Simultaneous occurrence of price jumps and changes in diffusive price volatility

Shuting Wei

Professor George Tauchen, Faculty Advisor

Acknowledgements

Abstract

This paper uses high frequency financial data to study the changes in diffusive stock price volatility when price jumps are likely to have occurred. In particular, we study this effect on two levels. Firstly, we compare diffusive volatility on jump and non-jump days. Secondly, we study the change in diffusive volatility in local windows before and after 5-minute intervals on which price jumps are likely to have occurred. We find evidence that market price jumps occur simultaneously with a change in diffusive volatility with negative dependence in the direction of the jump and the volatility change. However, a similar relationship is not detectable in individual stock price data.

1   Introduction

Modeling the behavior of asset and derivative price movements has many applications in financial economics, including asset pricing, portfolio management, risk management and hedging. Historically, asset prices were assumed to follow Brownian motion with time-varying drift and volatility with the implication that asset price movements are continuous in nature. Merton (1976) first proposed that discontinuities, or jumps, could theoretically exist in stock price behavior. The presence of discontinuities means that stock prices follow a smooth diffusive motion, but exhibit jumps from time to time. Since then, a number of empirical studies have demonstrated that the diffusive price model alone is unsatisfactory in modeling price behavior. For instance, Drost, Nijman, and Werker (1998) first noted that a continuous time diffusion model cannot explain the time series of dollar exchange rates clearly. Chernov, Gallant, Ghysels, and Tauchen (2003) found that the diffusive price model cannot approximate all the characteristics of daily observations on the S&P500 Index. Andersen, Benzoni, and Lund (2002) showed that in contrast, allowing for jumps in equity returns resulted in acceptable characterization of the actual data. These empirical studies have provided strong evidence in favor of including a jump component into stock price models.

Both financial theory and market practice have both increasingly recognized the economic importance of price discontinuities. Bakshi, Cao, and Chen (1997) showed that incorporating jumps into the price model reduces pricing errors for call options, whereas Naik and Lee (1990) found that dynamic hedging strategies require significant infusion of funds when price jumps occur. Liu, Longstaff, and Pan (2003) showed that optimal portfolio strategies can be significantly different when price jumps are taken into account in the form of event risk. Given the important of price discontinuities, recent financial literature models asset prices with two separate components: a continuous-time price diffusion and a jump component.

The volatility of stock prices is an important area of study since it represents the riskiness of an asset in standard risk-return considerations, with implications for the pricing of risk premiums. However, true volatility of the price process cannot be observed. Since high frequency minute-by-minute price data have become available, researchers have been able to make use of the data to develop estimators of volatility which converge to their

corresponding true values as the frequency of data observation becomes increasingly fine. Furthermore, modeling stock prices with both a continuous and jump component means that the total volatility of the price process can be similarly decomposed into a diffusive volatility component and a jump volatility component. Estimating price volatility using high frequency data enables the separate estimation of the diffusive and jump components of volatility so that their distinct characteristics can be studied.

At the same time, a number of statistical techniques have been developed to detect the presence of jumps in prices. The jump test developed by Barndorff-Nielsen and Shephard (2006) relies on the difference between an estimator of total price volatility and an estimator of the diffusive price volatility on the same day to classify the day as containing price jumps or otherwise. The difference between the two estimators of volatility represents the jump component of price volatility. Therefore, a larger difference indicates a more significant jump volatility component on the day, and a greater likelihood that price jumps were present. Jiang and Oomen (2008) used a similar intuition to develop a jump test based on the impact of jumps on the behavior of third and higher order moments of price returns. Lee and Mykland (2008) standardized discrete observations of price returns by their instantaneous diffusive volatility. When a price return is large relative to the instantaneous volatility, the observation is more likely to be a price jump. The development of these jump tests has enabled the identification of time intervals when jumps are likely to have occurred, therefore allowing further research on the characteristics of jump versus non-jump intervals.

Price jumps are commonly explained as the incorporation of large pieces of unanticipated and new information into stock prices by an efficient market. Barndorff-Nielsen and Shephard (2006) noted that most jumps in market prices as indicated by jumps in the Standard and Poor 500 Index (S&P 500 Index) that their jump test identifies are associated with governmental macroeconomic announcements. Lee and Mykland (2008) showed that individual stock jumps are associated with prescheduled earnings announcements and other company-specific news events, whereas S&P 500 Index jumps are associated with general market news announcements. In a more comprehensive study, Lee (2011) found that jumps are likely to occur shortly after macroeconomic information releases such as Fed announcements, nonfarm payroll reports, and jobless claims as well as market index jumps.

She also finds firm-specific jump predictors related to earnings releases, analyst recommendations, past stock jumps, and dividend dates.

The standard stock price model assumes independence between price jumps and diffusive price volatility. The implication of this assumption is that when market and firm-specific news announcements occur, price jumps fully capture the effect of the announcements on the price process, and no changes in diffusive price volatility result. However, researchers are currently finding evidence of possible correlation between price jumps and changes in diffusive volatility. Todorov and Tauchen (2011) studied co-jumps in the S&P 500 Index futures, a liquid financial instrument which is representative of movements in the S&P 500 Index and hence market price changes, and the VIX volatility index, which is based on close-to-maturity S&P 500 Index options and reflects market volatility. It was found that volatility jumps and market price jumps occur in most cases at the same time and exhibit high negative dependence. Jacod and Todorov (2010) developed a new test for deciding whether jumps in the price process are accompanied by simultaneous jumps in its volatility. When applied to the S&P 500 Index, it was found that at least 40 percent of price jumps are accompanied by volatility jumps. These results suggest common causes for the appearance of price jumps and changes in diffusive price volatility.

In the case that price jumps and volatility jumps systematically occur together, the standard price model where stochastic volatility is assumed to be independent of jumps can no longer fully characterize the price process. This can lead to economically important implications. In particular, dependence between price and volatility jumps will complicate hedging. If price jumps and volatility jumps were independent, separate hedging techniques could be employed against jump and volatility risks using common hedging instruments such as options. However, if they share a common origin, these instruments can no longer simultaneously hedge against both types of risks. Secondly, the risk premium associated with price jump and volatility risks should also be modeled jointly if high dependence exists between the two.

Motivated by these implications, this paper studies the relationship between price jumps and changes in diffusive price volatility. Previous studies have focused on co-jumps between price and diffusive volatility. However, since this is a relatively new area of study, there is no simple method of detecting jumps in volatility. Therefore, we will adopt the approach of

qualitatively studying the change in volatility when price jumps are detected. Specifically, we first ask the question of whether diffusive price volatility is impacted by the occurrence of price jumps in terms of a systematic increase or decrease following the occurrence of price jumps. If the former is true, then we look for a relationship between the direction of the price jumps and volatility changes; in particular we will find out whether the dependence is negative as found by Todorov and Tauchen (2010) in market price movements. We use existing jump tests to identify the intervals of time over which jumps are likely to have occurred. This will then enable us to study the changes in diffusive volatility between intervals of time when jumps are deemed to occur, as compared to when they are not. In addition to the existing literature, we will apply the study to both the S&P 500 Index as a proxy for market movements, as well as individual stock price movements.

The rest of the paper proceeds as follows. Section 2 introduces the theoretical framework for a standard model of returns, and methods to estimate the variance of the process. Section 3 explains the statistical methods used in this paper, including the jump tests that were employed and methods to compare diffusive volatility using jump test results. Section 4 introduces the empirical data that were used in the study, and handling of the data to remove erroneous components and account for contaminating noise. Section 5 explains the results that were obtained, and Section 6 concludes.

2   Theoretical Background

2.1   Stochastic Models for Returns

The standard model for an asset price movement is that the logarithmic price $p(t)$ follows the following stochastic differential equation

$$dp(t) = \mu(t)dt \qquad (1)$$
$$+ \sigma(t)dw(t),$$

where $\mu(t)$ represents the time-varying drift in prices, while $\sigma(t)dw(t)$ represents the time-varying volatility component in which $\sigma(t)$ is the instantaneous volatility of the process at time t and $w(t)$ indicates standardized Brownian motion. In the standard model, $\mu(t)$ and $\sigma(t)dw(t)$ are assumed to be independent of each other. Under this model, the price of an asset is assumed to follow Brownian motion with mean $\mu(t)$ and standard deviation $\sigma(t)$.

7

However, the model necessarily assumes that realizations of the price process are continuous in nature. Discontinuities, or "jumps" in prices, have been observed across asset classes since as early as Merton (1976). Jumps occur when new, unanticipated information becomes available and is incorporated into the price process by an efficient market. Including the price discontinuities therefore results in the model

$$dp(t) = \mu(t)dt + \sigma(t)dw(t) + \kappa(t)dq(t), \tag{2}$$

where $\kappa(t)dq(t)$ introduces a jump component into the price model, in which $\kappa(t)$ represents the magnitude of the jump at time t and $q(t)$ is a counting process for the number of jumps up until time t. The term $dq(t)$ is therefore the number of jumps within the infinitesimally small period of time dt.

2.2   Estimators for Variance

The asset price model suggests that volatility in the price process comes from two sources: the diffusive volatility $\sigma(t)$ and jumps $\kappa(t)$. The quadratic variation in the price process on day t can therefore be defined as the sum

$$QV_t = \int_{t-1}^{t} \sigma^2(s)ds + \sum_{j=1}^{M} \kappa^2(t_j). \tag{3}$$

Since the variance of a continuous process cannot be directly observed, Barndorff-Nielsen and Shephard (2004) proposed a nonparametric estimator for QV_t, the realized variance

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2 \rightarrow \int_{t-1}^{t} \sigma^2(s)ds + \sum_{j=1}^{M} \kappa^2(t_j), \tag{4}$$

where $r_{t,j} \equiv p_{t,j} - p_{t,j-1}$ is the intraday geometric return between the $(j-1)^{st}$ and j^{th} price observations on day t, and $M = 1/\Delta$ is the total number of returns observed on a day at sampling interval Δ. As the sampling frequency approaches zero, the realized variance converges in value to the quadratic variance of the price process in a day. Intuitively, the square of the geometric return is an estimator of variance in a particular interval. As the intervals between observations become infinitely small, the sum of the sequence of squared geometric returns estimates the total variance over the entire time period. Realized variance is therefore an asymptotically consistent estimator for the total variance in a day.

It is often useful to study the volatility of the diffusive and jump components separately. The integrated variance

$$IV_t = \int_{t-1}^{t} \sigma^2(s)ds \tag{5}$$

captures solely the volatility contribution by the continuous price process. To estimate the integrated variance, Barndorff-Nielsen and Shephard (2006) proposed the nonparametric estimator, bipower variation

$$BV_t = \mu_1^{-2}\left(\frac{M}{M-1}\right)\sum_{j=2}^{M}|r_{t,j}||r_{t,j-1}| \rightarrow \int_{t-1}^{t}\sigma^2(s)ds, \tag{6}$$

where $\mu_a = E(|Z|^a)$ for a standard normal Z. Andersen, Dobrev and Schaumburg (2010) introduced two additional measures of median variation and minimum variation to model integrated variance

$$MedV_t = \frac{\pi}{6 - 4\sqrt{3} + \pi}\left(\frac{M}{M-2}\right)\sum_{j=3}^{M}med(|r_{t,j}|,|r_{t,j-1}|,|r_{t,j-2}|)^2, \text{ and} \tag{7}$$

$$MinV_t = \frac{\pi}{\pi-2}\left(\frac{n}{n-1}\right)\sum_{i=2}^{M}\min(|r_{t,j}|,|r_{t,j-1}|)^2. \tag{8}$$

The intuition behind these estimators is based on the understanding that if a geometric return $r_{t,j}$ is considerably larger in magnitude than other returns within a day, it is more likely to be due to a jump in the interval than a continuous price movement. Since jumps are rare and occur instantaneously, large price movements in consecutive intervals are unlikely. Each of the three estimators seeks to reduce the contributions of these large price movements. Bipower variation reduces the effect of the large price movements by multiplying them with a small neighboring value. Minimum and median variations replace the large price movements with small neighboring values. All three estimators asymptotically approach the integrated variance as the sampling frequency approaches infinity.

Intuitively, the difference between realized variance and one of the jump-robust measures of variance measures the contribution of price jumps to total price volatility. The relative contributions of jumps to volatility on a particular day

$$RJ_t^{(CV)} = \frac{RV_t - CV_t}{RV_t},\qquad(9)$$

where CV represents one of the estimators of the continuous variance, is a standardized measure of the difference. The relative contribution of jumps therefore indicates the proportion of price volatility that can be attributed to jumps on that day.

3   Statistical Methods

3.1   Tests for price jumps

The estimators of variance in the previous section allow us to separate variation in the price process due to jumps and continuous price movements. However, it is often necessary to determine when these price jumps occur. To do so, a variety of jump tests based on different statistical methods have been developed. These tests attempt to identify intervals of time over which price jumps are likely to have occurred. The general intuition behind many of these tests is that significantly large price movements relative to local price volatility are likely to be jumps instead of diffusive movements. The use of high-frequency financial data is essential in the implementation of these jump tests in order for test statistics to converge towards asymptotic consistency.

Three different jump detection tests were used as part of this study. The tests proposed by Barndorff-Nielsen and Shephard (2004, 2006) and Aït-Sahalia and Jacod (2008) are two different methods to determine whether statistically-significant jumps have occurred in a specific time period, such as in a day. These two tests enable the classification of "jump days", days on which jumps are likely to have occurred, against "non-jump days". Using two different jump tests enables a check for robustness, since Schwert (2009) showed that different jump tests may be inconsistent with each other, and classify different days as containing jumps. However, the tests will not allow us to determine the number of jumps or the magnitude of jumps that have occurred during a day identified as a jump day. The final test we consider by Bollerslev, Todorov and Li (2011) proposed a method to classify each price movement observed discretely at a high frequency (e.g. a 5-minute interval) as either a jump or a diffusive price movement. This test offers an advantage over the former two since it enables individual price movements that are like to be jumps to be distinctly separated out. The direction and magnitude of the jump can also be identified.

3.1.1 Barndorff-Nielsen and Shephard (BNS) Test

Barndorff-Nielsen and Shephard (2004, 2006) developed a method for detecting the presence of jumps using the difference between a measure of quadratic variation and a jump-robust measure of variance, such as bipower variation. Intuitively, the presence of a jump will result in a significant difference between the two measures over an interval of time, such as on the day when the jump occurs. The significance of the measure of difference can be detected using a test of the statistical difference.

The BNS test uses the relative contribution of jumps based on bipower variation in (8) to construct the test statistic. Huang and Tauchen (2005) proposed a slight modification of the test statistic such that

$$
Z = \Delta^{-1/2} \frac{RJ_t^{BV}}{\sqrt{\left(\mu_1^{-4} + 2\mu_1^{-2} - 5 \right) \max\left\{ 1, \frac{\hat{Q}_t}{BV_t^2} \right\}}}, \tag{10}
$$

where $\Delta = \frac{1}{M}$ is the frequency of observation, $\mu_a = E(|Z|^a)$ and $\hat{Q}_t$ is a consistent estimator of the integrated quarticity $\int_{t-1}^{t} \sigma^4(s)ds$. Huang and Tauchen (2005) suggested that using the realized tripower quarticity as the estimator of integrated quarticity produces the best finite-sample properties as compared to statistics constructed on other measures of integrated quarticity. Therefore

$$
\hat{Q}_t = M\mu_{\frac{4}{3}}^{-3} \left(\frac{M}{M-2} \right) \sum_{i=3}^{M} |r_{t,i}|^{\frac{4}{3}} |r_{t,i-1}|^{\frac{4}{3}} |r_{t,i-2}|^{\frac{4}{3}} \rightarrow \int_{t-1}^{t} \sigma^4(s)ds \tag{11}
$$

is used to construct the test statistic.

The test statistic is asymptotically standard normal under the null hypothesis that no jumps are present in the price process. The standard normal distribution of the test statistic enables us to test for the statistical significance of the measure. The test is applied to daily data on the realized variance and bipower variation calculated using 5-minute data to detect the presence of jumps on each individual day. We use a standard one-sided t-test for the null hypothesis that a day contains no jumps at 5 percent significance level.

3.1.2   Aït-Sahalia and Jacod (AJ) Test

Aït-Sahalia and Jacod (2008) proposed a family of test statistics

$$\hat{S}(p,k,\Delta)_t = \frac{\hat{B}(p,k\Delta)_t}{\hat{B}(p,\Delta)_t} \text{ where } \hat{B}(p,\Delta)_t = \sum_{j=1}^{M} |r_{t,j}|^p, \tag{12}$$

for some constant k, which is based on the ratio of some measure of volatility at a certain high-frequency interval of observation, and a multiple of that frequency of observation. Intuitively, if a jump were to occur at a particular time, the volatility resulting from the jump will overwhelm any diffusive volatility in both the 5-minute and 10-minute intervals containing the jump. Hence, the ratio of volatilities in the two intervals will converge to 1 if there are jumps, and to another deterministic and known value if there are no jumps, as the frequency of observation $\Delta \to 0$. Under the null hypothesis that no jumps are present, a statistical test can be constructed to determine if the value of the statistic is significantly different from the value that it is known to converge to.

In order to make statistical inferences on the test statistic, it is necessary to derive the variance of the test statistic to construct statistical tests. The following estimator of the variance of the test statistic can be calculated as

$$\hat{V}_{M,t}^c = \frac{\Delta M(p,k)\hat{A}(2p,\Delta)_t}{\hat{A}(p,\Delta)_t^2}, \tag{13}$$

where

$$M(p,k) = \frac{1}{m_p^2}\left(k^{p-2}(1+k)m_{2p} + k^{p-2}(k-1)m_p^2 - 2k^{\frac{p}{2}-1}m_{k,p}\right), \tag{14}$$

$$\hat{A}(p,\Delta)_t = \frac{\Delta_n^{1-p/2}}{m_p}\sum_{i=1}^{M} |r_{t,j}|^p \mathbf{1}_{\{|r_{t,j}|\leq\alpha\Delta^\varpi\}}, \tag{15}$$

$$m_{k,p} = \mathrm{E}(|U|^p|U + \sqrt{k-1}V|^p) \text{ for U, V independent N(0,1) variables.} \tag{16}$$

In this equation, $\alpha>0$ and $\varpi\in(0, \frac{1}{2})$ are constants. The measure of volatility is then used to calculate the critical value

$$c_{M,t}^c = k^{\frac{p}{2}-1} + z_\alpha \sqrt{\hat{V}_{M,t}^c},$$ (17)

for z_α, the α-quantile of N(0,1). Then the critical (rejection) region takes the form

$$C_{M,t}^c = \{\hat{S}(p,k,\Delta)_t < c_{M,t}^c\}.$$ (18)

The choice of parameters is an important factor in conducting the jump test. This paper closely follows recommendations by the authors in our selections. Aït-Sahalia and Jacod (2008) recommended the selection of $k = 2$ and $p = 4$. The choice of k reflects the consideration that the larger the value of k, the more spaced out observations of data will be, representing loss of information between data points. Furthermore, larger values of k lead to a decrease in the effective sample size that can be used to estimate the numerator of the test statistic, representing a loss of efficiency. Hence, the smallest possible value of k is chosen. When choosing p, we consider the fact that the larger the value of p employed, the greater the emphasis is placed on larger price movements. Since smaller jumps are much harder to study, we try to keep p as small as possible. Nonetheless, p has to be reasonably large for the test statistic to follow the central limit theorem without bias. Therefore, $p = 4$ is chosen as a compromise. With these parameter selections, the asymptotic value of the test statistic when no jumps are present is 1, and the asymptotic value is 2 when jumps are present. Using $\alpha = 0.05$ and $\varpi = 0.05$ according to stimulation studies in the paper optimizes results.

3.1.3 Bollerslev, Todorov and Li (BTL) Test

Finally, Bollerslev, Todorov, and Li (2011) proposed a test that attempts to classify each discrete observation of price movement in a day as either a jump or a diffusive price movement. The results of this jump test are different from both previously-introduced tests, which classify days as either jump days or non-jump days. Since multiple price observations are made on each day, the previous tests do not allow the detection of specific times at which jumps are likely to have occurred on jump days. In contrast, the BTL test specifically detects jumps on each observation of price movement. For instance, if prices were observed at 5-minute intervals, the BTL test will then classify each 5-minute interval as a jump or a diffusive price movement.

For all jump tests, a threshold level above which large price movements can be classified as jumps must be determined. However, one complication to determining a suitable threshold level for intraday intervals is the U-shaped daily diurnal volatility pattern historically observed in asset prices. Wood, McInih and Ord (1985) observed that price volatility tends to be higher at the beginning and end of the trading day, meaning that large diffusive price movements are more likely during those periods. Andersen and Bollerslev (1997) confirmed this U-shaped intraday volatility pattern. This difference in intraday volatility can be dramatic, with the peak volatility during a day being twice the value of the minimum volatility on the same day. This volatility pattern distorts the classification of intraday intervals as jumps. If the volatility pattern were not taken into consideration, a large diffusive price movement during the beginning or the end of the day can be mistakenly classified as a price jump. Threshold levels must therefore be adjusted throughout the day to compensate for this diurnal volatility effect.

To take the diurnal volatility pattern into account, Bollerslev et al. (2011) use a Time-of-Day measure

$$TOD_i = \frac{n \sum_{t=1}^{T} |r_{i_t,n}|^2 \mathbf{1}_{\left(|r_{i_t,n}| \leq \tau \sqrt{BV_t \wedge RV_t} n^{-\varpi}\right)}}{\sum_{s=1}^{nT} |r_{s,n}|^2 \mathbf{1}_{\left(|r_{s,n}| \leq \tau \sqrt{BV_{[s/n]} \wedge RV_{[s/n]}} n^{-\varpi}\right)}}, \quad i_t = (t-1)n + 1, i \tag{19}$$

$$= 1, 2, \dots n,$$

where the indicator variable takes on the value of 1 when the absolute return is less or equal to τ standard deviations of a local estimator and 0 otherwise, and $\varpi \in (0, \frac{1}{2})$ is a constant. The Time-of-Day measure is essentially the ratio of the diffusive variation over a specific part of the day relative to the average diffusive volatility for the day. Using the Time-of-Day measure takes into account the local price volatility when calculating the threshold level for jumps. The truncation level used in separating jumps from the continuous price moves is then

$$\alpha_i = \tau \sqrt{BV_{[i/n]} \wedge RV_{[i/n]} * TOD_{i-[i/n]n}}, \quad i = 1, \dots, nT, \tag{20}$$

where observed price movements above the truncation level are classified as jumps. This study uses $\tau = 2.5$ and $\varpi = 0.49$ as used in the original paper. Intuitively, this means that all

the high-frequency price movements that are beyond two-and-a-half standard deviations of a local estimator of the corresponding stochastic volatility are classified as jumps.

3.2 Measures of changes in price volatility

The BNS and AJ jump tests enables the classification of days when jumps are likely to have occurred, whereas the BTL jump test enables the classification of specific time intervals on which jumps have occurred. The first set of information enables the comparison of diffusive price volatility on jump and non-jump days. The second set of information enables the comparison of diffusive price volatility in local windows of time before and after the price jump.

3.2.1 Change in Daily Volatility

The study commences with a comparison of price volatilities on days when jumps are likely to have occurred, as compared to days when it is likely that no jumps have occurred. A simple method to do so is to compare the average difference in diffusive price volatility between jump days and non-jump days as indicated by the BNS and AJ tests. The change in volatility variable is defined as

$$CIDV^{(V)} = \frac{1}{T_j} \sum_{t \in J} V_t \ - \frac{1}{T_c} \sum_{t \in C} V_t \ , \tag{21}$$

where V_t is the estimation of volatility as given by realized volatility in (5) or one of the three jump-robust measures of variance in (6), (7), and (8). J represents the set of all jump days that includes T_j days and C represents the set of all non-jump days with T_c days. In words, $CIDV^{(V)}$ is the difference in volatility between jump and non-jump days, where jump days are determined by either the BNS or AJ test.

The sign of the $CIDV^{(V)}$ variable is particularly interesting. When the variable is calculated using a jump-robust measure of variance – BV, $MedV$ or $MinV$, the resulting value will depend on the impact of price jumps on diffusive price volatility. In particular, price jumps can occur both in the upward or downward direction. Price jumps in different directions may be correlated with changes in diffusive volatility in different directions. A consistent sign in $CIDV^{(V)}$ values across different stocks would suggest that price jumps occur concurrently with a unidirectional change in diffusive price volatility, regardless of the

direction of the jump. However, if the $CIDV^{(V)}$ values have significant values but different signs between different stocks, then the direction of price jumps may affect the direction of the change in diffusive price volatility. The $CIDV^{(V)}$ value of a stock can then reflect the impact of a larger number of jumps in a particular direction overwhelming the impact of jumps in the opposite direction. The difference in signs of $CIDV^{(V)}$ values across different stocks can be attributed to each stock being characterized by a different number of jumps in the two directions. Unfortunately, since the BNS and AJ tests are unable to provide information on the direction of jumps on jump days, the nature of the relationship between the direction of price jumps and change in diffusive volatility cannot be made clear. Finally, if $CIDV^{(V)}$ were not significantly different from zero, we may postulate that price jumps do not occur concurrently with changes in diffusive volatility. An alternative possibility is that jumps in both directions are equally likely across all stocks and have opposite but the same magnitude of impact on diffusive price volatility.

When the variable is calculated using realized volatility, we may expect $CIDV^{(RV)}$ values to be positive in all cases. Since realized volatility is an estimator of quadratic variation which includes both a diffusive volatility and a jump volatility component, jump days are likely to have higher realized volatility than non-jump days, when jump volatility is zero. This is true if diffusive volatility is not significantly different between jump and non-jump days.

Since $CIDV^{(V)}$ values are a difference of the means of two sets of data with different sample sizes and possibly different volatilities, a Welch's t-test can be used to determine the significance of the values. The t-statistic is then defined by

$$t = \frac{CIDV^{(V)}}{\sqrt{\frac{s_j^2}{T_j} + \frac{s_c^2}{T_c}}},$$
(22)

where s_j^2 and s_c^2 are the standard errors of the volatility on jump days and the volatility on non-jump days respectively. The degrees of freedom associated with this estimate is approximated using the Welch-Satterthwaite equation

$$df = \frac{(\frac{s_j^2}{T_j} + \frac{s_c^2}{T_c})^2}{\frac{s_j^4}{T_j^2(T_j - 1)} + \frac{s_c^4}{(T_c)^2(T_c - 1)}} . \tag{23}$$

A two-sided t-test can then be conducted using the t-distribution at the above degrees of freedom to determine whether the $CIDV^{(V)}$ values are significantly different from zero.

3.2.2 Changes in Local Volatility

The Bollerslev, Todorov and Li (2011) Test enables the classification of individual observations of price movements as either jumps or diffusive movements at each high-frequency interval. This enables us to study whether the occurrence of price jumps is correlated with changes in diffusive volatility in a local window of time around the jump interval. The change in local volatility variable is defined as

$$CILV_i^{(V)} = \frac{1}{m} \sum_{m \in I_2(i)} V_m - \frac{1}{m} \sum_{n \in I_1(i)} V_n , \tag{24}$$

where I_1 is a local window of time before the jump with m intervals, and I_2 is a window of the same length after the jump. V_m and V_n are the individual elements used to calculate realized volatility in (5) or one of the three jump-robust measures of variance in (6), (7), and (8). It is important to note that individual values of V_i are not measures of instantaneous volatility, whereas sums of the elements over longer windows of time converge to the true value of volatility as the frequency of sampling increases. In words, $CILV_i^{(V)}$ is approximately the difference in volatility between two stipulated time intervals of the same length immediately before and after a price jump at interval i. The volatility on the jump interval itself is excluded from consideration so that the $CILV_i^{(V)}$ variable does not take the jump volatility on the jump interval into account. Instead, $CILV_i^{(V)}$ measures the change in diffusive volatility immediately before and after the jump.

To calculate $CILV_i^{(V)}$, only the intraday volatility is taken into account. This is because changes in volatility overnight is related to overnight price returns, which have different behavioral characteristics from intraday returns and are not considered in this study. The $CILV_i^{(V)}$ values are calculated for 15-minute, 30-minute, and 1-hour intervals before and after

the jumps indicated by the BTL test. Identified jump intervals which are too close to the start of the end of the day for sufficient volatility data to be aggregated are excluded from the study. It is also necessary to ensure that windows of time around each price jump do not overlap, as this would result in a price jump and its associated volatility impacts to be spilt over to the window considered for another price jump. With these considerations, the length of the local windows to be used faces a tradeoff. Since volatility cannot be directly observed, we use estimators which converge asymptotically to the true value of volatility as the number of data points goes to infinity. This means that the longer the window considered, the more data points are included in the estimator, and the better the estimation. However, the longer the window used, the more jump intervals have to be eliminated and the greater the likelihood that the window will contain another price jump. Hence, although $CILV_i^{(V)}$ can be calculated across a range of time intervals, we mainly study the 30-minute interval as the best compromise of the above considerations. The results of the BTL test show that using 30-minute windows largely preclude window overlaps between different price jumps.

In contrast to the $CIDV^{(V)}$ variable, we expected $CILV_i^{(V)}$ values to be similar for RV and the jump-robust measures of variation. This is because the jump volatility at the jump interval is excluded from consideration. RV then approximates diffusive price volatility in a similar way to the jump-robust volatility estimators in the local windows of time surrounding the price jump. Comparing the direction of price jump and the sign on the corresponding $CILV_i^{(V)}$ value will help to reveal the relationship between the directions of the price jump and corresponding change in diffusive volatility. Furthermore, an aggregate mean of the individual $CILV_i^{(V)}$ values, the $CILV^{(V)}$, could be considered separately for upward and downward price jumps.

Since $CILV^{(V)}$ values are measures of differences in paired samples of volatility, we can test for its statistical significance using a dependent t-test. The t-statistic is then defined by

$$t = \frac{CILV^{(V)}}{S_D/\sqrt{n}},$$

(25)

where S_D is the standard error of $CILV_i^{(V)}$ and n is the number of jump intervals found by the BTL test. A two-sided t-test can then be conducted using the t-distribution at the $n - 1$

degrees of freedom to determine whether the $CIDV^{(V)}$ values are significantly different from zero.

4   Data

We studied the S&P500 Futures (SPFU) index and two individual stocks: Fedex Corporation (FDX) and United Parcel Service, Inc. (UPS). The SPFU index is used as a good proxy to market price movements as it effectively trades the basket of 500 stocks in the S&P 500 index as a single bundle. Since the futures index is more liquid than the actual index, the SPFU is a cleaner measure of market price movements. Price data on every minute from 9:35 AM to 4:00 PM on trading days from Nov 11 1999 – Dec 30 2010 were obtained, giving a total of 2763 days of data for each stock. Although the trading day starts at 9:30 AM, the first 5 minutes are excluded from consideration due to excessive noise contamination. Overnight returns are also excluded, since overnight data tends to exhibit different dynamics as compared to intraday data.

All data is adjusted for stock splits and other abnormalities such as missing price observations. The three data sets are then aligned such that each time series contains the same set of data points on the same time intervals. Basic stock returns and volatilities are reported in Table 1.

4.1   Microstructure Noise

The asymptotic consistency of the estimators of variance suggests that sampling should be done as frequently as possible. However, when prices are observed over extremely short time intervals, the process becomes contaminated with "microstructure noise", or market frictions which cause a short-term deviation between the observed and efficient asset prices. The main reason for the observation of microstructure noise is that market prices are quoted on a discrete price grid with a spread between the asking price of sellers and the bidding price of buyers (the bid-ask spread). Therefore, different prices may be quoted simultaneously by competing market players due to heterogeneous beliefs, information and inventory positions. Consequently, any observed price does not represent the efficient market price but instead an underlying ideal price confounded by an error term reflecting the impact of the heterogeneities, or "noise". At high-frequency intervals, this error term becomes non-trivial.

Andersen, Bollerslev, Diebold, and Labys (2000) propose a "volatility signature plot" which graphs the realized variance against sampling frequency. Figure 1 shows a volatility signature plot for UPS. From the plot, it can be seen that variance increases at short time intervals (1-, 2-minutes), indicating noise contamination, and approach a flat, constant value for sufficiently long intervals. Using such a volatility signature plot, a 5-minute time interval was selected for this paper to maximize the use of available data (price information is available at every minute of the trading day) and limit the impact of microstructure noise.

5   Results

5.1   Jump Detection

The BNS Test[1] classified jumps to be likely on 22.8, 25.6 and 25.8 percent of days respectively for SPFU, FDX and UPS. The relatively lower jump contribution in the index as compared to individual stocks is likely due to the diversified nature of the index. Since individually-occurring stock jumps would not be significant enough to cause a jump in the index, the incidence of jumps in SPFU would reflect rarer market-wide co-jumps. Using the AJ test, we classified 24.3, 37.6 and 46.2 percent of days as jump days respectively for SPFU, FDX, and UPS. This corroborates with the paper's empirical finding when the test was applied to each of the 30 stocks in the Dow Jones Industrial Average (Aït-Sahalia and Jacod, 2008). The relatively lower jump contribution in the index as compared to the stocks is similarly observed. Both tests were conducted at the 5 percent significance level. From the BTL test, we identified the 5-minute intervals on which jumps are likely to have occurred. 1.8, 2.1 and 2.2 percent of all 5-minute intervals were classified as jumps for SPFU, FDX and UPS respectively. This corresponds to an approximate average of 1.5 jumps per day, although it appears that there may be some clustering of multiple jumps on days when jumps occur. Of the detected jump intervals, upward jumps represent 51 to 52 percent of jump intervals both SPFU and the two stocks and are therefore slightly more likely. Since the BTL test has a different objective from the other two jump tests, jumps detected from this test are not directly compared to the results of the other two tests.

The test results highlight a concern over the inconsistency between the BNS and AJ jump tests. Table 2 shows the percentage of jump days identified by the jump tests that corroborate

---

[1] See Appendix for a summary of the acronyms used in this paper

between the different jump tests used. Although the BNS and AJ tests similarly attempt to separate jump and non-jump days, the degree of corroboration between the two tests is low. The percentage of days classified as jump days is similar for both tests for SPFU. However, only around 20 percent of the jump days classified by each test have also been identified by the other test. For the two stocks, the percentage of days identified as jump days differ significantly between the two tests. The BNS test consistently classifies less days as jump days than the AJ test. Less than 30 percent of the AJ jump days were also detected by the BNS jump test for both stocks. Although the number of BNS jump days that are also detected by the AJ test is higher, this result is trivial given the larger number of jump days found by the AJ test. Since it is not possible to identify actual price jumps, it is not possible to check which of the tests are the most accurate. Schwert (2009) suggests that using different sampling frequencies for different tests could help in improving corroboration between the jump days classified by the BNS and AJ tests, although the exploration of the effect is beyond the scope of this paper.

The inconsistency across jump tests is not entirely surprising if "price jumps" were to be interpreted correctly. "True" price jumps cannot be observed in the asset price data, since the price process is continuous whereas observations are discrete. The inclusion of discontinuities in addition to a continuous price movement enables researchers to better model the observed price data. Hence, jump tests classify price movements which are likely to be jumps as such, based on characteristics such as the size of the jump relative to local volatility, but without necessarily being able to reflect the true discontinuities in prices. In particular, large continuous price movements and small jumps could be misclassified. We will therefore proceed with using the results from all three jump tests, while bearing in mind the possibility of inaccuracies in identified jumps by the jump tests.

5.2   Change in Daily Volatility

The change in daily volatility $CIDV^{(V)}$ values calculated from the BNS test are reported in Table 3 along with results of the corresponding t-tests for statistical significance of the values. $CIDV^{(RV)}$ values are not significantly different from zero for SPFU, FDX and UPS. Since realized volatility is an estimator of the sum of jump and diffusive volatility, it is expected to be higher on jump days if diffusive volatility is similar between jump and non-jump days. The results therefore suggest that diffusive price volatility is lower on jump days for all three

assets. The $CIDV^{(V)}$ values based on jump robust measures of volatility support this result. $CIDV^{(BV)}$, $CIDV^{(MedV)}$ and $CIDV^{(MinV)}$ values are all significantly negative for SPFU and the two stocks. Furthermore, the results appear to be robust if we look at the different measures of volatility on jump days separately from non-jump days. As expected, RV is significantly higher than the corresponding measures of jump-robust volatility on jump days whereas it is similar to the measures of diffusive volatility on non-jump days. To check for the robustness of the results between jump and non-jump days, we compare the $CIDV^{(V)}$ values from the BNS test with the AJ test.

$CIDV^{(V)}$ values from the AJ test are not significantly different from zero for all 3 assets and all measures of volatility (Table 4). However, the derived volatilities on jump and non-jump days detected by the test reflect an irregularity in the results of the AJ jump test. In particular, the differences between RV and the measures of jump-robust volatility on jump and non-jump days are similar. Since this difference represents jump volatility, this indicates that jump volatilities on jump and non-jump days detected by the AJ test are similar, contrary to expectations. Since there was an initial lack of corroboration between jump days classified by the AJ test and the BNS test, we will primarily rely on the $CIDV^{(V)}$ values from the BNS test.

The main finding from the $CIDV^{(V)}$ values based on the BNS test is therefore that diffusive price volatility appears to be lower on jump days. Since the BTL test revealed an approximately equal number of upward and downward jumps in SPFU and the two stocks, this result is likely to be unrelated to the direction of jumps. Instead, diffusive price volatility is systematically lower on jump days detected by the BNS test.

5.3 Change in Local Volatility

Given the inconsistencies in $CIDV^{(V)}$ values, we study the mean change in local volatility $CILV^{(V)}$ values to understand the relationship between price jumps and changes in volatility. Tables 5.1-3 show the $CILV^{(V)}$ values for SPFU, FDX and UPS at 15-, 30- and 60-minute intervals. We first examine $CILV^{(V)}$ values for market price movements, as indicated by the S&P 500 Futures Index. For downward jumps, $CILV^{(V)}$ are consistently positive across all the time intervals sampled and significant for all measures of diffusive volatility. For upward jumps, $CILV^{(V)}$ values are not significant at any time interval. However, they are negative

when calculated using 15-minute windows, but become more positive when the length of the window increases. There appears to be evidence that downward jumps in the market price occur together with an upward change in diffusive price volatility. In the case of upward jumps in the market price, the results could be explained by an initial downward change in diffusive price volatility, which is overwhelmed by an eventual increase in volatility as time passes following the jump. We see $CILV^{(V)}$ values tending towards zero as longer windows of time are considered for both upward and downward jumps, indicating a gradual return of volatility to original values. We therefore postulate that the price jumps occur concurrently with a change in volatility, and that a negative dependence exists between the two changes. However, volatility gradually tends to its original value as time passes following the price jump.

Figure 2 shows the scatter plots of individual $CILV_i^{(V)}$ values on the magnitude of the return at the price jump. The scatter for the 30-minute windows is shown, although the plots for the different time intervals are similar. We observe a clustering of $CILV_i^{(V)}$ values around zero, particularly when the magnitude of the jump is small. However, as the magnitudes of jumps increase for both upward and downward jumps, bigger $CILV_i^{(V)}$ values are observed. In particular, with increasing magnitude of jumps, a larger range of $CILV_i^{(V)}$ values is observed. However, the scatter plots appear to be roughly symmetric for upward and downward jumps, exhibiting no evidence of negative dependence between the direction of jumps and direction of change in diffusive price volatility.

For both UPS and FDX, $CILV^{(V)}$ values are consistently negative for upward jumps at the 30- and 60-minute intervals, although not statistically significant. The same pattern, however, is not observed when 15-minute intervals are used. We similarly observe the volatilities tending towards zero in the 60-minute versus the 30-minute windows. For downward jumps, the $CILV^{(V)}$ values are not very consistent in signs at the 15- and 30-minute intervals. However, they become more negative as time passes. The $CILV^{(V)}$ values are not statistically significant. We therefore find it difficult to draw any consistent conclusions about volatility changes during individual stock price jumps, a contrast to the relationships found in SPFU earlier.

Figure 3 shows the scatter plots of individual $CILV_i^{(V)}$ values for FDX on the magnitude of the return at the price jump. The scatter for the 30-minute windows is shown, although the plots for the different time intervals are similar. Results for UPS are similar. As with the scatter plots for SPFU, we observe increased $CILV_i^{(V)}$ values with larger magnitudes of jumps. However, there appears to be no evidence of dependence between the direction of jumps and direction of change in diffusive price volatility.

The results derived from $CILV_i^{(V)}$ values for both SPFU and the two stocks do not appear to directly explain the earlier result from $CIDV^{(V)}$ values, when we found diffusive volatility to be lower on jump days detected by the BNS test. Therefore, other factors apart from an immediate change in diffusive volatility following a price jump could account for the earlier result.

6   Conclusion

This paper was motivated by the question of whether the occurrence of price jumps is related to changes in diffusive volatility. To address this question, two approaches were adopted. First, two jump tests developed by Barndorff-Nielsen and Shephard (2006) and Aït-Sahalia and Jacod (2008) were used to classify days as jump or non-jump days. We then studied the difference in diffusive volatility between these jump and non-jump days. Second, an intraday jump test was used to classify each 5-minute price observation as either a jump or a diffusive price movement. We then compared the diffusive volatility in local windows of time immediately preceding and following intervals classified as containing jumps.

This paper represents a preliminary study of the relationship between price jumps and corresponding changes in diffusive price volatility. To further investigate the relationship between price jumps and changes in diffusive price volatility for individual stocks, additional stocks could be studied. The methodology of this paper is unable to detect the presence of volatility jumps in the presence of price jumps. A natural extension of this study is to examine the instantaneous volatility during price jump intervals. In particular, the test for co-jumps between price and volatility developed by Jacod and Todorov (2010) can be applied to individual stocks. Nonetheless, this study has resulted in some interesting findings which may inspire further research.

Using the BNS and AJ jump tests, we identified days during which jumps are likely to have occurred. The BNS jump test showed that jump days have lower diffusive price volatility than non-jump days. The direction of price jumps does not appear to factor into this effect. However, we found irregularities in the results of the AJ jump test since jump volatilities on jump and non-jump days classified by the test are similar, contrary to expectations.

Using the intraday jump test developed by Bollerslev et al. (2011), we identified 5-minute intervals when jumps are likely to have occurred. For market prices, we observe evidence that price jumps occur concurrently with a change in volatility, and that a negative dependence exists between the two changes. This is in line with the results of Todorov and Tauchen (2011). Furthermore, a price jump of greater magnitude is more likely to induce a larger change in diffusive price volatility. As time passes following the price jump, price volatility gradually tends to its original value. In contrast, for individual stock prices, there was no consistent change in volatility when a price jump occurs, suggesting that the relationship between price jumps and changes in diffusive price volatility are different for market prices and individual stock prices.

Tables and Figures

Tables

Table 1

	SPFU	FDX	UPS
RV	14.9828	24.9206	18.9037
BV	14.4576	23.8821	18.1116
MedV	14.2872	23.5663	17.9215
MinV	14.1716	23.6200	18.0231

Table 1 shows the mean values of various estimators of volatility for each stock and SPFU over the sample period.

Table 2

	SPFU	FDX	UPS
BNS jumps found in AJ test	0.2127	0.3994	0.5175
AJ jumps found in BNS test	0.2000	0.2714	0.2894

Table 2 shows the percentage of jump days found using the BNS or AJ jump test that are also found using the other test. The degree of corroboration between the two tests appears to be low.

Table 3

SPFU					
	Jump Days	Non-Jump Days	CIDV	t-statistic	p-value
RV	14.8626	15.0181	-0.1555	-0.3591	0.3598
BV	13.0809	14.8662	-1.7853	-4.5427	**6.1424e-06**
MedV	10.0120	11.6555	-1.6435	-5.2521	**1.7863e-07**
MinV	12.1956	14.7578	-2.5622	-6.8053	**1.5722e-11**
FDX					
	Jump Days	Non-Jump Days	CIDV	t-statistic	p-value
RV	25.5576	24.6963	0.8613	1.6488	0.0994
BV	22.2375	24.4435	-2.206	-4.6881	**3.0028e-06**
MedV	16.8194	18.7270	-1.9076	-5.1357	**3.1864e-07**
MinV	21.0469	24.4987	-3.4518	-7.4664	**1.3411e-13**
UPS					
	Jump Days	Non-Jump Days	CIDV	t-statistic	p-value
RV	19.2517	18.7772	0.4745	1.0568	0.2908
BV	16.7392	18.5879	-1.8487	-4.5239	**6.5290e-06**
MedV	12.5516	14.0815	-1.5299	-4.8042	**1.7068e-06**
MinV	16.0759	18.7004	-2.6245	-6.5689	**6.7772e-11**

Table 3 shows the mean values of various measures of volatility for each stock and SPFU, on days with or without jumps, as determined by the Barndorff-Nielsen and Shephard (2006) test, and the corresponding changes in volatility measures and t-test results. Statistically significant p-values for a two-sided t-test are highlighted in bold.

Table 4

SPFU					
	Jump Days	Non-Jump Days	CIDV	t-statistic	p-value
RV	15.4182	14.8431	0.5751	1.0249	0.3055
BV	14.9110	14.3142	0.5968	0.3580	0.7203
MedV	11.5867	11.1826	0.4041	0.2466	0.8052
MinV	14.6372	14.0248	0.6124	0.9334	0.3507
FDX					
	Jump Days	Non-Jump Days	CIDV	t-statistic	p-value
RV	23.8683	24.2072	-0.3389	-0.7197	0.4718
BV	22.7779	23.3638	-0.5859	-1.2834	0.1995
MedV	17.2389	17.9178	-0.6789	-1.9190	0.0551
MinV	22.3608	23.0932	-0.7324	-1.6000	0.1097
UPS					
	Jump Days	Non-Jump Days	CIDV	t-statistic	p-value
RV	19.1265	18.7052	0.4213	1.0249	0.3055
BV	18.1874	18.0449	0.1425	0.3580	0.7203
MedV	13.7275	13.6516	0.0759	0.2466	0.8052
MinV	18.2237	17.8507	0.3730	0.9334	0.3507

Table 4 shows the mean values of various measures of volatility for each stock and SPFU, on days with or without jumps, as determined by the Aït-Sahalia and Jacod (2008) test, and the corresponding changes in volatility measures and t-test results. Statistically significant p-values for a two-sided t-test are highlighted in bold.

Table 5

Table 5.1: 15-minute interval

SPFU						
	Upward jumps			Downward jumps		
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0027	-0.9830	0.3258	0.0027	0.6396	0.5226
BV	-0.0009	-0.6475	0.5174	0.0033	1.6663	0.0960
MedV	-0.0008	-0.8674	0.3858	0.0015	1.9890	**0.0470**
MinV	0.0014	1.1115	0.2666	0.0042	2.3022	**0.0216**
FDX						
	Upward jumps			Downward jumps		
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0019	-0.9849	0.3248	-0.0007	-0.1534	0.8782
BV	0.0003	0.3572	0.7210	-0.0001	-0.0227	0.9818
MedV	-0.0001	-0.1999	0.8416	-0.0022	-1.0356	0.3006
MinV	0.0021	2.1919	**0.0286**	0.0007	0.2707	0.7866
UPS						
	Upward jumps			Downward jumps		
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	0.0002	0.1632	0.8704	-0.0002	-0.0802	0.9362
BV	0.0014	1.7951	0.0730	0.0005	0.2910	0.7710
MedV	0.0002	0.4158	0.6776	-0.0015	-1.2186	0.2232
MinV	0.0023	2.8971	**0.0038**	0.0006	0.3504	0.7262

Table 5.1 shows the CILV values for each stock and SPFU calculated using 15-minute windows before and after jump intervals identified by the Bollerslev, Todorov and Li (2011) test, and the corresponding t-test results. Results are separated for upward and downward jumps, as indicated by the sign on the stock return during the jump interval. Statistically significant p-values for a two-sided t-test are highlighted in bold.

		SPFU				
		Upward jumps			Downward jumps	
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0019	-0.9849	0.3248	0.0034	1.2970	0.1950
BV	0.0003	0.3572	0.7210	0.0032	2.3513	**0.0190**
MedV	-0.0001	-0.1999	0.8416	0.0015	2.0157	**0.0442**
MinV	0.0021	2.1919	**0.0286**	0.0038	2.9624	**0.0032**
		FDX				
		Upward jumps			Downward jumps	
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0046	-1.6833	0.0926	-0.0010	-0.3337	0.7388
BV	-0.0025	-0.9316	0.3518	0.0000	-0.0202	0.9838
MedV	-0.0029	-2.2785	**0.0228**	-0.0020	-1.3007	0.1936
MinV	-0.0012	-0.4267	0.6696	0.0008	0.4194	0.6750
		UPS				
		Upward jumps			Downward jumps	
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0037	-1.7642	0.0780	0.0007	0.4146	0.6786
BV	-0.0025	-1.9023	0.0574	0.0015	0.8433	0.3992
MedV	-0.0018	-1.8525	0.0642	-0.0014	-1.4297	0.1532
MinV	-0.0023	-1.5760	0.1152	0.0015	0.9209	0.3574

Table 5.2 shows the CILV values for each stock and SPFU calculated using 30-minute windows before and after jump intervals identified by the Bollerslev, Todorov and Li (2011) test, and the corresponding t-test results. Results are separated for upward and downward jumps, as indicated by the sign on the stock return during the jump interval. Statistically significant p-values for a two-sided t-test are highlighted in bold.

Table 5.3: 60-minute interval

SPFU						
	Upward jumps			Downward jumps		
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	0.0002	0.1632	0.8704	0.0024	1.5633	0.1184
BV	0.0014	1.7951	0.0730	0.0025	2.7617	**0.0058**
MedV	0.0002	0.4158	0.6776	0.0011	2.2745	**0.0232**
MinV	0.0023	2.8971	**0.0038**	0.0027	3.2737	**0.0010**
FDX						
	Upward jumps			Downward jumps		
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0035	-1.8395	0.0662	-0.0037	-1.8011	0.0720
BV	-0.0017	-1.1179	0.2638	-0.0029	-1.8258	0.0682
MedV	-0.0025	-2.3599	**0.0184**	-0.0035	-3.2433	**0.0012**
MinV	-0.0010	-0.6338	0.5264	-0.0024	-1.5502	0.1214
UPS						
	Upward jumps			Downward jumps		
	CILV	t-statistic	p-value	CILV	t-statistic	p-value
RV	-0.0002	-0.1701	0.8650	-0.0007	-0.5357	0.5922
BV	-0.0016	-1.3233	0.1860	-0.0005	-0.4139	0.6790
MedV	-0.0013	-1.4287	0.1534	-0.0020	-2.4751	**0.0134**
MinV	-0.0024	-1.5975	0.1104	-0.0005	-0.4087	0.6828

Table 5.3 shows the CILV values for each stock and SPFU calculated using 60-minute windows before and after jump intervals identified by the Bollerslev, Todorov and Li (2011) test, and the corresponding t-test results. Results are separated for upward and downward jumps, as indicated by the sign on the stock return during the jump interval. Statistically significant p-values for a two-sided t-test are highlighted in bold.

Figures

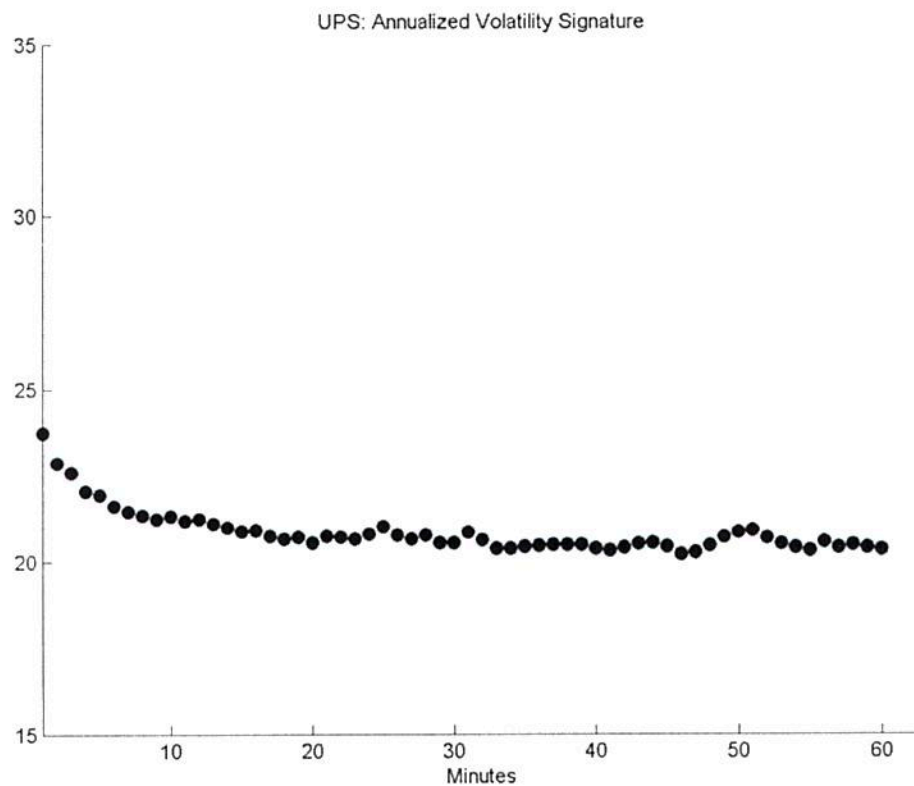*Figure 1*

UPS: Annualized Volatility Signature

Figure 1 shows the volatility signature plot for UPS. The horizontal axis shows the frequency of sampling, and the vertical axis shows the corresponding Realized Volatility measured using at the frequency. From the graph, it can be seen that volatility is higher at shorter intervals below 10-minutes due to contamination with microstructure noise, but flattens out as the frequency of sampling increases. The 5-minute frequency of sampling appears to give a reasonable measure of volatility while enabling relatively little wastage of data.
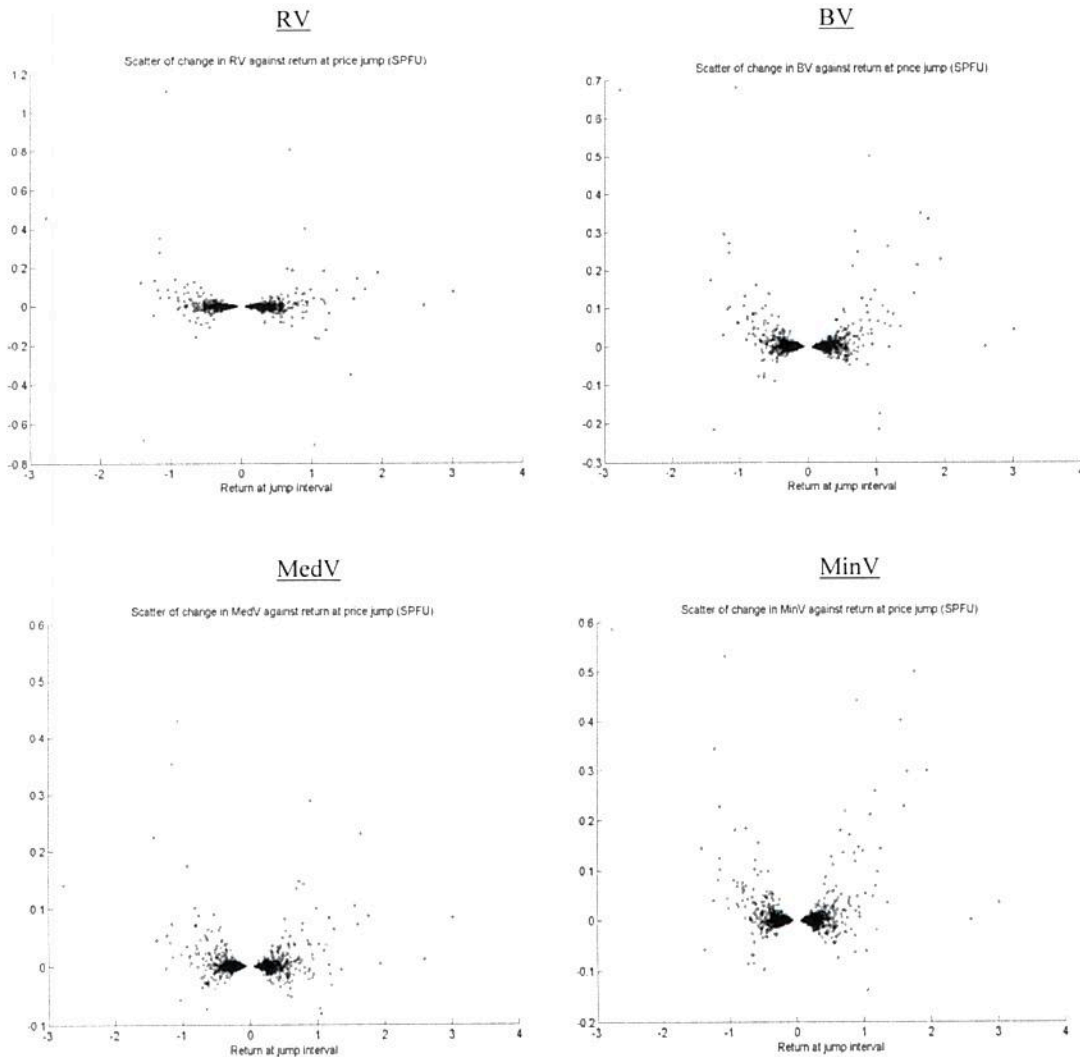
Figure 2

SPFU

Figure 2 shows the scatter plot of the CILV values on the magnitude of identified jumps for SPFU, measured at 30-minute windows before and after jump intervals identified by the BTL jump test. Scatter plot for CILV calculated using RV, BV, MedV and MinV are shown. There is no apparent relationship between the change in volatility and direction of the jump. Scatter plots at 15-, and 6s0-minute intervals are similar and therefore not shown.

Figure 3
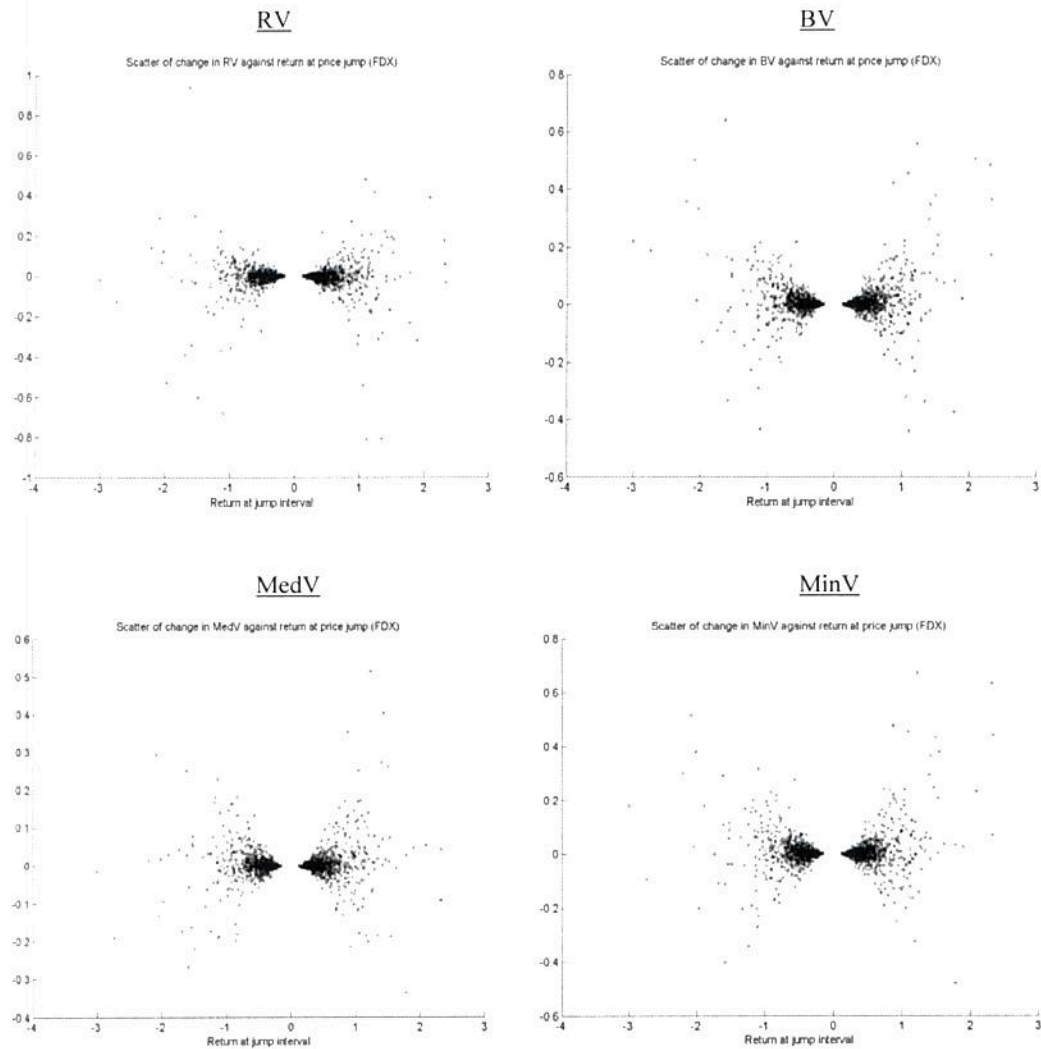
FDX

RV

BV

MedV

MinV

Figure 3 shows the scatter plot of the CILV values on the magnitude of identified jumps for FDX, measured at 30-minute windows before and after jump intervals identified by the BTL jump test. Scatter plot for CILV calculated using RV, BV, MedV and MinV are shown. There is no apparent relationship between the change in volatility and direction of the jump. Scatter plots at 15-, and 60-minute intervals are similar and therefore not shown. Results for UPS are similar.

References

[1]   Ait-Sahalia, Y., & Jacod, J. (2008). Testing for Jumps in Discretely Observed Process. *Annals of Statistics*, 35(1), 355-392.

[2]   Andersen, T., Benzoni, L., & Lund, J. (2002). An empirical investigation of continuous-time equity return models. *The Journal of Finance*, 57(3), 1239-1284.

[3]   Andersen, T., & Bollerslev, T. (1997). Intraday Periodicity and Volatility Persistence in Financial Markets. *Journal of Empirical Finance*, 4(2-3), 115-158.

[4]   Andersen, T., Bollerslev, T., Diebold, F., & Labys, P. (2000). Exchange Rate Returns standardized by Realized Volatility are (Nearly) Gaussian. *Multinational Finance Journal*, 4(3), 159-179.

[5]   Andersen, T., Dobrev, D., & Schaumburg, E. (2010). Jump-Robust Volatility Estimation using Nearest Neighbor Truncation. *Federal Reserve Bank of New York Staff Report 465.*

[6]   Bakshi, G., Cao, C., & Chen, Z. (1997). Empirical Performance of Alternative Option Pricing Models. *Journal of Finance*, 52, 2003-2049.

[7]   Barndorff-Nielsen, O., & Shephard, N. (2004). Power and Bipower Variation with Stochastic Volatility and Jumps. *Journal of Financial Econometrics*, 2(1), 1-37.

[8]   Barndorff-Nielsen, O., & Shephard, N. (2006). Econometrics of Testing for Jumps in Financial Economics Using Bipower Variation. *Journal of Financial Econometrics*, 4(1), 1-30.

[9]   Bollerslev, T., Todorov, V., & Li, S. Z. (2011). Jump Tails, Extreme Dependencies, and the Distribution of Stock Returns, accepted for publication in *Journal of Econometrics*.

[10]   Chernov, M., Gallant, A., Ghysels, E., & Tauchen, G. (2003). Alternative Models for Stock Price Dynamics. *Journal of Econometrics*, 116(1-2), 225-257.

[11]   Drost, F., Nijman, T., & Werker, B. (1998). Estimation and Testing in Models Containing Both Jumps and Conditional Heteroskedasticity. *Journal of Business Economics and Statistics*, 16(2), 237-243.

[12]   Huang, X., & Tauchen, G. (2005). The Relative Contribution of Jumps to Total Price Variance. *Journal of Financial Econometrics*, 3(4), 256-499.

[13]   Jacod, J. & Todorov, T. (2010). Do Price and Volatility Jump Together?. *Annals of Applied Probability*, 20, 1425-1469.

[14]   Jiang, G., & Oomen, R. (2008). Testing for Jumps When Asset Prices are Observed with Noise: A Swap Variance Approach. *Journal of Econometrics*, 144(2), 352-370.

[15]   Lee, S. (2011). Jumps and Information Flow in Financial Markets, accepted for publication in the *Review of Financial Studies.*

[16]   Lee, S., & Mykland, P. (2008). Jumps in Financial Markets: A New Nonparametric Test and Jump Dynamics. *Review of Financial Studies*, 21(6), 2535-2563.

[17]   Liu, J., Longstaff, F. & Pan, J. (2003). Dynamic Asset Allocation with Event Risk. *Journal of Finance,* 58, 231-259.

[18]   Merton, R. (1976). Option Pricing when Underlying Stock Returns are Discontinuous. *Journal of Financial Econometrics*, 3(1), 125-144.

[19]   Naik, V., & Lee, M. (1990). General Equilibrium Pricing of Options on the Market Portfolio with Discontinuous Returns. *Review of Financial Studies*, 3, 493-521.

[20]   Schwert, M. (2009). Hop, Skip and Jump - What Are Modern Jumps Tests Finding in Stock Returns?. *Duke University Economics Honors Thesis.*

[21]   Todorov, V., & Tauchen, G. (2011). Volatility Jumps. *Journal of Business & Economic Statistics*, 29(3), 356-371.

[22]   Wood, R., McInish, T. & Ord, J. (1985). An Investigation of Transaction Data for NYSE Stocks. *Journal of Finance*, 40(3), 723-739.

Appendix

Throughout the paper, the following acronyms were used:

AJ test: Jump test by Ait-Sahalia and Jacod (2008) (3.1.2)

BNS test: Jump test by Barndorff-Nielsen and Shephard (2004, 2006) (3.1.1)

BTL test: Jump test by Bollerslev et al. (2011) (3.1.3)

CIDV: Change in Daily Volatility (3.2.1)

CIDV: Change in Local Volatility (3.2.2)

RV: Realized Volatility (2.2)

BV: Bipower Variation (2.2)

MedV: Median Variation (2.2)

MinV: Minimum Variation (2.2)