Crisis Period Forecast Evaluation of the DCC-GARCH Model

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Abstract

The goal of this paper is to investigate the forecasting ability of the Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH). We estimate the DCC's forecasting ability relative to unconditional volatility in three equity-based crashes: the S&L Crisis, the Dot-Com Boom/Crash, and the recent Credit Crisis. The assets we use are the S&P 500 index, 10-Year US Treasury bonds, Moody's A Industrial bonds, and the Dollar/Yen exchange rate. Our results suggest that the choice of asset pair may be a determining factor in the forecasting ability of the DCC-GARCH model.

I. Introduction

Many of today's key financial applications, including asset pricing, capital allocation, risk management, and portfolio hedging, are heavily dependent on accurate estimates and well-founded forecasts of asset return volatility and correlation between assets. Although volatility and correlation forecasting are both important, however, existing literature has dealt more closely with the performance of volatility models – only very recently has the issue of correlation estimation and forecasting begun to receive extensive investigation and analysis. The goal of this paper is to extend research that has been undertaken regarding the forecasting ability of one specific correlation model, the Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedasticity (DCC-GARCH). Specifically, our study seeks to evaluate the DCC model's performance when used to estimate future correlations in volatile market environments (i.e. bull vs. bear markets) and across varying asset classes.

It is generally accepted that asset correlations increase during times of negative market returns; this motivates our analysis of DCC-GARCH as a forecasting tool during periods of "extraordinary" market activity – periods in which we know correlation dynamics are most likely to change significantly. We examine a number of periods of positive returns followed by extreme negative returns, namely the Savings and Loans Crisis from 1988-1991, the Dot-Com Boom/Crash from 1998-2001, and the Credit Crisis from 2004-2009. Specifically, we estimate the DCC-GARCH model during the "bull market" prior to each crisis, and then test the model parameters during the subsequent "bear market" periods. We choose these periods because we expect that the correlation dynamics during the stable in-sample periods are significantly different from the dynamics during the out-of-sample crisis

periods. We have included four different assets: the S&P 500 index, Dollar-Yen exchange rate, 10-year US Treasury Bonds, and Moody's A Industrial Bonds, attempting to determine the relationship between asset class and model performance. Another issue our study touches upon is whether different time lengths of sample periods will have a substantial impact on model performance. Because some of our crisis sample periods are of different lengths than others, our results may also provide some insight on whether these variations in sample lengths may contribute to variations in forecasting ability.

We evaluate the effectiveness of the out-of-sample forecasts by comparing them to a naïve unconditional correlation estimate over the in-sample period, and utilize mean-squared error analysis to quantify the outperformance of the DCC forecasts versus the unconditional correlation. Since the "crises" that we have selected relate particularly to the US equity markets, each of our tests evaluates the DCC model performance using the S&P 500 index relative to each other asset in turn.

The remainder of this paper is structured as follows: Section II gives a brief overview of some of the most relevant recent literature. Section III describes the data used in this study, and section IV presents DCC-GARCH model and the underlying methodology. Section V focuses on presenting and detailing the empirical results of our study, and Section VI concludes.

II. Literature Review

The dynamic conditional correlation generalized autoregressive conditional heteroskedasticity (DCC-GARCH) model was introduced by Robert Engle in 2002. The basis for the DCC model is Engle's original autoregressive conditional heteroskedasticity (ARCH) model (Engle 1982), Bollerslev's generalized autoregressive conditional heteroskedasticity (GARCH) model (Bollerslev 1986), and Bollerslev's constant conditional correlation (CCC) model (Bollerslev 1990). The DCC-GARCH model is an econometric tool used to model correlation between two or more data series. The basis of the model is to take standardized residuals from GARCH volatility estimations to estimate correlation (see Section IV. Data and Methodology).

The DCC model is especially relevant to the field of financial management, as correlations are critical inputs for many common tasks including hedging, pricing structured financial instruments, and managing risk through constructing an optimal asset portfolio based upon Markowitz portfolio theory (Markowitz 1952). Moreover, the ability to forecast these correlations is especially valuable for the same financial management tasks. Despite the importance of forecasting correlations, however, evaluating the performance of the DCC-GARCH and other correlation models in forecasting has not been extensively investigated until recently; instead, prior research has placed a stronger emphasis on the forecasting performance of volatility models such as the many variants of the ARCH/GARCH range of models.

The DCC-GARCH model has seen limited use for forecasting purposes, but has been utilized in a broad range of non-forecasting related evaluation studies and research. For instance, Kearney and Potì (2003) implement the DCC model in a study to capture the

behavioral differences between market-level correlations and firm-level correlations in European stock markets. Their study investigates patterns in the unconditional correlations on the market-level and firm-level, and then applies the DCC-GARCH model to evaluate the persistence of these correlations over time. Kearney and Potì find evidence of significant persistence in all their initial correlation estimates, and conclude that the aggregate European market-level correlation was influenced by a stochastic trend, or non-stationary market-level correlation process, possibly related to increased economic integration among the European Monetary Union (EMU) countries. On the other hand, the study finds that firm-level conditional correlation was not subject to the same trends as market-level correlation.

Frank et al. (2008) also use the DCC-GARCH model to track the transmission and persistence of certain market conditions. This study uses data from 2003 to 2008 to look at the transmission of the subprime mortgage crisis effects to other financial markets. The DCC-GARCH model is utilized to measure the strength of the transmission of liquidity shocks by looking at the correlation between five different market factors. The study's original DCC model indicates a large jump in correlation on July 24, 2007 across all markets, which suggests that there was a structural break in unconditional correlations on that day. In response to this finding, Frank et al. generate a revised DCC-GARCH model that accounts for the possibility of a structural break. Frank et al. report that the continued persistence of these correlations dynamics as measured by the DCC-GARCH indicates that the critical issues underlying the crisis may continue to exist even after the crisis has ended.

Research has also extended the DCC-GARCH model to further applications in financial risk management. Lee et al. (2006) implement the DCC model to evaluate the value-at-risk (VaR) of an asset portfolio, one of the earlier studies that uses the model to

forecast rather than simply evaluate trends in correlations. The study composes a portfolio of equally-weighted major stock indices of the G7, and uses DCC-GARCH to forecast the VaR both 1-day and 10-days into the future. In both the 1-day and the 10-day case, the DCC model outperforms more simple forecasts of VaR such as a simple moving average model or an exponential weighted moving average model. This study supports the use of DCC-GARCH as a forecasting tool in addition to its more frequent use, thus far, as an evaluation tool.

Other studies have also used DCC-GARCH to model correlation dynamics around crisis periods in equity markets. Chiang et al. (2007) study a similar dynamic of correlations across nine Asian stock-return series from January 1, 1990 to March 21, 2003, where the focus is on the financial crisis from 1997-1998. Chiang et al. find a strong increase in the equity correlations over the second half of 1997 and early into 1998, the early phase of the crisis, which they interpret to suggest a "contagion effect" followed by a "herding effect" (a contagion effect is described as the spread of a shock from one market to another, leading to increased correlations between markets, while a herding effect describes similar behavior of investors in individual markets that are highly correlated). The study also investigates the effects of sovereign credit-rating changes on various correlations. The results again show a significant negative impact on correlations between the equity indices of Thailand and other countries as a result of the downgrade in the sovereign credit rating of Thailand.

In addition to the behavior of correlation dynamics in different market conditions, a key question we seek to investigate is the forecasting behavior of the DCC-GARCH model when applied to different classes of assets. Cappiello et al. (2006) study this possibility for asymmetries in the correlation dynamics across asset classes and market conditions utilizing

the asymmetric generalized dynamic conditional correlation model (AG-DCC). Cappiello et al. utilize data from 21 international market and 13 government bond indices from 1987 to 2002, to evaluate asymmetries in both variance and covariance dynamics. One of the study's primary findings is that while negative shocks have more impact than positive shocks for both equities and bonds, bond returns do not experience as much asymmetry as equity returns.

A recent study by Skintzi and Xanthopoulos-Sisinis (2007) also examines correlation forecasting performance. The paper considers three asset classes: equally weighted portfolios in stocks, bonds, and currency. Skintzi and Xanthopoulos-Sisinis evaluate the forecasting performance of 11 correlation models (including the DCC-GARCH and AG-DCC) with the aid of statistical and risk management evaluation criteria, investigating the performance of not only each model, but also of all models across asset classes and in different time periods. Specifically, one of the key trends was that the forecast errors were smaller in long-term than in short-term models and decrease for longer forecast horizons. Moreover, the study's results show that the various correlation GARCH models (for instance DCC-GARCH and AG-DCC) outperform the other models according to all evaluation criteria and across all forecast horizons. Among these correlation models, Skintzi identifies the DCC and AG-DCC models as the most effective forecasting models, especially for the stock and bond portfolios. On the other hand, the currency portfolio results provide strong support in favor of simpler models such as the historical mean and the long-term moving average model. Skintzi and Xanthopoulos-Sisinis conclude that while complex correlation models such as the DCC may be a good fit for certain asset classes such as stocks and bonds,

opting for simple models such as covariance matrix specifications may be more suitable for other classes such as currencies.

III. Data

The daily price data on our four assets: the S&P 500 Index, Dollar-Yen exchange rate, 10-year US Treasury bonds, and Moody's A Industrial bonds[1], span from 1988 to 2009, and were obtained using the *Economagic* and *Global Financial Data* online financial databases. We evaluate the forecasting ability of the DCC-GARCH model when moving from relatively favorable equity market environments to more volatile, uncertain markets. We select three periods of relative market instability: the Savings and Loan (S&L) Crisis in 1990, the Dot-Com Crash in 2000, and the Credit Crisis in 2008. We estimate the DCC-GARCH model immediately prior to these events, and then test the estimated parameters out-of-sample, during the crises.

The following tables contain key summary statistics regarding the various asset classes in each of the three in-sample and out-of-sample time periods:

S&L Crisis Summary Statistics

	S&P 500 Index	**Moody's A Industrial Bonds**	**10-year US Treasury Bonds**	**Dollar-Yen Exchange Rate**
Mean Return (IS)	13.482%	1.302%	1.616%	1.237%
Mean Return (OoS)	8.188%	2.016%	3.219%	1.374%
Median Return (IS)	31.820%	0.000%	0.000%	22.051%
Median Return (OoS)	6.259%	0.000%	0.000%	19.138%
Vol. of Returns (IS)	13.018%	4.513%	6.847%	11.178%
Vol. of Returns (OoS)	18.472%	4.749%	6.704%	14.025%
Daily annualized return statistics: In-sample (IS): July 1, 1988 – July 31, 1990, Out-of-sample (OoS): August 1, 1990 – March 28, 1991				

[1] Both the Treasury and Industrial Bonds data are supplied in the form of daily yields to maturity. We convert these to a price series by assuming a bond with a 6% coupon, and converting yield to price using the standard price-yield function for semi-annual coupon bonds.

Dot-Com Boom/Crash Summary Statistics

	S&P 500 Index	Moody's A Industrial Bonds	10-year US Treasury Bonds	Dollar-Yen Exchange Rate
Mean Return (IS)	28.366%	-5.338%	-7.517%	-14.518%
Mean Return (OoS)	-21.969%	1.549%	8.219%	19.120%
Median Return (IS)	18.981%	0.000%	-0.742%	-9.891%
Median Return (OoS)	-32.304%	0.000%	7.782%	9.823%
Vol. of Returns (IS)	19.191%	6.111%	7.212%	14.692%
Vol. of Returns (OoS)	21.481%	4.819%	6.361%	9.898%
Daily annualized return statistics: In-sample (IS): October 1, 1998 – March 31, 2000, Out-of-sample (OoS): April 1, 2000 – March 30, 2001				

Credit Crisis Summary Statistics

	S&P 500 Index	Moody's A Industrial Bonds	10-year US Treasury Bonds	Dollar-Yen Exchange Rate
Mean Return (IS)	3.470%	-0.745%	0.993%	-1.732%
Mean Return (OoS)	-36.378%	-5.128%	5.413%	-4.732%
Median Return (IS)	18.636%	0.000%	0.000%	6.530%
Median Return (OoS)	4.725%	0.000%	0.000%	0.000%
Vol. of Returns (IS)	12.854%	5.351%	6.180%	9.415%
Vol. of Returns (OoS)	43.292%	9.811%	11.579%	17.154%
Daily annualized return statistics: In-sample (IS): March 1, 2004 – March 31, 2008, Out-of-sample (OoS): March 31, 2008 – March 26, 2009				

By looking at the returns statistics by period, we can gain some understanding of the market conditions before and during each of these crashes. Qualitatively, it appears that the S&L crisis was less severe than either of the other two periods of interest. Equity returns diminished marginally on average while returns on other assets either remained relatively stable or increased slightly from in-sample to out-of-sample. Return volatility did not increase noticeably in the "crisis period," and in fact decreased for US Treasury bonds. For all assets, the median return in the out-of-sample period was either positive or zero, indicating that at least half of the daily returns were positive. However, with the S&P 500 Index in the Dot-Com period, the median return in the crash period is significantly lower than the mean return in the same period, indicating that the S&P index had extremely negative returns for most of the crash. The statistics also indicate that the crash was primarily equity-

based, since in the crash period, both the mean and median returns of the other three assets either increased or remained the same while the mean and median return for the S&P index decreased by a large amount. Also, the return volatility increased from in-sample to out-of-sample for the S&P index, but decreased for the other three assets. It is reasonable to believe that during this period, people sought refuge in treasuries and industrial bonds. Finally, in the Credit Crisis, we see the impact of the crisis across all assets: the volatility of the returns increases from in-sample to out-of-sample and the median return either remains constant or decreases. The returns for all assets, except for US Treasuries, decrease as well. The most surprising relation in this crisis is that the median return for the S&P 500 index in the out-of-sample period is high positive (18.6%), while the mean return for this same period is highly negative (-36.4%). Such a negative mean return but positive median return indicates that there were a few days of extraordinarily negative returns during this period. The same phenomenon can be seen, but to a lesser extent, in the out-of-sample period for both Industrial Bonds and the Dollar-Yen exchange rate. We also note that volatility on all assets increases from in-sample to out-of-sample, even for the US Treasuries, which experienced a slight increase in mean return from in-sample to out-of-sample.

IV. Methodology

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Volatility Model

GARCH is one of a broad category of econometric models known as autoregressive conditional heteroskedasticity models, or ARCH models (Engle (1982); Bollerslev (1986)), used to model time-varying volatility for time series data. For our volatility analysis, we first utilize the GARCH (1,1) to derive standardized residuals, which are inputs for the DCC-GARCH model. Engle (2009) refers to this process as "DE-GARCHING" the data.[2]

GARCH (1,1) follows the below equation for modeling conditional variance:

$$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (1)$$

where σ^2 is the conditional variance of an asset, and $y_t = ln\left(\frac{P_t}{P_{t-1}}\right) - \mu$, with P_t representing the asset price at time t, $\mu = \frac{1}{n}\sum_{i=1}^{n} y_i$ (the sample mean of asset prices), and *n* being the number of days in the in-sample period. From this conditional variance, we derive the standardized residuals used in the DCC-GARCH model. The standardized residuals for a given asset are estimated as:

$$s_t = y_t/\sigma_t \quad (2)$$

As noted earlier, the estimation of a GARCH (1,1) model is an intermediate step in order to derive inputs for the DCC-GARCH model that we will use to model correlation between two assets. We estimate the GARCH parameters using maximum likelihood estimation (MLE); we maximize the log-likelihood function over the in-sample period:

[2] A wide range of GARCH-style models can be used for DE-GARCHING, including asymmetric volatility models such as TGARCH, or even completely different non-GARCH stochastic volatility models.

$$L = \sum_{t=1}^{n} log\left[\frac{1}{\sqrt{2\pi\sigma^2}} exp\left(-{y_t}^2 \Big/ 2{\sigma_t}^2\right)\right] \qquad (3)$$

For this paper, we define the in-sample period from time *t = 1* through *t = n*, and the out-of-sample period from time *t = n+1* through *t = n+m*.

After modeling individual asset conditional volatility using the GARCH (1,1) model, we can then use the standardized residual values for each asset as inputs into the DCC-GARCH.

Dynamic Conditional Correlation GARCH (DCC-GARCH) Correlation Model

The DCC-GARCH model is an econometric tool used to model correlations between two or more data series (Engle 2002). It takes the standardized residuals (see equation (2)), which are simply the data series residuals divided by the GARCH conditional standard deviation. The DCC-GARCH model then uses these standardized residuals to estimate DCC conditional correlations.

The standard DCC-GARCH model for two assets *i* and *j* at time *t* is the following, as detailed in Engle (2009):

$$\begin{cases} \rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}} \\ q_{i,j,t} = \bar{R}_{i,j}(1-\alpha-\beta) + \alpha s_{i,t-1} s_{j,t-1} + \beta q_{i,j,t-1} \\ q_{i,i,t} = \bar{R}_{i,i}(1-\alpha-\beta) + \alpha(s_{i,t-1})^2 + \beta q_{i,i,t-1} \\ q_{j,j,t} = \bar{R}_{j,j}(1-\alpha-\beta) + \alpha(s_{j,t-1})^2 + \beta q_{j,j,t-1} \\ \omega = \bar{R}_{i,j}(1-\alpha-\beta) \\ \bar{R}_{i,j} = \frac{1}{n}\sum_{t=1}^{n} s_{i,t}\, s_{j,t} \end{cases} \qquad (4)$$

In this model, $\rho_{i,j,t}$ is the DCC-model conditional correlation, $\bar{R}_{i,j}$ is the average realized correlation, $s_{i,t-1}$ and $s_{j,t-1}$ are the lagged GARCH standardized residuals, and the calculated *q* values are known as "quasi-correlations," which are then normalized to calculate

the conditional correlation $\rho_{i,j,t}$. In the three quasi-correlation formulas, the first term $\bar{R}_{i,j}(1-\alpha-\beta) \equiv \omega$ is restricted to be a constant that is generally stated along with α and β. This restriction is known as correlation targeting, which uses an estimate of unconditional correlations, $\bar{R}_{i,j}$, to reduce the number of unknown parameters to only two (α and β).

The model behaves as follows: the correlations evolve over time in response to new information regarding the returns, and as returns on both assets move in the same direction, the correlations will rise above their average level and remain at that elevated level for a brief period. Over time, this information will decay, and correlations will decrease back to the long-run average. A similar process holds for when asset returns move in opposite directions. The two parameters α and β of the model govern the "speed" of this adjustment to new information.

To estimate the DCC-GARCH parameters α and β, we use maximum likelihood estimation, similarly to the GARCH model. As noted by Engle (2009), the log-likelihood function in this case applies to a pair of assets, which is given by:

$$L_{2,i,j} = -\frac{1}{2}\sum_{t=1}^{n}\left(log\left[1-\rho_{i,j,t}^{2}\right] + \frac{s_{i,t}^{2}+s_{j,t}^{2}-2\rho_{i,j,t}s_{i,t}s_{j,t}}{[1-\rho_{i,j,t}^{2}]}\right) \qquad (5)$$

As per the convention mentioned earlier, the log-likelihood function applies to the in-sample period, and as such the time ranges from $t = 1$ through $t = n$.

Forecast Evaluation

After estimating the DCC-GARCH model, we obtain the model parameters α, β, and ω for any given pair of assets during an in-sample period and analyze the performance of this model in predicting the realized covariance between the two assets out-of-sample.

In order to have a benchmark for comparison, we utilize the product of the changes in price of the two assets as a proxy for our realized covariance, which for two assets *i* and *j* is defined as:

$$\sigma_{i,j,t} = E\left[y_{i,t} y_{j,t}\right] \qquad (6)$$

where $y_t = ln\left(\frac{P_t}{P_{t-1}}\right) - \mu$ as before[3]. We measure the effectiveness of the DCC model in predicting the realized covariance in comparison to a naïve unconditional average covariance over the in-sample period, which we then use as an estimate of covariance out-of-sample.

$$Cov\left(y_i, y_j\right) = \frac{1}{n}\sum_{t=1}^{n}\left(y_{i,t} - \mu_i\right)\left(y_{j,t} - \mu_j\right) \quad (7)$$

The key step in our forecasting analysis is to derive the DCC conditional correlations out-of-sample based on our in-sample DCC-GARCH parameters. From equation (4), we can estimate the DCC conditional correlations as:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t} q_{j,j,t}}} \; for\ t = n + 1\ to\ n + m \quad (8)$$

Again, the $q_{i,j,t}$ values are the quasi-correlations, and this process of standardizing the quasi-correlations serves to rescale them into DCC conditional correlations.

After applying the DCC in-sample parameters to the out-of-sample horizon, we now have the DCC conditional correlation estimate, which will serve as our first estimate, and we also have the naïve unconditional in-sample covariance, which will serve as our second estimate. As noted earlier, our benchmark for comparison is the realized covariance proxy for each day in our time series. In order to determine the relative effectiveness of our two

[3] This is based on the assumption that the expected return on each asset is 0, that is, $E[y_i]$ and $E[y_j] \approx 0$. Otherwise, $Cov\left(y_i, y_j\right) = E\left[y_i y_j\right] - E[y_i] \times E[y_j]$.

models in estimating correlations, we utilize mean-squared-error analysis to determine the difference between the realized covariance and the estimated covariances from the two models. The mean-squared error measures for the DCC versus realized covariance comparison and unconditional (UNC) versus realized covariance comparison are defined as follows:

$$\begin{cases} MSE_{DCC} = \sum_{t=n+1}^{n+m} \left(q_{i,j,t} - \sigma_{i,j,t}\right)^2 \\ MSE_{UNC} = \sum_{t=n+1}^{n+m} \left(\sigma_{i,j,t} - \overline{\sigma_{i,j,t}}\right)^2 \end{cases} \quad (9)$$

Here, $\overline{\sigma_{i,j,t}}$ is defined as the unconditional covariance in the in-sample period. Finally, we compare the performance of the DCC covariance estimates relative to the unconditional estimate by calculating the percentage difference between these two mean-squared-error values, explicitly:

$$DCC\ outperformance\ percentage = \frac{MSE_{UNC} - MSE_{DCC}}{MSE_{DCC}} \quad (10)$$

V. Results

The following tables summarize the results of our analysis. Overall, our analysis suggests that the performance of the DCC-GARCH model, relative to the in-sample unconditional covariance, does not depend on the time period, but rather the assets involved. In all three periods, the DCC estimate for the S&P 500 and US Treasury Bond pair (Tables 1-3) performed better as a forecast of future volatility than the unconditional method. For all other asset pairs, the DCC model did not demonstrate a meaningful improvement in forecasting ability in either the S&L Crisis or the Dot-Com Crash.

Asset Pair 1: S&P/US Treasury Bond

Table 1 (S&L Crisis)

Parameters	S&P GARCH	US Treasury GARCH	DCC-GARCH
ω	0.000	0.000	0.012
α	0.039	0.019	0.034
β	0.960	0.924	0.929
Unconditional Correlation (IS)		0.329	
Unconditional Correlation (OoS)		0.533	
Average Conditional Correlation (IS)		0.346	
Average Conditional Correlation (OoS)		0.418	
Volatility of Conditional Correlation (IS)		0.158	
Volatility of Conditional Correlation (OoS)		0.097	
Standard Error S&P Parameters		0.945	
Standard Error US Treasury Parameters		0.997	
Percent Difference		2.341%	

This table represents the parameters and statistics for the S&P/US Treasury Bond comparison for the S&L Crisis. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Table 2 (Dot-Com Boom/Crash)

Parameters	S&P GARCH	US Treasury GARCH	DCC-GARCH
ω	0.000	0.000	0.002
α	0.009	0.085	0.043
β	0.947	0.722	0.936
Unconditional Correlation (IS)		0.082	
Unconditional Correlation (OoS)		-0.258	
Average Conditional Correlation (IS)		0.109	
Average Conditional Correlation (OoS)		-0.061	
Volatility of Conditional Correlation (IS)		0.198	
Volatility of Conditional Correlation (OoS)		0.173	
Standard Error S&P Parameters		1.005	
Standard Error US Treasury Parameters		1.005	
Percent Difference		6.518%	

This table represents the parameters and statistics for the S&P/US Treasury Bond comparison for the Dot-Com Boom/Crash. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Table 3 (Credit Crisis)

Parameters	S&P GARCH	US Treasury GARCH	DCC-GARCH
ω	0.000	0.000	-0.009
α	0.047	0.049	0.129
β	0.940	0.946	0.794
Unconditional Correlation (IS)		-0.247	
Unconditional Correlation (OoS)		-0.422	
Average Conditional Correlation (IS)		-0.137	
Average Conditional Correlation (OoS)		-0.361	
Volatility of Conditional Correlation (IS)		0.267	
Volatility of Conditional Correlation (OoS)		0.203	
Standard Error S&P Parameters		1.002	
Standard Error US Treasury Parameters		0.712	
Percent Difference		9.015%	

This table represents the parameters and statistics for the S&P/US Treasury Bond comparison for the Credit Crisis. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

The DCC model outperformed the unconditional forecast for the S&P/US Treasury Bond correlation in all three periods, by at least 2.34% (S&L Crisis) and as much as 9.02% (Credit Crisis). In all three cases, the absolute value of the unconditional correlation is greater in the out-of-sample crisis period, which is consistent with previous research (Chiang et al. 2007). However, in the Dot-Com Crash, the correlation switched from positive to negative, which could indicate a movement to "safe haven" bond investments, and away from equities.

In the S&L Crisis and Dot-Com periods, the average conditional correlation in the out-of-sample period is lower than the out-of-sample unconditional correlation. While we might have expected this result, since the model is "trained" on a period of lower correlation, it still provides evidence that the DCC may not be able to reflect large changes in correlation from in-sample to out-of-sample. However, in the Credit Crisis, there does not appear to be a large difference in these two values. This could explain why the conditional model in the Credit Crisis period has a much smaller mean-squared error than the unconditional model. Based on the observation that correlations tend to increase in times of crisis, an out-of-sample average conditional correlation that is much less than the unconditional correlation in the same period could imply a model similar to the unconditional model. We also look at time series graphs of the out-of-sample realized covariance and the out-of-sample conditional covariance to investigate the volatility of covariances.

Chart 1 (S&L Crisis)

This chart shows a plot of the realized covariance and conditional covariance of the S&P 500/US Treasury Bond pair in the out-of-sample period (OoS). The covariances are multiplied by 10000 for clarity.

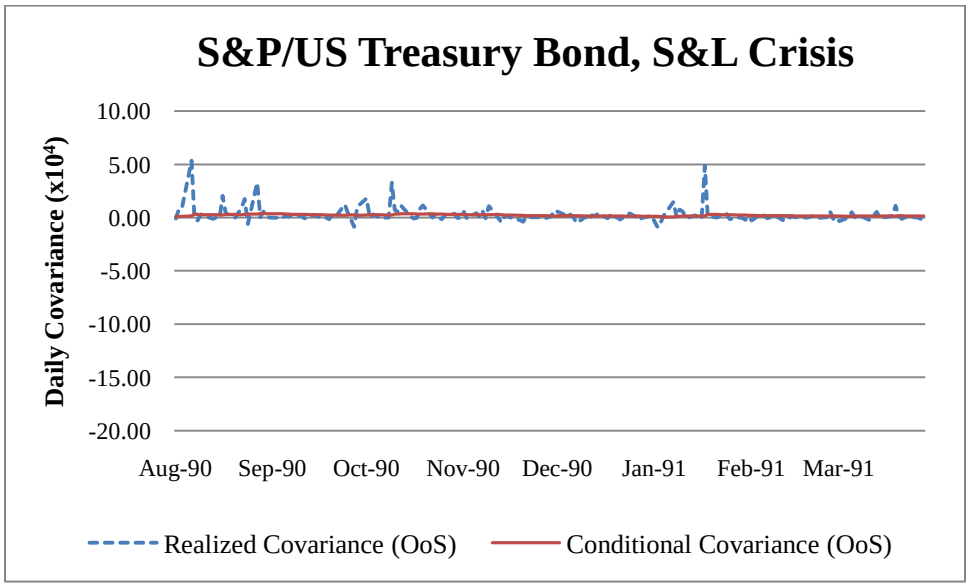


Chart 2 (Dot-Com Crash)

This chart shows a plot of the realized covariance and conditional covariance of the S&P 500/US Treasury Bond pair in the out-of-sample period (OoS). The covariances are multiplied by 10000 for clarity.

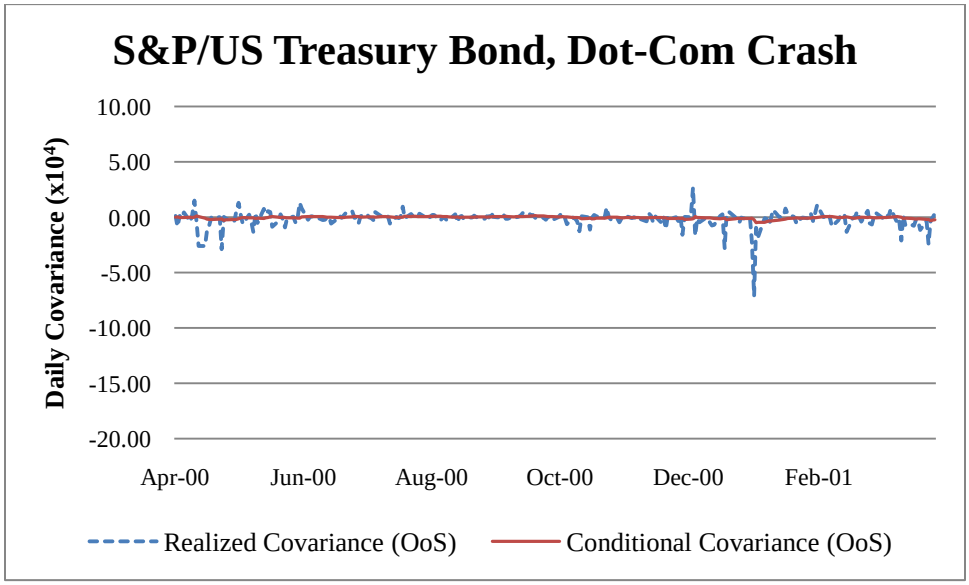
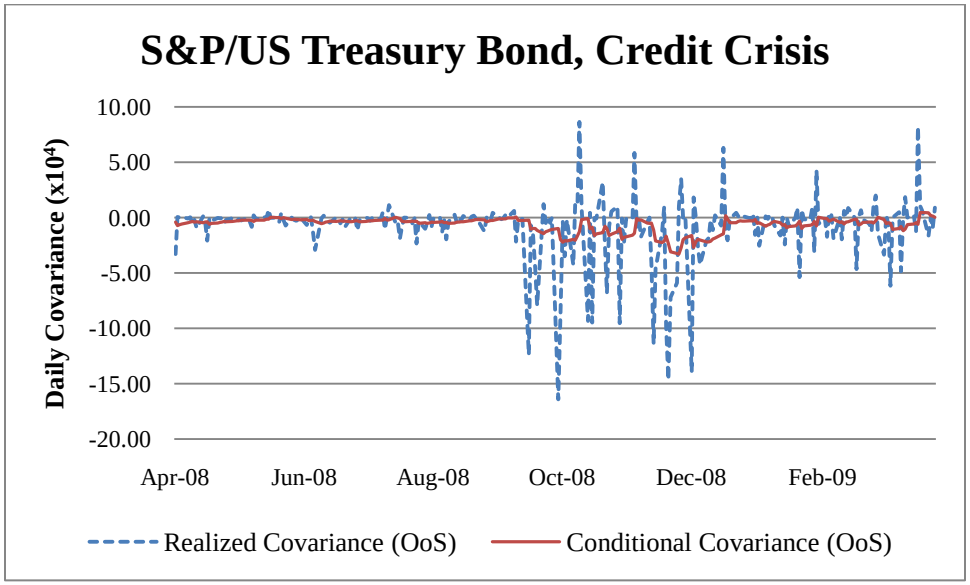


Chart 3 (Credit Crisis)

This chart shows a plot of the realized covariance and conditional covariance of the S&P 500/US Treasury Bond pair in the out-of-sample period (OoS). The covariances are multiplied by 10000 for clarity.



It is clear from these charts that the conditional covariance, although it seems to reflect the directional movements of the realized covariance, is much less volatile than the realized. Perhaps the fact that the Dot-Com and Credit Crisis comparisons performed the best relative to the unconditional model with this asset pair can be explained by the movement of the conditional covariance relative to the realized covariance. In the Dot-Com and Credit periods, there is very little observable movement of the conditional covariance in the direction of the movement of the realized covariance. However, even this slight volatility of conditional covariance may be what makes the DCC-GARCH model a good forecaster of correlation relative to an unconditional model. Since the unconditional model does not allow any volatility of covariance, even the slightest amount of volatility in covariance could lead the conditional model to forecast correlation better than the unconditonal model, as we see in these results.

Asset Pair 2: S&P/Industrial Bond

Table 4 (S&L Crisis)

Parameters	S&P GARCH	Industrial GARCH	DCC-GARCH
ω	0.000	0.000	0.001
α	0.039	0.169	0.023
β	0.960	0.645	0.948
Unconditional Correlation (IS)		0.038	
Unconditional Correlation (OoS)		0.151	
Average Conditional Correlation (IS)		0.041	
Average Conditional Correlation (OoS)		0.096	
Volatility of Conditional Correlation (IS)		0.076	
Volatility of Conditional Correlation (OoS)		0.060	
Standard Error S&P Parameters		0.945	
Standard Error Industrial Bond Parameters		0.999	
Percent Difference		0.261%	

This table represents the parameters and statistics for the S&P/Industrial Bond comparison for the S&L Crisis. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Table 5 (Dot-Com Boom/Crash)

Parameters	S&P GARCH	Industrial GARCH	DCC-GARCH
ω	0.000	0.000	0.041
α	0.009	0.058	0.029
β	0.947	0.879	0.000
Unconditional Correlation (IS)		0.021	
Unconditional Correlation (OoS)		0.052	
Average Conditional Correlation (IS)		0.042	
Average Conditional Correlation (OoS)		0.042	
Volatility of Conditional Correlation (IS)		0.028	
Volatility of Conditional Correlation (OoS)		0.032	
Standard Error S&P Parameters		1.005	
Standard Error Industrial Bond Parameters		1.005	
Percent Difference		0.060%	

This table represents the parameters and statistics for the S&P/Industrial Bond comparison for the Dot-Com Boom/Crash. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Table 6 (Credit Crisis)

Parameters	S&P GARCH	Industrial GARCH	DCC-GARCH
ω	0.000	0.000	0.000
α	0.047	0.068	0.014
β	0.940	0.905	0.976
Unconditional Correlation (IS)		0.025	
Unconditional Correlation (OoS)		-0.216	
Average Conditional Correlation (IS)		-0.031	
Average Conditional Correlation (OoS)		-0.012	
Volatility of Conditional Correlation (IS)		0.081	
Volatility of Conditional Correlation (OoS)		0.120	
Standard Error S&P Parameters		0.997	
Standard Error Industrial Bond Parameters		0.924	
Percent Difference		1.477%	

This table represents the parameters and statistics for the S&P/Industrial Bond comparison for the Credit Crisis. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

For the S&P/Industrial Bond comparison, the only period in which the DCC model's forecast of correlation has a mean-squared error of more than 1% less than the mean-squared-error of the unconditional forecast was in the Credit Crisis period. The Dot-Com results might be attributed to the $\beta = 0$ parameter, which reduces the DCC model to be similar to the unconditional model. However, the Credit Crisis has a similar relationship between in-sample unconditional correlation and out-of-sample average conditional correlation, but the model provides a small improvement relative to the unconditional model (Percent Difference = 1.48%). Perhaps what differentiates this period from the other two periods is the out-of-sample volatility of conditional correlation. This volatility is much higher in the Credit Crisis period than in either the S&L Crisis or the Dot-Com Crash, which makes the distribution of conditional correlations vastly different from the unconditional model.

Similar to the S&P/US Treasury Bond comparisons, the absolute value of the unconditional correlation is significantly greater out-of-sample. While the average

correlations are relatively weak between these assets (the period where these assets are most strongly correlated is in the out-of-sample period of the Credit Crisis where unconditional correlation = -.22), the differences in unconditional correlation from in-sample to out-of-sample suggest different relationships between these assets in each crisis. In the S&L and Dot-Com periods, the unconditional correlation between these assets became more positive from in-sample to out-of-sample. However, in the Credit Crisis, the unconditional correlation changed from positive in the in-sample period to negative in the out-of-sample period.

Asset Pair 3: S&P/Dollar-Yen Exchange

Table 7 (S&L Crisis)

Parameters	S&P GARCH	Dollar-Yen GARCH	DCC-GARCH
ω	0.000	0.000	0.057
α	0.039	0.038	0.069
β	0.960	0.898	0.001
Unconditional Correlation (IS)		0.061	
Unconditional Correlation (OoS)		-0.020	
Average Conditional Correlation (IS)		0.063	
Average Conditional Correlation (OoS)		0.057	
Volatility of Conditional Correlation (IS)		0.057	
Volatility of Conditional Correlation (OoS)		0.082	
Standard Error S&P Parameters		0.945	
Standard Error Dollar/Yen Parameters		1.014	
Percent Difference		0.941%	

This table represents the parameters and statistics for the S&P/Dollar-Yen Exchange comparison for the S&L Crisis. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Table 8 (Dot-Com Boom/Crash)

Parameters	S&P GARCH	Dollar-Yen GARCH	DCC-GARCH
ω	0.000	0.000	0.001
α	0.009	0.024	0.000
β	0.947	0.909	0.985
Unconditional Correlation (IS)		0.054	
Unconditional Correlation (OoS)		-0.130	
Average Conditional Correlation (IS)		0.051	
Average Conditional Correlation (OoS)		0.051	
Volatility of Conditional Correlation (IS)		0.000	
Volatility of Conditional Correlation (OoS)		0.000	
Standard Error S&P Parameters		1.005	
Standard Error Dollar/Yen Parameters		1.079	
Percent Difference		0.712%	

This table represents the parameters and statistics for the S&P/Dollar-Yen Exchange comparison for the Dot-Com Boom/Crash. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Table 9 (Credit Crisis)

Parameters	S&P GARCH	Dollar-Yen GARCH	DCC-GARCH
ω	0.000	0.000	-0.001
α	0.047	0.069	0.012
β	0.940	0.905	0.921
Unconditional Correlation (IS)		-0.008	
Unconditional Correlation (OoS)		-0.099	
Average Conditional Correlation (IS)		-0.017	
Average Conditional Correlation (OoS)		-0.026	
Volatility of Conditional Correlation (IS)		0.032	
Volatility of Conditional Correlation (OoS)		0.027	
Standard Error S&P Parameters		1.002	
Standard Error Dollar/Yen Parameters		1.003	
Percent Difference		-0.167%	

This table represents the parameters and statistics for the S&P/Dollar-Yen Exchange comparison for the Credit Crisis. The ω parameter represents the constant of the DCC-GARCH model while α and β represent the coefficients of the lagged standardized residuals term of the model and the lagged conditional covariance term respectively. IS refers to the in-sample period while OoS refers to the out-of-sample period. Percent difference refers to the percent reduction in mean-squared error of the conditional versus unconditional correlations relative to the realized correlation.

Through our mean-squared-error analysis, the DCC model does not perform more than 1% better than the unconditional model in all three periods. This result is consistent with the results of Skintzi and Xanthopoulos-Sisinis (2007). Their results showed that the DCC model is a more effective forecasting model for stock and bond portfolios than for

currency portfolios. In our observations, the results in the S&L and Dot-Com periods may be explained by the fact that the unconditional correlation switches from positive to negative from the in-sample period to the out-of-sample period, but the average conditional correlation in the out-of-sample period remains positive. In fact, in all three periods, the out-of-sample unconditional correlation is negative and larger in magnitude than the in-sample unconditional correlation, which may suggest an increased use of the Dollar-Yen exchange rate as a hedge against the poorly-performing S&P 500 index.

The results for the Dot-Com period seem to imply that there is very little to no volatility of correlation in the in-sample period since the model estimation gave a corner solution with $\alpha = 0$, so the model reverted to the unconditional case. The model also gave a near corner solution for the S&L Crisis period, but with this model, β was approximately 0. For the only period where the estimated DCC parameters were not corner solution or near to one (the Credit Crisis period), mean-squared-error of the dynamic correlation model was actually greater than that of the unconditional model. Although this difference was not large, it is still a strange result that raises questions about the effectiveness of the model when applied to this asset pair.

From our models, there were two discernable trends regarding the performance of the DCC model relative to the unconditional model. The most notable trend was the performance of the DCC model for the S&P/US Treasury Bond pair in all three periods of interest. For this asset pair, the DCC-GARCH model forecasted the correlation in the out-of-sample period greater than 2% better than the unconditional model in all periods. Since our study is restricted to forecasting correlations in crisis periods, it is difficult to say if this result can be generalized to all market conditions without further research.

The other observable trend was the performance of the DCC model relative to the unconditional model in the Credit Crisis period. In this period, the DCC model forecasted the out-of-sample correlation at least 1% better than the unconditional model for all asset pairs except for the S&P/Dollar-Yen. This may imply something unique to this period, at least relative to the other periods. This is supported by the fact that for the non-S&P/US Treasury asset pair comparisons, there were no comparisons where the DCC model performed more than 1% better than the unconditional model for either the S&L Crisis or the Dot-Com Crash. However, it may also be something unique about the models for the Credit Crisis. For example, the lengths of the in-sample periods for the S&L Crisis and Dot-Com Crash were significantly shorter than for the Credit Crisis (2 yrs vs. 4 yrs). To understand if this factor is significant, further research must be done concerning the relation between DCC forecasting ability and the length of the in-sample period.

VI. Conclusion

The goal of our study was to explore the forecasting ability of the DCC-GARCH model relative to a simple unconditional correlation estimate, especially when applied to extreme market environments (i.e. bear vs. bull markets) and across varying asset classes. Our results seem to indicate that the forecasting ability of the DCC-GARCH model relative to the unconditional model is largely determined by the asset pair investigated.

If further research indicates that the DCC model is more effective in forecasting correlations than an unconditional model either with particular asset pairs or in certain market conditions, then the model may be used to improve risk management techniques. Since, in portfolio theory, correlation plays a significant role in calculating the risk of a portfolio, then if we can better predict the correlation between assets, we can better allocate the assets of a portfolio to hit a certain risk target. Our study seems to show that a forecasting model that allows correlation to vary with time may be more effective in forecasting correlations than an unconditional model, at least in the case of some asset pairs. However, since the results are restricted to "crisis" periods, further research must be done investigating the forecasting ability of the DCC model with this asset pair in several other market conditions. This analysis is necessary to conclude if the DCC model will significantly improve correlation forecasts. Further study is also needed to conclude if the relatively stronger forecasting ability of the DCC-GARCH model relative to the unconditional model is based on product pair rather than time period, as our results seem to indicate. Finally, our results seem to show a strong performance relative to the unconditional model in the Credit Crisis period, which had a much longer in-sample period than the other

two periods. This suggests that the length of the in-sample period may contribute to the forecasting ability of the DCC-GARCH model.

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