Structural Estimation of FCC Bidder Valuation

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April 14th 2017

Honors thesis submitted in partial fulfillment of the requirements for Graduation with Distinction in Economics in Trinity College of Duke University

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Acknowledgements

We would like to thank our principal thesis advisor Professor Michelle Connolly of Duke University for her indispensable support, guidance, and intellectual input throughout the development of this paper. We would also like to thank fellow researchers in the Spectrum Lab and Honor Thesis Workshop for sharing with us findings of their own research, and in so doing, inspiring us to work harder. Lastly, we would also like to thank our seminar instructor Professor Tracy Falba and great friend Clement Lee for giving us valuable feedback throughout the research and writing process.

Abstract

We modify a method introduced in Fox and Bajari (2013) which structurally estimates the deterministic component of bidder valuations in FCC spectrum auctions based on a pairwise stability condition: two bidders cannot exchange two licenses in a way that increases the sum of their valuations, and we apply it to C block auctions 5, 22, 35 and 58. Our modifications improve the fit of the Fox and Bajari (2013)'s estimator especially in similar auctions involving big bidders. We find that there is evidence of significant "cross-auction" complementaries between licenses sold in a particular auction and those already owned by these endowed bidders.

JEL Codes : D44, D45, H82, L82

Keywords: auction theory, structural efficiency estimation, pairwise stability, telecommunications, regulation, spectrum auctions, welfare, FCC, broadband, small bidders, geographic complementarities

1. Introduction

Prior to the passing of the Omnibus Budget Reconciliation Act by Congress in 1993, the Federal Communications Commission (FCC) relied on comparative hearings and lotteries to allocate civilian rights to use non-overlapping segments of the electromagnetic spectrum in the United States. The Omnibus Budget Reconciliation Act gave the FCC the additional "authority to use competitive bidding to choose from among two or more mutually exclusive applications for an initial license."[2] Since 1994, the FCC has consistently exercised this authority by making auctions the primary means of distributing spectrum licenses. While these auctions are generally open to any eligible company or individual, the FCC imposes additional rules and access restrictions to fulfill the Congressional mandate to supporting smaller or underrepresented bidder[3]. The Balanced Budget Act of 1997[4] officiated the use of auctions as the default method of spectrum license allotment; it required that the FCC to use auctions to resolve mutually exclusive applications for initial licenses *unless* specific exemptions have been legally granted (FCC 2017).

These FCC auctions are ascending auctions which Klemperer (1999) defines as an allocative process where "the price is successively raised until only one bidder remains, and that bidder wins the object at the final price."[5] All auctions generally sell to the bidder with the highest signal. Insofar as an efficient auction can be described as one which allocates a resource to the individual

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[2] Omnibus Budget Reconciliation Act (1993), Sec. 6002

[3] These bidders are defined in the FCC's rules and regulations located in Title 47 of the Code of Federal Regulations (CFR) as "designed entities." Designed entities are "small businesses (including businesses owned by members of minority groups and/or women), rural telephone companies, and eligible rural service providers." The specific requirements to qualify as a "designed entity" are outlined clearly in CFR 1.2110.

[4] Balanced Budget Act (1997), Title III

[5] Klemperer (2007), pp 14.

able to extract the highest marginal revenue from that particular good[6] *and* an auction which is ascending, as compared to those which are Dutch, first-price sealed-bid or second-price seal-bid types, more greatly incentivizes players to bid their true values as signals (Klemperer 1999[7]; Smith 1987; Milgrom 1987), FCC auctions are designed to allocate spectrum licenses in a relatively efficient manner.

The FCC also employs a simultaneous multiple-round (SMR) auction structure designed to adequately capture the value of complementaries among licenses sold in an auction. Brunner et al (2009) explained that, as a result of these complementaries, "the value of a collection of spectrum licenses for adjacent areas can be higher than the sum of the values for separate licenses" and "bidders with value complementarities may have to bid more for some licenses than they are worth individually, which may result in losses when only a subset is won." [8] To address this, the simultaneous feature of FCC auctions allows for all licenses to remain available for bidding throughout the entire auction process. Bidders can vary their bidding strategies across their entire desired set of licenses according to public bids placed by other bidders.

Auction efficiency is important because it affects socioeconomic outcomes. In its 2016 Broadband Progress Report, the Federal Communications Commission (FCC) highlights that a digital divide remains prevalent.[9] There are two ways auction efficiency can help to reduce this digital divide.

[6] Myerson (1981) and Bulow and Roberts (1989).
[7] Klemperer (1999) argues that "it is a clearly dominant strategy to stay in the bidding until the price reaches your value […] the next-to-last person will drop out when her value is reaches, so the person with the highest value will win at the price equal to the value of the second-highest bidder". Based on such an auction design, "truth telling is a dominant strategy equilibrium".
[8] Brunner et al (2009) argues that geographical complementaries arise because "if a telecommunications company is already operating in a certain area, the cost of operating in adjacent areas tends to be lower […] and consumers may value larger networks that reduce the cost and inconvenience of roaming."
[9] 2016 Broadband Progress Report, FCC, 2016, p. 3.

Firstly, auction efficiency allows for the effective differentiation of spectrum access fees based on varied demands. For example, pricing can be adjusted to allow for access to advanced telecommunications capability from rural areas without necessarily underpricing spectrum for those living in urban areas. Secondly, by removing the need for license re-sales, an efficient auction eliminates transaction costs associated with secondary markets (Coase 1959) and hence improves overall access to telecommunication. Connolly, Lee and Tan (2016) argues that equal access to telecommunications is important to the extent that it allows for more equal access to economic opportunities and "unequal access to such technological tools can maintain or even worsen existing inequalities."

There are economic theories which posit that simultaneous English auctions can, under specific circumstances, fail to be perfectly efficient. Some potential sources of inefficiencies include: bidder intimidatory collusion and predatory pricing (Caillaud and Jehiel 1998; Engelbrecht-Wiggans and Kahn 2005),[10] unilateral demand reduction to order to drive prices lower (Ausubel and Cramton 2002[11]; Cramton and Schwartz 2001); difficulties in attaining aggregates of licenses which have complementaries[12] (Cramton 1998).

[10] Particularly disappointing were the results of the 1997 auction of supplemental wireless communication service spectrum, in which many licenses sold for only nominal amounts of money (Gruley 1997). The auction, which had been expected to raise $1.8 billion, raised only $13.6 million (Economist 1997). A common feature of these disappointing auctions is that a relatively small number of bidders competed against each other on a relatively larger number of items.

[11] Ausubel and Cramton (2002) pp. 0 argues "In auctions where bidders pay the market-clearing price for items won, large bidders have an incentive to reduce demand in order to pay less for their winnings. This incentive creates an inefficiency in multiple-item auctions."

[12] Cramton (1998) pp. 8 argues "Another source of inefficiency in the spectrum auctions comes from the difficulties firms may have in piecing together efficient sets of licenses."

To study these possible efficiency variations, Fox and Bajari (2013) proposed the use of a maximum rank correlation estimator first introduced in Fox (2010a) to examine the determinants of a bidder's valuation of a license. According to Fox and Bajari (2013), their new approach of efficiency evaluation improved upon existing economic analysis of spectrum auctions in these crucial ways– it

1. structurally estimates bidder valuation functions in a spectrum auction, enabling qualitative measurement of different valuation components;
2. does not determine a bidder's valuation using first-order condition which implicitly assumes that bids are already good reflections of valuations;
3. allows for the inclusion of unobserved heterogeneity in bidder valuations;
4. does not use bid values data;
5. accommodates a large choice set for all bidders;
6. examines pairwise stability which has a definite solution even though a noncooperative, dynamic game has multiple Nash equilibria; and
7. measures complementarities among licenses and does not assume additive separability in valuation.

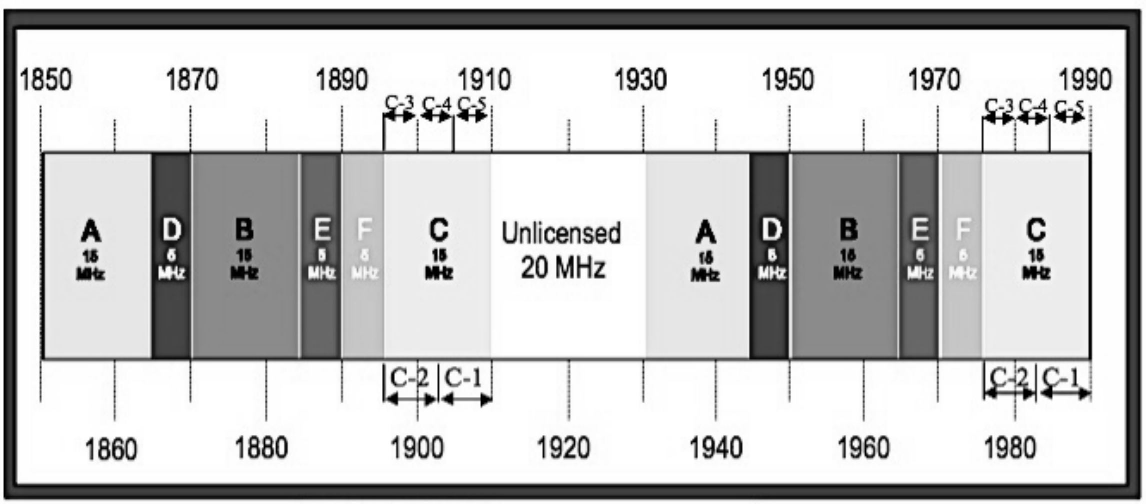
Fox and Bajari (2013) test this estimator in a single Broadband Personal Communication Services (PCS) C Block auction in 1995-1996. A recent extension to Fox and Bajari's work is the application of their model to the AWS-1 (2008) Auction in Canada by Hyndman and Parameter[13]

[13] Hyndman and Parmeter (2015), pp. 31 concludes "… in the absence of the set-aside was a spectrum allocation with no new entry and pre-existing market shares of the incumbents being largely unchanged. In this case, our results suggest an efficiency loss on the order of $400-500 million."

(2015), in which the authors find that in the absence of a set-aside rule, upwards of $400 million in efficiency loss could have been avoided. The objective of our paper is to extend the work of Fox and Bajari (2013) by applying the same estimator to four other PCS C Block auctions held by the FCC between 1996 and 2007. Our work contributes directly to the empirical analysis first initiated by Fox and Bajari (2013). Furthermore, it attempts to improve upon the structural estimation of bidder valuation proposed by them by including a dummy variable which, we argue, could relate to cross-auction complementaries between licenses in a current auction and licenses that a bidder already owns. In addition to reducing bias in unobserved heterogeneity, our new estimation allows for the analysis of the differences in valuation between big and small players within an auction.

2. Background of Chosen Auctions

Broadband PCS refers to spectrum in the 1850 MHz to 1990 MHz range that are most commonly used in mobile voice and data services, including cell phone, text messaging and Internet services. Broadband PCS spectrum is similar in utility to the 700 MHz Service, Advanced Wireless Service, 800 MHz Cellular and Specialized Mobile Radio spectrums. Broadband PCS auctions originated in 1993 when the FCC became aware of the need for rulemaking for the 1850 MHz to 1990 MHz spectrum. The FCC announced service rules for Broadband PCS and licensed 120 MHz of spectrum in 1993, with the remaining 20 MHz of unlicensed spectrum in the 1910 – 1915 and 1990 – 1995 MHz range later becoming available under Block G for licensed used in 2015.



Channel Block	Bandwidth (MHz)	Frequencies
C	30	1895-1910, 1975-1990
C1	15	1902.5-1910, 1982.5-1990
C2	15	1895-1902.5, 1975-1982.5
C3	10	1895-1900, 1975-1980
C4	10	1900-1905, 1980-1985
C5	10	1905-1910, 1985-1990

Note: Some of the original C Block licenses (Originally 30 MHz each) were split into multiplelicenses (C-1 and C-2, 15 MHz; C-3, C-4, and C-5 10 MHz).

Figure 1. Block Classifications by the FCC based on Spectrum Frequencies

Our paper applies Fox (2010a)'s maximum rank correlation estimator to determine the relative significance of different components of bidder valuation for Auction 5, Auction 22, Auction 35 and Auction 58. We have chosen to specifically examine only C-block auctions so as to hold constant both the type of spectrum being auctioned and the scope of the market being considered. It is particularly important that the latter is unchanged because part of Fox and Bajari (2013)'s structural estimation of bidder valuation is based on within-auction complementaries. For example, in Auction 5, this within-auction complementaries would be that among C-block frequencies. If we had not taken out F-block license from Auction 35, then estimated values of within-auction complementaries in Auction 35 could include those between C-Block and F-Block licenses since a bidder could bid and win licenses from both blocks within the same auction. In these cases, the relative importance of within-auction complementaries in Auction 35 would be artificially inflated.

Auction	Dates	Number of Licenses	Number of Bidders	Blocks	Total Auction Revenue
5	December 18th 1995 – May 6th 1996	493	89	C	\$13.4B
22	March 23rd 1999 – April 15th 1999	294	56	C	\$410M
35	December 12th 2000 – January 26th 2001	355	33	C	\$13.9B
58	January 26th 2005 – February 15th 2005	173	19	C	\$1.86B

Table 1. Information on the four PCS Broadband Auctions examined in this paper

A. Auction No. 5 (1995-1996)

Fox and Bajari (2013) apply their model solely on Auction 5. In this auction, all the licenses were specifically reserved for bidders that are classified as entrepreneurs, and to qualify as such, these bidders were typically small businesses required to have gross revenues of not more than $125 million in each of the previous two years and total assets of not more than $500 million at point in time when the bidders file their FCC Form 175.[14] Bidders were eligible for a bidding credit, which is a subsidy that depends on the average gross revenues for the previous three years of the bidder and installment payments were offered. A total of 255 bidders qualified for the auction, of which, 89 of them won all 493 licenses in a total of 184 rounds, raising $13.4 billion. The largest single winner was NextWave Personal Communication, winning 56 licenses for a total of $4.2 billion.[15]

B. Auction No. 22 (1999)

Auction No. 22 was a Broadband PCS C, D, E and F Block auction that began on 23 March 1999 and closed on 15 April 1999, offering a total of 347 licenses (206 30 MHz C block, 133 15MHz C block, six 10 MHz E block and two 10MHz F block) for ten-year terms. The C and F block spectrum licenses were classified as Entrepreneur's Blocks and specifically reserved for those who met the entrepreneur requirements, similar to Auction No. 5. Bidding credits were offered only for the C and F license blocks and no installment payment plans were offered. A total of 67 bidders qualified for the auction, of which, 57 of them won 302 licenses in a total of 78 rounds, raising

[14] In this case, the FCC Form 175, which requires bidders to disclose basic information about themselves, was due on November 6, about a month before the commencement of the auction.

[15] Kwerel and Rosston (2000) points out that all "top three bidders in the auction (NextWave, Pocket and GWI) have declared bankruptcy and the fourth largest bidder (BDPCS) failed to make the initial down payment. Their bids represented 75% of the $10 billion in C block net bids [in Auction 5]."

$533 million. Of these 57 winning bidders, 48 of them are small bidders. The largest single winner was VCook Inlet/VoiceStream PCS, winning 28 licenses for a total of $192 million.

C. Auction No. 35 (2000-2001)

Auction No. 35 was a Broadband PCS C and F Block auction that began on 12 December 2000 and closed on 26 Jan 2001, offering a total of 422 licenses (312 10 MHz C block, 43 15MHz C block and 67 10MHz F block) for a ten-year term. These license blocks were divided according to the nature of bidding (open or closed to large bidders) and its tiers (Tier 1 blocks were those with population more than 2.5 million and Tier 2 blocks were the remaining), which is shown in Table 2.

Channel Block	**Eligibility Status**	
	Tier 1	**Tier 2**
C1	Open	Closed
C2	Open	Closed
C3	Closed	Closed
C4	Open	Closed
C5	Open	Open

Table 2. Breakdown of all C blocks based on tiered eligibility status (“Open” or “Close”)

Some licenses were reserved only to entrepreneurs in “closed” bidding, while the rest were open to all bidders in “open bidding.” Similar to Auction No. 5 and 22, bidding credits were offered to small businesses that satisfied stipulated gross revenues requirements and no instalment payment plans were offered. A total of 87 bidders qualified for the auction, of which, 35 of them won 422 licenses in a total of 101 rounds, raising $17.6 billion. The largest single winner was Cellco Partnership, winning 113 licenses for a total of $8.8 billion.

D. Auction No. 58 (2005)

Auction No. 58 was a Broadband PCS A, C, D, E and F Block auction that began on 26 January 2005 and closed on 15 February 2005, offering a total of 242 licenses (two 30 MHz A block, 188 10 and 15MHz C block, eleven 10 MHz D block, 20 10 MHz E block and 21 10MHz F block) for ten-year terms. These license blocks were divided according to the nature of bidding (open or closed) and their population size (Tier 1 blocks were those with population more than 2.5 million and Tier 2 blocks were the remaining).

Specific C block licenses were reserved for entrepreneurs (qualifying requirements similar to previous auctions) in a “closed” bidding, while the rest are available to all bidders in “open” bidding. Bidding credits were offered only for the C and F license blocks and no instalment payment plans were offered. A total of 35 bidders qualified for the auction, of which, 24 of them won 217 licenses in a total of 91 rounds, raising $2.3 billion. The largest single winner Cellco Partnership, winning 26 licenses for a total of $365 million.

3. Structural Estimation of Bidder Valuation

A. Fox and Bajari (2013)'s Valuation Function[16]

We now introduce the component of a bidder's profit function as proposed in Fox and Bajari (2013). Bidders of an auction are defined as:

$$x = 1, \dots, X \ \in \mathbb{X}$$

and licenses for sale are noted as

$$j = 1, \dots, L \ \in \mathbb{L}.$$

Any subset of licenses can be rewritten as

$$j = 1, \dots, J \ \in \mathbb{J}$$

where $\mathbb{J} \subset \mathbb{L}$. A bidder x's valuation of any set of licenses $\mathbb{J}$ is dependent on its own bidder characteristics and also characteristics of those licenses within the package, and will be written generally as $\pi_x(\mathbb{J})$. This can be expressed as

$$(1) \qquad \pi_x(\mathbb{J}) = \pi_{\vec{\beta}}(x, \mathbb{J}) + \sum_{j \in \mathbb{J}} \xi_j + \sum_{j \in \mathbb{J}} \epsilon_{x,j}$$

where $\pi_{\vec{\beta}}(x, \mathbb{J})$ is parameterized by a finite vector of parameters $\vec{\beta}$ constant within each auction and is depended deterministically on characteristics of the bidder x and the set of licenses being considered $\mathbb{J}$. ξ_j represents the additive separable fixed effect which captures the component of valuation assigned to the each license common to all bidders, and $\epsilon_{x,j}$ is a private idiosyncratic value specific to license j and bidder x not known to the researcher.

[16] See Fox and Bajari (2013) Sec. II B for a list of assumptions and their corresponding justifications implicit in their model.

In particular, Fox and Bajari (2013) chose for $\pi_{\vec{\boldsymbol{\beta}}}(x,\ \mathbb{J})$ to be described as follows

(2) $$\pi_{\vec{\beta}}(x,\ \mathbb{J}) = \beta_0.elig_x.(\textstyle\sum_{j\in\mathbb{J}} pop_j) + \beta_1.geocomp_{\mathbb{J}} + \beta_2.travelcomp_{\mathbb{J}}$$

where $elig_x$ is the population that a bidder is eligible to bid for based on their eligibility downpayment, pop_j is the population size of each license, $geocomp_{\mathbb{J}}$ is a variable which measures the total within-auction geographical complementaries of license set $\mathbb{J}$, and $travelcomp_{\mathbb{J}}$ is a similar variable which measures the total within-auction ease-of-travel complementaries of license set $\mathbb{J}$.

Fox and Bajari (2013) find that initial eligibility strongly correlates to the size of the bidder and the likelihood of the bidder winning a license. Initial eligibility is therefore chosen as a variable to interact with $\sum_{j\in\mathbb{J}} pop_j$ in order to "…capture [the] assortive matching between bidders with higher values and packages of licenses with more population."[17] $geocomp_{\mathbb{J}}$ and $travelcomp_{\mathbb{J}}$ are nonlinear and non-additive constructions of license and bidder characteristics as follow–

(3) $$geocomp_{\mathbb{J}} = \sum_{i\,\in\,\mathbb{J}}\left[pop_i\,\frac{\sum_{j\,\in\,\mathbb{J},\mathrm{j}\,\neq\mathrm{i}}\frac{pop_i pop_j}{dist_{i,j}^2}}{\sum_{j\,\in\,\mathbb{L},\mathrm{j}\,\neq\mathrm{i}}\frac{pop_i pop_j}{dist_{i,j}^2}}\right]$$

and

(4) $$travelcomp_{\mathbb{J}} = \sum_{i\,\in\,\mathbb{J}}\left[pop_i\,\frac{\sum_{j\,\in\,\mathbb{J},\mathrm{j}\,\neq\mathrm{i}}\ trips(origin\ is\ i,destination\ is\ j)}{\sum_{j\,\in\,\mathbb{L},\mathrm{j}\,\neq\mathrm{i}}\ trips(origin\ is\ i,destination\ is\ j)}\right]\ ,$$

[17] Fox and Bajari (2013), pp. 10

where $dist_{i,j}$ is the geographical distance between the population-weighted centroid of each license, and $trips(origin\ is\ i,\ destination\ is\ j)$ is the total number of air-travel from geographical region of license i to geographical region of license j in a given year.[18]

B. Omission of Travel Complementaries

Our paper has omitted the $travelcomp_{\mathbb{J}}$ in the application of Fox and Bajari (2013)'s structural estimation and pairwise stability estimator on Auction 5, 22, 35 and 58. This is justified on three grounds. Firstly, we found no reliable and complete data on American travel for periods after 1998 to accurately measure travel frequencies. Secondly, existing air travel data are based on locales which do not correspond exactly to the geographical boundaries of spectrum licenses; this means that it is possible for a license located 100 kilometers from the airport to have the same travel complementaries measure as one located right next to the airport. Lastly, when Fox and Bajari omitted $travelcomp_{\mathbb{J}}$ and reapplied their amended model on the same dataset, i.e. Auction 5, there was no significant change in maximum pairwise stability score attained;[19] this implied that a large part of the variation in $travelcomp_{\mathbb{J}}$ which relates to variations in bidder valuation was

[18] An intuitive way to think about geographical complementaries is to think of it as a cumulative total of licenses in a set, each multiplied by a respective weight. This weight gives a proxy of the valuation of that license derived from its "geographic synergy" with other licenses that same set. For example, we would expect a similar license to be of highest "geographic synergy" when this license is owned in conjunction with all other licenses and of lowest "geographic synergy" if this license is owned in isolation. As explained by Fox and Bajari (2013), the complementary proxy can be motivated as follows. "Consider a mobile phone user in a home market *i*. That mobile phone user potentially wants to use his phone in all other markets. He is more likely to use his phone if there are more people to visit, so his visit rate is increasing in the population of the other license, *j*. The user is less likely to visit *j* if *j* is far from his home market *i*, so we divide by the distance between *i* and *j*. We care about all users equally, so we multiply the representative user in *i*'s travel experience by the population of *i*.

[19] With the inclusion of $travelcomp_{\mathbb{J}}$, Fox and Bajari found a maximum rank correlation estimate of 0.960. Without $travelcomp_{\mathbb{J}}$, the estimate was 0.956.

already explained by variations in either $elig_x . (\sum_{j \in \mathbb{J}} pop_j)$ or $geocomp_{\mathbb{J}}$, and we do not lose much explanatory power when omitting $travelcomp_{\mathbb{J}}$.

C. Replacement of eligibility with downpayment

Our paper has chosen to replace $elig_x$, expressed in terms of population in Fox and Bajari (2013), with the downpayment amount $down_x$ required to attain that $elig_x$. In Auction 5, the required downpayment amount is calculated as 1.5 cents per MHz-individual for any license, and all licenses are fixed at 30MHz. As such, in Auction 5, $down_x$ and $elig_x$ are related to each other by a fixed scalar multiple. However, in subsequent auctions, it is difficult to convert the downpayment number into a population figure. In these later auctions, licenses are segmented into 10Mhz, 15Mhz and 30MHz blocks, and there is no way to know *ex-ante* for which licenses a bidder is putting his downpayment. This replacement of variable is appropriate for us to compare across auctions, without losing the intended effects captured by initial eligibility in the first place.

D. Modified Valuation Function

While Fox and Bajari (2013)'s description of $\pi_{\bar{\beta}}(x, \mathbb{J})$ is consistent in providing a deterministic structural estimation for bidder valuation in Auction 5, we anticipate that it would have difficulties accommodating open auctions which involve bigger players. This is because Fox and Bajari (2013) considers within-auction geographical complementaries, but not those complementaries between licenses sold in the particular auction and those already owned by the bidder. In Auction 5 which is open only to small bidders, the impact of these cross-auction complementaries is likely to be small, and could therefore be absorbed into the private exogenous term $\epsilon_{x,j}$ without creating much problem.

Our paper posits that such a setup would however have lower maximum pairwise stability matching when used to estimate valuation functions in auctions involving bigger players. This is because, based on Fox and Bajari (2013)'s model, a big bidder who does not buy much from an auction but nonetheless values a license because it complements many of their *pre-existing* spectrum ownership would be assigned a low bid valuation $\pi_{\bar{\boldsymbol{\beta}}}(x,\ \mathbb{J}\ )$. If the big bidder eventually wins the license following a high bid that accurately signals the high cross-auction complementaries, Fox and Bajari (2013)'s scoring estimator would still likely count such an outcome as pairwise unstable.

Expanding the existing $geocomp_{\mathbb{J}}$ variable to include the set of licenses already owned by the bidder would most accurately endogenize the cross-auction complementaries. However, while this is conceptually simple, it is empirically extremely difficult. This is because pre-auction ownership is a mixed result of bids in multiple past auctions, private purchases in secondary spectrum license markets and corporate mergers or acquisitions–all of which are difficult to accurately keep track of. Instead, we introduce a dummy variable $\delta_{x,j}$ which is bidder- and license-specific as a proxy term indicating the existence of significant cross-auction complementaries. It is defined as follows

(5)
$$\delta_{x,j} = \begin{cases} 1 & \text{if bidder x made a bid for license j and is not eligibile for bid credit} \\ 0 & \text{otherwise} \end{cases} .$$

The modified valuation function then becomes

$$(6) \qquad \pi_{\vec{\beta}}(x, \mathbb{J}) = \beta_0 . down_x . (\textstyle\sum_{j\in\mathbb{J}} pop_j) + \beta_1 . geocomp_{\mathbb{J}} + \beta_2 . \textstyle\sum_{j\in\mathbb{J}} \delta_{x,j}$$

in which the cross-auction complementaries dummy $\delta_{x,j}$ can be motivated as such. A big bidder, one who is not eligible for bid credit, would more likely have existing ownership of spectrum. This means that, compared to small player, a big bidder is likely to enjoy a significant degree of cross-auction complementaries. Still, a bidder does not enjoy cross-auction complementaries for all licenses simply by virtue of being big. Instead, similarly to within-auction complementaries, a bidder only enjoys complementaries for licenses which are geographically close to those it already owns. We make the assumption that one key way in which that big bidder reveals the existence of these license-specific cross-auction complementaries is when it places some bid for that particular license. Therefore, we assign $\delta_{x,j}$ to be 1 only if both conditions are met. Consequently, we think that a significant part of the estimation of $\beta_2 . \sum_{j\in\mathbb{J}} \delta_{x,j}$ would theoretically be composed of these cross-auction complementaries.

4. Data and Methodology

A. Data for the Covariates

Our paper uses FCC auction data obtained from the Center for Study of Auctions, Procurements and Competition Policy at Pennsylvania State University[20] to generate license- and bidder- specific variables such as $down_x$ [in dollars], pop_j [in persons] and $\delta_{x,j}$ for all $j \in \mathbb{L}$ and $x \in \mathbb{X}$. We have also intentionally removed all licenses that are not in the C block for each of the auctions for reasons detailed earlier in this paper. To generate geographical complementary $geocomp_{\mathbb{J}}$, the latitude and longitude coordinates for each of the licenses are generated using the Christos Samaras's VBA Google Geocoding Functions[21] based on market descriptions provided by the FCC, and distances $dist_{i,j}$ [in kilometers] are calculated using a Haversine function we wrote to account for the Earth's spatial curvature.

For the purpose of testing pairwise stability, it is necessary to find the respective license sets won by each bidder *x*. We define the license set won by bidder *x* to be a specific $\mathbb{J}$ noted as $\mathbb{J}_{\boldsymbol{x}} \subset \mathbb{L}$. Accordingly, $\mathbb{J}_{\boldsymbol{x}}$ are non-overlapping for all *x*, and if all licenses are sold, then

$$\cup_{x \in \mathbb{X}} \mathbb{J}_{\boldsymbol{x}} = \mathbb{L}\,.$$

[20] These FCC auction data can be accessed online directly at http://capcp.psu.edu/data-and-software/fcc-spectrum-auction-data.
[21] Christo Samaras's VBA Google Geocoding Functions is published for public use at http://www.myengineeringworld.net/2014/06/geocoding-using-vba-google-api.html.

Utilizing this new notation, $geocomp_{\mathbb{J}_x}$ would be defined as the specific geographical complementaries of the set of licenses won by bidder x. The summary statistics of the three key covariates in the proposed bidder valuation function– $down_x$, pop_j, and $geocomp_{\mathbb{J}_x}$ is outlined in Table 3.[22]

Variable	Auction	Mean	StandardDev	Minimum	Maximum
$down_x$	5	305 million	8.35×10^8	0.83 million	79.23 million
	22	5.08 million	1.06×10^7	33,000	50.0 million
	35	39.0 million	6.35×10^7	6,400	239 million
	58	11.6 million	1.19×10^7	0.14 million	36.9 million
$\sum_{j \in \mathbb{J}_x} pop_j$	5	2.83 million	1.08×10^6	2,500	93.8 million
	22	1.89 million	4.13×10^6	316,700	18.9 million
	35	10.41 million	2.46×10^7	687,000	118.9 million
	58	10.41 million	1.14×10^7	989,200	37.0 million
$geocomp_{\mathbb{J}_x}$	5	0.92 million	4.75×10^6	0	42.6 million
	22	0.48 million	1.50×10^6	0	8.24 million
	35	2.06 million	7.24×10^6	0	41.3 million
	58	1.36 million	1.98×10^6	0	7.09 million

Table 3. Summary statistics of $down_x$, $\sum_{j \in \mathbb{J}_x} pop_j$ and $geocomp_{\mathbb{J}_x}$ across winning bidders in Auctions 5, 22, 35 and 58

To normalize the data, we divide values of $down_x$ by the total downpayment required to obtain all licenses in each specific auction. This number is released by the FCC in an information package prior to every auction. Next, we divide all values of pop_j by the total population size in each specific auction. We divide $geocomp_{\mathbb{J}_x}$ by the geographical complementaries exhibited for a set

[22] The statistics based on the (x x j) matrix for $\delta_{x,j}$ can be found in the Appendix.

encompassing all licenses each specific auction, i.e. $geocomp_{\mathbb{L}}$; it is useful to note that $geocomp_{\mathbb{L}}$ in each specific auction is also equivalent to the total population size of that auction. Lastly, we divide $\delta_{x,j}$ by the total number of licenses for sale in each auction. The summary statistics of the three normalized key covariates in the proposed bidder valuation function– $down_x$, pop_j, and $geocomp_{\mathbb{J}_x}$ is outlined in Table 4.[23]

Variable	Auction	Mean	StandardDev	Minimum	Maximum
$down_x$	5	0.040	0.110	0.000	0.697
	22	0.032	0.066	0.000	0.314
	35	0.194	0.315	0.000	1.187
	58	0.114	0.117	0.001	0.362
$\sum_{j \in \mathbb{J}_x} pop_j$	5	0.0112	0.00426	0.00001	0.3714
	22	0.0179	0.0391	0.003	0.1786
	35	0.0303	0.0716	0.002	0.3462
	58	0.0526	0.0576	0.005	0.1868
$geocomp_{\mathbb{J}_x}$	5	0.004	0.019	0.000	0.169
	22	0.005	0.014	0.000	0.078
	35	0.006	0.021	0.000	0.120
	58	0.007	0.010	0.000	0.036

Table 4. Summary statistics of normalized values of $down_x$, $\sum_{j \in \mathbb{J}_x} pop_j$ and $geocomp_{\mathbb{J}_x}$ across winning bidders in Auctions 5, 22, 35 and 58

[23] The statistics based on the normalized (*x* x *j*) matrix for $\delta_{x,j}$ can be found in the Appendix.

B. Pairwise Stability

Consider two bidders, $x_1, \ x_2 \ \in \mathbb{X}$, who won license sets $\mathbb{J}_{\boldsymbol{x_1}}$ and $\mathbb{J}_{\boldsymbol{x_2}}$ respectively. Their valuation of their won sets are defined as $\pi_{x_1}(\mathbb{J}_{\boldsymbol{x_1}})$ and $\pi_{x_2}(\mathbb{J}_{\boldsymbol{x_2}})$. The allocation of resources between these two bidders is defined to be pairwise stable if–

(7) $$\pi_{x_1}(\mathbb{J}_{x_1}) + \pi_{x_2}(\mathbb{J}_{x_2}) \geq \pi_{x_1}\left((\mathbb{J}_{x_1} \setminus \{i\}) \cup \{j\}\right) + \pi_{x_2}\left((\mathbb{J}_{x_2} \setminus \{j\}) \cup \{i\}\right)$$

for all $i \in \mathbb{J}_{\boldsymbol{x_1}}$ and $j \in \mathbb{J}_{\boldsymbol{x_2}}$.[24] This means that there must be no possible way for these two bidders to exchange *one-for-one* their licenses in order to increase their collective profits. Since fixed effects ξ_j are additively separable and common to both bidders, and since further private values $\epsilon_{x,j}$ are assumed to be random, they cancel out on both sides of the inequality. (7) can therefore be equivalently expressed as

(8) $$\pi_{\vec{\beta}}(x_1, \mathbb{J}_{\boldsymbol{x_1}}) + \pi_{\vec{\beta}}(x_2, \mathbb{J}_{\boldsymbol{x_2}}) \geq \pi_{\vec{\beta}}(x_1, (\mathbb{J}_{\boldsymbol{x_1}} \setminus \{i\}) \cup \{j\}) + \pi_{\vec{\beta}}(x_2, (\mathbb{J}_{\boldsymbol{x_2}} \setminus \{j\}) \cup \{i\})$$

We note that pairwise stability only considers the pairwise exchange of one license and not multiple licenses simultaneously. It does not consider the possibility of a bidder "donating" a license to another bidder without receiving a license in return. It also does not allow for more than two players to swap licenses among themselves simultaneously. Hence, pairwise stability has been sometimes criticized as a weak concept.[25]

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[24] The definition follows from Jackson and Wollinsky (1996).

[25] Jackson (2003), pp. 19 "First, it is a weak notion in that it only considers deviations on a single link at a time. This is part of what makes it easy to apply. However, if other sorts of deviations are viable and attractive, then pairwise stability may be too weak a concept."

While there are stronger definitions of stability or efficiency that allow for larger groups of players to deviate from the allocation such as those mentioned by Dutta and Mutuswami (1997), it is not computationally feasible to consider every configuration of licenses among the bidders of each auction[26]. Jackson (2003) considered the interactions between pairwise stability and different forms of efficiency and found cases in which networks that are pairwise stable are also efficient. While pairwise stability is certainly a weaker condition than allocative efficiency, Fox (2010b) proved that nonparametric identification of the valuation function can work equally well from conditions of pairwise stability as conditions of efficiency.

C. Scoring Algorithm and Structural Estimator

Using this definition of pairwise stability, for each auction, we find the vector of parameters $\vec{\beta}$ that maximizes the total number of pairwise stable configurations among all bidders. Similar to how a regression finds the optimal parameterization of covariates to best-fit a set of observable outcomes, Fox (2010a)'s structural estimator finds the optimal parameterization of valuation components to best-fit the observed auction pairwise outcomes. In the case of a regression, "best-fit" could be defined, say, as the minimization of sum of the squares of the difference between an observed value and the value by the structural estimation. In the case of Fox (2010a)'s estimator, "best-fit" is defined as the maximization of pairwise stable configurations.

[26] See Appendix C for a breakdown of estimated runtimes for different definitions of pairwise stability.

Specifically, in Fox and Bajari (2013), the scoring objective function to be maximized is defined as–

(9) $$F(\vec{\beta}) = \sum_{x_1=1}^{H-1}\sum_{x_2=x_1+1}^{H}\sum_{i\in \mathbb{J}_{x_1}}\sum_{j\in \mathbb{J}_{x_2}} \mathbf{1}\left[\sum_{j\in(\mathbb{J}_{x_1}\setminus\{i\})\cup\{j\}} pop_j \le elig_{x_1}, \sum_{j\in(\mathbb{J}_{x_2}\setminus\{j\})\cup\{i\}} pop_j \le elig_{x_2}\right].\mathbf{1}\left[\pi_{\vec{\beta}}(x_1,\mathbb{J}_{x_1}) + \pi_{\vec{\beta}}(x_2,\mathbb{J}_{x_2}) \ge \pi_{\vec{\beta}}(x_1,(\mathbb{J}_{x_1}\setminus\{i\})\cup\{j\}) + \pi_{\vec{\beta}}(x_2,(\mathbb{J}_{x_2}\setminus\{j\})\cup\{i\})\right]$$

where **1[.]** is an indicator function and H is the number of winning bidders. The objective function counts the number of pairwise stable configurations which do not violate the initial population eligibility limit of each bidder. Cases where, say, both bidder x_1 and bidder x_2 are not able to be collectively better off if they swap one-to-one, but where at least one of them is unable to accommodate the new license due to initial eligibility constraints, would not count as a stable pair. This logical interaction of $\mathbf{1}\left[\sum_{j\in(\mathbb{J}_{x_1}\setminus\{i\})\cup\{j\}} pop_j \le elig_{x_1}, \sum_{j\in(\mathbb{J}_{x_2}\setminus\{j\})\cup\{i\}} pop_j \le elig_{x_2}\right].\mathbf{1}\left[\pi_{\vec{\beta}}(x_1,\mathbb{J}_{x_1}) + \pi_{\vec{\beta}}(x_2,\mathbb{J}_{x_2}) \ge \pi_{\vec{\beta}}(x_1,(\mathbb{J}_{x_1}\setminus\{i\})\cup\{j\}) + \pi_{\vec{\beta}}(x_2,(\mathbb{J}_{x_2}\setminus\{j\})\cup\{i\})\right]$ is presented in Figure 2.

	Both bidder better off after swap [0]	**Both bidder worse off after swap [1]**
Eligibility limits allow for swap to happen [0]	0	1
Eligibility limits do not allow for swap to happen [1]	0	0

Figure 2. Value of Fox and Bajari (2013)'s interacting indicator functions under different cases

In Figure 2, a value of 1 signifies pairwise stability and a value of 0 signifies the lack thereof. In our paper, we have chosen instead to relax the latter assumption, and consider bottom-right cases in Figure 2 as pairwise stable. Intuitively, we think this new methodology allows for a fairer assessment of pairwise stability; if a pair of bidders was already not going to swap their licenses because the new collective valuation is poorer, an external imposition of a restriction which prevent them from doing what they do not want to do in the first place should not make them pairwise unstable.

Therefore, we use a modified scoring objective function defined as follows–

(10) $$F(\vec{\beta}) = \sum_{x_1=1}^{H-1} \sum_{x_2=x_1+1}^{H} \sum_{i\in \mathbb{J}_{x_1}} \sum_{j\in \mathbb{J}_{x_2}} \mathbf{1}\,[\pi_{\vec{\beta}}(x_1, \mathbb{J}_{x_1}) + \pi_{\vec{\beta}}(x_2, \mathbb{J}_{x_2}) \geq \pi_{\vec{\beta}}(x_1, (\mathbb{J}_{x_1} \setminus \{i\}) \cup \{j\}) + \pi_{\vec{\beta}}(x_2, (\mathbb{J}_{x_2} \setminus \{j\}) \cup \{i\})]$$

and the logical interactions of $\mathbf{1}\,[\pi_{\vec{\beta}}(x_1, \mathbb{J}_{x_1}) + \pi_{\vec{\beta}}(x_2, \mathbb{J}_{x_2}) \geq \pi_{\vec{\beta}}(x_1, (\mathbb{J}_{x_1} \setminus \{i\}) \cup \{j\}) + \pi_{\vec{\beta}}(x_2, (\mathbb{J}_{x_2} \setminus \{j\}) \cup \{i\})]$ is represented in Figure 3.

<table>
<tr><th></th><th>Both bidder better off after swap</th><th>Both bidder worse off after swap</th></tr>
<tr><th>Eligibility limits allow for swap to happen</th><td rowspan="2">0</td><td rowspan="2">1</td></tr>
<tr><th>Eligibility limits do not allow for swap to happen</th></tr>
</table>

Figure 3. Value of this paper's pairwise scoring indicator function under different cases

Computationally, we design an algorithm which considers each distinct pair of owners, and iterates through every possible one-for-one exchange between the owners' license sets. After the swap is conducted, the new result obtained from the valuation function is compared to the value before the swap. Whenever we find that the new result does no better than the old one, we add one to the score counter. The final value of the score counter after all swaps are considered is then divided by the total number of swaps to give us a score between 0 and 1 that reflects the pairwise stability–or degree of "best-fit"–of the actual allocation of licenses.

The algorithm is run repeatedly for different guesses of $\vec{\beta}$. We normalize β_0 to be ± 1 so as to limit the range that the estimator $\vec{\beta}$ can be found. This does not materially change the validity of our estimator as the scaling factor would be canceled out on both sides of the inequality in (8). A noteworthy observation is that the scores were neither strictly decreasing nor increasing with changes in guesses of $\vec{\beta}$, so it is possible to have a range of non-unique values of $\vec{\beta}$ which maximizes the objective function.

There are a few things to note when designing $\pi_{\vec{\beta}}(x, \mathbb{J})$. It is prudent to ensure that each parameterized component of the valuation function varies to *both* bidder- and license-characteristics in a nonlinear manner. For instance, suppose that we design a structural estimation valuation function to be $\pi_{\vec{\beta}}(x, \mathbb{J}) = \beta_0.down_x$ such that the component is not dependent on license characteristics. Then $\beta_0.down_x$ would not change for all possible one-to-one swaps. It follows that if $down_{x_1} > down_{x_2}$, then $\pi_{\vec{\beta}}(x_1, \mathbb{J}_{x_1}) > \pi_{\vec{\beta}}(x_2, \mathbb{J}_{x_2})$ always; we will find that there are infinitely many possible estimation of $\widehat{\beta_0}$. Now, suppose that the valuation function is

$\pi_{\vec{\beta}}(x, \mathbb{J}) = \beta_0 . \sum_{j \in \mathbb{J}} pop_j$ instead. Then, $\beta_0 . \sum_{j \in \mathbb{J}} pop_j$ becomes a license fixed effect common to all bidders; this means that all bidders will value the license equally. This fixed effect would be canceled out on both sides of the inequality (8). Obviously, both problems would still exist even if we define the valuation function as $\pi_{\vec{\beta}}(x, \mathbb{J}) = \beta_0 . \xi_{x, \mathbb{J}}$, where $\xi_{x, \mathbb{J}}$ is some covariate that is a linear combination of a bidder-only characteristic and a license-only characteristic, e.g. $\xi_{x, \mathbb{J}} = down_x + 2 \sum_{j \in \mathbb{J}} pop_j$.

5. Results

A. Estimating Fox and Bajari (2013)'s model[27]

Our paper tests the explanatory power of the following adapted structural estimation first proposed by Fox and Bajari (2013):

$$\pi_{\vec{\beta}}(x,\ \mathbb{J}\,) = \ \pm 1.\, down_x \,.\, (\textstyle\sum_{j \in \mathbb{J}} pop_j) + \ \beta_1 .\, geocomp_{\mathbb{J}} . \tag{11}$$

The results of the estimation are outlined in Table 5. The parameters are those which best optimizes the scoring objective function. The pairwise stability score is the corresponding percentage of pairwise stable configuration based on the optimized parameters.

Auction	$\pm\mathbf{1}$	$\widehat{\boldsymbol{\beta}_1}$	**Percentage of Pairwise Stable Configurations**
5	+1	13.4	96.3%
22	+1	9.3	90.6%
35	+1	10.3	72.2%
58	+1	1.4	67.3%

Table 5. Results of Fox and Bajari (2013) estimator when applied on Auctions 5, 22, 35 and 58

[27] See Fox and Bajari (2013) Sec. II B for a list of assumptions and their corresponding justifications implicit in their model.

B. Estimating our Modified Valuation Function

We then test the explanatory power of the modified structural estimation proposed earlier in this paper:

(12) $$\pi_{\vec{\beta}}(x,\ \mathbb{J}\,) = \pm 1.\,down_x\,.\left(\sum_{j\in\mathbb{J}} pop_j\right) + \beta_1.\,geocomp_{\mathbb{J}} + \beta_2.\sum_{j\in\mathbb{J}} \delta_{x,j}$$

Similarly, the results of the new estimation are outlined in Table 6.

Auction	$\pm\mathbf{1}$	$\widehat{\boldsymbol{\beta_1}}$	$\widehat{\boldsymbol{\beta_2}}$	Percentage of Pairwise Stable Configurations
5	+1	13.4	-[28]	96.3%
22	+1	6.8	20.0	92.1%
35	+1	7.4	19.2	87.0%
58	+1	1.2	10.2	79.5%

Table 6. Results of our modified estimator when applied on Auctions 5, 22, 35 and 58

[28] Note that Auction 5 has a $\widehat{\beta_2}$ value that is undefined because the entire auction is closed, meaning only small bidders are allowed in the auction.

6. Discussion

A. Modifying Scoring Algorithm

As discussed earlier, our paper modifies the scoring objective function used in Fox and Bajari (2013) in order to employ a less restrictive definition of pairwise stability. We explain in Section 4 that this would better capture the true valuation of the bidder–one which is independent of auction mechanics. As a result, even without the inclusion of the dummy variable $\delta_{x,j}$, our estimates for $\beta_1$in Auction 5 differs from that found in Fox and Bajari (2013).

Objective Function Used	± 1	$\widehat{\beta_1}$	Percentage of Pairwise Stable Configurations
With eligibility restrictions	+1	1.06	0.956
Without eligibility restrictions	+1	13.4	0.963

Table 7. Comparing estimator results for Auction 5 with and without eligibility restrictions in scoring

Both scoring objective functions give roughly similar maximum pairwise stability scores. However, our methodology results in a higher coefficient for geographical complementaries as a component in bidder valuation. We can interpret the higher $\widehat{\beta_1}$ to be indicating the existence of pairs consisting of bidders who (i) derive significant valuation from geographical complementaries such that they (ii) would not collectively do better after swapping one-for-one with another bidder, and (iii) could not swap even if they had wanted to due to eligibility constraints. Dissimilar to Fox and Bajari (2013), we think that these bidder pairs should count towards the the pairwise stability score.

Since we normalized the coefficient of $down_x . (\sum_{j \in \mathbb{J}} pop_j)$ to be ± 1, $\widehat{\beta_1}$ represents the relative importance of the geographical complementaries valuation component $geocomp_{\mathbb{J}}$ vis-à-vis that of the assortive matching between bidders who paid higher levels of downpayment and packages with more population, $down_x . (\sum_{j \in \mathbb{J}} pop_j)$. Using the new scoring objective function, our estimated $\widehat{\beta_1}$ is 13 times that of Fox and Bajari (2013)'s. We think that Fox and Bajari (2013)'s methodology of scoring pairwise stability may have understated the importance of geographical complementaries in determining Auction 5 bidder valuation.

B. Relationship of Auction Estimator to the Number of Licenses

Based on the replicated Fox and Bajari (2013) structural estimation of bidder valuation (without the inclusion of dummy variable $\delta_{x,j}$), as the number of licenses in an auction decreases, the importance of $geocomp_{\mathbb{J}}$ falls. This is observed from the positive correlation between $\widehat{\beta_1}$ and the number of licenses.

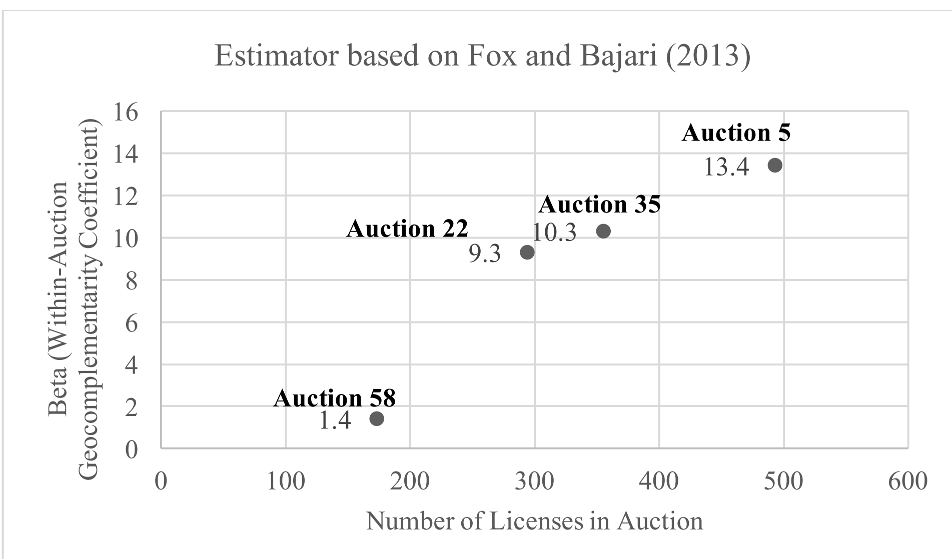

Figure 4. Relationship of Fox and Bajari (2013) within-auction geographical complementarity estimator $\widehat{\beta_1}$ to the number of licenses in Auctions 5, 22, 35 and 58

We normalize our data by dividing $down_x$, $pop_j, geocomp_{\mathbb{J}}$ by their respective theoretically-attainable maxima specific to each auction, in order to be able to interpret the relative importance of variables across auctions. It is to be expected that the average value of within-auction complementaries would fall when the number of licenses decreases; this is because in auctions where only a small subset of total license set is up for sales, there are going to be many missing pieces when a bidder tries to build geographical complements. In other words, the average quality of within-auction geographical complementaries falls.

We posit that much of this missing within-auction geographical complementaries would instead be consigned to what our paper classifies as cross-auction geographical complementaries. An analysis of the estimators obtained in our modified structural estimator gives support to this hypothesis, assuming that $\delta_{x,j}$ is indeed correlated to the unobserved cross-auction complementaries valuation component. Figure 5 shows the relationship between the ratio of the estimated coefficient of the big bidder dummy, $\delta_{x,j}$, to that of $geocomp_{\mathbb{J}}$ and the number of licenses across the auctions we have tested.

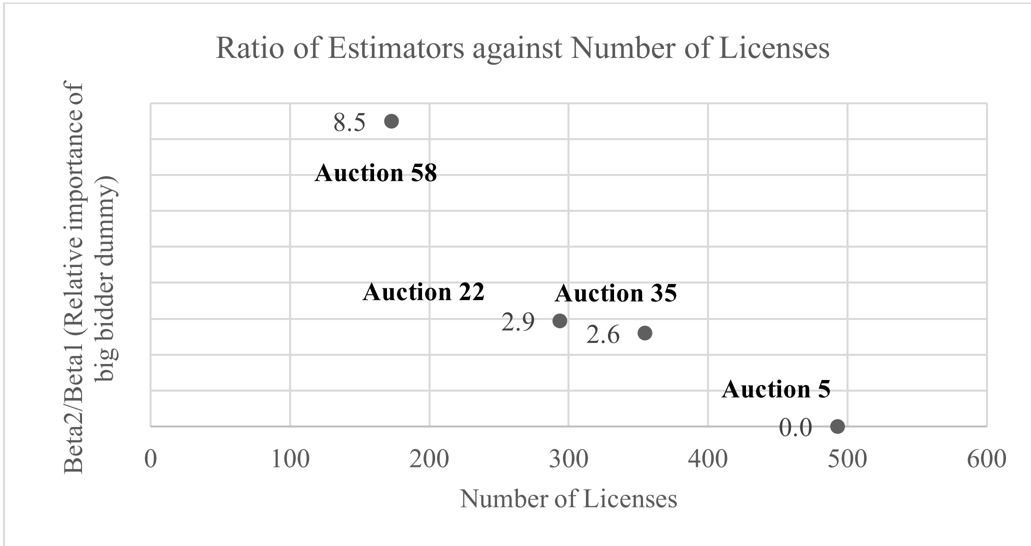

Figure 5. Illustration of how the $\frac{\widehat{\beta_2}}{\widehat{\beta_1}}$ changes with the number of licenses in an auction

As shown, the relative importance of cross-auction complementaries increases as the number of licenses decreases. In Auction 58, for a theoretical big bidder who bid and bought all 173 licenses, their total valuation based on our model is 1*1*1 + 1.2*1 + 10.2*1 = 12.4. Out of the bidder's total valuation, 82% comes from the supposed cross-auction complementaries and about 10% comes from within-auction complementaries. In contrast, in the bigger Auction 35, a theoretical big bidder who bid and bought all 355 licenses, would have a total valuation of 1*1*1 + 7.4*1 + 19.2*1 = 27.6, of which only approximately 70% comes from the supposed cross-auction complementaries and 27% comes from within-auction complementaries.

C. Goodness-of-fit Improvement

While the replicated structural estimation used in Fox and Bajari (2013) is strong in explaining much of the pairwise configurations observed in Auction 5, it was considerably weaker in explaining subsequent auctions which included big bidders, i.e. Auction 22, 35 and 58. We observe that the weakness of the model worsens as the proportion of big bidders to small bidder's increases.

Auction	Percentage of Big Bidders	Maximum Pairwise Stability
5	0%	96.30%
22	13.79%	90.60%
35	18.18%	72.20%
58	26.32%	67.30%

Table 8. Percentage of big bidders and pairwise stability score based on Fox and Bajari (2013)

The inclusion of the dummy variable $\delta_{x,j}$ which supposedly correlates to the existence of cross-auction complementaries increases the strength of the structural estimation. The improvement in maximum pairwise stability score is presented graphically in Figure 6.

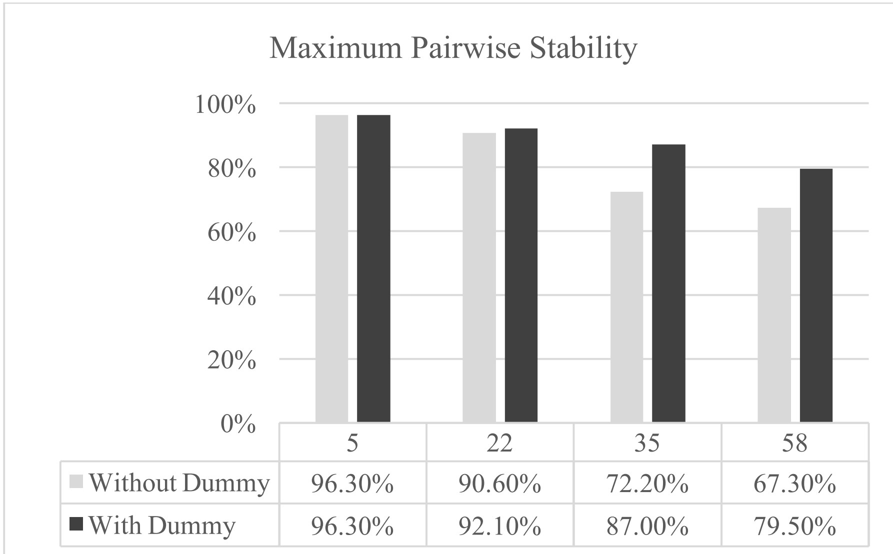

Figure 6. Pairwise stability improvements upon inclusion of big bidder dummy variable

Our results show significant variation in the inherent valuation structure between small and big bidders in C block auctions. In our discussion, we have presented some evidence that these valuation differentials could be due to cross-auction complementaries. However, more work needs to be done to verify if the same trends can be observed in other spectrum auctions and to test competing hypotheses of what the big bidder dummy could represent. Where these differentials come from has particularly consequential implications for FCC auction design. If big bidders are valuing licenses more because they have better access to cross-auction complementaries or wider economies of scale, then the FCC should ideally design auctions to provide incentives for these bigger players to signal their value differentials. In this way, licenses can be allocated to these bigger players such that cross-auction complementaries would not be lost from total welfare. However, if big bidders are valuing licenses more as means of intimidatory pricing or to unilaterally enact high barriers of entry in the spectrum market to further their own profit motives, then the FCC might want to eliminate these differentials from auction bids so as to countervail monopoly power and protect end consumers.

To supplement policy analysis, we run counterfactual simulations on the auctions involving big bidders (i.e. Auctions 22, 35 and 58) where we assume that a tax on big bidders is imposed by the FCC. Note that this is isomorphic to small bidders receiving bid credits in open auctions including big bidders. In these simulations, we use the estimated parameterizations of valuation components for each auction to generate numerical bidder valuations before subtracting a tax amount. A single unit of tax is measured as a percentage α of the maximum valuation of a monopolist bidder who bids and wins all licenses in an auction, divided by the total population represented in all licenses won by big bidders–

(13) $$unitTax = \alpha \; \frac{\pm 1 + \widehat{\beta_1} + \widehat{\beta_2}}{\sum_{x \in \mathbb{X}_{\boldsymbol{big}}} \sum_{j \in \mathbb{J}_x} pop_j}$$

where $\mathbb{X}_{big}$ is the set of all big bidders in an auction.

For all bidders, their new post-tax valuation function then becomes as follows

(14) $$\pi_{\overline{\beta}}'(x,\ \mathbb{J}\,) = \ \pi_{\overline{\beta}}(x,\ \mathbb{J}\,) - \ unitTax\ \textstyle\sum_{j \in \mathbb{J}} pop_j\,.\,\mathbf{1}[\text{if bidder is big}]$$

The tax, which only a big bidder has to pay, is the $unitTax$ multiplied by the population size of the license set in which it wins. The percentage of pairwise stable configurations using the new post-tax valuation function, for different values of tax rate α between 0 and 1, are shown in Figure 7.

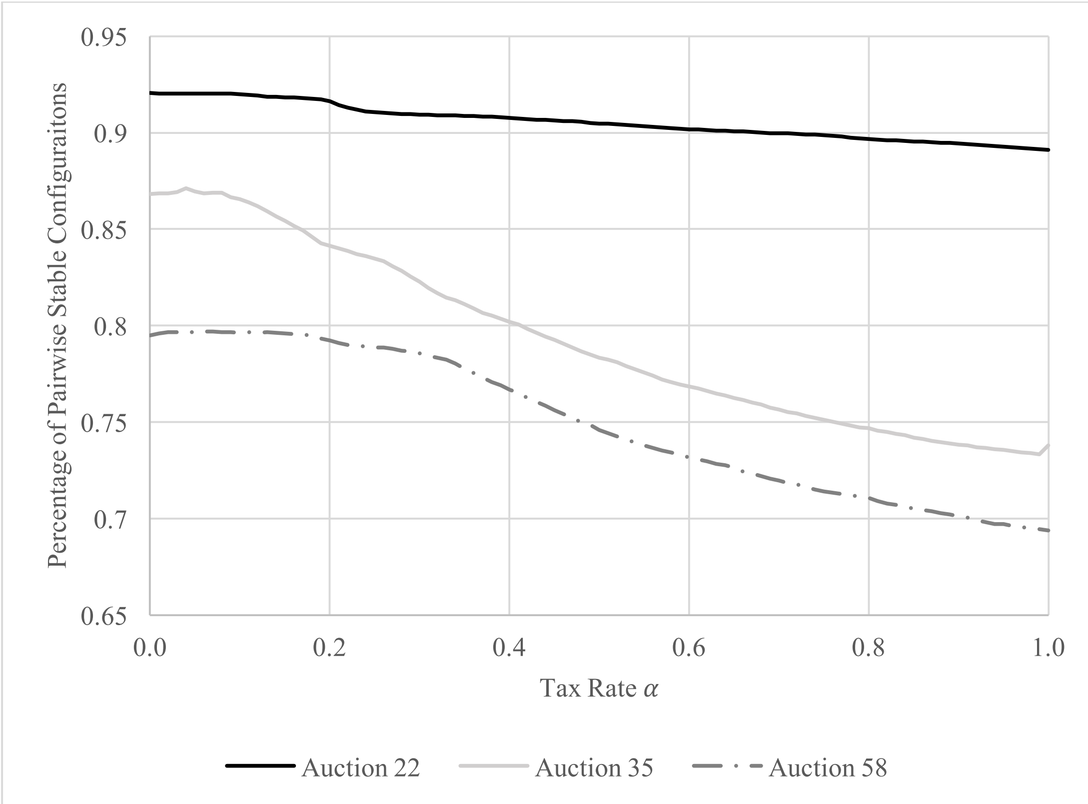

Figure 7. Counterfactual simulations of the impact of taxation of big bidders on pairwise stability

The results can be interpreted as the relative sensitivities of the auction allocation outcomes to changing degrees of taxation on big bidders. When the tax rate $\alpha = 0$, the post-tax bidder valuation for all bidders is exactly the same as their initial bidder valuation; at this pre-tax level, a large part of observed auction outcomes can be explained by the model. As the tax rate increases, the deviations between observed auction and the outcome predicted by the new post-tax valuation increase. The results of these counterfactual simulations are particularly useful in informing policymakers of the varied disruptive effective associated with the taxation of big bidders (or equivalently the subsidization of small bidders).

Appendix A: Graphs of Results and Variables

In the first part of this section, we show the plotted graphs of pair-wise stability scores $F(\hat{\beta})$ against estimated β_1 under the Fox and Bajari's model across Auction 5, 22, 35 and 58. The structural estimation is restated below-

$$\pi_{\vec{\beta}}(x, \mathbb{J}) = \pm 1. down_x . (\textstyle\sum_{j \in \mathbb{J}} pop_j) + \beta_1 . geocomp_{\mathbb{J}}.$$

Figure A1 – Relationship of Fox and Bajari (2013) estimator to estimated betas for Auction 5

Figure A2 – Relationship of Fox and Bajari (2013) estimator to estimated betas for Auction 22

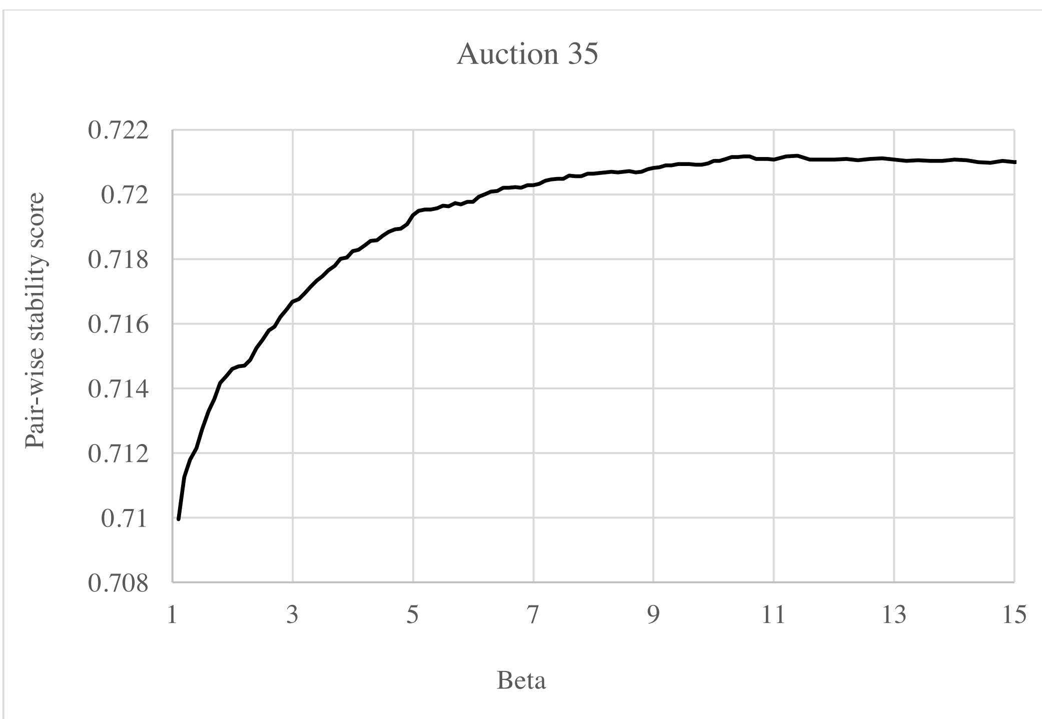

Figure A3– Relationship of Fox and Bajari (2013) estimator to estimated betas for Auction 35

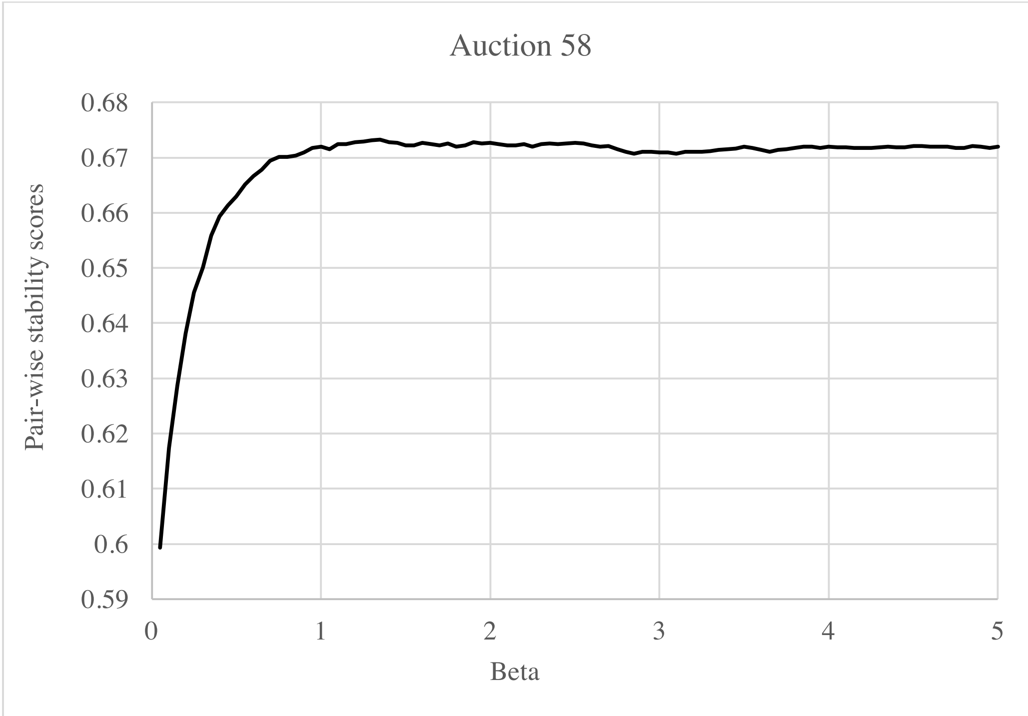

Figure A4 – Relationship of Fox and Bajari (2013) estimator to estimated betas for Auction 58

In the second part of this section, we show the plotted graphs of pair-wise stability scores $F(\hat{\beta})$ against β_1 and β_2 under the modified structure estimation across Auction 5, 22, 35 and 58. The modified structural estimation is restated below-

$$\pi_{\vec{\beta}}(x, \mathbb{J}) = \pm 1. down_x . (\sum_{j \in \mathbb{J}} pop_j) + \beta_1 . geocomp_{\mathbb{J}} + \beta_2 . \sum_{j \in \mathbb{J}} \delta_{x,j}$$

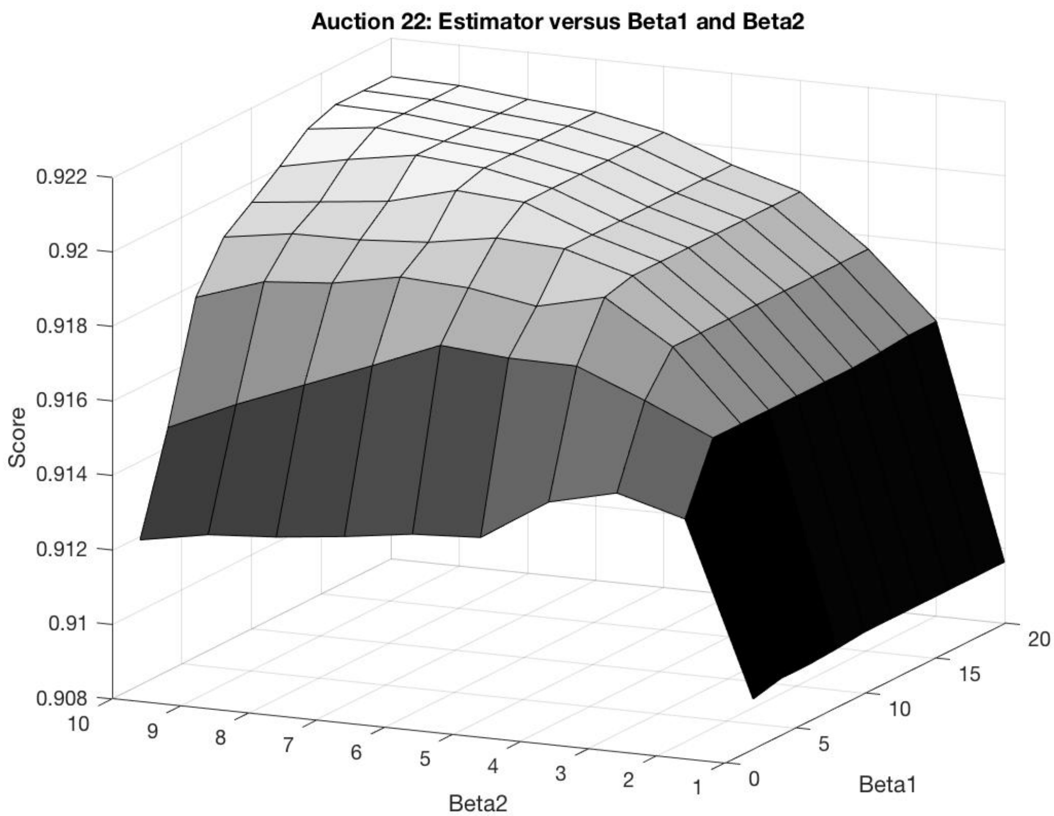

Figure A6 – Relationship of modified estimator to estimated betas for Auction 22

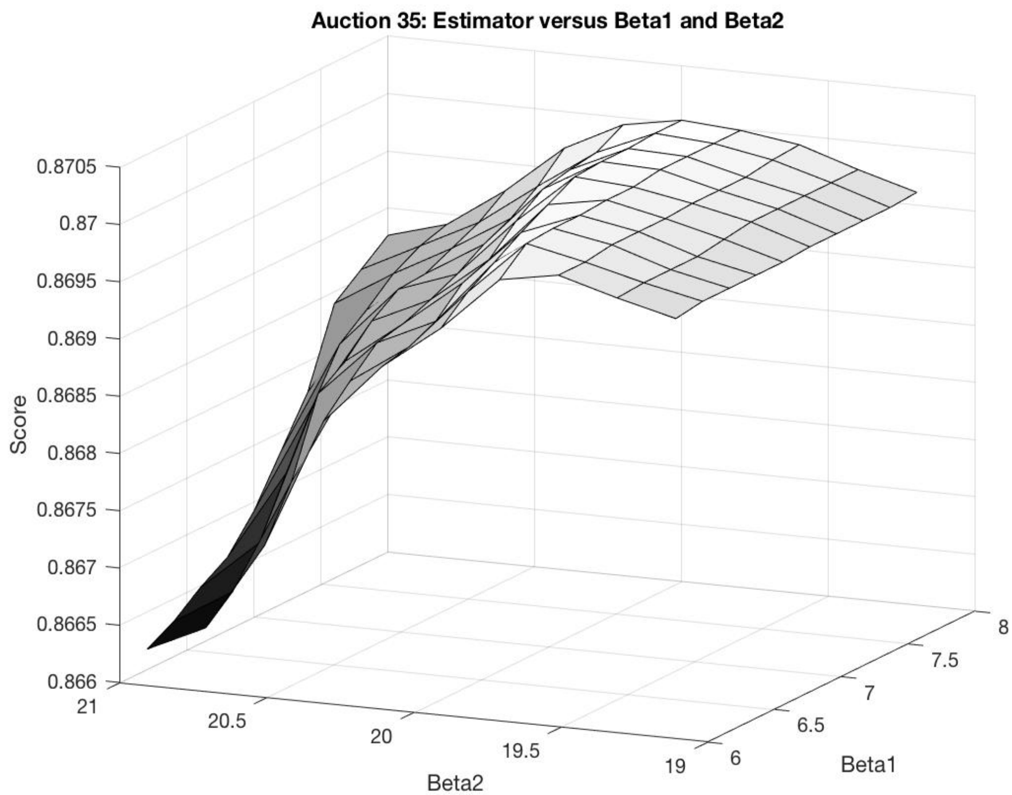

Figure A7 – Relationship of modified estimator to estimated betas for Auction 35

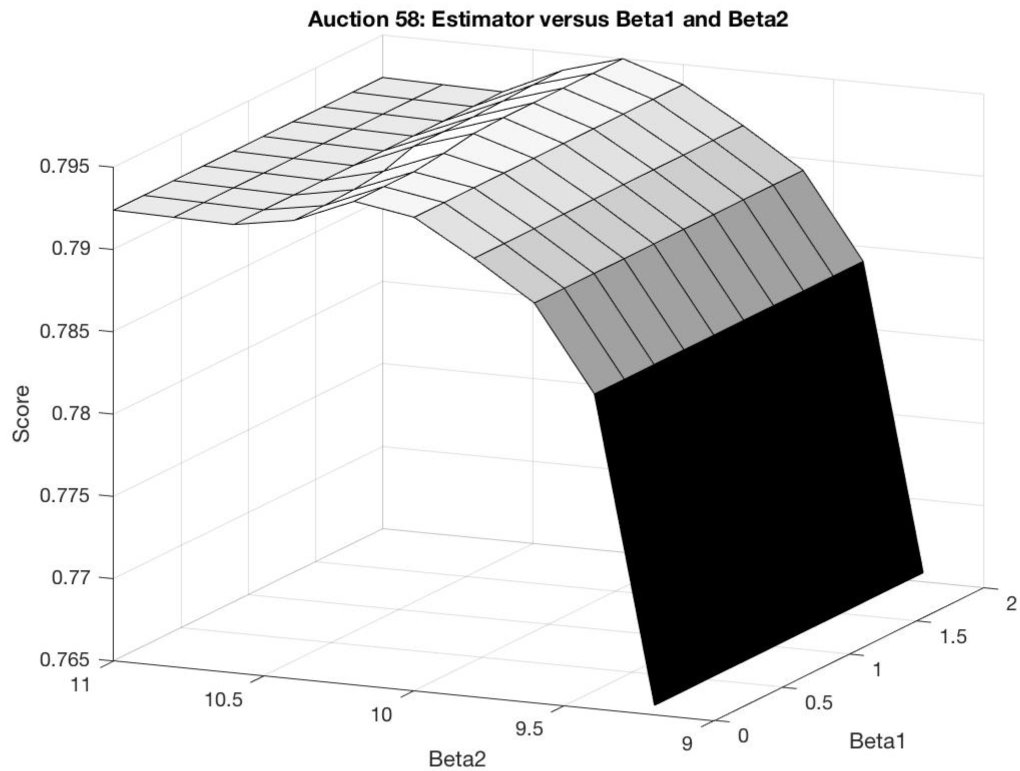

Figure A8 – Relationship of modified estimator to estimated betas for Auction 58

In the third part of this section, we show the scatter plots of each of our normalized covariates ($down_x$, $\sum_{j\in\mathbb{J}} pop_j$, $down_x$. $(\sum_{j\in\mathbb{J}} pop_j)$, $geocomp_{\mathbb{J}}$, $\sum_{j\in\mathbb{J}} \delta_{x,j}$) across Auction 5, 22, 35 and 58. For $down_x$, $\sum_{j\in\mathbb{J}} pop_j$, $down_x$. $(\sum_{j\in\mathbb{J}} pop_j)$, we use logarithmic plots to address skewness towards large values.

Scatter Plots of $down_x$

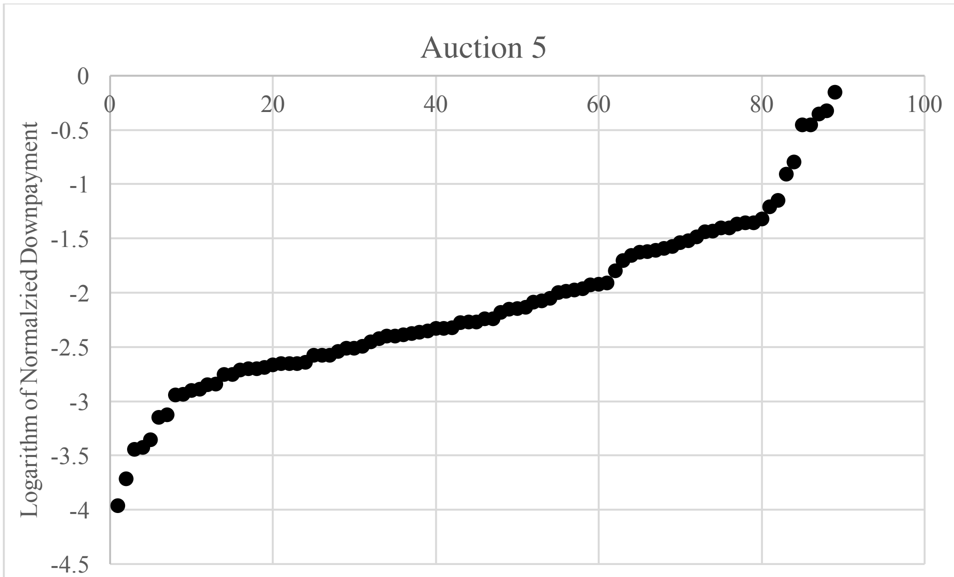

Figure A9 – Scatter plot of log (normalized down payment) for Auction 5

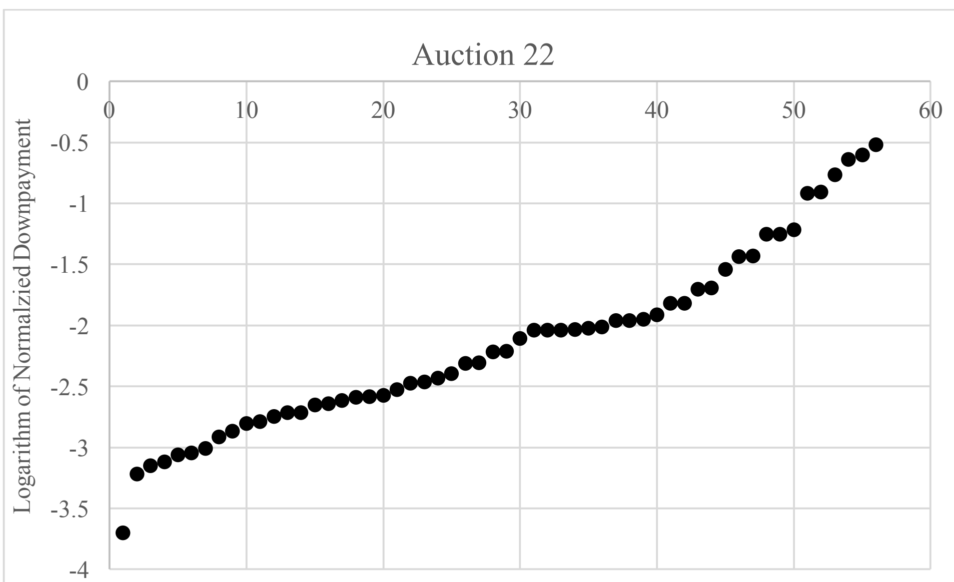

Figure A10 – Scatter plot of log (normalized down payment) for Auction 22

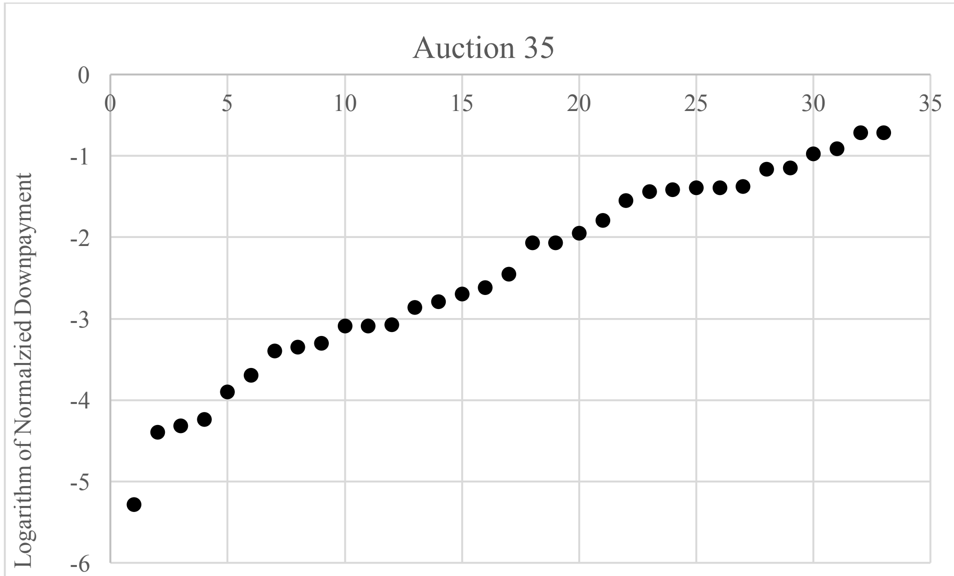

Figure A11 – Scatter plot of log (normalized down payment) for Auction 35

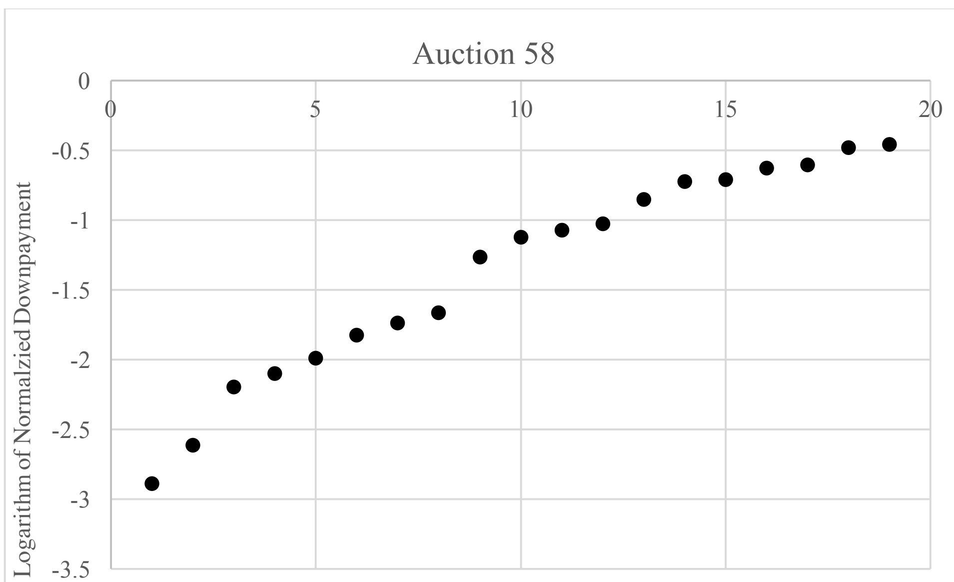

Figure A12 – Scatter plot of log (normalized down payment) for Auction 58

Scatter Plots of $\sum_{j\in\mathbb{J}} pop_j$

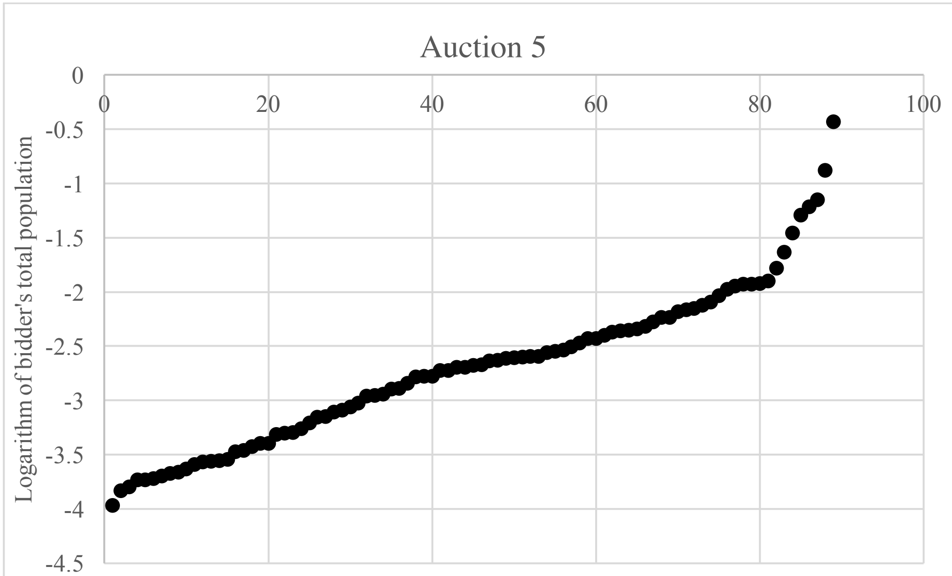

Figure A13 – Scatter plot of log (bidder's normalized total population) for Auction 5

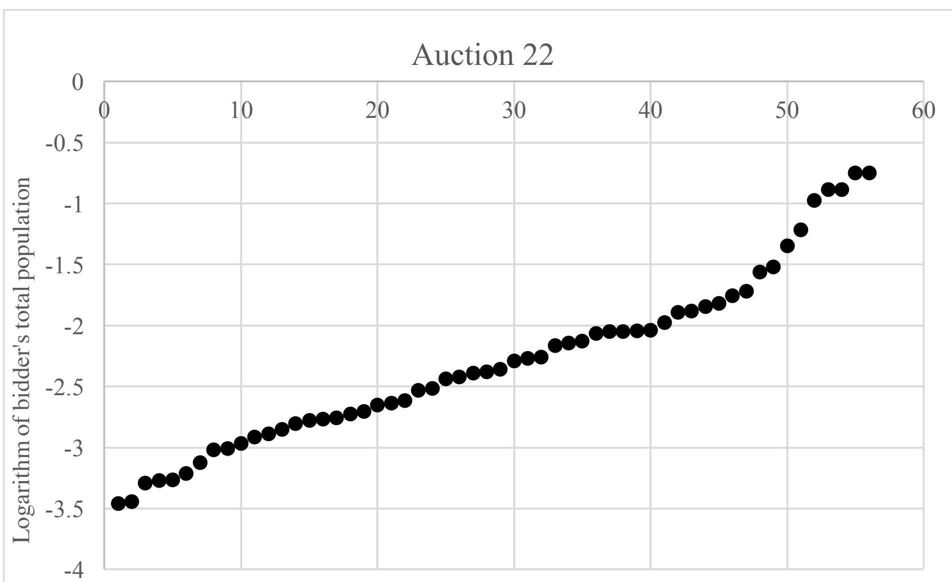

Figure A14 – Scatter plot of log (bidder's normalized total population) for Auction 22

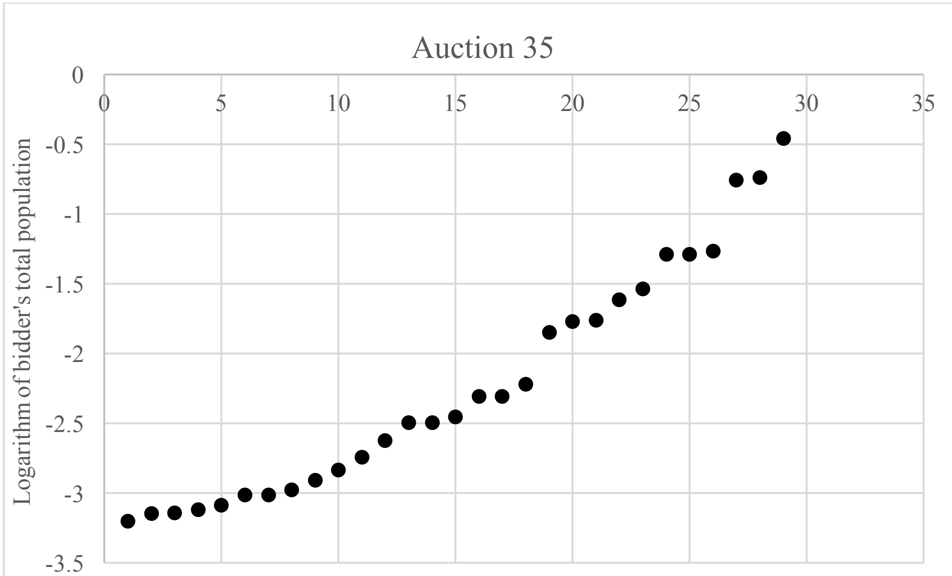

Figure A15 – Scatter plot of log (bidder's normalized total population) for Auction 35

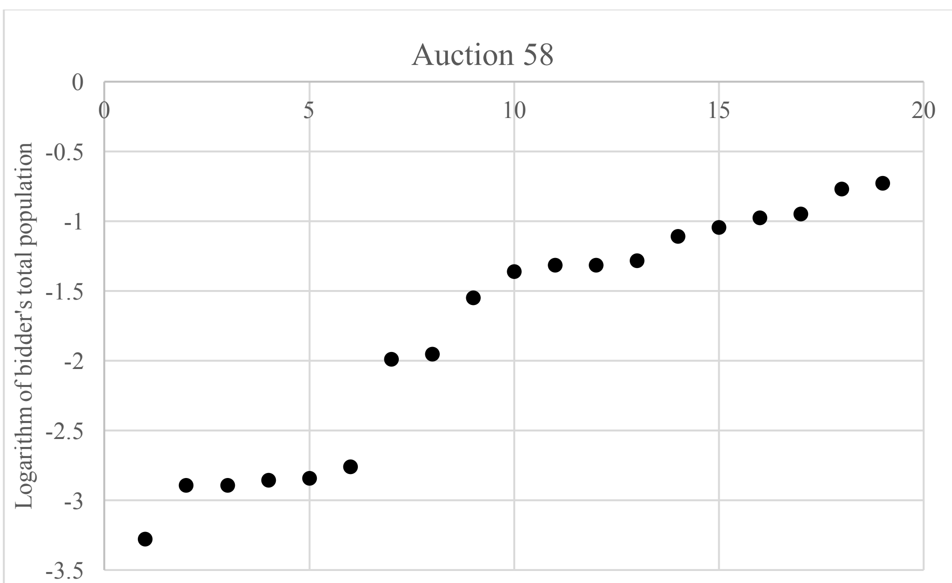

Figure A16 – Scatter plot of log (bidder's total population) for Auction 58

Scatter Plots of $\boldsymbol{down_x} \cdot \sum_{j \in \mathbb{J}} \boldsymbol{pop_j}$

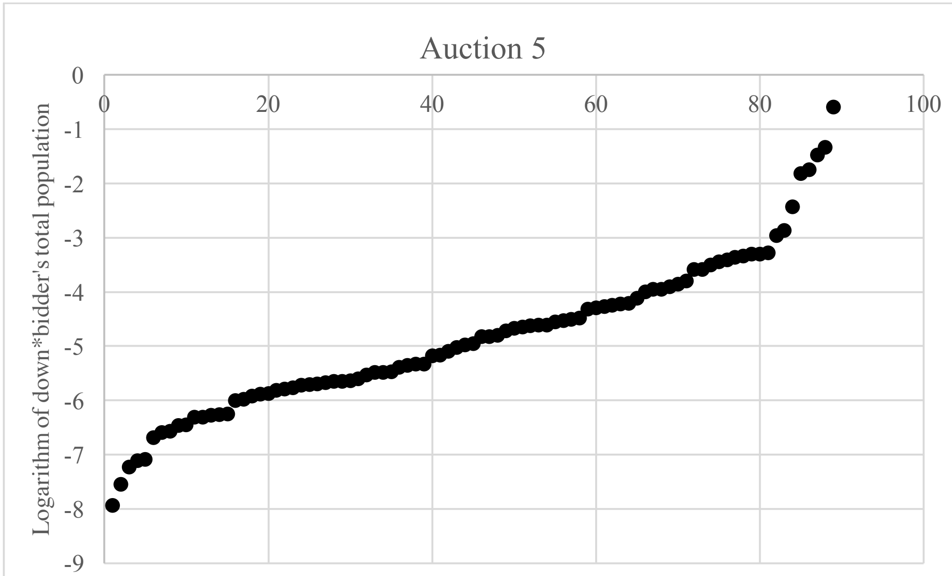

Figure A17 – Scatter plot of log (normalized down * bidder’s total population) for Auction 5

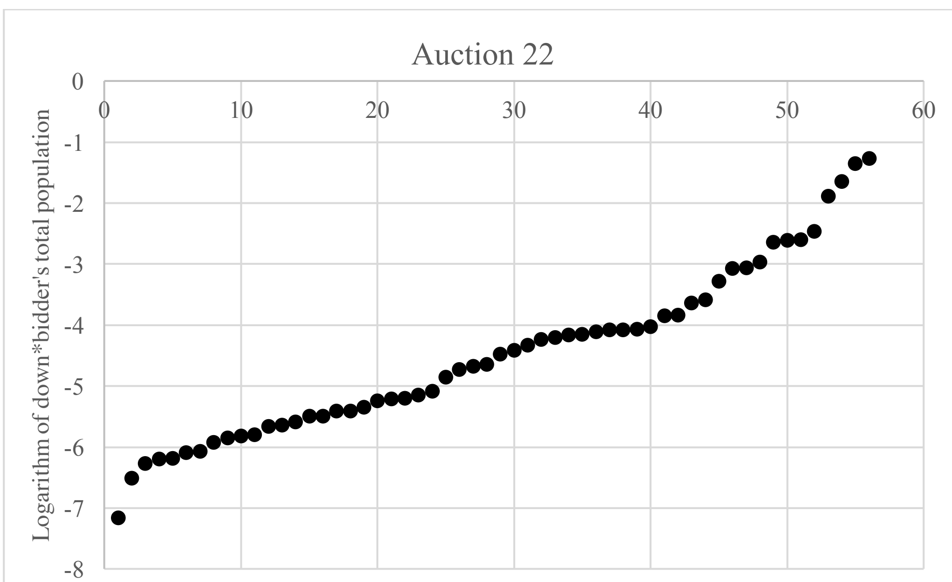

Figure A18 – Scatter plot of log (normalized down * bidder’s total population) for Auction 22

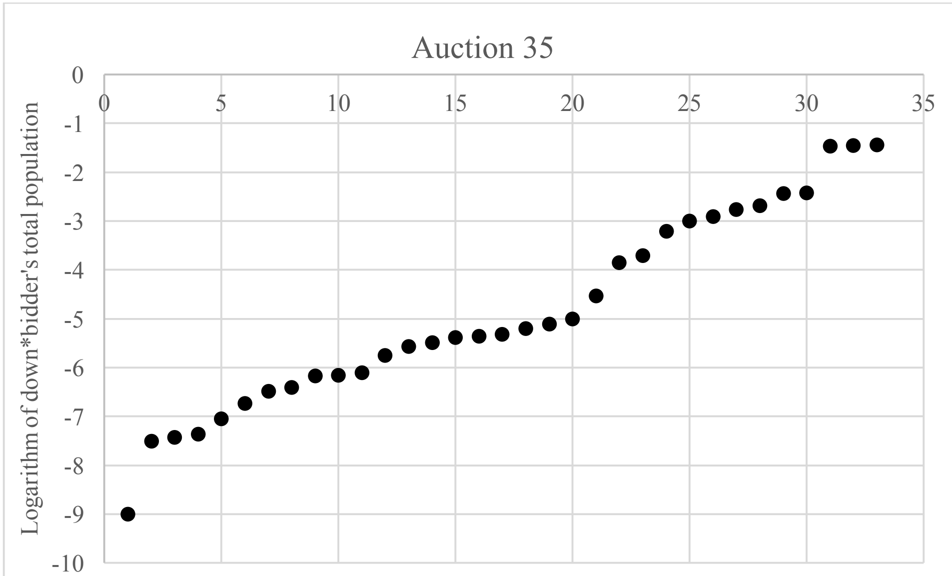

Figure A19 – Scatter plot of log (normalized down * bidder's total population) for Auction 35

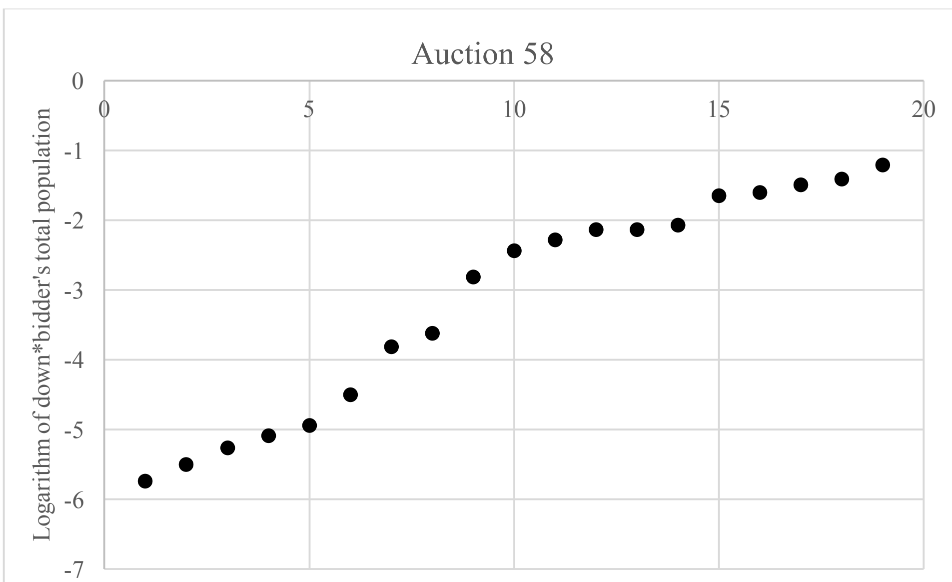

Figure A20 – Scatter plot of log (normalized down * bidder's total population) for Auction 58

Scatter Plots of $geocomp_{\mathbb{J}}$

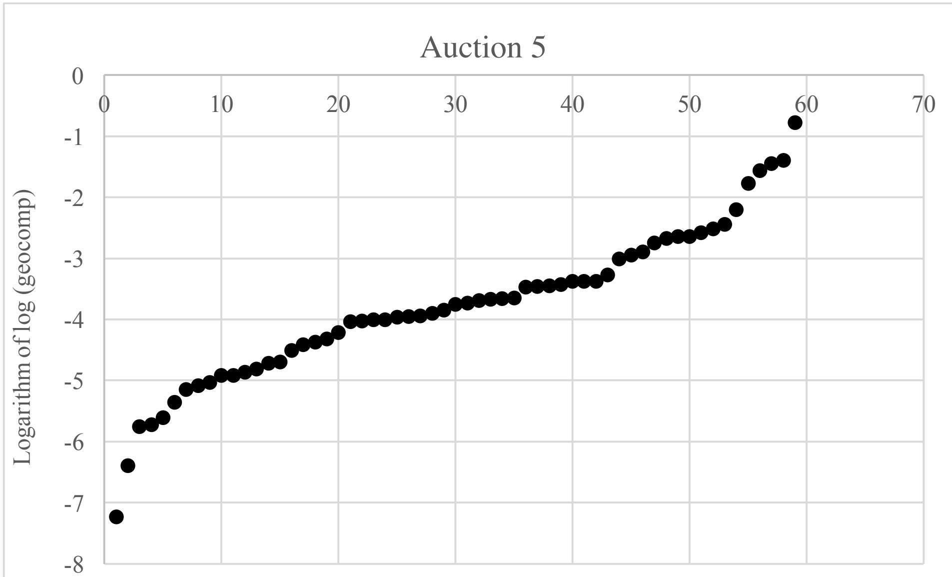

Figure A21 – Scatter plot of log (normalized geocomp) for Auction 5

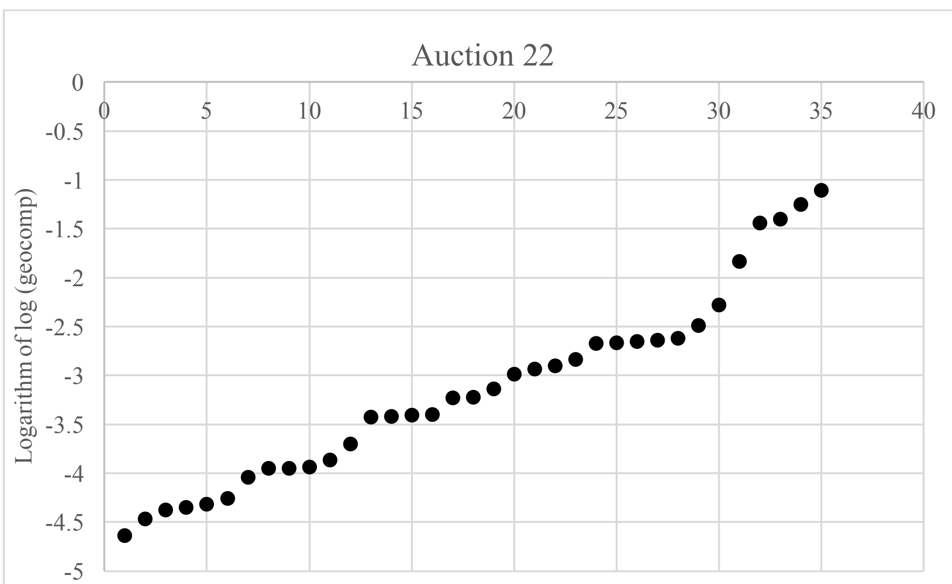

Figure A22 – Scatter plot of log (normalized geocomp) for Auction 22

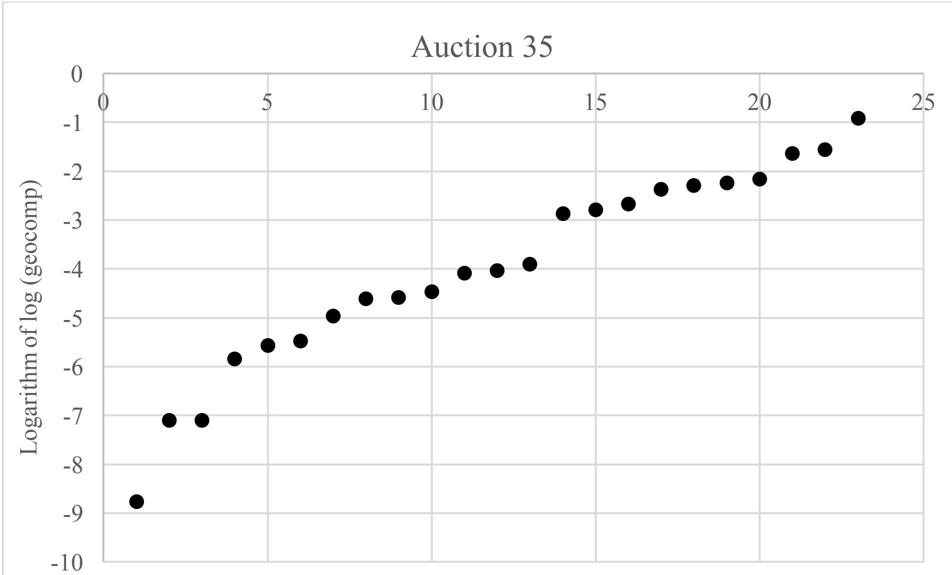

Figure A23 – Scatter plot of log (normalized geocomp) for Auction 35

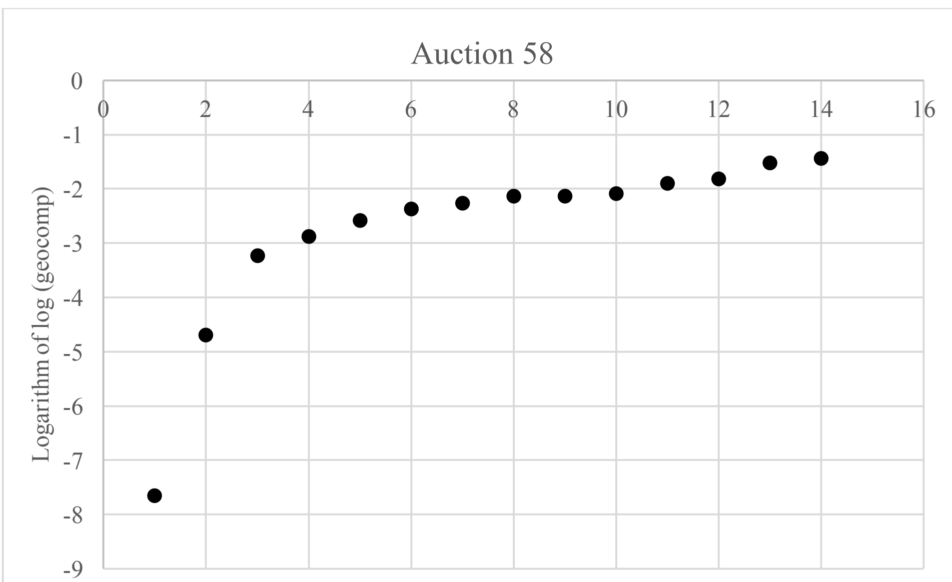

Figure A24 – Scatter plot of log (normalized geocomp) for Auction 58

Bar Graphs of $\sum_{j \in \mathbb{J}} \boldsymbol{\delta}_{x,j}$

Lastly, we show graphs of big bidders and their corresponding total $\boldsymbol{\delta}_{x,j}$. There will be no graph for Auction 5 as there is no big bidder present.

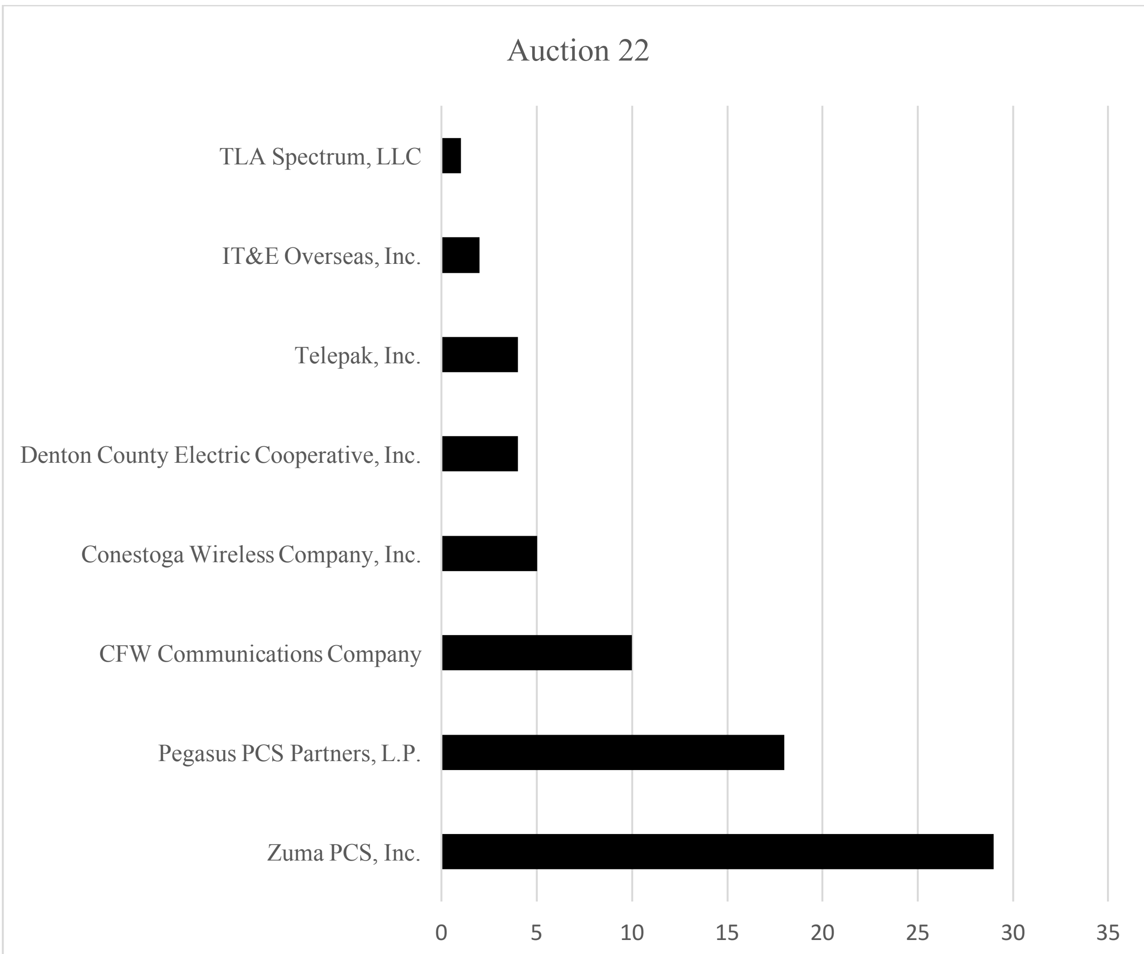

Figure A26 – Bar graph of big bidder’s total delta for Auction 22

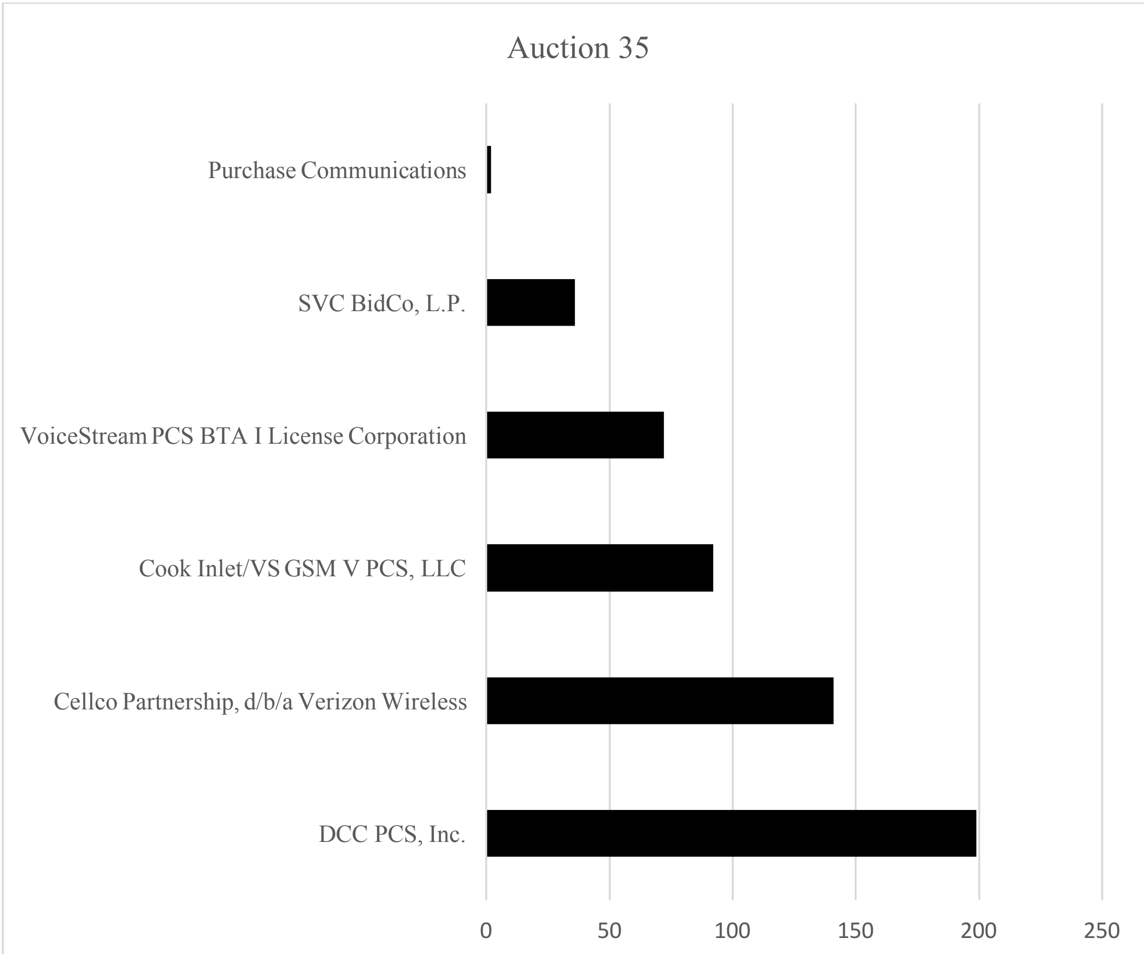

Figure A27 – Bar graph of big bidder’s total delta for Auction 35

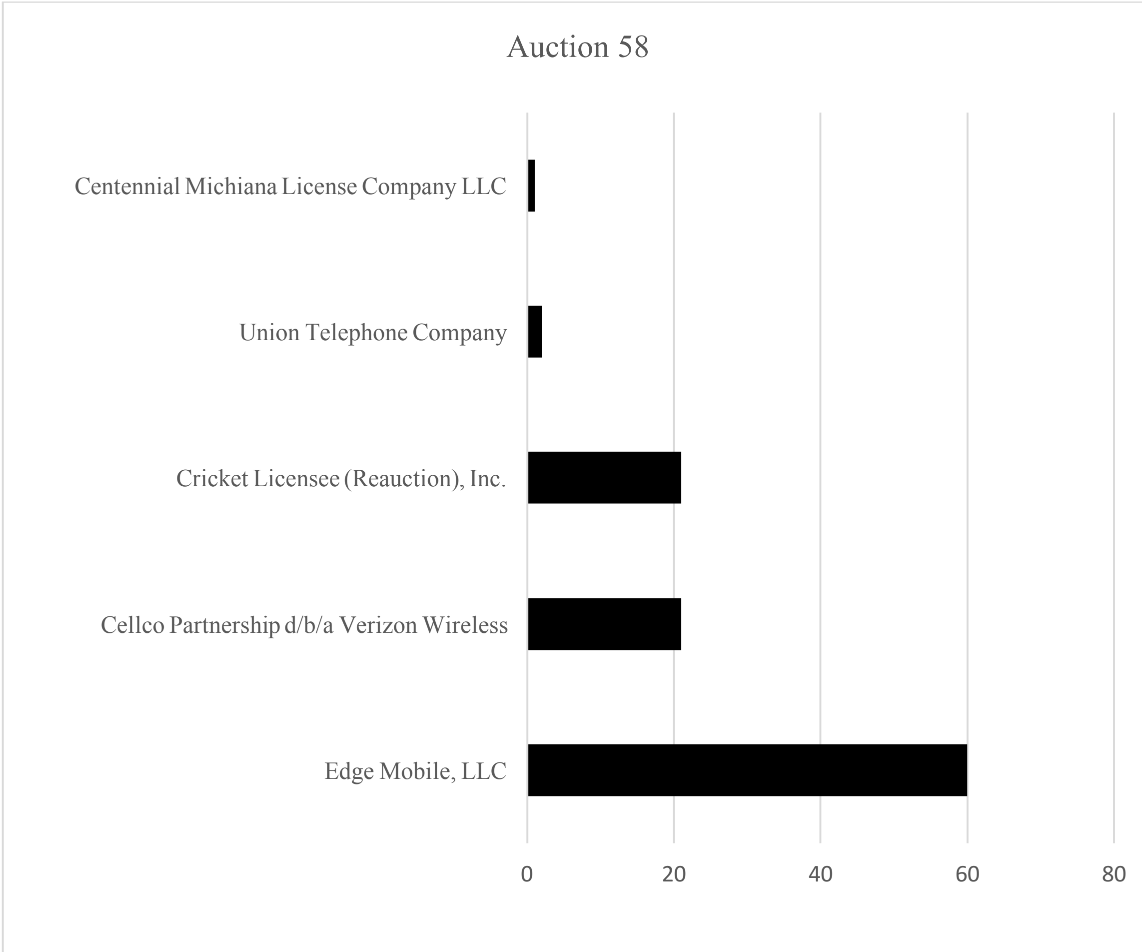

Figure A28 – Bar graph of big bidder’s total delta for Auction 58

We present the summary statistics of the $\sum_{j \in \mathbb{J}} \delta_{x,j}$ below. There will be no data for Auction 5 since there are no big bidders in that auction.

	Average	**Standard Deviation**	**Min**	**Max**
5	n/a	n/a	n/a	n/a
22	0.0310	0.0309	0.0034	0.0986
35	0.256	0.185	0.006	0.564
58	0.121	0.123	0.006	0.347

Table A1 – Summary Statistics for $\sum_{j \in \mathbb{J}} \delta_{x,j}$ of big bidders

Appendix B: Documentation of Algorithm Code

Script 1. Data

```
% License Data
Let = Data(:,3+Adj); % Latitude of license location
Lat = deg2rad(Lat); % Converting to radians
Long = Data(:,4+Adj); % Longitude of license location
Long = deg2rad(Long); % Converting to radians
Pop = Data(:,5+Adj); % Population of license location
LicenseElig = Data(:,6+Adj); % Eligibility required for license
NumLicense = length(Lat); % Total number of licenses


% Bidder Data
Elig = Data(:,12); % Eligibility of better
Elig = Elig(Elig>0); % Removing entries with non-valid eligibility
```

Notes

The data we imported were categorized into license specific data and bidder specific data. The Latitude and Longitude numbers were generated using Christos Samaras's VBA Google Geocoding Functions in Microsoft Excel.

Script 2. Indexing

```
% The following indexing allow us to identify ownership of licenses
IndexStart = Data(:,10);
IndexStart = IndexStart(IndexStart>0);
IndexEnd = Data(:,11);
IndexEnd = IndexEnd(IndexEnd>0);
IndexPure = zeros(length(Elig),2); % IndexPure for non-repeated indexes
Index = zeros(length(Lat),1); % Index for repeated indexes
NumOwner = length(IndexPure); % Number of owners
% Creation of IndexPure
for k = 1:length(Elig)
    IndexPure(k,1) = IndexStart(k);
    IndexPure(k,2) = IndexEnd(k);
end
counter = 1;
% Creation of Index
for l = 1:length(Elig)
    NumLicEachBidder = IndexPure(l,2)-IndexPure(l,1)+1;
    for m = 1:NumLicEachBidder
        Index(counter,1) = IndexPure(l,1);
        Index(counter,2) = IndexPure(l,2);
        counter = counter +1;
    end
end
```

<u>Notes</u>

IndexPure has m rows and two columns, with m representing the number of bidders winning at least one license in the auction. (Row J, column 1) tells us the first license on the list of licenses won by bidder J, and (Row J, column 2) tells us the last license on the list of licenses won by bidder J.

Index has n rows and two columns, with n representing the number of licenses in the auction. (Row j, column 1) tells us the first license on the list of licenses won by the bidder who won license j and (Row J, column 2) tells us the last license on the list of licenses won by the bidder who won license j.

Script 3. Matrix Construction

```
% PopMatrix construction
PopMatrix = meshgrid(Pop)';
for g=1:NumLicense
    PopMatrix(g,g)=0;
end
DistanceCheck = zeros(NumLicense);


% GeoMatrix Construction
GeoMatrix = zeros(NumLicense);
for k=1:NumLicense
   for l=1:NumLicense
       DistanceNum = haversine(Lat(k),Long(k),Lat(l),Long(l));
       DistanceSquared = power(DistanceNum,2);
       DistanceCheck(k,l) = DistanceNum;
       if DistanceNum == 0
           GeoMatrix(k,l) =0;
       else
           GeoMatrix(k,l) = Pop(k).*Pop(l)./DistanceSquared;
end end end
for m=1:NumLicense
    GeoMatrix(m,m)=0;
End
% Bigsmall Matrix Data
BigSmallMatrix = Data(:,14:14+NumOwner-1);
```

Notes

PopMatrix is constructed as a symmetrical matrix with the diagonals having a value of zero. It is used later on in the calculation of geocomplementarity. GeoMatrix is a helper matrix constructed to help us generate two more matrices, the Numerator and the Denominator matrices that will help us calculate the $\frac{\sum_{j \in \mathbb{J}, j \neq i} \frac{pop_i pop_j}{dist_{i,j}^2}}{\sum_{j \in \mathbb{L}, j \neq i} \frac{pop_i pop_j}{dist_{i,j}^2}}$ term in the geocomp function. BigSmallMatrix helps us calculate the $\sum_{j \in \mathbb{J}} \delta_{x,j}$ term in the valuation function.

Script 4. Geocomplementarity calculation

```
Numer = zeros(NumLicense);
for n=1:NumLicense
    Numer(n) = sum(GeoMatrix(Index(n,1):Index(n,2),n));
end
% Transposing Numer to operate on it later
Numer = Numer(:,1)';
Denom = sum(GeoMatrix);

% GeoComp by license j
GeoCompj = zeros(1,NumLicense);
for p=1:NumLicense
    GeoCompj(p) = Pop(p).*Numer(p)./Denom(p);
end
% GeoComp by bidder J
GeoCompJ = zeros(1,NumOwner);
for q=1:NumOwner
    GeoCompJ(q) = sum(GeoCompj(IndexPure(q,1):IndexPure(q,2)));
end
```

Notes

In Numer we sum across the set J for each bidder, with set J representing the licenses the particular bidder owns. In Denom we sum across the set L for each bidder, which set L representing the universal set of licenses in the auction. GeoComp J is the array containing all the geocomplementarity values for each bidder that we want to find:

$$geocomp_{\mathbb{J}} = \sum_{i \in \mathbb{J}} \left[pop_i \, \frac{\sum_{j \in \mathbb{J}, j \neq i} \frac{pop_i pop_j}{dist_{i,j}^2}}{\sum_{j \in \mathbb{L}, j \neq i} \frac{pop_i pop_j}{dist_{i,j}^2}} \right]$$

Script 5. Loop Construction

```
for run =1:1
    count = 0;
    Score = 0;
    Beta1 = 0+0.2*(run1);
    Beta2 = 9+0.2*(run2);
    for a = 1:NumOwner-1
        for b = a+1:NumOwner
            Owner1 = IndexPure(a,:);
            Owner2 = IndexPure(b,:);
            % Size tells us how many license each owner has
            Size1 = Owner1(2)-Owner1(1)+1;
            Size2 = Owner2(2)-Owner2(1)+1;
            for c =1:Size1
                for d = 1:Size2
                    count = count +1;
                    % k for bidder 1, l for bidder 2, k and l indicate the
                    % index of the cth license of owner a and dth license of
                    % owner b,
                    k = IndexPure(a,1)+c-1;
                    l = IndexPure(b,1)+d-1;
                    …
```

<u>Notes</u>

The number of runs to guess Beta is determined by the loop containing the index run. Different Beta 1 and Beta 2 values are guessed with each iteration of the loop. Loops with indices a and b consider all the possible combinations of bidder pairs. Loops with indices c and d consider all the possible combinations of license pairs to be swapped between any two bidders.

Script 6. Eligibility Check

```
% Eligibility Check
Bidder1Elig = Elig(a);
Bidder2Elig = Elig(b);
License1Elig =LicenseElig(k);
License2Elig = LicenseElig(l);

% Current Elig minus Elig being considered
Bidder1RemainingElig = Elig(a)-
sum(LicenseElig(IndexPure(a,1):IndexPure(a,2)))+License1Elig;
Bidder2RemainingElig = Elig(b)-
sum(LicenseElig(IndexPure(b,1):IndexPure(b,2)))+License2Elig;
```

Notes

The number of runs to guess Beta is determined by the loop containing the index run. Different Beta 1 and Beta 2 values are guessed with each iteration of the loop. Loops with indices a and b consider all the possible combinations of bidder pairs. Loops with indices c and d consider all the possible combinations of license pairs to be swapped between any two bidders.

Script 7. Swapping Mechanism

```
%Create temporary matrix for geocomp
GeoMatrixTemp = GeoMatrix;
%Swapping of rows for geocomp
TempGeoRow1 = GeoMatrixTemp(k,:);
TempGeoRow2 = GeoMatrixTemp(l,:);
GeoMatrixTemp(k,:) = TempGeoRow2;
GeoMatrixTemp(l,:) = TempGeoRow1;

%Swapping of columns for geocomp
TempGeoCol1 = GeoMatrixTemp(:,k);
TempGeoCol2 = GeoMatrixTemp(:,l);
GeoMatrixTemp(:,k) = TempGeoCol2;
GeoMatrixTemp(:,l) = TempGeoCol1;

%Create temporary matrix for population column vector
PopTemp = Pop;
%Swapping of rows for pop
TempPopRow1 = PopTemp(k);
TempPopRow2 = PopTemp(l);
PopTemp(k) = TempPopRow2;
PopTemp(l) = TempPopRow1;

%Create temporary matrix for bigsmall column vector
BigSmallTemp = BigSmallMatrix;
%Swapping of rows for pop
TempBidRow1 = BigSmallTemp(k,:);
TempBidRow2 = BigSmallTemp(l,:);
BigSmallTemp(k,:) = TempBidRow2;
BigSmallTemp(l,:) = TempBidRow1;
```

Notes

Swapping licenses translates to swapping rows l and k and swapping columns l and k in the script for the matrices we constructed. We performed it for the geocomp matrix, pop matrix and bigsmall matrix. The temp matrices represent the values after the swap took place.

Script 8. Calculations after swapping

```
NewNumer = Numer;
NewDenom = Denom;
NewGeoCompjTemp = GeoCompj;
NewGeoCompJTemp = GeoCompJ;

for n=Owner1(1):Owner1(2)
      NewNumer(n) = sum(GeoMatrixTemp(Index(n,1):Index(n,2),n));
      NewDenom(n) = sum(GeoMatrixTemp(1:NumLicense,n));
      NewGeoCompjTemp(n) = PopTemp(n).*NewNumer(n)./NewDenom(n);
end

for n=Owner2(1):Owner2(2)
      NewNumer(n) = sum(GeoMatrixTemp(Index(n,1):Index(n,2),n));
      NewDenom(n) = sum(GeoMatrixTemp(1:NumLicense,n));
      NewGeoCompjTemp(n) = PopTemp(n).*NewNumer(n)./NewDenom(n);
end

NewGeoCompJTemp(a) =sum(NewGeoCompjTemp(IndexPure(a,1):IndexPure(a,2)));
NewGeoCompJTemp(b) =sum(NewGeoCompjTemp(IndexPure(b,1):IndexPure(b,2)));

%Making Comparisons, stating new variables
Bidder1OrigGC = GeoCompJ(a)/USPOP;
Bidder2OrigGC = GeoCompJ(b)/USPOP;
Bidder1NewGC = NewGeoCompJTemp(a)/USPOP;
Bidder2NewGC = NewGeoCompJTemp(b)/USPOP;
Bidder1Elig = Elig(a)/ELIG;
Bidder2Elig = Elig(b)/ELIG;
Bidder1OldBig =
sum(BigSmallMatrix(IndexPure(a,1):IndexPure(a,2),a))/NumLicense;
Bidder2OldBig =
sum(BigSmallMatrix(IndexPure(b,1):IndexPure(b,2),b))/NumLicense;
Bidder1NewBig =
sum(BigSmallTemp(IndexPure(a,1):IndexPure(a,2),a))/NumLicense;
Bidder2NewBig =
sum(BigSmallTemp(IndexPure(b,1):IndexPure(b,2),b))/NumLicense;
Bidder1OldPop = sum(Pop(IndexPure(a,1):IndexPure(a,2)))/USPOP;
Bidder2OldPop = sum(Pop(IndexPure(b,1):IndexPure(b,2)))/USPOP;
Bidder1NewPop = sum(PopTemp(IndexPure(a,1):IndexPure(a,2)))/USPOP;
Bidder2NewPop = sum(PopTemp(IndexPure(b,1):IndexPure(b,2)))/USPOP;
```

Notes

Adjustments were first made to recalculate the Numer, Denom and GeoComp matrices after the swapping took place. The GeoComp (GC), Eligibility (Elig), Dummy variable (Big) and Population (Pop) terms were calculated for both bidder 1 and bidder 2 for the cases before the swap and after the swap.

Script 9. Scoring Mechanism

```
%Making Valuation Function
Bidder1OldValue=
Bidder1Elig.*Bidder1OldPop+Beta1.*Bidder1OrigGC+Beta2.*Bidder1OldBig;
Bidder2OldValue=
Bidder2Elig.*Bidder2OldPop+Beta1.*Bidder2OrigGC+Beta2.*Bidder2OldBig;
Bidder1NewValue=
Bidder1Elig.*Bidder1NewPop+Beta1.*Bidder1NewGC+Beta2.*Bidder1NewBig;
Bidder2NewValue=
Bidder2Elig.*Bidder2NewPop+Beta1.*Bidder2NewGC+Beta2.*Bidder2NewBig;

if (Bidder1OldValue + Bidder2OldValue) >= (Bidder1NewValue + Bidder2NewValue)
                            Score = Score +1;
                        end
                end
            end
        end
    end


ScorePercent = Score/count;
Results(run1+1,1) = Beta1;
Results(1,run2+1) = Beta2;
Results(run1+1,run2+1) = ScorePercent;
end
end
```

<u>Notes</u>

The combined valuation of bidder 1 and bidder 2 before the swap was compared to the combined valuation of bidder 1 and bidder 2 after the swap. If the pre-swap combined valuation was larger than or equal to the post-swap valuation, we added one to the score.

The percentage score was calculated by taking the number of cases in which pre-swap valuation did at least as well as post-swap valuation divided by the number of swaps that were considered overall.

Appendix C: Analysis of Alternative Estimator

In our paper, our estimator examines the case of a one-to-one matching transfer, where bidder *a* adds a single license *j* to its license set J after giving up a single license *i* to bidder *b*. It is defined as a matching transfer because the quantity of licenses exchanged is equal in both parties. It is possible to examine alternative estimators by using inequalities based on different theoretical conditions so as to better reflect bidders' actions and improve upon the definition of pair-wise stability. As an extension, our paper will provide a precursory understanding to the expected computational intensity of using these alternative estimators, namely, a non-matching transfer and a multi-degree transfer. We show a summary of expected inequalities and corresponding runtime for a single guess of beta for each alternative estimator.

It is important to note that the algorithm runtime is largely determined by the number of inequalities compared in each auction, which is further determined by the number of owners and the number of licenses each owner owns respectively. By extrapolating the number of inequalities expected when an alternative estimator is used, one can arrive at a predicted runtime.

The table below provides a summary of one-to-one matching swap, the case we have studied.

Auction	Number of Inequalities	Approximate runtime for 1 loop
5	117,300	~38s
22	39,407	~12s
35	54,535	~26s
58	13,150	~3s

Table C1 – Number of inequalities and approximate runtime of Auction 5, 22, 35 and 58 for one-to-one swap only

In the case of a two-to-two matching swaps, where bidders exchange a set of two licenses instead of a single license, we found out that the number of inequalities and runtime increased exponentially.

Auction	Expected number of Inequalities	Predicted runtime for 1 loop
5	6,258,786	~34min
22	4,495,587	~24min
35	18,620,859	~100min
58	1,074,801	~6min

Table C2 – Number of inequalities and approximate runtime of Auction 5, 22, 35 and 58 for two-to-two matching swap

The primary reason behind this is that a single owner now has a larger number of possible combinations of a set of two licenses, specifically, for an owner holding *x* number of licenses, he will have a total of $\frac{x!}{2!(x-2)!}$ possible combinations of a set of two licenses. It is important to note that the presence of owners with a large set of licenses will greatly increase the number of inequalities being considered because there are now much more possible combinations of a set of two licenses.

Non-matching swaps

The imposition of a strictly matching swap can also be relaxed to allow owners to exchange licenses with one another for a non-equal return. A non-matching swap is thus defined as an exchange of licenses between owners, where the quantity given and received for each owner is not the same. In this scenario, we would explore the case where each owner is allowed to give one license to another without anything in return and the case where each owner is allowed to give two license to another without anything in return. The results are reported in the table below:

Auction	Expected number of Inequalities	Predicted runtime for 1 loop
5	43,384	~14s
22	16,170	~5s
35	11,649	~3s
58	3,460	~1s

Table C3 – Number of inequalities and approximate runtime of Auction 5, 22, 35 and 58 for give-one only

Auction	Expected number of Inequalities	Predicted runtime for 1 loop
5	230,724	~75s
22	128,240	~41s
35	182,232	~59s
58	25,920	~8s

Table C4 – Number of inequalities and approximate runtime of Auction 5, 22, 35 and 58 for give-two only

n-degree swap

Lastly, we allow for the possibility of a n-degree swap. A n-degree swap is defined as an exchange of licenses among a party of *n+1* owners where n > 1. As a form of simplification, we study the case of a two-degree swap of a single license. A two-degree swap will then be an exchange between three owners where Owner 1 first swaps a specific license X with Owner 2, who then accept the swap, knowing that he can further swap it with Owner 3. The results are reported in the table below:

Auction	Number of Inequalities	Approximate runtime for 1 loop
5	36,909,493	~3hrs
22	6,990,689	~37min
35	672,327	~3min
58	1,360,113	~7min

Table C5 – Number of inequalities and approximate runtime of Auction 5, 22, 35 and 58 for a two-degree swap of a single license

The primary reason for this large increase in runtime is that the possibilities compound every time an owner transfers the license to the next. Given this, a n-degree swap will be the most computationally intensive as n increases.

The algorithm employed in this paper is brute-force in nature and works by exhausting all possibilities to determine the estimator. Given this, it is unlikely that such an algorithm will be feasible to solve for the estimator in the case of a n-degree swap, especially for large auctions.

Appendix D: Valuations of bidders in different auctions

In the section, we show cumulative distribution plots for the normal and modified valuation functions of bidders across all auctions. We use the betas reported in the Results to generate a cumulative plot of bidders' valuations.

We first show the plots for the normal valuation function, which is restated below-

$$\pi_{\vec{\beta}}(x,\ \mathbb{J}) = \pm 1.down_x\,.(\textstyle\sum_{j\in\mathbb{J}} pop_j) + \beta_1.geocomp_{\mathbb{J}}.$$

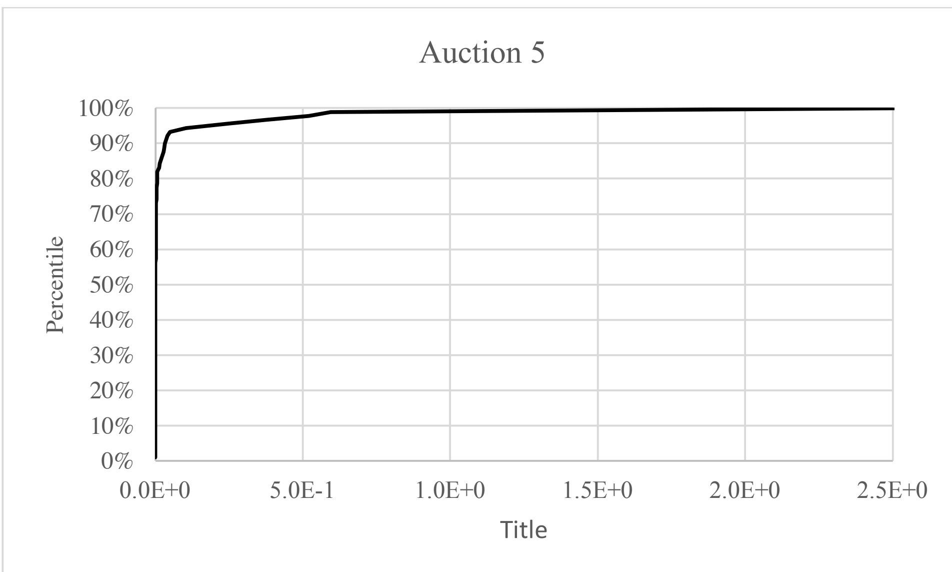

Figure D1 – Cumulative distribution plot for valuation functions of bidders in Auction 5

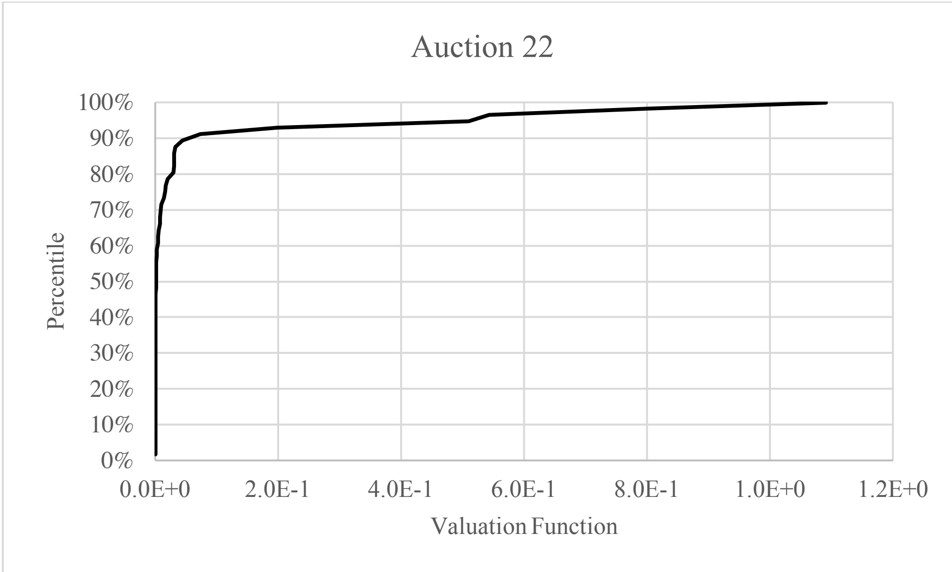

Figure D2– Cumulative distribution plot for valuation functions of bidders in Auction 22

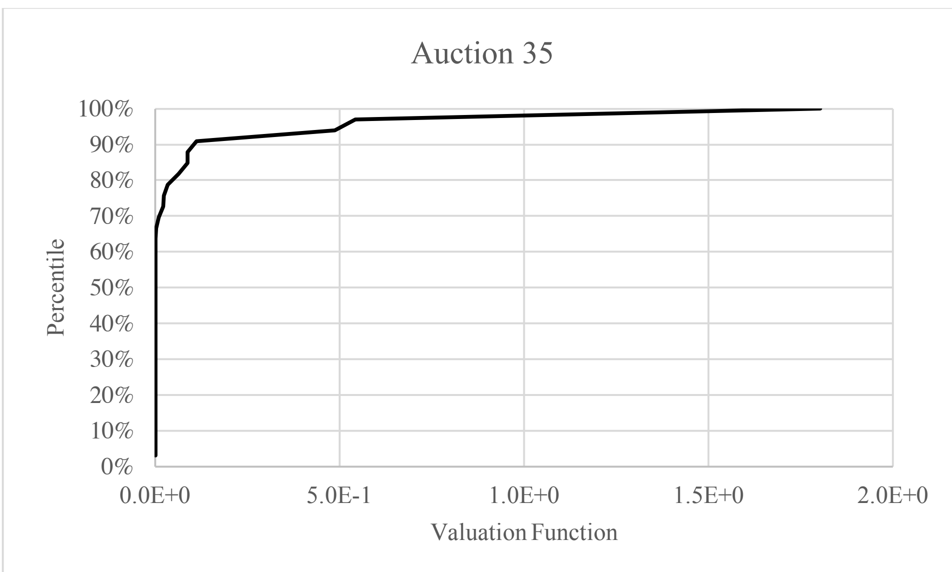

Figure D3 – Cumulative distribution plot for valuation functions of bidders in Auction 35

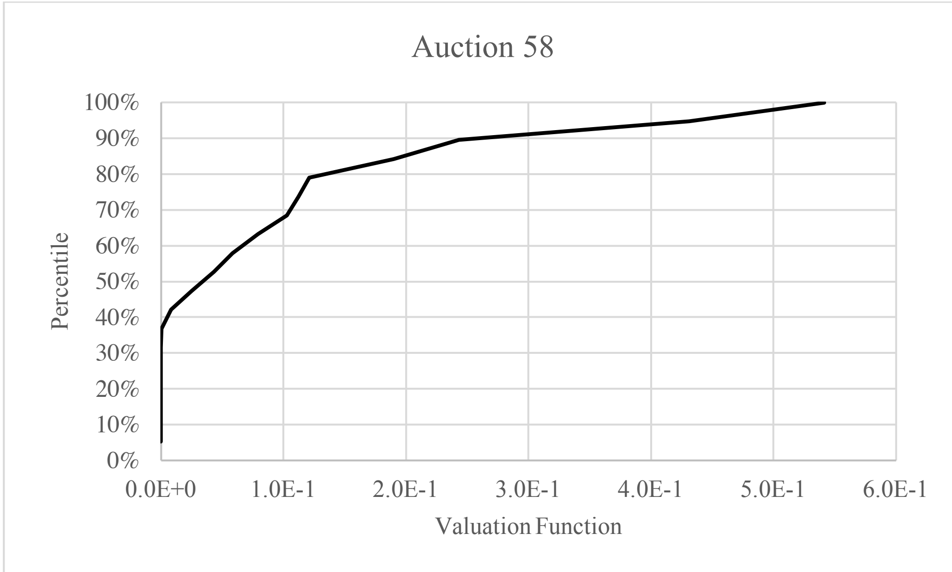

Figure D4 – Cumulative distribution plot for valuation functions of bidders in Auction 58

We then show the cumulative plots for the modified valuation function, which is restated below-

$$\pi_{\vec{\beta}}(x, \mathbb{J}) = \pm 1. down_x . (\sum_{j \in \mathbb{J}} pop_j) + \beta_1 . geocomp_{\mathbb{J}} + \beta_2 . \sum_{j \in \mathbb{J}} \delta_{x,j}$$

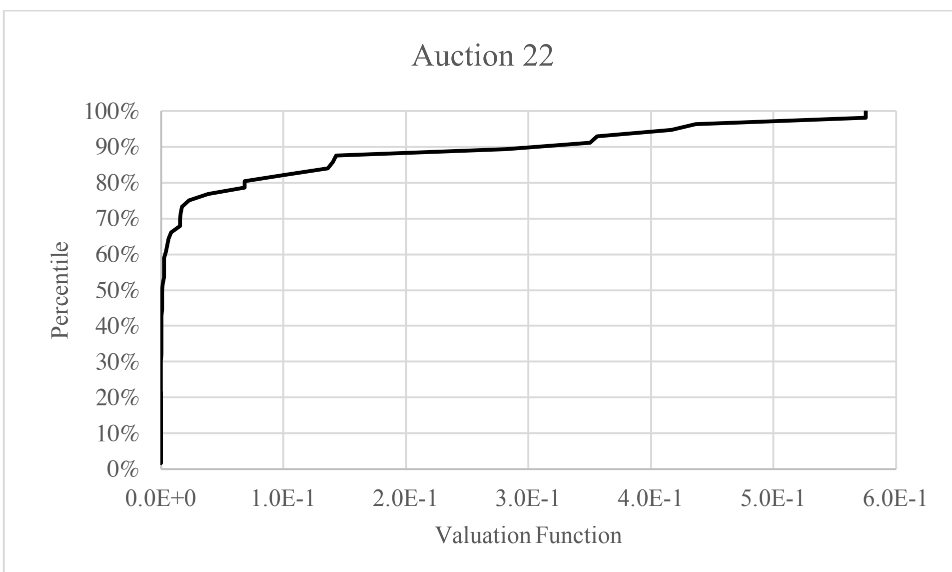

Figure D5 – Cumulative distribution plot for modified valuation functions of bidders in Auction 22

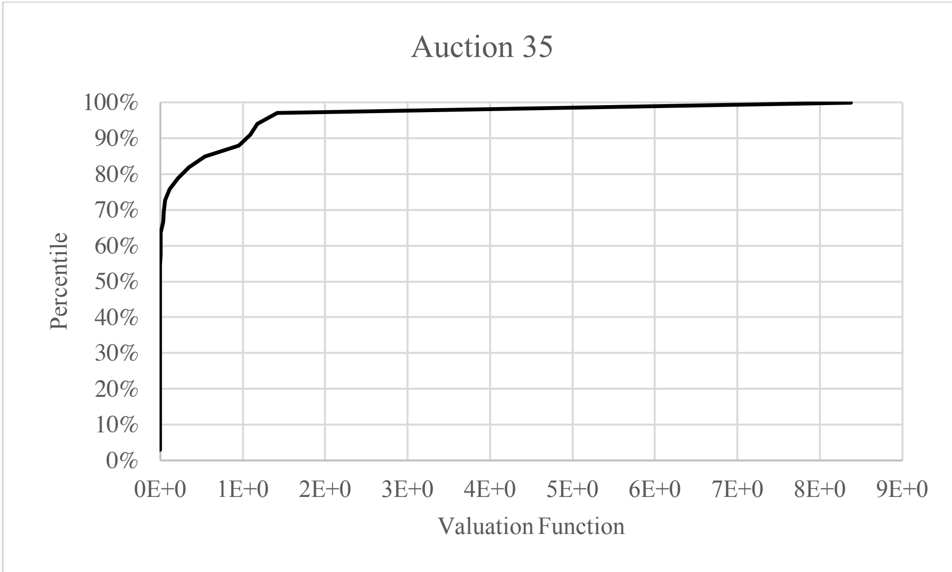

Figure D6– Cumulative distribution plot for modified valuation functions of bidders in Auction 35

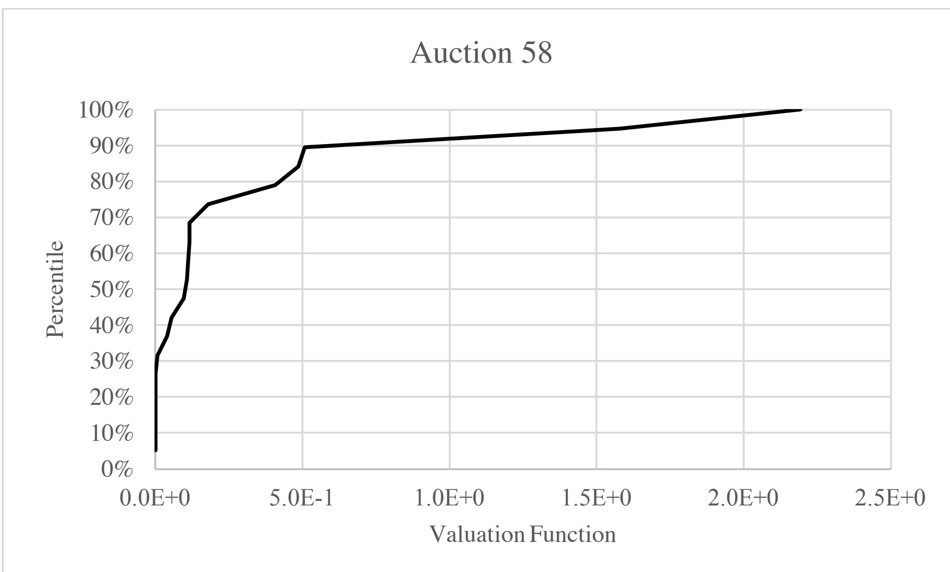

Figure D7 – Cumulative distribution plot for modified valuation functions of bidders in Auction 58

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