneurial startup and firm exit rates, and then construct a parsimonious general-equilibrium incomplete markets model of entrepreneurship and uncertainty shocks to explain these facts. Using this model, I then argue that a combination of perceived heightened uninsurable risks faced by entrepreneurial households, and an extensive margin of adjustment for entrepreneurial firms, can magnify and prolong recessions driven by uncertainty shocks.

I first begin by providing some new stylized evidence on the effect of uncertainty shocks on the extensive margin of adjustment for entrepreneurs. In figures 1 and 2, I plot a time series of the entry and exit rates of entrepreneurial firms, i.e., startup formation and firm exit, along with the economic policy uncertainty index constructed by Baker, Bloom, and Davis (2016). Two stylized facts emerge: First, we see that entry is highly negatively correlated with economic policy uncertainty, regardless of whether we are computing entry using the aggregated data from the BDS (correlation of -0.64), or microdata from the PSID (correlation of -0.46); Second, we see that exit is only very weakly correlated with economic policy uncertainty, a statement which again holds true when we compute exit using the BDS (correlation of 0.04) or the PSID (correlation of 0.04).

In fact, looking to the microdata in the PSID, panel probit regressions reveal that a one standard deviation increase in (log) policy uncertainty (evaluated at the mean) leads to a statistically significant decrease of 0.28 percentage points in the entry propensity. Given that entry rate in the PSID is only 3% on average, this is an economically significant number. In contrast, the same increase in economic policy uncertainty has only statistically insignificant effects on exit propensities.

Why does the entry and exit margin vary so differently with respect to an uncertainty shock? A contribution of this paper is to explain these facts using a model of endogenous entrepreneurial choice, where entrepreneurial capital is illiquid as in Tan (2018). As heavily emphasized by the literature, when capital adjustment faces a significant non-convex (fixed) cost element, the real options effect generated by the interaction of the non-convex adjust-

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1 The entry and exit rates are computed using both the BDS and the PSID. In the case of the BDS, “entrepreneurial firms” are defined as firms with employment sizes of less than 250 employees, and the entry and exit rates pertain to these firms. In the case of the PSID, which is a survey of households, an “entrepreneurial firm” is synonymous with an individual who runs a business. For further discussion of their exact definitions, I refer the reader to Appendix A.

2 The entry propensity refers to the probability that an individual enters into entrepreneurship.
ment costs and uncertainty can induce a strong “wait-and-see” effect\(^3\). Transient shocks to uncertainty about future profitability can therefore greatly decrease the value of starting a business, leading to a fall in entry propensities. However, along the exit margin, uncertainty shocks exert two opposing forces. Along one dimension, heightened uncertainty generates a similar wait-and-see effect that decreases exit propensities for some entrepreneurs. This happens because the real options effect increases the value of delaying disinvestment decisions

\(^3\)See, for instance, Bloom (2009), Bachman and Bayer (2014) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)
(of which exit is a form of). However, low performing entrepreneurs also perceive a higher probability of receiving negative profitability shocks if they stay in business. In contrast, exiting their business allows entrepreneurs to draw fresh ideas in the next period that are uncorrelated with their current business prospects. As a result, the continuation value of entrepreneurship falls relative to their outside option, driving up exit propensities. At the aggregate level, these two competing forces manifest themselves as an ambiguous effect on exit, thus leading to a low correlation between exit and uncertainty.

The adjustment along the extensive margin, in response to an uncertainty shock, has important implications for business cycle dynamics. The entrepreneurial sector shrinks after an uncertainty shock, decreasing overall labor demand; in contrast, the labor supply increases as more individuals pursue labor work. As a consequence, this generates a much sharper fall in the wage rate, relative to a model where the extensive margin of adjustment was shut down. Moreover, as it takes time for the entrepreneurial sector to rebuild and return to its steady-state size, output from the entrepreneurial sector is persistently depressed for a long period of time. The combined fall in the size of this sector, along with the long recovery time, generates a much deeper and longer recession than a similar model without an extensive margin of adjustment. Qualitatively, this suggests that models that abstract from the extensive margin might under-estimate the impact of uncertainty shocks.

1.1 Related literature

This paper relates to several strands of literature that study the effect of uncertainty and uncertainty shocks on entrepreneurial risk taking, household savings, and firm investment. First, this paper is most directly related to the firm dynamics literature that has studied the impact of uncertainty shocks on firm investment. Similar to Bloom (2009), Bachman and Bayer (2013), Bachman and Bayer (2014), Gilchrist, Sim, and Zakrajsek (2014), Basu and Bundick (2017), and Bloom et al. (2018), I find that uncertainty shocks can generate a steep recession and sharp declines in investment. Unlike their paper, my results pertain to an economy where firms are fully owned by individual households, and where these business risks are uninsurable. As a result, I find that the precautionary savings motive that arises during an uncertainty shock also depresses consumption during impact, and does not feature the “volatility overshoot” documented in Bloom et al. (2018). Moreover, this outcome also arises due to a standard consumption smoothing mechanism, rather than due to nominal rigidities as in Basu and Bundick (2017).

Second, this paper also relates to a recent set of literature that has emphasized the importance of including an extensive margin of adjustment in otherwise standard heterogeneous
firm dynamics models. As in recent papers such as that by Clementi and Palazzo (2016) and Lee and Mukoyama (2018), interactions between the extensive and intensive margin of adjustment have an important channel in magnifying and propagating business cycle shocks. Similar to the prior literature, I find that a model with an active extensive margin of adjustment generates a recession that is 20% deeper than one without such a margin of adjustment. Unlike this literature, however, the main focus of this paper is on the impact of uncertainty shocks in a model of endogenous entry and exit, while the prior literature explores only the impact of first moment shocks (such as TFP shocks). Moreover, this paper emphasizes a new mechanism in which uncertainty affects entry and exit — specifically, the interaction of an innovation to the spread of outside options with partial irreversibility of investment. Finally, this paper emphasizes the role of precautionary savings and incomplete markets, which departs from the prior literature such as Clementi and Palazzo (2016), which looks at primarily a model of complete markets and risk-neutral firms.

Finally, this paper also relates more broadly to the macroeconomics literature that has emphasized the role of small or entrepreneurial firms in business cycles, such as that by Gertler and Gilchrist (1994), Rampini (2004), and Bassetto, Cagetti, and De Nardi (2015). The main focus of these papers is the reaction of the entrepreneurial sector to first moment shocks or financial shocks, while this paper primarily addresses the impact of shocks to idiosyncratic uncertainty (i.e., a second moment shock).

The rest of this paper is organized as follows. In Section 2, I describe the model environment which I will use to analyze the impact of an uncertainty, and in Section 3, I briefly discuss the model calibration. In Section 4, I present and discuss the model results. Section 5 concludes this paper, and discusses future work.

2 Model

This section describes the baseline model which I will use to analyze the effect of an uncertainty shock. The model is an incomplete markets entrepreneurship model, following the setup in Tan (2018)\(^4\). The uncertainty shock will be modeled as an unexpected (temporary) one time shock that increases the level of uncertainty as it pertains to the entrepreneur’s profits.

\(^4\)I follow this setup given its ability to successfully capture the investment behavior of entrepreneurs.
2.1 Households and Production

The economy is populated by a continuum of households, each indexed by \( i \in [0,1] \), as well as a continuum of representative corporate firms. Households and firms are infinitely lived, and time is discrete. As in Bewley (1977), individuals are subject to idiosyncratic shocks; moreover, individuals are also subjected to aggregate shocks in the form of uncertainty shocks.

Preferences and Discounting

All households are endowed with identical time-separable utility function with constant relative risk aversion (CRRA), and discount future utility at rate \( \beta \). Households value non-durable consumption \( c \), a choice which depends on the household’s endowment of other factors and savings choices. The household’s lifetime expected utility can therefore be written as

\[
V_0 \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_{i,t})
= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{1,t}^{1-\gamma}}{1-\gamma}
\]

where \( \gamma \) is the coefficient of relative risk aversion. The household’s objective is to maximize expected lifetime utility by choosing a sequence of consumption \( \{c_{i,t}\}_{t=0}^{\infty} \).

Production Technology

An individual begins each period as either a worker (W) or an entrepreneur (E), as well as a portfolio of liquid risk-free assets \( b_{i,t} \) and illiquid physical capital \( k_{i,t} \). Occupational type and asset allocations are chosen in the last period. In this section, I will focus only on the production technologies available to the households and firms, delaying a discussion of the asset structure to the next section.

Workers  If the individual is a worker, she supplies inelastic labor efficiency units \( \theta_{i,t} \) to a spot market, and receives a wage income of \( w_t \theta_{i,t} \). \( \theta_{i,t} \) is an idiosyncratic shock to labor productivity, and follows an AR(1) process that depends on the individual’s last period

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\(^5\)As discussed in Tan (2018), the entrepreneurs in fact hold a mix of liquid bonds, liquid stocks, and illiquid capital. However, the lack of aggregate risk in this economy implies that bonds and stocks are perfect substitutes, and for the sake of brevity, I will collectively refer to them as liquid risk-free assets.
occupation. Specifically, if the individual was also a worker in the last period, then $\theta$ evolves as follows:

$$\log \theta_{i,t} = \mu^\theta (1 - \rho^\theta) + \rho^\theta \log \theta_{i,t-1} + \sigma^\theta \epsilon_{i,t}$$  \hspace{1cm} (1)$$

where $\theta_{i,t-1}$ is the worker’s productivity last period; and if the individual was an entrepreneur, then it evolves as

$$\log \theta_{i,t} = \mu^\theta (1 - \rho^\theta) + \rho^\theta \log \psi_{i,t-1} + \sigma^\theta \epsilon_{i,t}$$  \hspace{1cm} (2)$$

where $\psi_{i,t-1}$ is an initial condition drawn by the individual prior to switching into labor work. $\psi_{i,t-1}^\theta$ is assumed to be drawn from the mean-shifted invariant distribution of $\theta$:

$$\log \psi_{i,t-1}^\theta \sim N \left( \mu_{\psi^\theta}^\theta, \frac{(\sigma^\theta)^2}{1 - (\rho^\theta)^2} \right)$$  \hspace{1cm} (3)$$

In other words, $\psi_{i,t-1}^\theta$ is drawn from a normal distribution with the same variance as that of $\theta$, but a (potentially) different mean.

**Entrepreneurs** If the individual is an entrepreneur, she operates a production technology that combines external labor hired from a spot market ($l$), endowed labor ($\bar{l}$), capital ($k$), common productivity $A^z_t$ and idiosyncratic entrepreneurial productivity $z_{i,t}$ to produce homogeneous output $y_{i,t}$ as given by the production function below:

$$y_{i,t} = A^z_t z_{i,t} \left( k_i^\alpha e_{i,t} (\bar{l} + l_{i,t})^{1-\alpha e} \right)^\nu$$

where $\alpha_e \leq 1$ is the capital intensity, and $\nu < 1$ is the span-of-control parameter that determines the degree of decreasing returns to scale\(^6\). $A^z_t$ is a common shock to entrepreneurial productivity. In the baseline model, this is assumed to be constant. $z_{i,t}$ is an idiosyncratic shock to entrepreneurial productivity, and follows an AR(1) process that depends on the last period occupational type of the household. If the individual was also an entrepreneur in the last period, then $z$ evolves as follows:

$$\log z_{i,t} = \mu^z_t (1 - \rho^z) + \rho^z \log z_{i,t-1} + \sigma^z \epsilon_{i,t}$$  \hspace{1cm} (4)$$

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\(^6\)As in Lucas (1978), this captures the idea that managerial skills become stretched over larger and larger projects. The endowed labor $\bar{l}$ is a parameter and assumed to be constant across households, and is a perfect substitute for externally hired labor $l$, which costs $w$ per unit of labor efficiency units hired.
where \( \log z_{i,t-1} \) was the entrepreneur’s productivity in the last period; and if she was a worker, then it evolves as

\[
\log z_{i,t} = \mu_z^* (1 - \rho^z) + \rho_z \log \psi^*_{i,t-1} + x^z_i \sigma^z \epsilon_{i,t} \tag{5}
\]

where \( \psi^*_{i,t-1} \) is an initial condition drawn by the individual prior to becoming an entrepreneur. \( \psi^*_{i,t-1} \) is assumed to be drawn from the following distribution:

\[
\log \psi^*_{i,t-1} \sim N \left( \mu_{\psi^*}, \left( \frac{x^z_{t-1} \sigma^z}{1 - (\rho^z)^2} \right)^2 \right) \tag{6}
\]

**Uncertainty shocks**

Uncertainty shocks appear in this model through two channels: \( x^z_i \), as it appears in equation 4 and 5, and \( x^\psi_{i,t} \), as it appears in equation 6. These two shocks have different roles. \( x^z_i \) increases the conditional volatility of the entrepreneur’s and entrant’s productivity, which makes continuation or entry into entrepreneurship riskier when it increases in the current period. On the other hand, \( x^\psi_{i,t} \) increases the spread of the distribution of initial conditions that the potential entrants can draw prior to entry. This means that workers who decide to wait one period before entering also face greater risk; likewise, current entrepreneurs who decide to exit also face greater risks.

Note that \( \mu^*_z \), the unconditional mean of the \( z \) processes, is also allowed to vary with time. In the baseline model, \( \mu^*_z \) is simply adjusted to ensure that the shocks to \( z \) are mean-preserving spreads in levels; that is,

\[
\mu^*_z = \frac{1}{2} (x^z_i - x^z_0)^2 \sigma^2_z. \]

This avoids the effect where an uncertainty shock, especially a large shock, drastically increases the average productivity of the entrepreneur.\(^7\)

**Representative corporate firms**

In the real economy, a substantial fraction of investment and hiring are done by large corporate firms. Therefore, following Cagetti and De Nardi (2006), I model this sector as a second sector of production populated by a large number of homogeneous firms operating a

\(^7\) This issue arises due to the log-normal assumption. The mean of the \( \psi^z \) process, however, does not need to be adjusted. This is because what matters for the worker is simply the relative starting positions of the initial conditions, rather than the actual levels.
constant returns to scale production technology given by

\[ Y_t^C = A_t \left( K_t^C \right)^\alpha \left( L_t^C \right)^{1-\alpha} \]

where \( A_t \) is aggregate TFP and \( \alpha \) is the capital share of this sector. \( A_t \) is allowed to potentially vary over time, but the process is assumed to be deterministic. As a result, there is no aggregate risk in this economy. As in the entrepreneur’s choices, the corporate firm’s capital stock \( K^C \) is determined from last period’s choices, while labor \( L^C \) is decided in the current period. The corporate firm funds its investment by issuing claims to its dividend flows (i.e. shares). Finally, I also assume that the corporate firms produce the same homogeneous outputs as entrepreneurial firms.

**Asset Structure**

As discussed earlier, households have the option to hold a portfolio of illiquid physical capital and liquid risk-free assets, conditioned on their occupational choice.

**Illiquid physical capital** Only households who elect to become (or stay) entrepreneurs the next period can save in illiquid physical capital \( k \), which depreciates at rate \( \delta_k \). Capital is illiquid because of buying and selling frictions associated with adjusting the capital stock. On the resale side, I assume that the entrepreneur faces two types of downsizing frictions which are associated with the entrepreneur’s future occupational choice. When the entrepreneur decides to downsize, but still continue in entrepreneurship, she faces a per-unit transaction cost \( \lambda \), such that she recoups \((1 - \lambda)\) of the transacted asset. If the entrepreneur decides to fully exit the business, she has to pay an additional proportional selling cost \( \zeta \) on top of the earlier transaction cost. As a result, the net return from selling a unit of capital when exiting is \( = (1 - \zeta)(1 - \lambda) \). This implies that small scale changes along the intensive margin are less costly than large changes along the extensive margin. This asymmetry therefore also influences how entrepreneurs react to an uncertainty shock in terms of exit.

On the investment side, entrepreneurs face a proportional fixed cost when they choose to expand their capital stock. This modeling assumption is similar to that taken by Bloom et al (2018), and in the face of an uncertainty shock, generates an additional wait-and-see attitude with regards to investment.

**Liquid risk-free assets** All individuals can trade liquid risk-free assets \( b \) to smooth inter-temporal consumption. For savers, they cost one unit of consumption today, and return a risk free rate of \( r_{t+1} \) the next period; for borrowers, every unit of debt taken on costs the
individual an interest rate of $r_{t+1} + \phi_d$ tomorrow. $\phi_d$ is an intermediation cost that borrowers have to bear. For individuals who are borrowing, they have two options to borrow. Firstly, all individuals are allowed to borrow via unsecured debt, where they face an ad-hoc borrowing constraint $b$. Secondly, individuals that are entrepreneurs also own capital, and they are allowed to use the liquid value of capital as collateral to borrow a larger volume. This debt can be used to finance more investment into their firm, or simply for smoothing consumption. Here, the liquid value of capital is simply the resale value of capital, i.e. $(1 - \lambda) (1 - \delta_k) k$. Therefore, all households face the following borrowing constraint

$$b_{i,t+1} \geq -\varphi (1 - \lambda) (1 - \delta_k) k_{i,t+1} - b$$

where $\varphi \in [0, \infty)$, with $\varphi = 0$ representing no collateralized borrowing, and $\varphi \to \infty$ representing no collateral required for borrowing. As is common in this literature\(^8\), $\varphi$ capture the idea of limited enforceability of debt contracts, while as discussed in Tan (2018), $(1 - \lambda)$ further tightens the individual's borrowing constraints due to the lower resale value of capital.

\section*{2.2 Individuals’ Problems}

\subsection*{The household’s problem}

At the beginning of the period, an individual is characterized by her occupational type $h_{i,t} \in \{W, E\}$ and her asset holdings $(k_{i,t}, b_{i,t})$. If the individual is an entrepreneur, she also starts the period with entrepreneurial productivity shock $z_{i,t}$, and an outside option shock $\psi_{\theta i,t}$ as discussed earlier. If the individual is a worker, she starts the period with a labor productivity shock $\theta_{i,t}$, and an outside option shock $\psi_{z i,t}$. For brevity, denote the vector of idiosyncratic state variables for the entrepreneurs as $\vec{s}^{e i,t} \equiv (\psi_{\theta i,t}, z_{i,t}, k_{i,t}, b_{i,t})$, the vector of idiosyncratic state variables for the workers as $\vec{s}^{w i,t} \equiv (\psi_{z i,t}, \theta_{i,t}, b_{i,t})$, and the vector of aggregate state variables as $\Omega_t \equiv (x_t^z, x_t^{\psi z}, r_t, w_t)$.

Let the value functions of entrepreneurs and workers be represented by $V^e$ and $V^w$ respectively, and the adjustment cost function for capital (as discussed earlier) by $C(k_{t+1}, k_t, h_{t+1})$. Entrepreneurial households solve the following recursive problem

$$V^e (\vec{s}^{e i,t}; t) = \max \{V^{ee} (\vec{s}^{e i,t}; t), V^{ew} (\vec{s}^{e i,t}; t)\}$$

where $V^{ee}$ and $V^{ew}$ are the value functions of the entrepreneur conditioned on choosing to stay in entrepreneurship, or exit into labor work. Note that the time subscripts are made

\(^8\)See for instance, Buera and Shin (2013).
explicit, as the distribution is not stationary due to the presence of aggregate (deterministic) shocks.

The entrepreneur who decides to stay in entrepreneurship solves

\[
V^{ee}(\tilde{s}_{i,t}^e; t) = \max_{k_{i,t+1}, b_{i,t+1}} U(c_{i,t+1}) + \beta \int_{\psi_{i,t+1}} \int_{z_{i,t+1}} V^{e}(\tilde{s}_{i,t+1}^e; t + 1) dP_{z_{i,t+1}} dF_{\psi_{i,t+1}}
\]

s.t.
\[
\hat{\pi}_{i,t} \equiv y_{i,t} - w_{t} l_{i,t} + (1 + r_{t} \times 1_{\{b_{i,t} \geq 0\}} + r_{d} \times 1_{\{b_{i,t} < 0\}}) b - C(k_{i,t+1}, k_{i,t}, h_{i,t+1})
\]
\[
k_{i,t+1} > 0
\]
\[
c_{i,t} = \hat{\pi}_{i,t} - k_{i,t+1} - b_{i,t+1} \geq 0
\]
\[
b_{i,t+1} \geq -\phi (1 - \lambda) (1 - \delta_{k}) k_{i,t+1} - \bar{b}
\]

, while the entrepreneur who exits entrepreneurship solves

\[
V^{ew}(\tilde{s}_{i,t}^e; t) = \max_{b_{i,t+1}} U(c_{i,t}) + \beta \int_{\psi_{i,t+1}} \int_{\theta_{i,t+1}} V^{w}(\tilde{s}_{i,t}^w; t + 1) dP_{\theta_{i,t+1}} dF_{\psi_{i,t+1}}
\]

s.t.
\[
\hat{\pi}_{i,t} \equiv y_{i,t} - w_{t} l_{i,t} + (1 + r_{t} \times 1_{\{b_{i,t} \geq 0\}} + r_{d} \times 1_{\{b_{i,t} < 0\}}) b - C(k_{i,t+1}, k_{i,t}, h_{i,t+1})
\]
\[
k_{i,t+1} = 0
\]
\[
c_{i,t} = \hat{\pi}_{i,t} - k_{i,t+1} - b_{i,t+1} \geq 0
\]
\[
b_{i,t+1} \geq -\phi (1 - \lambda) (1 - \delta_{k}) k_{i,t+1} - \bar{b}
\]

where \(1(\cdot)\) is an indicator function.

For workers, they solve

\[
V^{w}(\tilde{s}_{i,t}^w; t) = \max \left\{ V^{we}(\tilde{s}_{i,t}^w; t) , V^{ww}(\tilde{s}_{i,t}^w; t) \right\}
\]

where \(V^{we}\) and \(V^{ww}\) are the value functions of the worker conditioned on entering into entrepreneurship, or choosing to stay in labor work.
The worker who decides to enter into entrepreneurship solves

$$V_{we} (s_{i,t}; t) = \max_{k_{i,t+1}, b_{i,t+1}} U (c_{i,t}) + \beta \int_{\psi_{i,t+1}} \int_{\theta_{i,t+1}} V^e (s_{i,t+1}; t + 1) dP_{\theta_{i,t+1}} dF_{\psi_{i,t+1}}$$

subject to

1. $$c_{i,t} = \theta_{i,t} \omega + (1 + r_t \times 1_{\{b_{i,t} \geq 0\}} + r_d \times 1_{\{b_{i,t} < 0\}}) b_{i,t} - k_{i,t+1} - b_{i,t+1}$$
2. $$k_{i,t+1} > 0$$
3. $$b_{i,t+1} \geq -\nu (1 - \lambda) (1 - \delta_k) k_{i,t+1} - b$$

while the worker who decides to stay in labor work solves

$$V_{ww} (s_{i,t}; t) = \max_{b_{i,t+1}} U (c_{i,t}) + \beta \int_{\psi_{i,t+1}} \int_{\theta_{i,t+1} \in \Theta} V^w (s_{i,t}; t + 1) dP_{\theta_{i,t+1}} dF_{\psi_{i,t}}$$

subject to

1. $$c_{i,t} = \theta_{i,t} \omega + (1 + r_t \times 1_{\{b_{i,t} \geq 0\}} + r_d \times 1_{\{b_{i,t} < 0\}}) b_{i,t} - b_{i,t+1}$$
2. $$b_{i,t+1} \geq -b$$

The corporate firm’s problem

The corporate firm objective is to maximize lifetime dividend flows for its shareholders. It decides independently on how much physical capital to invest, and how much labor to hire at the prevailing wage $$\omega$$. To raise capital for investment, the representative firm can issue equity. Let $$\Pi$$ denote the value function of the corporate firm. The representative firm solves the following recursive problem

$$\Pi (K^c_{t}) = \max_{K^c_{t+1}} \pi + \frac{1}{1 + r_{t+1}} \Pi (K^c_{t+1})$$

subject to

$$\pi = Y^c_t - (K^c_{t+1} - (1 - \delta) K^c_t) - w_t L^c_t$$

where $$\pi$$ represents current period dividends paid out to the firms’ investors, and the firm discounts future profits at rate $$\frac{1}{1 + r_{t+1}}$$. This market arrangement leads to the standard first
order condition for capital and labor demand:

\[
    r_t + \delta = \alpha A_t \left( \frac{K^c_t}{L^c_t} \right)^{a-1} \tag{14}
\]

\[
w_t = (1 - \alpha) A_t \left( \frac{K^c_t}{L^c_t} \right)^{a} \tag{15}
\]

2.3 Equilibrium

Uncertainty shocks: Pure uncertainty and volatility effects

As discussed in Bloom (2009), uncertainty shocks exert their forces on the aggregate economy through two distinct channels: First through a pure uncertainty channel which work through the individual’s expectations (i.e. individuals face higher uncertainty); and second through a realized volatility effect where the higher dispersion in shocks pushes individuals towards both tails of the distribution. As there is sparse evidence regarding how much of uncertainty faced by entrepreneurs amount to subjective uncertainty (i.e. expectations that may not align fully with the actual dispersion of the stochastic process) or real uncertainty (i.e. real volatility effects), moving forward, I will discuss in detail both scenarios. To fix ideas and notation, I will define both equilibriums in this section first, and then discuss in greater detail in the next section the setup of these shocks.

Subjective uncertainty equilibrium

Using the notation from earlier, that is letting \( s^e_t \) and \( s^w_t \) denote the state vector of idiosyncratic states faced by the entrepreneur and worker respectively, and \( \Omega_t = \{r_t, w_t, x^z_t, x^{\psi z}_t\} \) the state vector of aggregate variables; and letting \( \tilde{x}^z_t \) and \( \tilde{x}^{\psi z}_t \) denote the individual’s perception of the shock to the volatility of \( z \) such that the aggregate state vector perceived by the household is \( \tilde{\Omega}_t = \{r_t, w_t, \tilde{x}^z_t, \tilde{x}^{\psi z}_t\} \), the equilibrium is defined as follows

Definition 1. A sequential equilibrium with subjective uncertainty is defined by

1. A sequence of realized shocks to volatility \( x^z_t \) and \( x^{\psi z}_t \)

2. A sequence of shocks to the individual’s perception of the path of the actual shocks: \( \tilde{x}^z_t \) and \( \tilde{x}^{\psi z}_t \), where in general, \( \tilde{x}^z_t \neq x^z_t \) and \( \tilde{x}^{\psi z}_t \neq x^{\psi z}_t \)

3. A sequence of interest rates and wage rate \( \{r_t, w_t\}_{t \geq 0} \)

4. A sequence of value functions: \( \{V^e_{i,t} \left( s^e_t, \tilde{\Omega}_t \right), V^w_{i,t} \left( s^w_t, \tilde{\Omega}_t \right), \Pi_t (r_t, w_t)\}_{t \geq 0, i \in [0,1]} \)
5. A sequence of policy functions: \(\left\{ k_{i,t} \left( s^e_t, s^w_t, \tilde{\Omega}_t \right), b_{i,t} \left( s^e_t, s^w_t, \tilde{\Omega}_t \right), h_{i,t} \left( s^e_t, s^w_t, \tilde{\Omega}_t \right) \right\} \) \(t \geq 0, i \in [0, 1]\)

6. A sequence of labor demand \(\left\{ l_{i,t} \left( s^e_t, \tilde{\Omega}_t \right) \right\} \) \(t \geq 0, i \in [0, 1]\) from entrepreneurs, and labor supply \(\left\{ \theta_{i,t} \right\} \) \(t \geq 0, i \in [0, 1]\) from workers

7. A sequence of factor demand \(\left\{ K^c_t (r_t, w_t), L^c_t (r_t, w_t) \right\} \) \(t \geq 0\) from the corporate sector

8. A sequence of distributions of individuals \(\left\{ \Lambda_t \left( s^e_t, s^w_t, \tilde{\Omega}_t, \Omega_t \right) \right\} \) \(t \geq 0\)

such that

1. Taking \(\left\{ r_t, w_t \right\} \) \(t \geq 0\) as given, the households’ decision rules and value functions, as in equations 7, 8, 9, 10, 11, and 12, solve the individual problems.

2. Taking \(\left\{ r_t, w_t \right\} \) \(t \geq 0\) as given, the representative corporate firm’s decision rules and value function, as given in equation 13, 14, and 15, solve the firm’s problem.

3. Factor markets clear, where for all \(t\)

   (a) Bonds: \(\int b_{i,t} d\Lambda_t = K^c_t\)

   (b) Labor: \(\int \theta_{i,t} 1_{\{h_{i,t}=E\}} d\Lambda_t = \int l_{1,\{h_{i,t}=W\}} d\Lambda_t + L^c_t\)

4. The aggregate resource constraint is satisfied, where

\[
\int c_{i,t} + k_{i,t+1} 1_{\{h_{i,t+1}=E\}} + b_{i,t+1} + C(k_{i,t+1}, k_{i,t}, h_{i,t+1}) 1_{\{h_{i,t+1}=E\}} d\Lambda_t \\
= \int \pi^*_{i,t} 1_{\{h_{i,t}=E\}} + \theta_{i,t} w_t 1_{\{h_{i,t}=W\}} + (1 + r_t) b_{i,t+1} + (1 - \delta_k) k_{i,t} 1_{\{h_{i,t}=E\}} d\Lambda_t
\]

5. The decision rules of the households, along with the exogenous Markov and iid processes, generate the sequence of Markov transition kernels \(\Gamma_t\) which, given any initial distribution of households \(\Lambda_0\), generates the sequence of distributions \(\{\Lambda_t\}_{t \geq 0}\); that is,

\[\Lambda_{t+1} = \Gamma_t (\Lambda_t)\]

where in particular, in steady state, the distribution \(\Lambda\) is time invariant, and prices are constant.

At this point, it is important to point out that the individual’s decision rule depend on \(\tilde{\Omega}_t\), while the exogenous processes themselves depend on \(\Omega_t\); as a result, the transition kernel
\( \Gamma_t \) arises as a combination of both the individual’s perception of \( z \), as well as the actual evolution of \( z \).

**Real uncertainty equilibrium**

The equilibrium under real uncertainty is defined similarly, with the only modification being that \( \tilde{x}_t^* = x_t^* \); that is, the beliefs of the entrepreneurs fully align with the actual realized volatility process. As a result, the only modification to the equilibrium definition is that \( \tilde{\Omega}_t = \Omega_t \). For the sake of brevity, I will defer the formal definition of equilibrium to the appendix.

### 3 Calibration

#### 3.1 Steady-state model

Given that the goal of this paper is to study the impact of uncertainty shocks on entrepreneurial choices, it is crucial that the model is able to simultaneously replicate the micro-level investment behavior of entrepreneurs and entrants, as well as the broader distribution of wealth across entrepreneurs and workers.

In particular, as discussed in Tan (2018), entrepreneurs face very stark disinvestment frictions that make downsizing difficult; that is, entrepreneurial investment is very illiquid. The same effect also means that in steady-state, the distribution of productivity is decreasing in wealth. This implies that the effect of a subjective uncertainty shock can potentially have extremely large effects on the aggregate economy. For instance, as discussed in Bloom et al. (2018), the real options value effect generated by these non-convexities in capital adjustment cost can lead to sudden drops in investment during periods of heightened uncertainty. In the context of entrepreneurs who face incomplete markets, this effect could be even more severe since an increase in uncertainty corresponds to an increase in the extent of uninsurable risks. This in turn drives up the illiquidity risks that potential entrants and entrepreneurs face. Moreover, the joint distribution of wealth and productivity means that high productivity entrepreneurs are also hit hardest by the uncertainty shock, since these are also individuals with low buffer stock of savings.

The calibration therefore follows from Tan (2018), and are reported in tables 1a and 1a respectively. I refer the reader to Tan (2018) for the exact strategy which was used to calibrate these parameters.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_\theta$</td>
<td>0</td>
<td>Unconditional mean of labor productivity</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>0.90</td>
<td>Persistence of labor productivity</td>
<td>Floden and Linde (2001), Storesletten et al (2004)</td>
</tr>
<tr>
<td>$\sigma_\theta$</td>
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<td>Conditional variance of labor productivity</td>
<td>Floden and Linde (2001), Storesletten et al (2004)</td>
</tr>
<tr>
<td>$\mu_z$</td>
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<td>Mean of business productivity process</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Capital share of corporate sector</td>
<td>Fixed to value in Cagetti and De Nardi (2006)</td>
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<td>$\lambda$</td>
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<td>Resale transaction cost</td>
<td>Probability that firm stays in quintile 1 of ARPK distribution</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.314</td>
<td>Exit friction</td>
<td>Skewness of ARPK</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.930</td>
<td>Collateral constraint</td>
<td>Skewness / Probability that firm stays in quintile 5 of ARPK distribution</td>
</tr>
<tr>
<td>$f_s$</td>
<td>0.032</td>
<td>Investment fixed cost</td>
<td>Rate of positive investment reported</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>0.810</td>
<td>Autocorrelation of productivity shock</td>
<td>Autocorrelation of investment rates</td>
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<tr>
<td>$\sigma_c$</td>
<td>0.350</td>
<td>Volatility of productivity shock</td>
<td>Coefficient of variation of investment, firm size distribution</td>
</tr>
<tr>
<td>$l$</td>
<td>0.211</td>
<td>Entrepreneur’s endowed labor</td>
<td>% of firms that are employers</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>0.510</td>
<td>Mean of entrepreneurial prospects signal shock</td>
<td>Fraction of households that are entrepreneurs in steady-state</td>
</tr>
<tr>
<td>$\rho_{\theta}$</td>
<td>0.707</td>
<td>Mean of labor prospects signal shock</td>
<td>Exit rate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.9185</td>
<td>Discount factor</td>
<td>Interest rate of 3.5%</td>
</tr>
</tbody>
</table>

(a) Fixed and estimated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>$\beta$</td>
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<td>Discount factor</td>
<td>Interest rate of 3.5%</td>
</tr>
</tbody>
</table>

(b) Calibrated parameters

Table 1: Parameter values
3.2 Uncertainty shocks

Evidence on the extent of an uncertainty shock faced by entrepreneurs is relatively sparse. In light of this fact, I consider three calibration strategies.

First, I consider an innovation to the individual’s perceived uncertainty that matches the percentage growth in economic policy uncertainty between 2007 and 2008; this amounts to an innovation of 46%. In other words, I assume that individuals face a one time shock to $\tilde{x}^z$ and $\tilde{x}^\psi$, where $\tilde{x}^z$ and $\tilde{x}^\psi$ rise from their (normalized) steady-state value of 1 to 1.46 within a single period, before reverting back to unity for the rest of the experiment. This process is reflected in figure 3 below. However, as the uncertainty shock is only subjective, $x^z$ and $x^\psi$ remain at their steady-state. Given that the evidence provided in the introduction pertains to the effect of individual perception of uncertainty (i.e., the EPU index captures an individual’s uncertainty about the future rather than realized volatility), I believe that this is an appropriate assumption.

![Path of uncertainty shock](image)

**Figure 3:** Path of uncertainty shock

In an alternate calibration, I also consider a subjective uncertainty shock, but utilize the calibration in Bloom et al. (2018). In this case, the authors assume that individuals face a 410% elevation of their idiosyncratic uncertainty process; that is $\tilde{x}^z_t$ spikes from a steady-state level of 1 to 4.1 for a year. I find that the results are qualitatively similar, but leads to unrealistic predictions with respect to its impact on the real economy. For instance, gross positive investment falls by 100% upon impact. The results from this experiment are therefore relegated to the appendix.

In a third calibration, I consider the case of a shock where uncertainty is “real”, as defined
in the earlier section. In this calibration, the large volatility effect produces a large increase in the spread of realized productivities, which in turn generates a large increase in the population of very high productivity entrepreneurs. As aggregate output (and hence labor demand) is convex in the spread of productivities (i.e., the Abel-Hartman-Ooi effect), the large increase in realized volatility means that the entrepreneurial sector dominates all labor demand in the economy, therefore shutting down the corporate sector. Given this unrealistic outcome, the results are therefore also relegated to the appendix.

In future work, a goal of this paper is to document a better calibration with regards to the uncertainty shock.

4 Results

Figure 4 reports the impact of a subjective uncertainty shock on the aggregate economy for various outcomes. First directing the reader to figures 4a and 4b, we see that a positive innovation to subjective uncertainty leads to a large fall in the entry rate (of around 17.6%), while a relatively low increase in the exit rate (around 2.3%). This is consistent with the evidence provided earlier, where we saw that entry rates were strongly and negatively correlated with economic policy uncertainty, whereas exit rates were only weakly (and positively correlated) so.

The model is able to replicate this feature of the data due to a combination of the investment real options effect driven by the partial irreversibility of entrepreneurial investment, and an outside option effect relating to how entrants draw their fresh ideas (the initial condition shock $\psi^z$). In the case of entry, partial irreversibility of investment implies that uncertainty shocks lower the continuation value of entrepreneurship from the viewpoint of potential entrants. This effect means that some potential entrants might choose to delay entry when uncertainty is higher — a real options effect that has been well documented in the literature. However, this is not the only channel in the model. Compounding the real options effect is that fact that uncertainty shocks imply that potential entrants who wait might potentially receive an even better outside option draw in the next period. This effect increases the value of being a worker in the next period\(^9\). The two effects therefore complement each other, leading to a large decline in the entry rate during a transient uncertainty shock.

Uncertainty shocks have a much weaker effect on the exit margin, because the real options effect now compete with the outside option effect. When uncertainty is higher, entrepreneurs

\(^9\)This effect is actually very similar to the “locally risk loving effect” that Vereshchagina and Hopenhayn (2009) discusses in their paper. In the context of this paper, the marginal entrant delays entry because the increase in uncertainty over future signals increases the value of waiting.
delay disinvestment decisions, as the value of holding on to capital is higher than selling it. This implies that some marginal entrepreneurs who might have exited in steady-state now choose to delay exit. In contrast, the outside option effect means that exiting now becomes a more attractive prospect for very low productivity entrepreneurs. While these entrepreneurs might have chosen to stay in entrepreneurship in steady-state, the uncertainty shock implies that if they shut down their business, they might be able to obtain a much better entrepreneurial idea (as a worker) in the next period\textsuperscript{10}. As a result, the uncertainty shock has a purging effect that pushes very low productivity entrepreneurs towards exit. Since an uncertainty shock simultaneously decreases exit propensity for some entrepreneurs while increasing it for others, the net impact is much weaker. In the baseline calibration, we see that the latter effect is slightly stronger, therefore generating a weak increase in exit rates.

Figure 4c reports the impact of a subjective uncertainty shock on GDP. As in Bloom et al. (2018), an uncertainty shock has a large impact on GDP; at the trough of the recession, GDP falls by about 0.6\%\textsuperscript{11}. As we can see from figure 4d, the large fall in aggregate GDP is driven by a steep decline of output in the entrepreneurial sector; at the trough of the recession, entrepreneurial output falls by almost 8.6\%. As a result, although entrepreneurial output in this economy accounts for only about 21\% of GDP, the large fall in output of this sector generates a large recession.

Moreover, notice in figure 4e that consumption falls upon impact by about 0.5\%, and stays depressed for a long period before converging to the steady-state. This is very different from the result in Bloom et al. (2018), who document that an uncertainty shock generates a small “consumption overshoot” at the initial stages of the shock. The result here documents why it is important to model uninsurable risks, as well as a two asset model. Unlike in models of complete markets and single assets, the sharp fall in investment (which also happens in this economy, as we can see in figure 4f) does not have to translate directly into an increase in consumption. Instead, the uncertainty shock increases the precautionary savings motives of the household. This leads households to cut back on investment into the risky asset — as we see in figure 4f, where net investment plunges by about 58\% — while simultaneously shifting their income into the risk-free asset — as we see in figure 4i, where the liquid asset holdings per entrepreneur spike by 6.7\%\textsuperscript{12}. As a result, consumption falls upon impact, and

\textsuperscript{10}This effect, while also present even if entrepreneurs stay in business, is much weaker, since productivity conditioned on staying is persistent, whereas an exiting entrepreneur can draw a completely new idea in the next period from the unconditional distribution.

\textsuperscript{11}Remarkably, if the calibration of Bloom et al. (2018) was used, then GDP falls by about 2.5\%, which almost matches the finding in their paper (see figure 6 of their paper).

\textsuperscript{12}In fact, this effect is similar to that documented in Bayer, Luetticke, Lien, and Tjaden (Forthcoming).
Figure 4: Effect of an uncertainty shock on the aggregate economy.

stays depressed for a long period of time.

Why does the entrepreneurial sector respond so strongly to an uncertainty shock? A part of the reason stems from the standard investment real options effect, which leads to sharp declines in investment and overall capital reallocation. For instance, if we look to figure 4g and 4h, we see that the net investment per surviving firm\textsuperscript{13} plunges by about 43% upon

\textsuperscript{13}Going forward, all variables will be aggregate variables, unless I explicitly note that they are normalized variables. Due to the changes in the measure of active entrepreneurs, some aggregate variables such as investment change due to both the extensive and intensive margins. Therefore, it becomes very important
impact, while gross disinvestment also falls. While investment quickly increases after the shock, the desire to smooth consumption means that entrepreneurs take time to accumulate capital. In contrast to a risk-neutral firm — for instance, as in Bloom et al. (2018) — risk-averse households do not immediately pivot their portfolio of assets towards the risky asset even after the shock has dissipated. As a result, as we see in figures 4g and 4i, while entrepreneurs do draw down on their liquid savings to finance their investments after the shock, they do so slowly.

However, the investment real options effect, while powerful in magnifying and propagating the effects of an uncertainty shock, is not the only channel through which uncertainty shocks play out. Instead, the interaction of the uncertainty shock with the extensive margin also has a large impact in amplifying the recessionary effects of uncertainty shocks. This channel is the focus of the next subsection.

4.1 The Role of the Extensive Margin

In figure 5a, I plot the change in GDP for the baseline economy (solid blue line) against a counterfactual economy where individuals do not have an occupational choice (dashed red line) — that is to say, an economy where the measure of workers and entrepreneurs stay constant. In the figure, we see that when there is no extensive margin of adjustment, the uncertainty shock generates a shallower recession that also recovers faster after the shock has passed.

![Figure 5: Effect of an uncertainty shock: The extensive margin](image)

To understand the role of the extensive margin, I plot in figure 5b the change in aggregate GDP if the output in the corporate sector was held constant — in other words, this is the to be clear which of the changes are due to intensive margins of changes (as in this example), and extensive margins of adjustment.
contribution of the entrepreneurial sector to the fall in aggregate GDP. We can see clearly that the entrepreneurial sector has a much larger impact under the baseline model. If we look to figure 5c, we see that the impact of an uncertainty shock on the entrepreneurial sector itself is almost the same for both models. However, as the entrepreneurial sector constitutes a larger share of output under the baseline calibration (21% vs 15%), this translates into a sharper and more prolonged recession.

Figure 6: Effect of the uncertainty shock on labor and wages.

An additional dimension through which the extensive margin of adjustment exacerbates the recessionary effect of an uncertainty shock is through its effect on the labor markets. Along the intensive margin, the uncertainty shock leads to a “misallocation” of capital due to the wait-and-see effect on investment. As a result, labor demand from the entrepreneurial sector declines, as labor is complementary with capital. This in turn leads to a decline in wages in equilibrium, as we see in figure 6c. This effect is present in both the model with and without endogenous occupational choice, as we can see in 6b. However, because more individuals now pursue labor work in the baseline model, labor supply shifts out (figure 6a), which amplifies the decline in wages. As a result, the initial decline in wages in the baseline model is more than double that of the counterfactual model14.

While uncertainty shocks result in sharper recessions when the extensive margin of adjustment is active, it is important to note that this impact is an economically efficient outcome.

---

14 As the corporate sector serves to absorb the residual labor supply, and households do not have an intensive margin of adjustment for labor, there is in fact no structural unemployment in this model. Moreover, because savings also increase, this then implies that the corporate sector undergoes a small expansion upon impact of the shock, and interest rate rises. In ongoing work, I consider two extensions to this basic model. First and foremost, I incorporate uncertainty shocks that also affect the corporate sector, where I extend the model to a heterogeneous corporate sector composed of entrepreneurs who are able to “take their firm public”. In this extension, I therefore break the direct link between wages and interest rates imposed by a frictionless representative corporate firm (i.e. the aggregate capital-labor ratio is no longer a sufficient statistic for market clearing.). Second, I extend the model to include frictional employment.
To see this, I plot in figures 7a and 7b the response of output per entrepreneur and entrepreneur sector TFP respectively. We see that for the counterfactual model, the average output per entrepreneur plunges by about 8.4%, while TFP falls by about 1.5%, upon impact of the uncertainty shock. In contrast, the impact for the baseline model is much more tempered.

![Diagram](image)

Figure 7: Uncertainty shocks lead to sharp falls in average output per entrepreneur, but has a cleansing effect when the extensive margin is taken into account.

The reason why this happens is because the extensive margin of adjustment serves as an outside option for entrepreneurs to fall back on. This means that on impact, the uncertainty shock has a cleansing effect that drives out low productivity entrepreneurs to utilize their outside option. While this leads to a sharp decline in entrepreneurial output due to the exit of entrepreneurs, this also means that surviving entrepreneurs are in fact more productive than if they did not have the option to exit. For instance, if we look to figure 7a, we see that the fall in GDP per entrepreneur is only about half of that of the counterfactual economy; likewise, if we look to the change in aggregate TFP in the entrepreneurial sector, the measure-adjusted TFP of the baseline economy falls by only about 0.5%, in contrast to the counterfactual which falls by around 1.5%.

### 4.2 Aggregate welfare

Uncertainty shocks have a counterintuitive effect on welfare. In table 2, I report the change in welfare for entrepreneurs and workers due to the uncertainty shock. The welfare changes are computed with reference to a counterfactual scenario where the economy remains in steady-state, reported in percent consumption equivalent variation terms.

In column 1, I report the welfare change for the baseline economy where movements along the extensive margins are active. Here, we see a surprising outcome — uncertainty
shocks generate a recession, but increases welfare for entrepreneurs and workers. In other
words, households actually prefer being hit by an uncertainty shock. In column 2, I report
the welfare change for the economy where the extensive margin of adjustment is inactive.
Here, we see that entrepreneurs still benefit (in welfare terms) from the uncertainty shock,
but workers now would pay up to 0.05% of lifetime consumption to avoid the shock.

Column 3 reveals the reason why we see such a counterintuitive result, and also explains
why the two economies have such different responses. In column 3, I recompute the welfare
changes assuming that the real economy evolves as in under the baseline calibration, but
that the uncertainty shock never hits the households’ expectations formations process. In
other words, this reports the effect on welfare due to real changes in the economy. Here, we
see that both workers and entrepreneurs are hurt (in a welfare sense) by the recessionary
effects of the uncertainty shock. This tells us that the actual impact of the shock is large
and negative, but due to a corresponding change in the perception held by entrepreneurs
and workers, individuals in the economy actually perceive higher welfare.

<table>
<thead>
<tr>
<th></th>
<th>w/ endo occ</th>
<th>w/o endo occ</th>
<th>w/ endo occ, Prob adjusted</th>
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</thead>
<tbody>
<tr>
<td>Entrepreneurs</td>
<td>0.35</td>
<td>0.25</td>
<td>-0.05</td>
</tr>
<tr>
<td>Workers</td>
<td>0.39</td>
<td>-0.05</td>
<td>-0.1</td>
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</table>

Table 2: Effects of an uncertainty shock on welfare. Columns 1 and 2 report the average change
in welfare for each sub-group of households, in percent consumption equivalent variation. Col-
umn 3 reports the change in welfare under the baseline calibration, with the household’s perceived
uncertainty adjusted to their pre-shocked levels, but keeping the responses of the economy constant.

This outcome relates to the non-convexity arising from the extensive margin of adjust-
ment. For individuals who are on the margin of adjustment, the value function is locally
convex, which generates a locally risk-loving region\textsuperscript{15}. For workers who are on the verge of
entry, the larger uncertainty regarding future signals implies that workers who delay entry
have a higher probability of drawing an even better signal tomorrow, whereas the worst case
scenario is that the worker simply remains a worker. In other words, since the downsize
risk is bounded below whereas the upside risk is fully unbounded, an increase in the spread
of the shock strictly improves the worker’s perceived welfare. A similar effect plays out for
low productivity entrepreneurs who are on the exit margin. For them, their downsize risk
is simply taking home their labor income, while the upside risk is that they get to start
afresh with a potentially good new business idea. This outcome in fact explains why workers
are worse off under the counterfactual model without endogenous occupational choice, and
entrepreneurs also see a lower welfare gain. Since workers do not have the opportunity to

\textsuperscript{15}This is similar to the mechanism discussed in Vereshchagina and Hopenhayn (2009)
enter into entrepreneurship, they simply bear the full brunt of the recession without the benefit of the change in the upside risk. Entrepreneurs also benefit less, since they are not able to exit and draw a fresh business idea.

Finally, we see that entrepreneurs perceive higher welfare with or without an extensive margin of adjustment. This arises because entrepreneurial production is convex with respect to the realization of entrepreneurial productivity, and therefore expected output in the next period is increasing in the volatility of productivity. Here, the downsize risks for surviving entrepreneurs is bounded below by 0 (the lowest possible state of \( z \), given the assumption of a log-normal process), while the upside risk is unbounded. As a result, entrepreneurs perceive that expected future business income is in fact increasing in the volatility of entrepreneurial productivity, and hence have higher welfare.

5 Conclusion and Future Work

This paper provides an analysis of the role of uncertainty shocks for entrepreneurs, and show how the recessionary effects of uncertainty shocks are transmitted from the entrepreneurial sector to the broader aggregate economy. In particular, it argues that the empirical relationship of entry rates, exit rates, and economic and policy uncertainty can be rationalized through a model where entrepreneurial investment is partially irreversible, and entrepreneurs and workers face time-varying outside options. Moreover, it documents the importance of the extensive margin in amplifying and propagating the recessionary effects of a positive innovation to uncertainty. In future drafts, I will consider three key extensions.

First, the evidence I present is relatively stylized. As such, in ongoing work, the goal is to better identify what constitutes empirically an uncertainty shock from the perspective of entrepreneurs.

Second, the subjective uncertainty shock in this paper is modeled as an adhoc shock to the perceived risks faced by individuals. In ongoing work, I explore variations of this framework by including a process of learning a la David, Hopenhayn, and Venkateswaran (2016) and David and Venkateswaran (2018), and modeling the uncertainty shock as a shock to the posterior variance of beliefs as in Senga (2018). The goal of this extension is to endogenize the adhoc assumption made in this draft. Moreover, in this draft, the uncertainty shock arises as a symmetric increase of the spread of productivity risks. As documented recently by Bloom, Guvenen, and Salgado (2016), recessions are typically characterized by a negative skewness shock — in other words, entrepreneurs should face a higher probability of negative idiosyncratic draws than positive draws. Future extensions of this paper will incorporate this insight.
Finally, the assumption of a representative corporate firm with frictionless capital and labor adjustment leads to an overly tight link between interest rates and wages, as the (corporate) capital-labor ratio is the only market clearing object. In ongoing work, I extend my model to allow entrepreneurs to endogenously incorporate their firms, and as a result, the corporate sector is composed of heterogeneous corporate firms. In this framework, the direct link between wages and interest rates are broken, as corporate capital and labor are now two distinct market clearing objects.
References


