

# Monetary Unions and Long-Run Growth

Levi G. Crews

*Professor Pietro Peretto, Faculty Advisor*

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## Abstract

This paper develops two complementary models of monetary unions and long-run growth. The key result is that a reduction in foreign exchange costs via monetary unification provides a positive growth effect for member nations. This growth effect may come through increased knowledge spillovers in the deterministic model or through the migration of funds to higher-yield investments in the stochastic model. Empirical evidence is presented that generally supports both of these channels of growth.

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*“It is . . . the costs of transacting that are the key obstacles that prevent economies and societies from realizing well-being.”*

– Douglass North, “Institutions and Economic Growth: A Historical Introduction”

## 1 Introduction

Recent crises in Europe have led to the reconsideration of the viability of monetary unions. With persistent youth unemployment in the Mediterranean, asymmetric shocks from austerity, and the reemerging possibility of Grexit in the wake of Brexit, the costs of membership in a monetary union seem high. But the benefits could far outweigh the costs if joining a union provides not only a level increase in income but a sustained rise in the growth rate of income.

The potential for economic growth to transform the income of a region is a historical certainty: consider the Four Asian Tigers of the late twentieth century or the subsequent feline economies of the Baltics and Southeast Asia. In the case of monetary unions, however, this potential dynamic gain has gone almost wholly unconsidered. The goal of this paper is to take a step towards modeling this potential growth effect by establishing a sort of upper bound on growth gains from forming a monetary union. These gains will flow from the diminished transaction costs implicit in the formation of a union. The theory I develop is distinct from traditional optimum currency area theory, which deals almost exclusively with static models and sticky prices. It also provides a more complete picture of the importance of transaction costs than much of the open-economy endogenous growth literature.

The description of a monetary union that I adopt is largely due to Bayoumi (1994). In a two-region world, each region chooses its exchange rate regime with respect to the other. If region A chooses a separate currency from region B, the exchange rate is allowed to vary. This flexibility results in a transaction cost in all trades that require an exchange of currency. I will refer to this as a foreign exchange cost. If regions A and B choose to form a monetary union, the exchange rate is fixed at unity and foreign exchange costs are null.

I will present two dynamic models: one deterministic with trade in capital goods and one stochastic with trade in assets. In both growth is driven by the steady invention of specialized intermediate goods that are used in the production of the final consumption good. The deterministic model supposes that these intermediates are traded internationally at a cost, while the stochastic model instead presumes that claims to the profits from the sale of these intermediates are what is traded.

The open-economy dynamics of the deterministic model are adapted from Rivera-Batiz and Romer (1991). An open world economy comprised of two identical regions is considered. The production technology for final output, common across regions, is a function of labor and specialized intermediates. These intermediates are produced by intentional R&D and sold in a monopolistically-competitive and globally-integrated market. R&D is knowledge-driven and subject to spillovers. Open-economy growth rates

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are strictly higher than autarkic growth rates because of increased spillovers.

From this base my model makes its first key deviation: it supposes that knowledge spillovers are not complete across borders in the presence of foreign exchange costs. In particular, knowledge spillovers are assumed to be dependent on the volume of trade, which is necessarily dependent on the costs of foreign exchange. Insofar as foreign exchange costs limit knowledge spillovers and R&D, the formation of a monetary union between two suitable economies should boost growth.

But I aim to support an even stronger claim: Insofar as the formation of a monetary union *simply lowers foreign exchange costs*, it will boost growth. That is, the assumed connection between foreign exchange costs and knowledge spillovers is not necessary for our result. To support the claim, I extend the model to asset markets for claims to the profits of the aforementioned R&D firms. This section is necessarily stochastic in order to motivate trade in assets. The dynamics are borrowed from Obstfeld (1994), in which households shift their portfolio from safe, low-yield assets to risky, high-yield ones as asset markets integrate globally. Monetary unification, by diminishing foreign exchange costs, spurs this shift in assets. Growth results from this migration of funds to high-yield investments.

For simplicity, this model is entirely populated with real variables. In particular, I have assumed that the essence of a monetary union can be captured without considering a nominal instrument. It would not be difficult to add nominal variables to the extant model, though it may be notationally cumbersome. Moreover, I have not considered the political institutions that are so deeply intertwined with the decision to unify under a common currency. A natural extension would be to populate each region with a decision maker who chooses to pursue unification based on some criteria.<sup>1</sup>

A key assumption in the model is that the two countries are already well-suited for monetary unification. Therefore, I make no attempt to recommend monetary unification in the real world. I remain silent with regards to what regions or nations, including those in Europe, are suitable enough to satisfy the model. Furthermore, if two regions happen to be suitable enough, the model does not suggest that unification is imminent. The costs of unifying could be prohibitive or at least high enough to still outweigh the growth gains. In this respect the model is merely a small step towards a much richer but fallow field of inquiry.

The paper proceeds as follows. In the remainder of Section 1, I review the relevant literature. In Section 2, I develop and solve the deterministic model with trade in capital goods and briefly present empirical evidence. In Section 3, I do the same with the stochastic model. Section 4 concludes and posits paths for further work.

## 1.1 Literature Review

This paper is at the intersection of two distinct strands of literature: optimum currency area theory and endogenous growth theory. In this section, I review the papers from each

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<sup>1</sup>Stiglitz (2016) argues that the problems in the euro zone mentioned earlier are almost entirely due to inadequate institutions. With the right institutions, then, it is increasingly likely that the growth effect from unification would outweigh the remaining costs of inflexibility in monetary policy and the like.

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strand that are most pertinent for the present analysis. I will then discuss the little work that has been done at their intersection to date.

**Optimum Currency Area Theory:** The first strand consists of the definition and analysis of optimum currency areas. Mundell (1961) is the first to define an optimum currency area: a region in which it would maximize some measure of economic efficiency if the region had a single currency. He bucks the traditional idea that currencies must be bound by national borders, instead proposing that the appropriate domain of a currency area could be strictly within or across borders. He argues that while transaction costs are lower in a currency union, a fixed exchange rate could be costly in the face of asymmetric disturbances and price rigidities. Therefore, the optimum currency area must have a high internal factor mobility (limiting the costs of asymmetric shocks) but low external factor mobility (capturing all the gains from lower transaction costs). Hence Mundell considered factor mobility the main criterion for determining an optimum currency area.

Subsequent papers expanded the menu of criteria. McKinnon (1963) argues that the openness of the economy, i.e., the ratio of tradable to non-tradable goods, should be another important criterion. In particular, he argues that the volume of trade within an optimum currency area should be high but the volume with respect to the outside should be low. Kenen (1969) argues that the degree of product diversification must also be considered. Regions with high product diversification, he suggests, would be better able to maintain a currency union than those with low diversification since the latter are subject to larger disturbances.

Together these three papers represent the core theory of optimum currency areas and have been the benchmark for all subsequent work. Bayoumi (1994) and Melitz (1996) have worked to formalize these merely verbal arguments with general equilibrium models of optimum currency areas. Both describe the benefits of monetary unification as a reduction in iceberg costs. The costs of unification, however, are described differently: Bayoumi (1994) considers a rise in unemployment from wage rigidities while Melitz (1996) considers a decrease in the speed of adjustment of the terms of trade to their long-run equilibrium level.

Both these models are static and therefore able to determine optimality by weighing current benefits against costs. In contrast, the present paper presents a dynamic model that suggests the possible magnitude of long-run gains from accelerated growth. This is necessary for an accurate discussion of the optimality of a currency area. Since any costs and benefits considered by Bayoumi (1994) and Melitz (1996) would be dwarfed by even modest gains in growth, however, I at present lose the ability to pin down an optimum. Nevertheless, the potential benefits from accelerated growth are too massive to ignore.

As a final note, Tavlas (1993, 1994) review the resurgence of optimum currency area research in the wake of European integration. Here Tavlas classifies levels of monetary integration that will be useful to consider. He defines monetary integration as having irrevocably fixed exchange rates, full convertibility of currencies, financial market integration, and complete liberalization of movements on current transactions. Monetary unification, then, is monetary integration plus a single currency and common central bank. This is the definition of a monetary union that I am implicitly considering in later sections.

**Endogenous Growth Theory:** This strand explores mechanisms through which

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growth rates of output are endogenously determined by market parameters and consumer preferences. Romer (1986) presents the first such mechanism: knowledge spillovers from production. Disembodied and non-rival technological knowledge is assumed to be proportional to the extant capital stock so that productivity and income grow perpetually.

Romer (1990) employs another mechanism: expanding varieties of specialized intermediate goods used in production of the consumption good. This production technology is assumed to be additively-separable in the intermediates so that there is always positive demand for a new intermediate. The pace of creation of intermediates is assumed to be proportional to the existing stock of intermediates as a form of knowledge spillovers. In this way Romer (1990) can be considered as a more micro-founded version of Romer (1986).

Rivera-Batiz and Romer (1991) consider an open-economy version of this model. Here two different specifications of R&D are considered: the knowledge-driven and the so-called “lab equipment” specifications. The former is simply the assumption that the pace of intermediate creation is proportional to intermediates created. The latter, on the other hand, assumes that the pace of intermediate creation is proportional to final goods production. Rivera-Batiz and Romer show that trade in goods alone will not raise the growth rate under the knowledge-driven specification but will under the lab equipment one. If knowledge is allowed to flow unabated across borders, however, both will increase growth.

The model in Section 2 uses the knowledge-driven specification of R&D. While Rivera-Batiz and Romer consider only completely-free or completely-prohibited flows of knowledge, however, I will consider the case in which the flow of knowledge can be partial. In particular, I will assume that the flow of knowledge is inversely related to the foreign exchange cost. So long as this cost is not complete, the growth rate will increase in both economies.

It is important to note that other open-economy extensions of expanding-varieties models have shown that integration does not boost growth absolutely. Grossman and Helpman (1989), for example, show that an increase in growth does not necessarily lead to an increase in welfare if growth is boosted by R&D subsidies. Similarly, Grossman and Helpman (1990) show that positive shocks in consumer demand and labor could lead to lower worldwide growth if they occur in the nation with a comparative disadvantage in R&D. These results suggest that the present model may be sensitive to the assumption that the two economies are identical. Even if this is the case, however, that may simply imply further criteria for determination of an optimum currency area.

Each of the previous three models considers how international trade in goods may affect growth by shifting resources among alternative productive uses. The following works extend this reasoning to financial markets: a pure expansion of opportunities for trade across states of nature, that is, for trade of market assets, may itself promote research allocations favorable to long-run growth. The simplest example is Pagano (1993). He presents an AK model of growth in which a constant fraction of saving is lost in the process of financial intermediation. The growth rate, which depends on the saving rate, thereby becomes dependent on the cost of financial intermediation. Insofar as equity trades, for example, are costly, that cost limits growth.

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Stochastic models, as presented in Section 3, provide deeper intuition concerning the benefits of developed financial institutions. The general hypothesis is this: financial institutions accelerate growth insofar as they facilitate the migration of funds to the best user, that is, the one that will yield the highest expected returns. King and Levine (1993), for example, consider financial institutions that evaluate prospective entrepreneurs and fund the most promising ones. They find that the rate of innovation and growth is negatively proportional to the level of financial sector distortions. Others argue that intermediaries, through their ability to mitigate risk and pool funds, draw funds to higher-risk investments with higher returns. Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Saint-Paul (1992), Obstfeld (1994), and Acemoglu and Zilibotti (1997) present models with this line of analysis.

The model in Section 3 fits somewhat neatly into this literature insofar as it shows that foreign exchange costs lead to inefficient financial markets. Their alleviation should boost the average return on investment in productive capital and thereby drive growth. This structure necessarily deviates from the perfect capital markets purported in the literature.

**Work at the Intersection:** This strand consists of a few pieces of empirical work with positive but weak results. Baldwin (1989) predicts that European unification in 1992 would raise the long-run growth rate by about 0.8% across member nations. He does so by calibrating two endogenous growth models: the Romer (1986) model and an R&D-driven model akin to Romer (1990). He discounts his results, however, citing a lack of strong data.

In a series of studies, Schnabl has investigated the impact of exchange rate volatility on growth in the developing world. Schnabl (2007) examines the small open economies at the EMU periphery. Schnabl (2009) and Hoffman and Schnabl (2011) consider cross-country panels from Emerging Europe, East Asia, and Latin America. Each time the estimations provide evidence for a negative impact of exchange rate volatility on growth. That is, having fixed exchange rates is predicted to boost growth. Schnabl's explanation is that fixed exchange rates may provide a more stable framework for the adjustment of asset and labor markets of countries in the economic catch-up process thereby accelerating growth. My model provides a theoretical base upon which future empirical work of this sort may be completed.

## 2 The Deterministic Model

The model presented here builds from Romer (1990). Growth is driven by the perpetual invention of specialized intermediate capital goods. The rate at which these goods are invented depends on the amount of labor engaged in research and the extent of knowledge spillovers from international trade. Trade is presumed to increase as foreign exchange costs fall.

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## 2.1 Deterministic Growth in a Closed Economy

In order to develop intuition, I first solve the deterministic model in a closed economy. The economy is populated by three types of agents. First, a continuum of mass one of producers hire labor and specialized intermediate goods to produce a final good that is sold at unit price. Second, research firms pay a fixed cost to develop new intermediates, which are sold at a constant mark-up above marginal cost in a monopolistically-competitive market. Third, a representative consumer maximizes household utility.

### 2.1.1 Producers of the Final Good

Following Romer (1987, 1990), a representative producer of the final good combines labor and a number of specialized intermediate goods according to the production technology

$$Y = (\phi L)^{1-\alpha} \sum_{i=1}^N X_i^\alpha \quad (2.1)$$

where  $Y$  is output,  $\phi L$  is labor input, each  $X_i$  is a specialized intermediate good, and  $N$  is the total number of intermediates. The population  $L$  is assumed to be constant;  $\phi$  is the fraction of the population that works in the final goods sector.<sup>2</sup> Labor is supplied inelastically, with each worker's time endowment normalized to unity.

All intermediates are imperfect substitutes for one another since the marginal product of each  $X_i$  is independent of that of  $X_j$  for all  $i \neq j$ . Therefore there will always be positive demand for a new intermediate. Moreover, I assume that the intermediates are nondurable and so cannot be accumulated by the firm.

The final goods sector is perfectly competitive, so each factor is paid its marginal product. The final goods wage is then

$$w_G = (1 - \alpha)(\phi L)^{-\alpha} \sum_{i=1}^N X_i^\alpha \quad (2.2)$$

while the price paid for each intermediate good is

$$p_i = \alpha(\phi L)^{1-\alpha} X_i^{\alpha-1}. \quad (2.3)$$

Producers of the final good therefore demand each intermediate good according to

$$X_i = \phi L \left( \frac{\alpha}{p_i} \right)^{\frac{1}{1-\alpha}}. \quad (2.4)$$

### 2.1.2 Research Firms

Each intermediate good is invented by a research firm. For simplicity, I assume that each research firm invents only one intermediate good, so that the number of research firms is

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<sup>2</sup>I am implicitly assuming that the proportion of labor that works in the final goods sector will be constant over time. This will prove to be true in equilibrium.

also  $N$ . The number of research firms is considered a proxy for the level of technology in the economy. Following Romer (1990), research is knowledge-driven according to

$$\frac{\dot{N}}{N} = \gamma(1 - \phi)L. \quad (2.5)$$

That is, the growth of the number of firms is linear in the fraction of the population,  $(1 - \phi)L$ , that works in research. I follow Barro and Sala-i-Martin (2004) in thinking of  $N$  as continuous for analytical convenience.

A research firm pays a fixed cost  $P_N$  to invent each new kind of intermediate good. To determine the size of this fixed cost, we must first determine the price that a firm would require to supply its intermediate good to a producer of the final good. Suppose a research firm has just invented a new intermediate good  $X_z$ . Assume that this firm maintains a perpetual monopoly over the production and sale of units of  $X_z$ . The present value of inventing  $X_z$  is then the discounted value of all future profit flows from its sale, which is given by

$$V_z(t) = \int_t^\infty \pi_z(v) \cdot e^{-\bar{r}(t,v) \cdot (v-t)} dv \quad (2.6)$$

where  $\pi_z$  is the profit flow and  $\bar{r}$  is the average real interest rate. To maximize the value of the invention, firms maximize the profit flow at every time  $v$ . The profit flow is given by

$$\pi_z(v) = [p_z(v) - r\eta]X_z = [p_z(v) - r\eta] \cdot L \left( \frac{\alpha}{p_z(v)} \right)^{\frac{1}{1-\alpha}} \quad (2.7)$$

where  $r$  is the rental rate and  $\eta$  is the cost in units of forgone consumption to make one unit of capital good. It follows that the price for each intermediate good will be

$$\bar{p}_i = \frac{r\eta}{\alpha}.$$

Each research firm will therefore supply

$$\bar{X}_i = (\phi L) \cdot \left( \frac{\alpha^2}{r\eta} \right)^{\frac{1}{1-\alpha}}$$

to the representative producer of the final good. The profit flow for each research firm simplifies to

$$\pi_i = (1 - \alpha)\bar{p}_i\bar{X}_i. \quad (2.8)$$

A research firm will pay the fixed cost to develop a new intermediate good so long as the cost is less than or equal to this present value. Assume free entry and the constant fixed cost  $P_N$  so that

$$V_i(t) = P_N \quad (2.9)$$

for all  $i$  actually produced. Differentiating the condition in (2.9) with respect to time, Leibniz's rule implies

$$r(t) = \frac{\pi}{V(t)} + \frac{\dot{V}(t)}{V(t)} = \frac{\pi}{P_N}. \quad (2.10)$$

Therefore the rental rate is constant in equilibrium.

### 2.1.3 Households

The representative household maximizes lifetime utility

$$U = \int_0^{\infty} e^{-\rho t} \left( \frac{C(t)^{1-\theta} - 1}{1-\theta} \right) dt \quad (2.11)$$

where  $\rho$  is the household's subjective discount rate and  $\theta$  is the inverse of its elasticity of intertemporal substitution. The household therefore satisfies the standard Euler equation

$$\frac{\dot{C}}{C} = \frac{1}{\theta} \cdot (r - \rho). \quad (2.12)$$

### 2.1.4 Equilibrium

An equilibrium in this economy requires equal wages and constant labor shares across the two sectors of production. Recall from Equation (2.2) that the wage in the final goods sector is

$$w_G = (1 - \alpha)(\phi L)^{-\alpha} \sum_{i=1}^N X_i^\alpha.$$

The wage in the research sector is simply

$$w_R = P_N \gamma N$$

since all income from the sale of intermediates is used to pay for research labor. Setting these two wages equal gives

$$P_N = \frac{(1 - \alpha)}{\gamma} (\phi L)^{-\alpha} \bar{X}_i^\alpha. \quad (2.13)$$

On the other hand, combining Equations (2.8) and (2.10) yields

$$P_N = \frac{(1 - \alpha)}{r} \alpha (\phi L)^{1-\alpha} \bar{X}_i^\alpha. \quad (2.14)$$

Setting these two expressions equal to each other allows us to solve for  $\phi L$ :

$$\phi L = \frac{r}{\alpha \gamma}. \quad (2.15)$$

Now we can solve for the equilibrium growth rate. First, define an accounting measure of total capital  $K$  as cumulative forgone output  $N\eta\bar{X}_i$ . Thus  $K(t)$  evolves according to the following rule

$$\dot{K}(t) = Y(t) - C(t)$$

where  $C(t)$  is aggregate consumption at time  $t$ . Since  $K = N\eta\bar{X}_i$  and  $\bar{X}_i$  is constant, total capital must grow at the same rate as  $N$ . Let  $g$  denote the growth rate of  $N$ ,  $Y$ , and  $K$ . Since  $K/Y$  is constant, the ratio

$$\frac{C}{Y} = 1 - \frac{\dot{K}}{Y} = 1 - \frac{\dot{K}}{K} \frac{K}{Y}$$

must also be constant. The growth rate  $g$  is therefore

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{N}}{N} = \gamma(1 - \phi)L. \quad (2.16)$$

Using the relationship in Equation (2.15), we find that

$$g = \gamma L - \frac{r}{\alpha}.$$

We can substitute the expression  $r = \theta g + \rho$  from the Euler equation and solve for  $g$  to get our final result:

$$g = \frac{\alpha\gamma L - \rho}{\alpha + \theta}. \quad (2.17)$$

Growth is therefore increasing in the population size and the relative patience of the representative household.<sup>3</sup>

## 2.2 Deterministic Growth in an Open Economy with Foreign Exchange Costs

I now consider two small open economies, East and West, that engage in trade. Each country has its own currency, and foreign exchange costs arise from this difference. Foreign exchange costs are modeled as iceberg costs: for each unit of a good traded from East to West, only  $\tau$  units arrive in West, where  $0 \leq \tau \leq 1$ . The same holds for trade from West to East. If  $\tau = 0$ , foreign exchange costs are prohibitively high and East and West do not engage in trade. The results from the previous section therefore hold for each economy. If  $\tau = 1$ , the cost of exchanging currencies must be zero. I equate this with the formation of a monetary union.

### 2.2.1 Producers of the Final Good

As in autarky, a representative producer of the final good employs labor and a number of specialized intermediate goods as inputs. Assuming transaction costs are not prohibitive, however, intermediates are now traded so that each producer uses the production technology

$$Y = (\phi L)^{1-\alpha} \sum_{i=1}^{2N} X_i^\alpha \quad (2.18)$$

where intermediates  $1, \dots, N$  are produced in East and  $N + 1, \dots, 2N$  are produced in West.

For convenience, assume that the same final good is produced in both countries using the same production technology. Moreover, assume that East and West both have populations of size  $L$  and that the number of intermediates produced in each is  $N$ . Finally,

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<sup>3</sup>Backus, Kehoe, and Kehoe (1992) find little empirical evidence of a relation between the growth rate of GDP per capita and the size of the labor force. Restricting attention to the manufacturing sector, however, they find a significant relation between the growth rate of output per worker and the number of workers.

assume that the final good is not traded: consumers in East only buy output from East; likewise for consumers in West. This allows us to continue to set the price of output as our numeraire.

There is full employment of domestic labor and no movement of labor across borders. The final goods wage in both East and West will therefore be

$$w_G = (1 - \alpha)(\phi L)^{-\alpha} \sum_{i=1}^{2N} X_i^\alpha. \quad (2.19)$$

The price paid for each intermediate depends on whether that intermediate is produced domestically or imported. If it is produced domestically, the price is

$$p_i = \alpha(\phi L)^{1-\alpha} X_i^{\alpha-1}. \quad (2.20)$$

If it is imported, however, the price is

$$p_i^* = \frac{p_i}{\tau} = \frac{1}{\tau} \cdot \alpha(\phi L)^{1-\alpha} X_i^{\alpha-1} \quad (2.21)$$

which must be equivalent to

$$p_i^* = \alpha(\phi L)^{1-\alpha} (X_i^*)^{\alpha-1} \quad (2.22)$$

where  $X_i^*$  is demand for imports of  $X_i$ .

The equality in (2.21) holds because paying  $p_i$  will only result in  $\tau$  units reaching the firm. On the other hand, the equality in (2.22) holds because each intermediate is paid its marginal product. It follows from Equations (2.21) and (2.22) that firms will demand a greater amount of domestic intermediates than imported ones whenever  $0 \leq \tau < 1$ :

$$X_i^* = \tau^{\frac{1}{1-\alpha}} X_i. \quad (2.23)$$

When a monetary union is formed ( $\tau=1$ ), however, domestically-produced and imported intermediates are demanded in equal quantities.

### 2.2.2 Research Firms

Again, the number of research firms in each country is considered a proxy for the level of technology in the global economy. Recall from Equation (2.5) that the rate of firm creation is assumed to be proportional to the number of existing firms. In autarky, this simply means  $\dot{N}$  is proportional to  $N$ . In the open-economy setting, however, in which  $2N$  firms exist and foreign exchange costs are present, the relationship between  $\dot{N}$  and  $N$  ought to be more nuanced. If foreign exchange costs are prohibitive ( $\tau = 0$ ), for example, we want  $\dot{N}$  to be proportional to  $N$  as in autarky. If foreign exchange costs are null ( $\tau = 1$ ), on the other hand, we want  $\dot{N}$  to be proportional to the total number of firms,  $2N$ , since the global economy can then be thought of as a single, large market with  $2N$  firms. For  $0 < \tau < 1$ , we desire something in between.

To satisfy these requirements, let  $\psi : [0, 1] \rightarrow [0, 1]$  be a function of  $\tau$  that satisfies  $\psi'(\cdot) > 0$ . That is,  $\psi$  is a monotone increasing function between  $\psi(0) = 0$  and  $\psi(1) = 1$ . The simplest example is the identity function:  $\psi(\tau) = \tau$ . The function  $\psi(\tau)$  can be interpreted as a proxy for the level of integration between East and West when foreign exchange costs are  $1 - \tau$ . The previous subsection showed that the volume of trade is increasing in  $\tau$ , so we can think of  $\psi$  as a normalized measure of the volume of trade in particular.

Assuming that knowledge is embodied in capital goods and that capital goods are traded with foreign exchange cost  $1 - \tau$ , it is natural to then assume that

$$\frac{\dot{N}}{N} = \gamma[1 + \psi(\tau)](1 - \phi)L. \quad (2.24)$$

That is, the rate of firm creation is proportional to  $[1 + \psi(\tau)]N$ . Considering the bounds on  $\psi$ , we have that  $N \leq [1 + \psi(\tau)]N \leq 2N$  for all allowable values of  $\tau$ . Therefore (2.24) not only describes the autarkic ( $\tau = 0$ ) and costless trade ( $\tau = 1$ ) rates of firm creation, but also describes the intermediate case in which knowledge spillovers occur ( $\tau > 0$ ) but are not complete ( $\tau < 1$ ).

As in autarky, firms pay a fixed cost  $P_N$  to invent a new intermediate whose value is the present value of future profit flows from its sale. To maximize the present value of future profit flows, research firms again maximize the profit flow at every time  $v$ . The profit flow for each firm is now

$$\pi_i(v) = p_i \cdot X_i + \frac{p_i}{\tau} \cdot X_i^* - r\eta(X_i + X_i^*). \quad (2.25)$$

From Equations (2.20) through (2.22), both  $X_i$  and  $X_i^*$  can be written in terms of  $p_i$ :

$$X_i = (\phi L) \cdot \left( \frac{\alpha}{p_i} \right)^{\frac{1}{1-\alpha}}$$

$$X_i^* = (\phi L) \cdot \left( \frac{\tau\alpha}{p_i} \right)^{\frac{1}{1-\alpha}}.$$

Substituting these expressions into (2.25) and solving for the monopoly price  $\bar{p}_i$  gives

$$\bar{p}_i = \frac{r\eta}{\alpha} \cdot \left[ \frac{1 + \tau^{\frac{1}{1-\alpha}}}{1 + \tau^{\frac{\alpha}{1-\alpha}}} \right]. \quad (2.26)$$

Each research firm will therefore supply

$$\bar{X}_i = (\phi L) \cdot \left[ \frac{\alpha^2(1 + \tau^{\frac{\alpha}{1-\alpha}})}{r\eta(1 + \tau^{\frac{1}{1-\alpha}})} \right]^{\frac{1}{1-\alpha}}$$

to domestic firms and

$$\bar{X}_i^* = (\phi L) \cdot \left[ \frac{\alpha^2\tau(1 + \tau^{\frac{\alpha}{1-\alpha}})}{r\eta(1 + \tau^{\frac{1}{1-\alpha}})} \right]^{\frac{1}{1-\alpha}}$$

to foreign firms. The equilibrium profit flow for each research firm simplifies to the constant

$$\pi_i = [1 + \tau^{\frac{\alpha}{1-\alpha}}](1 - \alpha)\bar{p}_i\bar{X}_i. \quad (2.27)$$

Notice that each research firm makes its autarkic profit when foreign exchange costs are prohibitive and double that when a monetary union is formed.

A research firm will pay the fixed cost to develop a new intermediate good so long as the cost is less than or equal to this present value. Assume free entry and the constant fixed cost  $P_N$  so that

$$V_i(t) = P_N$$

for all intermediates  $i$  actually produced. As in Section 2.2, Leibniz's rule implies

$$r(t) = \frac{\pi}{P_N}. \quad (2.28)$$

Therefore the rental rate is constant in equilibrium.

### 2.2.3 Equilibrium

Assume that households continue to consume only their domestic final good and therefore obey Equations (2.11) and (2.12). We can now proceed to solve the model. An equilibrium in this economy requires equal wages and constant labor shares across all four sectors of the economy (Eastern and Western final goods and research). I have already argued that wages and labor shares are identical across countries for each sector. To show equal wages and constant labor shares across sectors within a country, we can solve the model as we did in Section 2.4.

Without loss of generality, consider the Eastern final goods and research sectors. Recall from Equations (2.19) and (2.23) that the wage in the Eastern final goods sector will be

$$w_G = (1 - \alpha)(\phi L)^{-\alpha} \sum_{i=1}^N (1 + \tau^{\frac{1}{1-\alpha}})\bar{X}_i^\alpha.$$

The wage in the Eastern research sector is simply

$$w_R = P_N\gamma[1 + \psi(\tau)]N$$

since all income from the sale of intermediates is used to pay for research labor. Setting these two wages equal gives

$$P_N = \frac{(1 + \tau^{\frac{1}{1-\alpha}})(1 - \alpha)}{\gamma[1 + \psi(\tau)]}(\phi L)^{-\alpha}\bar{X}_i^\alpha. \quad (2.29)$$

On the other hand, combining Equations (2.27) and (2.28) yields

$$P_N = \frac{(1 - \alpha)}{r} [1 + \tau^{\frac{\alpha}{1-\alpha}}] \alpha (\phi L)^{1-\alpha} \bar{X}_i^\alpha. \quad (2.30)$$

---

Setting these two expressions equal to each other allows us to solve for  $\phi L$  in East:

$$\phi L = \frac{r[1 + \tau^{\frac{1}{1-\alpha}}]}{\alpha\gamma[1 + \psi(\tau)][1 + \tau^{\frac{\alpha}{1-\alpha}}]}. \quad (2.31)$$

Now we can solve for the equilibrium growth rate. First, we define an accounting measure of total domestic capital  $K$  as cumulative forgone output  $N\eta(\bar{X}_i + \bar{X}_i^*)$  for each country. Thus  $K(t)$  evolves according to the following rule

$$\dot{K}(t) = Y(t) - C(t) \quad (2.32)$$

where  $C(t)$  is aggregate domestic consumption at time  $t$ . Since  $K = N\eta(\bar{X}_i + \bar{X}_i^*)$  and  $\bar{X}_i$  is constant, total capital must grow at the same rate as  $N$ . Let  $g$  denote the growth rate of  $N$ ,  $Y$ , and  $K$ . Since  $K/Y$  is constant, the ratio

$$\frac{C}{Y} = 1 - \frac{\dot{K}}{Y} = 1 - \frac{\dot{K}}{K} \frac{K}{Y}$$

must also be constant. The growth rate  $g$  is therefore

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{N}}{N} = \gamma[1 + \psi(\tau)](1 - \phi)L.$$

Using the relationship in Equation (2.31), we find that

$$g = \gamma[1 + \psi(\tau)]L - \frac{r[1 + \tau^{\frac{1}{1-\alpha}}]}{\alpha[1 + \tau^{\frac{\alpha}{1-\alpha}}]}.$$

We can substitute the expression  $r = \theta g + \rho$  from the Euler equation and solve for  $g$  to get our main result for this section:

$$\hat{g} = \frac{\alpha\gamma L[1 + \psi(\tau)][1 + \tau^{\frac{\alpha}{1-\alpha}}] - \rho[1 + \tau^{\frac{1}{1-\alpha}}]}{\alpha[1 + \tau^{\frac{\alpha}{1-\alpha}}] + \theta[1 + \tau^{\frac{1}{1-\alpha}}]}. \quad (2.33)$$

When foreign exchange costs are prohibitive ( $\tau = 0$ ), this is simply the autarkic rate of growth. When a monetary union is formed ( $\tau = 1$ ), on the other hand, the growth rate becomes

$$\hat{g}_{MU} = \frac{\alpha\gamma L[1 + \psi(1)] - \rho}{\alpha + \theta} = \frac{2\alpha\gamma L - \rho}{\alpha + \theta}, \quad (2.34)$$

which is unambiguously higher than in autarky. For reasonable parameter values, (2.33) is increasing in  $\tau$  on the unit interval, suggesting that the growth rate in (2.34) is maximal for allowable values of  $\tau$  as desired.<sup>4</sup> That is, monetary unification raises growth unambiguously.

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<sup>4</sup>Derivatives computed using random selections from the following parameter values were found to be positive on the unit interval:  $\alpha \in [.25, .75]$ ,  $\rho \in [.005, .5]$ , and  $\theta \in [1, 20]$ . Both  $\gamma$  and  $L$  were normalized to one, and  $\psi(\tau) \equiv \tau$  for simplicity. This range of parameter values is not exhaustive but represents reasonable bounds for each parameter. All derivatives were computed using Mathematica.

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## 2.3 Empirical Evidence

Here I will briefly recount the extant empirical work that is related to the model above. The bulk of the work has been done in one of two categories: estimating the effect of monetary unions on trade, or estimating the effects of trade on growth.

Rose (2000) presents evidence of the long-run gains in trade from monetary unification. He estimates a gravity model of international trade with a dummy variable for currency union, using panel data from 1970 to 1990 over 186 countries. He obtains a statistically significant effect that two countries sharing the same currency trade three times as much as they would with different currencies. In a companion paper, Rose and van Wincoop (2001) assert that the result from Rose (2000) is robust with respect to the model of trade used: estimates from multiple international trade models with a currency union dummy return a statistically significant effect of monetary unification on trade. Frankel and Rose (2002) take this claim one step further and estimate that every 1 percent increase in a country's overall trade from monetary unification boosts growth by roughly one-third of a percent. Nitsch (2002) and Baldwin (2006), however, temper these claims. Both find statistically-significant positive effects of monetary unification on trade between members but estimate these effects to be of a lower magnitude. De Grauwe (2007) goes so far as to argue that Rose's results are grossly overestimated.

As for the effect of trade on growth, Coe and Helpman (1995) is most closely related to the present analysis. Trade is presumed to affect growth through increased international R&D spillovers. Their estimates indicate that foreign R&D has beneficial effects on domestic productivity and that these are stronger the more open an economy is to foreign trade. Keller (2004) reviews other literature in this vein.

Finally, Kovač and Spruk (2016) provide direct estimates of the impact of transaction costs on long-run growth. Their evidence from cross-country panel data regression analysis reveals a persistent and robust negative effect of increasing transaction costs on the path of economic growth. Their results imply that transaction costs, including foreign exchange costs, may be central to the study of cross-country productivity differences.

## 3 The Stochastic Model

In Section 2, monetary unification increased growth by lowering foreign exchange costs and thereby increasing trade in intermediate capital goods. These goods were presumed to contain technological knowledge so that increased trade brought increased knowledge spillovers. In sum: the lower the foreign exchange costs, the more trade, the more spillovers, the faster growth would be. The empirical evidence that was just presented, however, suggests that there is reason to doubt the strength of this chain.

I will now present a stochastic model of monetary unions and long-run growth that is tied intimately to this framework but does not rely on its chain of reasoning. The model builds from a simplified version of Obstfeld (1994) in which households invest either in safe, low-yield capital or risky, high-yield capital and trade assets in an international market with foreign exchange costs. This setup is a stylized rendition of the idea in

Section 2 that ongoing growth depends on investments in supplying specialized, hence inherently risky, intermediate capital goods. Because risky capital is presumed to have higher returns, lower foreign exchange costs in asset trading will allow households to hold a more globally diversified portfolio of risky assets, encouraging all households to shift toward risky, high-yield investments, thereby driving growth.

### 3.1 Household Choice in a Closed Economy

Analagous to Section 2.1, I first solve the stochastic model in a closed economy. The economy is populated by identical infinitely-lived households who face the choice between consuming or investing a single good. The economic decision interval has length  $h$ . At time  $t$  a representative household maximizes the intertemporal objective  $U(t)$  defined by the recursion

$$[1 - R]U(t) = C(t)^{1-R}h + e^{-\delta h}[1 - R]E_t U(t + h) \quad (3.1)$$

where  $E_t$  is mathematical expectation conditional on information at time  $t$ ,  $C(t)$  is consumption at time  $t$ ,  $\delta > 0$  is the subjective rate of time preference, and  $R$  is the household's measure of relative risk aversion. This is the standard state- and time-separable expected-utility setup.

Individuals save by accumulating capital and by making risk-free loans that pay real interest at the instantaneous rate  $i(t)$ . Units of consumption and capital can be transformed into one another freely at a one-to-one rate. Capital comes in two varieties: riskless capital offering a sure instantaneous yield of  $r$  (a constant) and risky capital offering a random instantaneous yield with constant expected value  $\alpha > r$ . Therefore individuals face a portfolio decision—how to allocate their wealth among the two types of capital and loans—as well as a saving decision. There is no nondiversifiable income (such as labor income), so asset markets are complete.

A no-arbitrage condition reduces the individual's portfolio to a choice over two assets: risky capital and a composite safe asset offering the sure instantaneous real return  $i(t)$ .<sup>5</sup> For simplicity, assume that this real interest rate is constant at level  $i$ . The next section shows that  $i$  is indeed constant in equilibrium.

Let  $V^B(t)$  denote the cumulative time- $t$  value of a unit of output invested in safe assets at time 0. Similarly, let  $V^K(t)$  denote the cumulative time- $t$  value of a unit of output invested in risky capital at time 0. Clearly  $V^B(0) = V^K(0) = 1$ . With payouts reinvested and continuously compounded,  $V^B(t)$  obeys the ordinary differential equation

$$dV^B(t) = iV^B(t)dt. \quad (3.2)$$

The value  $V^K(t)$  follows the geometric diffusion process

$$dV^K(t) = [\alpha dt + \sigma dz(t)]V^K(t) \quad (3.3)$$

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<sup>5</sup>If  $i(t) > r$ , individuals hold no safe capital and cannot go short in that asset. If  $i(t) = r$  the division between safe capital and loans is indeterminate. Finally,  $i(t) < r$  cannot hold in equilibrium because it implies a sure arbitrage profit from borrowing for investment in safe capital.

where  $dz(t)$  is a Brownian motion process such that  $z(t) = z(0) + \int_0^t dz(s)$  and  $\sigma^2$  is the instantaneous variance of returns.

Per capita wealth  $W(t)$  is the sum of per capita holdings of the composite safe asset,  $B(t)$ , and per capita holdings of risky capital,  $K(t)$ :

$$W(t) = B(t) + K(t). \quad (3.4)$$

Let  $\omega(t)$  denote the fraction of wealth invested in risky capital. That is,  $K(t) = \omega(t)W(t)$  and  $B(t) = [1 - \omega(t)]W(t)$ . Then Equations (3.2), (3.3), and (3.4) imply that

$$dW(t) = \{\omega(t)\alpha + [1 - \omega(t)]i\}W(t)dt + \omega(t)\sigma W(t)dz(t) - C(t)dt. \quad (3.5)$$

Our next step is to maximize the continuous-time limit of  $U(t)$  in (3.1) subject to the law of motion for wealth in (3.5) and an initial wealth endowment  $W(t) = W_t$ . Let  $J(W_t)$  denote the maximum feasible level of lifetime utility when wealth at time  $t$  equals  $W_t$ . Itô's lemma shows that in continuous time the stochastic Bellman equation for this maximization problem is

$$0 = \max_{\omega, C} \{C^{1-R} - \delta(1-R)J(W) + (1-R)[J'(W)(\omega\alpha W + [1-\omega]iW - C) + \frac{1}{2}J''(W)\omega^2\sigma^2W^2]\} \quad (3.6)$$

where time indices are henceforth suppressed unless necessary.<sup>6</sup> The first-order conditions with respect to  $\omega$  and  $C$  are

$$\begin{aligned} \omega : J'(W)(\alpha - i) + J''(W)\omega\sigma^2W^2 &= 0 \\ C : (1-R)C^{-R} - (1-R)J'(W) &= 0. \end{aligned}$$

Following Obstfeld (1994), we guess that maximized lifetime utility  $U$  is given by  $J(W) = (mW)^{1-R}/(1-R)$  for some constant  $m > 0$ . Given this functional form for  $J(W)$ , the first-order conditions simplify. The former now implies that demand for the risky asset is a constant fraction of wealth:

$$\omega = (\alpha - i)/R\sigma^2. \quad (3.7)$$

The latter becomes  $C = (m^{1-1/R})W$ , so that the consumption-wealth ratio is also a constant, denoted  $\mu$ . To pin down  $\mu$ , first notice that  $\mu = m^{1-1/R}$  by definition. Then substitute the optimal values of  $C$ ,  $\omega$ , and  $J(W)$  determined above into (3.6) and solve for  $\mu$  to get

$$\mu = \frac{C}{W} = \frac{1}{R} \left\{ \delta - (1-R) \left[ i + \frac{(\alpha - i)^2}{2R\sigma^2} \right] \right\}. \quad (3.8)$$

This confirms that the value function is

$$J(W) = \frac{[\mu^{R/(R-1)}W]^{1-R}}{1-R}. \quad (3.9)$$

<sup>6</sup>Continuous-time methods for solving problems of this class are provided in Svensson (1989).

## 3.2 Closed-Economy Equilibrium

We can now determine equilibrium growth in this closed economy. Because the two types of capital goods can be interchanged freely at a one-to-one rate, the equilibrium asset demand given by (3.7) can always be accommodated. There are two types of equilibrium: one in which both types of capital are held and one in which only risky capital is held.

The first type of equilibrium occurs when  $(\alpha - r)/R\sigma^2 \leq 1$ . In this case the interest rate satisfies  $i = r$ , and the share of the economy's wealth held in the form of risky capital is  $\omega = (\alpha - i)/R\sigma^2 \leq 1$ . The second type of equilibrium occurs when  $(\alpha - r)/R\sigma^2 > 1$ . The no-arbitrage condition implies that  $i$  rises above  $r$  until only risky capital is held, that is, until  $\omega = (\alpha - i)/R\sigma^2 = 1$ . The implied equilibrium interest rate is  $i = \alpha - R\sigma^2 > r$ , which confirms the constancy of  $i$  that was assumed in the last section.

The equilibrium interest rate helps pin down an equilibrium rate of economic growth. Equations (3.5) and (3.8) imply that wealth accumulates according to

$$dW = [\omega\alpha + (1 - \omega)i - \mu]Wdt + \omega\sigma Wdz. \quad (3.10)$$

By Equations (3.8) and (3.10), per capita consumption obeys

$$dC = [\omega\alpha + (1 - \omega)i - \mu]Cdt + \omega\sigma Cdz. \quad (3.11)$$

Define  $g$  as the instantaneous expected growth rate of consumption. Then (3.11) shows that  $g$  is endogenously determined as the average expected return on wealth,  $\omega\alpha + (1 - \omega)i$ , less the ratio of consumption to wealth,  $\mu$ . Combining this result with (3.7) and (3.8) yields a closed-form expression for the expected consumption growth rate:

$$g = \frac{1}{R}(i - \delta) + \frac{(1 + 1/R)(\alpha - i)^2}{2R\sigma^2}. \quad (3.12)$$

When no riskless capital is held in equilibrium, the growth rate can be expressed as

$$g = \frac{1}{R}(\alpha - \delta) + \frac{(1 - 1/R)R\sigma^2}{2} \quad (3.13)$$

which follows upon substitution of  $\alpha - R\sigma^2$  for  $i$  in (3.12).

## 3.3 Open Economy with Foreign Exchange Costs

We now return to a two-country world as in Section 2.2, with countries named East and West. For parsimony in notation, index countries by  $j = 1, 2$ , where  $j = 1$  is East and  $j = 2$  is West. Each country has a representative household with preferences of the form specified in Equation (3.1). The representative households have identical measures of relative risk aversion,  $R$ , and rates of time preference,  $\delta$ .

The rate of return on safe capital,  $r$ , is common to both countries. For risky capital, however, assume that households repatriate only a fraction  $\tau \in (0, 1]$  of the returns they earn abroad. The remaining  $1 - \tau$  fraction can be thought of as the cost of foreign

exchange.<sup>7</sup> For simplicity, assume that expected before-cost returns on risky capital are the constant value  $\alpha$  worldwide. Therefore the representative Eastern household can expect to earn  $\alpha$  from investing in Eastern risky capital but only  $\tau\alpha$  from Western risky capital.

The cumulative value of a unit investment in country  $j$ 's risky capital by an investor from country  $k$  follows the geometric diffusion process

$$dV_{jk}^K(t) = [\alpha_{jk}dt + \sigma dz_j(t)]V_{jk}^K(t) \quad \text{for } j, k = 1, 2 \quad (3.14)$$

where  $\alpha_{jk} = \alpha$  if  $j = k$  and  $\alpha_{jk} = \tau\alpha$  otherwise. I have implicitly assumed that country-specific technology shocks have the same variance  $\sigma^2$ . These shocks display the instantaneous correlation structure

$$dz_j dz_k = \rho_{jk}. \quad (3.15)$$

The symmetric  $2 \times 2$  covariance matrix  $\mathbf{\Omega} \equiv [\sigma^2 \rho_{jk}]$  is assumed to be invertible.

Households can invest in both of the risky technologies described by (3.14) and (3.15) and the global, safe asset. Let  $\mathbf{1}$  denote the  $2 \times 1$  column vector with all entries equal to 1, let  $\boldsymbol{\alpha}_j$  denote the  $2 \times 1$  column vector whose  $k$ th entry is  $\alpha_{jk}$ , and let  $\boldsymbol{\omega}_j$  denote the  $2 \times 1$  column vector whose  $k$ th entry is the demand for country  $k$ 's risky capital by a household of country  $j$ . A generalization of the argument in Section 3.1 shows that a household from country  $j$  has the following vector of portfolio weights for the two risky assets:

$$\boldsymbol{\omega}_j = \frac{\mathbf{\Omega}^{-1}(\boldsymbol{\alpha}_j - i^*\mathbf{1})}{R} = \frac{1}{R\sigma^2(1 - \rho^2)} \begin{bmatrix} \sigma^2(\alpha(1 - \rho\tau) - i^*(1 - \rho)) \\ \sigma^2(\alpha(\tau - \rho) - i^*(1 - \rho)) \end{bmatrix} \quad (3.16)$$

where  $i^*$  is the world real interest rate that all countries face and  $\rho$  is short for  $\rho_{jk}$  with  $j \neq k$ . Notice that  $\omega_{jj} > \omega_{jk}$  for  $j \neq k$  so long as  $1 - \rho\tau > \tau - \rho$ . This clearly holds since the previous inequality is equivalent to  $1 + \rho > \tau(1 + \rho)$ . That is, households display a home bias in equities that depends on the size of the foreign exchange costs  $1 - \tau$ . I will discuss empirical evidence for this home bias in Section 3.4.

We can collapse each household's investment in the two types of risky capital into an investment in a country-specific mutual fund. The  $2 \times 1$  vector of portfolio weights for mutual fund  $j$  is

$$\boldsymbol{\theta}_j \equiv \frac{\mathbf{\Omega}^{-1}(\boldsymbol{\alpha}_j - i^*\mathbf{1})}{\mathbf{1}'\mathbf{\Omega}^{-1}(\boldsymbol{\alpha}_j - i^*\mathbf{1})} \quad (3.17)$$

where a prime denotes matrix transposition. Since the portfolio weights are constant, the mean return on country  $j$ 's mutual fund will be  $\alpha^* \equiv \boldsymbol{\theta}_j' \boldsymbol{\alpha}_j$  with return variance  $\sigma^{*2} \equiv \boldsymbol{\theta}_j' \mathbf{\Omega} \boldsymbol{\theta}_j$ . The symmetry of the model implies that both the Eastern and Western mutual funds will have this mean return  $\alpha^*$  and return variance  $\sigma^{*2}$ .

We can now find the equilibrium growth rate. Assume that households are risk averse enough that they always hold some risk-free capital. This assumption implies  $i^* = r$

<sup>7</sup>We could also have a proportional cost to repatriating earnings from investment in safe capital abroad, but then foreign risk-free capital would be return-dominated by its domestic counterpart. That is, no one would invest in foreign safe capital. It is therefore easier to simply consider a global, risk-free asset.

globally. An equilibrium must satisfy the conditions

$$\begin{aligned} K_j/(K_1 + K_2) &= \theta_j \quad \text{for } j = 1, 2 \\ K_1 + K_2 &= \frac{\alpha^* - i^*}{R\sigma^{*2}}(W_1 + W_2). \end{aligned}$$

Note that Equations (3.7) and (3.8) remain valid, with  $\alpha$  replaced by  $\alpha^*$  and  $\sigma^2$  replaced by  $\sigma^{*2}$ . Solving the same way as in Section 3.2, the mean growth rate in both East and West is

$$g^* = \frac{1}{R}(i^* - \delta) + \frac{(1 + 1/R)(\alpha^* - i^*)^2}{2R\sigma^{*2}}. \quad (3.18)$$

A quick argument will show that  $g^*$  is maximized at  $\tau = 1$ . Only  $\alpha^*$  and  $\sigma^*$  in (3.18) are functions of  $\tau$ , so  $g^*$  is maximized at  $\tau = 1$  if and only if  $(\alpha^* - i^*)^2/\sigma^{*2}$  is. Clearly  $\frac{\partial \alpha^*}{\partial \tau} \geq 0$  on the unit interval since the mean return must (weakly) rise as a greater fraction of earnings abroad are able to be repatriated. Moreover,  $\sigma^*$  is minimized at  $\tau = 1$  since risk is diversified evenly over the two risky assets. Therefore  $(\alpha^* - i^*)^2/\sigma^{*2}$  is indeed maximized at  $\tau = 1$ . We have now proven the main result: monetary unification raises the growth rate in an economy with foreign exchange costs.

### 3.4 Empirical Evidence

One key trait of the previous model is that households display a portfolio home bias. French and Poterba (1991) were the first to notice this phenomenon in the data. They found that each of the world's five largest stock markets displayed a severe bias towards domestic equities at the time of writing. This portfolio home bias has persisted; possible explanations for it include informational asymmetries and transport costs. Fidora, Fratzscher, and Thimann (2007) cite exchange rate volatility as another possible cause. They find that reduction in monthly real exchange rate volatility from its sample mean to zero reduces bond home bias by up to 60 percentage points and equity home bias by 20 percentage points.

Coeurdacier and Rey (2013) support this conclusion with data from Europe, where the degree of home bias has fallen from about 0.7 to 0.5 since monetary unification in 1999. This is significantly lower than the worldwide average of 0.63. Moreover, there is reason to believe that this reduction in home bias comes from a reduction in transaction costs as modeled above. Coeurdacier and Martin (2009) have found that the euro implies lower transaction costs on the purchase of euro zone assets. For other euro zone members, this reduction in cost was somewhere between 24% and 34%. Balta and Delgado (2009) caution that this reduction in home bias is uneven across Europe and has largely leveled off.

How do reduced transaction costs in financial markets affect growth? Levine and Zervos (1996) use cross-country regressions to suggest that stock market development is positively and robustly associated with long-run growth. Risk diversification is one of their measures of stock market development, so reduced transaction costs will positively affect growth insofar as they reduce home bias and therefore increase risk diversification.

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The evidence just presented suggests that the link between transaction costs and home bias is indeed quite strong.

Lastly, the data suggests that the the models in Sections 2 and 3, when combined, may provide an extra boost of growth. Martínez-San Román, Bengoa, and Sánchez-Robles (2016) find a positive and robust correlation between trade integration and foreign direct investment, implying that reduced foreign exchange costs could cause both increased knowledge spillovers via trade and greater investment in risky capital goods.

## 4 Conclusion and Suggestions for Future Work

This paper has developed two complementary models of monetary unions and long-run growth. The key result is that a reduction in foreign exchange costs via monetary unification provides a positive growth effect for member nations. This growth effect may come through increased knowledge spillovers or the migration of funds to higher-yield investments. A swath of empirical evidence generally supports both of these channels of growth.

One avenue for future work is to explore possible negative growth effects from monetary unification. For instance, monetary unification often eliminates the ability of a member country to use monetary policy to combat a recession. Instead, surplus labor may be forced to migrate to other member countries to find work. Depending on the sectors that suffer the loss of and that absorb this labor surplus, this redistribution of labor may lower aggregate productivity and retard growth.

Another possible avenue for further research is a more comprehensive empirical investigation of the model's validity. Trade and per capita growth data from a monetary union comprised of well-suited member nations would be ideal, but data from the EMU or UEMOA may suffice.

Finally, we ultimately wish to develop a framework that allows us to determine optimum currency areas. Static models were able to do exactly that by weighing present benefits against present costs. When dynamic gains and losses are considered, however, the task becomes much tougher. Any progress on this front could have important policy implications.

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