Conditional Beta Models for Asset Pricing by Sector in U.S. Equity Markets

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Abstract

In finance, the beta of an investment is a measure of the risk arising from exposure to general market movements as opposed to idiosyncratic factors. Therefore, reliable estimates of stock portfolio betas are essential for many areas in modern finance, including asset pricing, performance evaluation, and risk management. In this paper, we investigate Static and Dynamic Conditional Correlation (DCC) models for estimating betas by testing them in two asset pricing context, the Capital Asset Pricing Model (CAPM) and Fama-French Three Factor Model. Model precision is evaluated by utilizing the betas to predict out-of-sample portfolio returns within the aforementioned asset-pricing framework. Our findings indicate that DCC-GARCH does consistently have an advantage over the Static model, although with a few exceptions in certain scenarios.

JEL Classification: C32; C51; G12; G17

Keywords: Beta, Asset Pricing, Dynamic Correlation, Equity, U.S. Markets

1 Introduction

Beta measures the systematic risk, or the exposure of each individual asset to the fluctuations in the returns of the market portfolio, usually represented by a properly weighted and well-diversified market index. Thus, accurate forecasting of stock or portfolio betas plays a crucial role in finance, including asset pricing, portfolio allocation, and risk management. For example, often investment funds are promoted based on their risk inherent to the entire market (systematic or undiversifiable risk) features, with the funds' forecasted beta being the weighted average of the beta forecasts of the individual assets that comprise the fund. A fund with a beta forecast of one is said to follow the broad market, whereas a fund with a beta forecast of zero is said to market neutral.

Since true betas are not directly observable, mathematical estimations of betas were developed using measures of variance and covariance between the asset or portfolio and the market. Thus reliable estimates of variance and covariance are vital, and differences between these methods can have significant impact on betas. Academic and market practitioners have used a range of static and dynamic models for estimation purposes, and new models (or extensions of existing models) continue to be explored for improved accuracy.

In this thesis, we estimate the daily betas for industry portfolios of U.S. equities using a range of variance and covariance models in the context of two pricing models, the Capital Asset Pricing Model (CAPM) and Fama-French Three Factor Model. We then examine the goodness-of-fit of the estimated betas and their trends and movements during U.S. economic booms and recessions. Lastly, we assess these models by utilizing the betas out-of-sample to predict portfolio returns within the aforementioned asset-pricing framework. This allows us to measure the effectiveness of the two models in a practical application.

The CAPM, independently developed in Sharpe (1964) and Lintner (1965a, b) simplifies measuring risk to only the non-diversifiable market (or systematic) risk. The CAPM beta is then defined as the covariance between the stock return and the market return divided by the variance of the market return, since idiosyncratic (or firm-specific) risk is not correlated with market return. This is equivalent to estimating the beta by regressing asset or portfolio return on the market return, i.e. the return on a market index used as a proxy for the market portfolio. However, gradually over the next decade, studies began to document what appeared to be violations of the CAPM for certain portfolios of securities. Since that was the case, then a potential remedy would be to include new pervasive risk factors into the risk-return model.

Later, a more advanced multifactor model was developed by Fama and French (1993) as they observed that small-cap stocks and high book-to-market ratio stocks tend to have higher returns than the market as a whole. Empirical analysis states that value stocks and small stocks have considerably higher cash-flow than growth stocks and large stocks (Campbell and Vuolteenaho 2003). Lakonishok et. al. (1994) propose that the book-to-market effect is related to a cognitive bias on behalf of investors that arises as they extrapolate firms' future earnings and growth potential from past values. Fama-French Three Factor Model expands on the CAPM by adding the size and value factors in addition to the market risk factor. The size factor, usually known as SMB (Small Minus Big), is designed to measure the additional return investors have historically received by investing in stocks of companies with relatively small market capitalization. The value factor, which is High Minus Low (HML), has been constructed to measure the value premium provided to investors for investing in companies with high book-to-market values. The corresponding coefficients of SMB and HML factors, b_s and b_v, together with beta are determined by linear regression.

In terms of variance and covariance estimation, different models propose different weightings on past information. For example, a rolling historical model has a fixed termination point in the past where data prior to that point is deemed uninformative. Other models use declining weights without a fixed termination point. A similar distinction is made between conditional and unconditional models. An unconditional perspective is adopted by popular approaches (Andersen et. al. 2006) such as rolling historical (Officer 1973) and extreme-value theory (Wiggins 1992). However, these models have mostly given way to more dynamic conditional models (Campbell et. al. 2001), as the unstable nature of volatilities and correlations can lead to poor forecasting performance using simple unconditional risk measures (Solnik et. al. 1996).

Various dynamic models have been proposed in recent years. The Dynamic Conditional Correlation (DCC) developed by Engle (2002a) is the most prominent among the dynamic models. DCC incorporates a variance estimation step by using, for example, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Volatility model (Bollerslev 1986) which allows for greater flexibility. Engle and Patton (2001) argue that a good volatility model should have the ability to forecast exhibit persistence, and display mean-reversion, among other characteristics. And, they show that GARCH volatility models developed by Bollerslev (1986) and others adequately capture these features.

In this paper, we select two existing methods for beta estimation: the constant ordinary least squares (OLS) betas from simple regressions on portfolio return over market return and the more frequently used DCC/GARCH model for time-varying betas. For each of the unconditional and DCC/GARCH model, we investigate precision of the estimates and trends of industry beta movements in booms and recessions. We then examine which model best reflects the changing cycle of betas during bull and bear markets. Further to examine the beta trends, we then utilize these beta estimates out-of-sample as inputs into the asset pricing models for the subsequent period. We measure the implied expected returns, compare them to the actual realized returns, and evaluate the effectiveness between two models.

The rest of the paper is organized as follows. Section 2 presents a literature review that more fully describes the subject and discusses our research focus in the context of the current existing literature. Section 3 presents the theoretical framework underlying the research and discusses the statistical methodology employed in the study. Section 3 introduces the dataset and data treatment used in the study. Section 5 presents our findings on beta movements during bullish and bearish time periods along with the comparison of two models. Finally, Section 6 concludes the paper and discusses further research direction.

2 Literature Review

Our topic is an extension to recent research on the time-varying betas (Mandelker 1974), which can be traced back to definition of beta (Sharpe 1963, 64; Lintner 1965a, b). The most common way to estimate a stock beta is to compute the covariance and variance from the most recent 5 years of monthly returns, following Fama and MacBeth (1973). However, recent advances in financial econometrics have demonstrated substantial improvements in beta measurement and forecasting by computing betas from returns measured at a higher frequency than monthly (Andersen et. al. 2006). These studies exploit the time variation of beta and suggest the use of simple autoregressive time series models for forecasting purposes.

Since then an enormous literature in financial econometrics on modeling and forecasting time-vary volatility has formed. The idea of more accurate variance and covariance measurements from the summation of squares and cross-products of higher frequency within period returns was noted by Merton (1980), though it was Andersen and Bollerslev (1998) that demonstrated the usefulness of these measures, called 'realized volatility' and 'realized covariance' in model building and evaluation. Fleming et. al. (2003) show superior performance in portfolio optimizing strategies that utilize realized volatility and realized covariance measures. These models allow market volatility, portfolio-specific volatility, and beta to respond asymmetrically to positive and negative markets (Braun et. al. 1995). However, Ghysels (1998) argues that if the beta risk is inherently misspecified, serious pricing errors can be committed, potentially larger than with a constant traditional beta model.

While much comparison and research have been done on the effects of constant and time-varying beta models to asset pricing, much of the literature was focusing on single factor asset pricing models. Little literature focuses on applying beta models to the more accurate and practical multivariate factor asset pricing, but we are filling the gap by extending the research in this direction. Little literature focuses on estimating industry-specific betas relative to a market index. Moreover, even less literature estimate industry-specific betas relative to a market index or try to discover beta movements during different economic time frames, i.e. recessions, booms, etc, across various industry sectors.

The closest precursors to our work are the papers by Brooks et. al (2000) and Ozoguz (2009). The former paper generates time-varying estimates of Australian industry betas relative to an Australian market index and a world market index using the Kalman filter approach. These conditional estimated betas are used to forecast each industrys return in-sample as a means of comparison. The latter paper investigates empirically the dynamics of investors beliefs and Bayesian uncertainty about the state of the economy as state variables that describe the time-variation in investment opportunities.

3 Methodology

Our goal is to evaluate the performance and forecasting ability of the two volatility and correlation models in the context of asset pricing models. In the Static model, we simply calculate sample standard deviations of sector returns and correlations between sectors and the market. In the DCC model, we compute conditional volatility and correlation that account not only for returns, but also past volatilities. Section 3.1 introduces the Static model in more details and Section 3.2 specifies the DCC-GARCH model. Section 3.3 presents the out-of-sample forecast evaluation for the Static and DCC model under the two asset pricing models and introduces measures of evaluating model precision. Finally, Section 3.4 briefly touches on the methodologies in evaluating beta trends during crises or booms.

3.1 Static Model[1]: Rolling Window

In this paper, Static beta is simply the ordinary least squares (OLS) coefficient based off of simple regression on portfolio return over market return under a rolling time window. Rolling window uses only the most recent N observations, so as time progresses we discard older observations and include more recent observations. We let $N = 60$ in our case. Such beta calculations can be extend to the multivariate level to estimate several coefficients at once.

Betas can also be calculated directly by using portfolio and market returns from the start of the sample period to the end of the sample period. For a given period $\tau = t$ - t_0 that consists of N observations, beta is as follows:

$$\beta_{i,t} = \frac{\sigma^2_{i,m,\tau}}{\sigma^2_{m,\tau}} \tag{1}$$

where σ^2_m is the variance of the market and $\sigma^2_{i,m}$ is the covariance between sector i and relevant market index.

3.2 DCC-GARCH Model

The DCC-GARCH model proposed by Engle (2002a) parameterized conditional correlations directly and the estimation is carried out in three steps. The first step is a preliminary step to estimate the mean return to be used for the DCC model. The second step involves the estimation of conditional volatility. Finally in the last step of DCC, we estimate conditional correlation.

3.2.1 Mean for DCC Model: ARMA Process

To implement the DCC model, we estimate the mean return by an autoregressive moving average (ARMA) model. An ARMA process captures the return series' dependence on its own previous value plus a combination of current and previous values of a white-noise error term. We use an ARMA(1,1) model as below:

$$R_{i,t} = \zeta + \varphi R_{i,t-1} + \theta\epsilon_{i,t-1} + \epsilon_{i,t} \tag{2}$$

[1]Theoretically speaking, a rolling window should not be considered as a static model since the time frame of estimation changes. However, in our paper, for simplicity and contrast, we call it the Static model.

At current day t, $R_{i,t}$ is our daily excess portfolio return data for sector i and $\epsilon_{i,t}$ is the ARMA(1,1) residual for sector i.

3.2.2 GARCH Model: Conditional Volatility Estimation

We use Bollerslevs GARCH(1,1), which allows the conditional variance of the current period to be a function of both lagged conditional variance and past residuals:

$$\sigma_{i,t}^2 = \omega + \beta\sigma_{i,t-1}^2 + \alpha\epsilon_{i,t-1}^2 \tag{3}$$

The GARCH parameters ω, β, and α are estimated using Maximum Likelihood Estimation (MLE); we maximize the log-likelihood function over the in-sample period:

$$L = \sum_{t=1}^{T} log(\frac{1}{\sqrt{2\pi\sigma_t^2}} exp\frac{-\epsilon_{i,t}}{2\sigma_t^2}) \tag{4}$$

3.2.3 DCC Model: Conditional Correlation Estimation

From the previous conditional variance estimation, we derive the standardized residuals for the DCC-GARCH model. The standardized residuals or volatility-adjusted returns s_i are calculated from GARCH volatilities:

$$s_{i,t} = \frac{\epsilon_{i,t}}{\sigma_{i,t}} \tag{5}$$

For the CAPM model, we use two returns (portfolio i and the market); for the Fama-French model, we use 4 returns: portfolio i, the market, SMB factor and HML factor.

These standardized residuals are used as inputs in estimating quasi-correlations $q_{i,j,t}$ between series i and j in the DCC model:

$$q_{i,j,t} = \mu_{i,j} + \eta s_{i,t-1}s_{j,t-1} + \phi q_{i,j,t-1} \tag{6}$$

where

$$\mu_{i,j} = (1-\eta-\phi)\overline{R}$$
$$\overline{R} = \frac{1}{T}\sum_{t=1}^{T} s_{i,t}s_{j,t}$$

$\overline{R}$ is the averaged realized average, $s_{i,t-1}$ and $s_{j,t-1}$ are the lagged GARCH standard residual. $\mu_{i,j} = (1-\eta-\phi)\overline{R}$, known as correlation targeting, is restricted to be a constant that is generally stated along with η and ϕ. T represents the total number of days in the sample.

These quasi-correlations are then re-scaled to be between -1 and 1 and thus $\beta_{i,j,t}$ is the conditional correlation at time t between the two series i and j:

$$\rho_{i,j,t} = \frac{q_{i,j,t}}{\sqrt{q_{i,i,t}q_{j,j,t}}} \tag{7}$$

Similarly to those parameters in the GARCH Model, parameters η and ϕ are estimated using the MLE. As noted by Engle (2009), the log-likelihood function in this case applies to a pair of assets/portfolios, which is given by:

$$L_{i,j} = -\frac{1}{2}\sum_{t=1}^{T}(log(1-\rho_{i,j,t}^2) + \frac{s_{i,t}+s_{j,t}-2\rho_{i,j,t}s_{i,t}s_{j,t}}{1-\rho_{i,j,t}^2}) \tag{8}$$

3.3 Application to Asset Pricing

We use the volatility and correlation estimates derived in the previous section as inputs for betas under the CAPM and Fama-French Three Factor model in order to calculate the daily expected returns from 1960 - 2012. Beta outputs in day t are applied as the coefficients for day $t+1$ to achieve out-of-sample prediction . We then examine the Root Mean Square Error (RMSE) of each model to determine the precision of realized estimated return to actual returns.

3.3.1 CAPM Return Prediction

The Capital Asset Pricing Model (CAPM) can be used to determine a theoretically required return of individual sectors. Under the CAPM model, return on sector i is calculated as follows:

$$R_{i,t} = \beta_{i,t}R_{m,t} + \varepsilon_{i,t} \tag{9}$$

where $R_i = r_i - r_f$ is the excess return on sector i (i.e. return on sector i minus the risk-free rate), R_m is the excess return on a market index m, and ϵ_i is the idiosyncratic risk of sector i, which is uncorrelated with R_m or the idiosyncratic risk of any other sector under CAPM assumptions. With $\hat{\beta_{i,t}}$ as the estimated beta from t, the out-of-sample predicted return for $t+1$ is then:

$$\hat{R}_{i,t+1} = E[R_{i,t+1}|\hat{\beta}_{i,t}, R_{m,t+1}] = \hat{\beta}_{i,t}R_{m,t+1} \tag{10}$$

3.3.2 Fama-French Three Factor Model Return Prediction

Fama-French model expanded the CAPM by adding two variables in addition to the returns of the market as a whole. Fama and French started with the observation that two classes of stocks have tended to do better than the market as a whole: (i) small caps and (ii) stocks with a high book-to-market ratio (customarily called value stocks, in contrast with growth stocks). They then added two factors to CAPM to reflect a portfolio's exposure to these two classes:

$$R_{i,t} = \beta_{i,t}R_{m,t} + b_{s,i,t} \times \text{SMB}_{i,t} + b_{v,i,t} \times \text{HML}_{i,t} + \varepsilon_{i,t} \tag{11}$$

where R_i is the portfolio's excess expected rate of return and R_m is the excess return of the market portfolio. The "three factor" β is analogous to the classical β but not equal to it, since there are now two additional factors to do some of the work. SMB stands for "Small Minus Big" (by market capitalization) and HML for "High Minus Low" (by book-to-market ratio); they measure the historic excess returns of small caps over big caps and of value stocks over growth stocks. These factors are calculated with combinations of portfolios composed by ranked stocks and available historical market data. Similarly, the out-of-sample predicted returns for the Fama-French model on sector i in day $t+1$ is as such:

$$\begin{aligned}\hat{R}_{i,t+1} &= E[R_{i,t+1}|\hat{\beta}_{i,t}, R_{m,t+1}, \hat{b}_{s,i,t}, \text{SMB}_{i,t+1}, \hat{b}_{v,i,t}, \text{HML}_{i,t+1}] \\ &= \hat{\beta}_{i,t}R_{m,t+1} + \hat{b}_{s,i,t} \times \text{SMB}_{i,t+1} + \hat{b}_{v,i,t} \times \text{HML}_{i,t+1}\end{aligned} \tag{12}$$

3.3.3 Forecast Evaluation

After applying the Static and DCC in-sample parameters to the out-of-sample horizon, we now have predicted returns by sector, which will serve as our first parameter. In addition, we have the actual realized return as our second parameter. We then examine the Mean Square Error (MSE) from each model to determine the relative effectiveness and precision of our two models in estimating betas.

Let $e^{(S)}_{i,t+1}$ and $e^{(D)}_{i,t+1}$ be the forecast errors of Static and DCC-GARCH model respectively. We have:

$$\begin{aligned}e^{(S)}_{i,t+1} &= r^{(S)}_{i,t+1} - \hat{r}^{(S)}_{i,t+1} \\ e^{(D)}_{i,t+1} &= r^{(D)}_{i,t+1} - \hat{r}^{(D)}_{i,t+1}\end{aligned} \tag{13}$$

Then, we conduct the Diebold-Mariano (DM) test as our way of comparing the accuracy of two forecasts. In this paper, we compare the forecasts by comparing the difference in the squared errors from two forecasts:

$$d_{i,t} = (e^{(S)}_{i,t})^2 - (e^{(D)}_{i,t})^2 \tag{14}$$

The null hypothesis of the DM test assumes that the two forecasts are equally good, and thus we test,

$$\begin{aligned}H_0 &: E[d_{i,t}] = 0 \\ \text{vs. } H_1 &: E[d_{i,t}] > 0 \\ \text{and } H_2 &: E[d_{i,t}] < 0\end{aligned}$$

3.4 Beta Trends in Crisis or Booms

Trends in beta from 1963 to 2012 will be compared both graphically and analytically. Figures are created where t is the x-axis while beta is the y-axis. The graphs make it easier for us to distinguish any movements in eras of financial crises. Analytically, we first create a binary indicator variable $\mathrm{I(Regression)_t}$ defined below:

$$\mathrm{I(Regression)_t} = \begin{cases} 0 & \text{if in Recession at time t} \\ 1 & \text{if in Booms at time t} \end{cases} \tag{15}$$

where recession and boom periods data are provided from the National Bureau Economics Research (NBER) Business Cycles section (Appendix B).

Then, we would add the $\mathrm{I(Regression)_t}$ variable to our forecasted asset pricing models in order to specifically segregate returns into two sets, i.e. regression time returns and boom time returns:

- For CAPM:

$$\begin{aligned} \hat{R}^r_{i,t+1} &= \mathrm{I(Regression)_t}(\hat{\beta}_{i,t}R_{m,t+1}) \\ \hat{R}^b_{i,t+1} &= (1 - \mathrm{I(Regression)_t})(\hat{\beta}_{i,t}R_{m,t+1}) \end{aligned} \tag{16}$$

- For Fama French Three Factor Model:

$$\begin{aligned} \hat{R}^r_{i,t+1} &= \mathrm{I(Regression)_t}(\hat{\beta}_{i,t}R_{m,t+1} + \hat{b}_{s,i,t} \times \mathrm{SMB}_{i,t+1} + \hat{b}_{v,i,t} \times \mathrm{HML}_{i,t}) \\ \hat{R}^b_{i,t+1} &= (1 - \mathrm{I(Regression)_t})(\hat{\beta}_{i,t}R_{m,t+1} + \hat{b}_{s,i,t} \times \mathrm{SMB}_{i,t+1} + \hat{b}_{v,i,t} \times \mathrm{HML}_{i,t+1}) \end{aligned} \tag{17}$$

  where $\hat{R}^r_{i,t+1}$ and $\hat{R}^b_{i,t+1}$ represent forecasted returns at recession era for sector i and at boom era for sector i, respectively.

Our interests lie in the absolute and relative difference in beta averages in periods of recessions and booms and perform DM test to compare model precision under the contraction or expansionary economic background.

4 Data

4.1 Data Source

We use daily industry portfolio and market returns acquired from Kenneth French Data Library. Price data are calculated as daily net logarithmic (continuously compounded) returns as such:

$$r_{t+1} = log(\frac{P_{t+1}}{P_t}) = log(P_{t+1}) - log(P_t) \tag{18}$$

Therefore, our data comprise of daily value weighted returns on 10 US sectors, as well as daily value-weighted returns on the overall market index from January 1960 to December 2012. We believe year 1960 to be an appropriate starting point because this time period is long enough to capture beta trends in recessions and/or booms.

Returns reported by industry sectors are designated by Kenneth French. The ten industries are: Consumer Non-Durables, Consumer Durables, Manufacturing, Energy, Technology, Media & Telecom, Stores & Services, Utilities, and Others[2]. They assign each NYSE (New York Stock Exchange), AMEX (American Stock Exchange), and NASDAQ (NASDAQ Stock Market) stock to an industry portfolio at the end of June of year t based on its four-digit SIC (Standard Industrial Classification) code at that time. They use Compustat SIC codes[3] for the fiscal year ending in calendar year $t-1$. Whenever Compustat SIC codes are not available, they use CRSP SIC codes[4] for June of year t.

Market return is compiled using the 201402 CRSP database. The T-bill return is the simple daily rate that, over the number of trading days in the month, compounds to 1-month T-Bill rate from Ibbotson and Associates, Inc. T-Bill rate is used as the risk-free rate to calculate return premium.

The choice of daily sampling frequency is determined based on the trade-off between additional information and data source restrictions. While more information is obtained when sampling frequency is high, the additional information only has marginal and diminishing effects on beta. That is, intraday returns are highly correlated with daily returns, meaning forgoing any more information would not significantly improve our information base. For instance, our data size would be seven times our current dataset if hourly returns were used but this does not indicate that we would increase our precision of beta measurements by seven times as well. Thus, daily returns offer a good trade-off between additional information and data restrictions.

4.2 Data Analysis

Table 1 presents summary statistics for daily returns data for 10 US industry portfolios and market index covering the period from January 1960 to December 2012. All series have mean returns very close to zero. Consumer Non-Durables and Utilities have lower standard deviation compared to others while Technology has the highest standard deviation among all sectors. In the context of the economy, such results indicate that Technology sector to be the most volatile in the market and Non-Durables and Utilities sectors the least volatile. In addition, minimum returns in absolute value for all sectors (including market index) are bigger than their maximum returns, indicating that all stocks are more volatile in recessions than booms.

[2]Full descriptions and SIC codes of each industry portfolio can be found in Appendix A

[3]SIC codes provided by Standard & Poors Financial Services LLC

[4]SIC codes provided by Center for Research in Security Prices (CRSP)

Table 1: Analysis of Industry Portfolio & Market Returns (1960 - 2012)

US Industry	Mean	Stdev.	Min	Max	Skewness	Kurtosis
Non-Durables	0.03061	0.8644	-17.06	10.28	-0.6235	22.282
Durables	0.01986	1.3095	-18.38	9.72	-0.2147	11.788
Manufacturing	0.02253	1.0337	-20.03	10.74	-0.6701	21.280
Energy	0.03315	1.2695	-19.46	19.33	-0.1917	19.858
Technology	0.02405	1.4266	-20.01	16.01	-0.0295	11.905
Media & Telecom	0.02291	1.1026	-16.71	14.51	-0.0218	17.118
Stores & Services	0.02745	1.0507	-16.60	10.99	-0.2678	13.982
Healthcare	0.02831	1.0681	-17.92	11.09	-0.4002	14.403
Utilities	0.02063	0.8184	-12.89	14.43	-0.0045	29.265
Others	0.02298	1.1259	-15.33	11.26	-0.2464	18.114
Market	0.02180	0.9852	-17.44	11.35	-0.5203	19.235

All returns are excess returns. For simplicity and clarity of the table, all values besides kurtosis are represented in percentages (%).

Figure 1 plots the cumulative time-series returns for industry sectors and we have seen continuous growth through the 1960s till present. In addition, we have observed a dip in 1973-1975 (period of economic stagnation in the Western World that puts an end to post-World War II economic boom), and a drop-off following the dot-com burst, followed by a slow yet steady improvement until the credit crisis from late 2007 and sovereign debt crisis from late 2009. In addition, All sectors and market index show a huge sudden dip in returns around October 1987, which is what we generally referred to as “Black Monday” (October 19th, 1987) when stock markets around the world crashed.

A sharp rally through early 2000 can be seen especially in the Technology and Media & Telecom sectors and a rally in Energy, Manufacturing, and Others after 2004 compared to other sectors. According to SIC Codes in Appendix A, Others include Financial Services, Entertainment, Hotels, etc. Therefore, we contribute the rally largely to booms in Financial Services and Entertainment throughout 2003 - 2007.

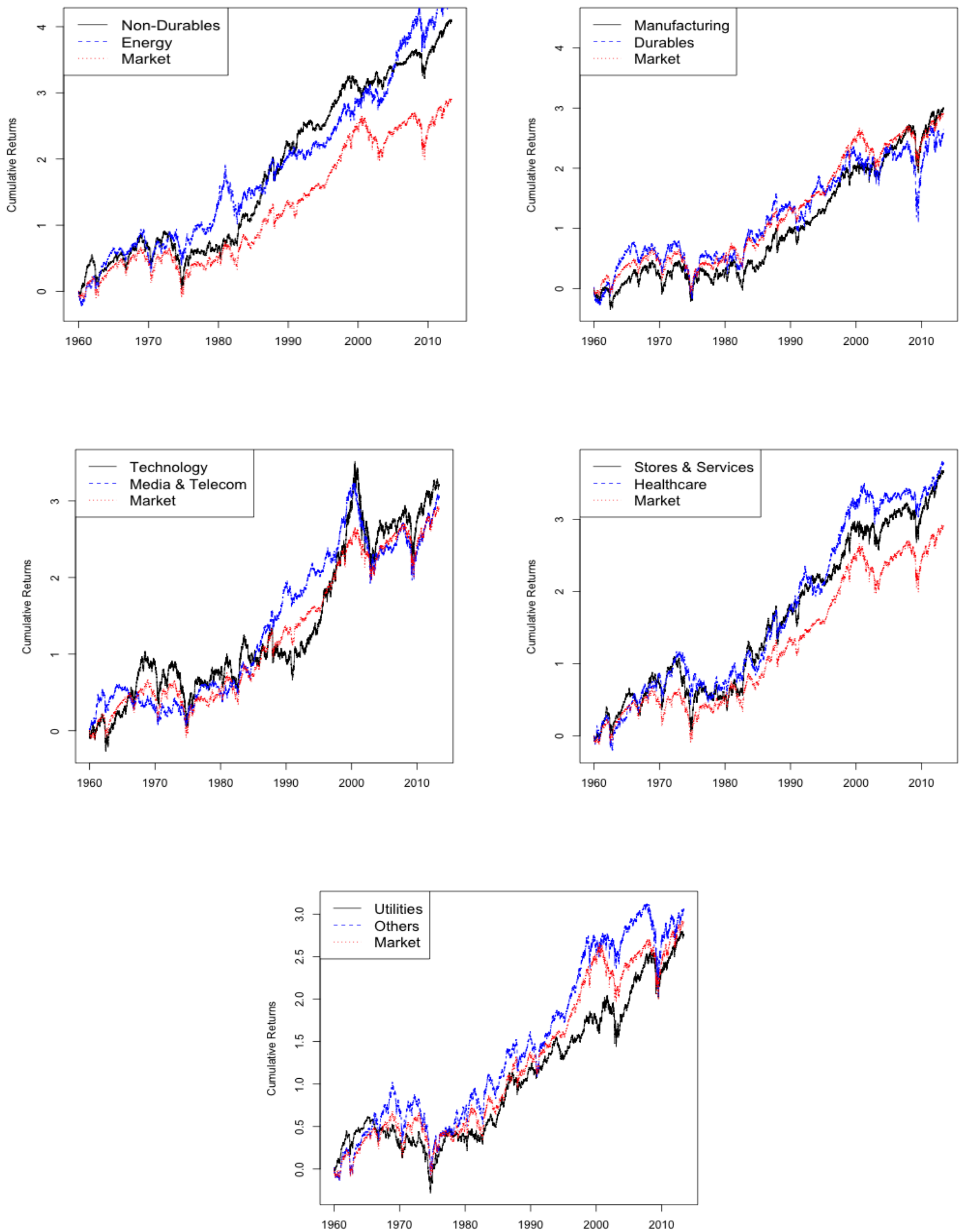


Figure 1: US Excess Cumulative Returns by Sector

5 Results

For the sake of brevity, we select to report the average industry beta under different pricing and beta valuation models across the entire 52 years from 1960 to 2012 (Table 2). Then we show the betas during contraction periods and expansion periods as defined by the NBER under each of the asset pricing model - CAPM (Table 5) and Fama French model (Table 6). By definition, contraction periods yielded significant negative returns due to the bursting of many financial crises and expansion periods produced significant positive returns as economic growth spiked. Therefore, we select contractions as representation of bear market and and expansions as bull market.

Furthermore, we select two industries, Technology and Stores & Services, to illustrate comparison of beta estimates between models. These two industries are representative among all industries in terms of scale and fluctation. Beta estimation comparison for all 10 industries are shown in Appendix D. The ARMA, GARCH, and DCC parametrs for the DCC-GARCH model are reported in Appendix C.

5.1 Industry Average Beta Findings

Table 2 shows the average industry beta estimation using the two beta models for the CAPM and Fama-French Three Factor model[5] across the entire sample period (1960 2012). Estimates from the two circumstances differ but not significantly. Across all sectors, Utilities has the lowest beta value, indicating its low exposure to market volatility as a fairly conservative and consistent industry. Technology ranks among the most volatile sector by its nature of business and, potentially, partially due to the Internet boom and its eventual bust. Other relative stable industries, that is, industries with lower betas include No-Durables and Media & Telecom, while Durables demonstrates a comparatively volatile returns to the market.

Average beta estimates under similar economic environment are very similar and rather close to 1, the market beta average. Such finding stands as mean averages out any anomalies with a short period of time across a long time-span. The GARCH beta estimates are consistently slightly lower than Static betas between the two beta models used. Moreover, from figures in Appendix D, it can be found that beta curves from GARCH model are smoother and in most cases have smaller maximum and minimum beta values than the Static model, due to the mean reversion property and momentum effect for GARCH. More interestingly, when comparing between the two asset pricing models using identical beta modeling, Fama-French betas are slightly higher than those of CAPM for each industry.

[5]Fama-French betas present in all tables below correspond to the market beta term

Table 2: Average Industry Betas (1960 - 2012)

U.S. Industry	CAPM		Fama-French	
	Static	DCC	Static	DCC
Non-Durables	0.7982	0.7966	0.8043	0.7957
Durables	1.1558	1.1506	1.1789	1.1510
Manufacturing	1.0165	1.1048	1.0471	1.0143
Energy	0.9133	0.9103	0.9946	0.9084
Technology	1.3296	1.3171	1.1178	1.3174
Media & Telecom	0.8462	0.8438	0.9173	0.8458
Stores & Services	0.9669	0.9620	0.9913	0.9625
Healthcare	0.9711	0.9636	0.8766	0.9634
Utilities	0.5476	0.5407	0.7033	0.5411
Others	1.0122	1.0101	1.0899	1.0098

Figure 2 and 3 illustrate daily Static and GARCH betas from 1960 to 2012 in the context of two asset pricing models for the Technology and Stores & Services Industry. Both figures show similar movements in betas and draw an identical conclusion that beta movements generally give indication to great changes in the economy such as economic recessions or booms. Yet the necessary correlation between the direction of beta movement and good or bad economic changes has to be established case by case. We will discuss fluctuations of betas in relationship with contractions and expansion in more detail in a later section.

It is interesting to note that CAPM betas offer much clearer trend of movement than Fama-French betas. This is mainly caused by absorbing effect of the other two factors in the Fama-French model, making the market beta less indicative of the general market trend. Moreover, GARCH beta estimates in general, for both CAPM and Fama French, show less swing in beta values than Static betas. This demonstrates GARCH's mean reversion nature, which ensures stationarity in the beta values to fluctuate within a reasonable range around 1.

- Case 1: Technology Industry

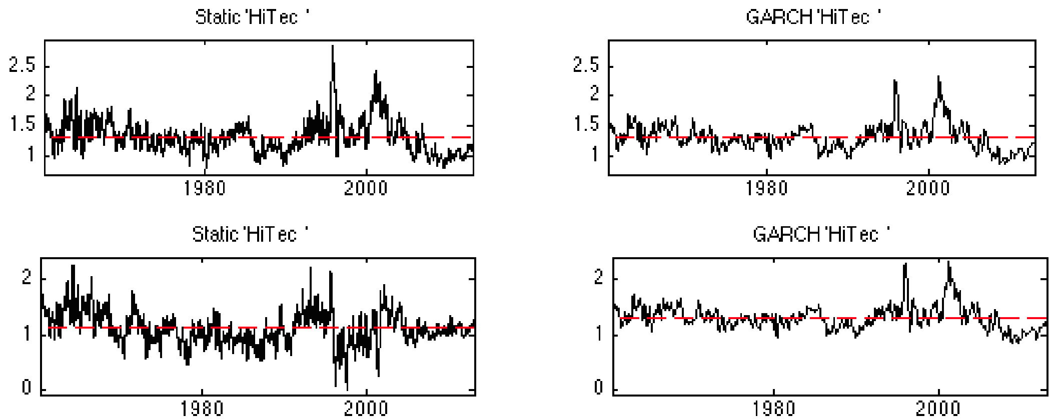


Figure 2: Technology Industry CAPM & Fama-French Betas (1960 - 2012)

- Case 2: Stores & Services Industry

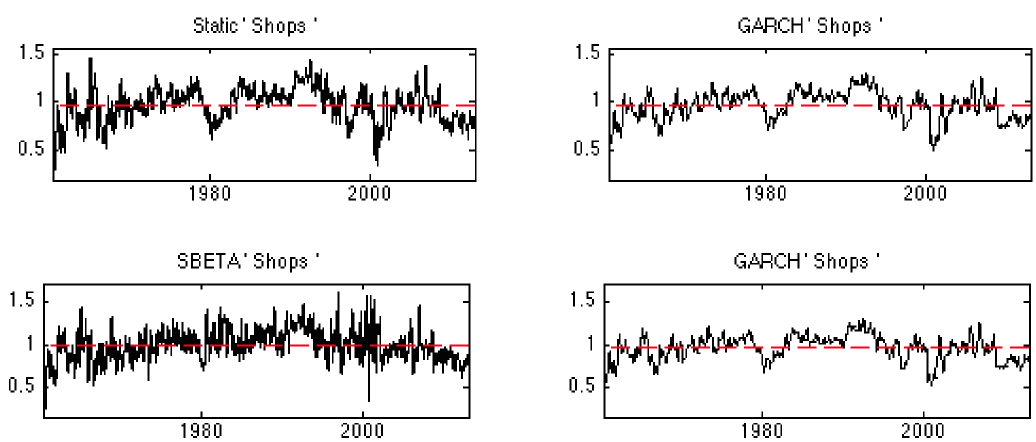


Figure 3: Stores & Services Industry CAPM & Fama-French Betas (1960 - 2012)

5.2 Beta Forecast Comparison

Table 3 and 4 below summarize the results of the Diebold-Mariano (DM) test over the entire sample period and presents level of significance between two beta methods and asset pricing models using the DM test.

5.2.1 Static vs. DCC-GARCH Beta Comparison

Table 3 shows accuracy of beta estimation using the two methods by the same asset-pricing model. When comparing Static beta modeling with GARCH(1,1) model under the CAPM framework, Energy, Healthcare, and Other sectors are estimated with non-signifcant accuracy than using GARCH(1,1) model while Non-Durables, Durables, Utilities, and Media & Telecom sector demonstrate much more precision with Static beta modeling. Other sector return estimations demonstrated similar goodness-of-fit and accuracy using either method.

However, under the Fama-French framework, advantages of the GARCH(1,1) model are much more significant across multiple vectors, namely, Durables, Manufacturing, Technology, Media & Telecom, Stores & Stores, Healthcare, and Utilities. Differences in the results can be explained by the fact that conditional volatility and correlations respond more flexibly to changes at multivariate level.

Table 3: DM Test: Static vs. GARCH Model Precision (1960 - 2012)

U.S. Industry	CAPM	Fama-French	U.S. Industry	CAPM	Fama-French
Non-Durables	1.945*	1.448	Media & Telecom	3.112***	3.713***
Durables	3.692***	2.767***	Stores & Services	1.026	2.482***
Manufacturing	0.684	2.667***	Healthcare	-0.523	2.297***
Energy	-0.788	1.069	Utilities	1.892*	1.951**
Technology	3.112	3.713***	Other	-0.746	0.908

where *** < 1% significance level; ** < 5% significance level; * < 1% significance level

5.2.2 CAPM vs. Fama-French Framework Comparison

Table 4 compares beta estimates vertically between the CAPM and Fama-French model under identical beta modeling methodology. Fama French betas prove to be better that CAPM betas in most sectors. Our conclusion aligns with past research that argued CAPM fail to demonstrate accuracy in predicting asset of portfolio returns.. There is one exception to our conclusion - the Utilities sector. Under the Static model, the CAPM model performs marginally better than the Fama-French model in predicting Utilities sector returns. One possible explaination for the result is that neither of the two asset pricing models offer good predictions given the low exposure of this industry to the overall market.

Table 4: DM Test: CAPM vs. Fama-French Model Precision (1960 - 2012)

U.S. Industry	Static	GARCH	U.S. Industry	Static	GARCH
Non-Durables	1.4581	2.0516**	Media & Telecom	3.7186***	3.7373***
Durables	2.0909**	3.3898***	Stores & Services	3.2912***	3.0551***
Manufactoring	1.0407	2.3173**	Healthcare	3.7795***	2.8676***
Energy	0.6791	2.0162**	Utilities	-0.7011	1.7468**
Technology	1.7499	2.9500***	Other	2.3595**	-0.7937

where *** < 1% significance level; ** < 5% significance level; * < 1% significance level

5.3 Beta in Recessions and Booms

Table 5 and 6 show the industry average betas estimated in recessions and booms under the CAPM and Fama-French framework respectively. Findings in Section 5.1 still apply regardless of the economic environment. DM test is conducted to parse the best combination of beta and asset pricing models in economic recessions or booms.

5.3.1 Beta Fluctuations

In general, CAPM betas in the Energy, Utilities, and Others sector tend to increase in recessions while decrease in economic booms. All other sectors show the opposite trend. Specifically, Non-Durables, Durables, Manufacturing, Technology, Media & Telecom, Stores & Services, and Healthcare are likely to decrease in beta values during contraction periods. That is, these sectors become more resistant to market turmoil during economic downturn. Minimal change in beta is seen for the Manufacturing industry while maximum in Energy sector between contractionary and expansionary eras.

Fama French betas in 6 - Non-Durables, Manufacturing, Media & Telecom, Healthcare, Utilities, and Other - out of 10 sectors betas go up in contractions and go down in expansions when applying the Rolling Window beta estimation methodology. It is interesting to see contrasting results of beta trends when applied in different pricing frameworks. The discrepancy in beta performance could be due to the estimation mechanisms (i.e. SMB and HML factors) as well as different equity behaviors for valued and growth stocks for the Fama-French model. Yet, we see consistent results for the GARCH model regardless of asset pricing models. This find is consistent with our assumption that GARCH inherently carry the robustness through incorporating conditional volatility.

Table 5: CAPM Average Industry Betas In Business Cycles (1960 - 2012)

U.S. Industry	Rolling Window		DCC-GARCH	
	Contraction	Expansion	Contraction	Expansion
Non-Durables	0.7481	0.8077	0.7413	0.8071
Durables	1.1102	1.1644	1.0882	1.1624
Manufactoring	1.0175	1.0163	1.0037	1.0169
Energy	0.9932	0.8981	0.9942	0.8945
Technology	1.2796	1.3391	1.2721	1.3256
Media & Telecom	0.7905	0.8567	0.7666	0.8584
Stores & Services	0.9162	0.9765	0.9071	0.9723
Healthcare	0.9210	0.9806	0.9021	0.9753
Utilities	0.5632	0.5447	0.5559	0.5378
Others	1.0458	1.0058	1.0426	1.0040

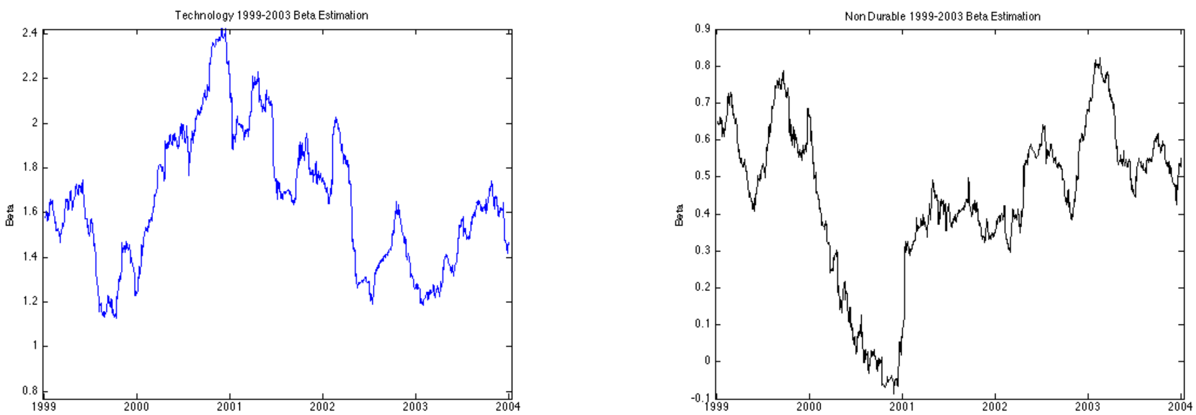
Table 6: Fama-French Average Industry Betas In Business Cycles (1960 - 2012)

U.S. Industry	Rolling Window		DCC-GARCH	
	Contraction	Expansion	Contraction	Expansion
Non-Durables	0.8254	0.8003	0.7416	0.8060
Durables	1.1507	1.1843	1.0855	1.1634
Manufactoring	1.0445	1.0163	1.0029	1.0164
Energy	0.9816	0.9970	0.9896	0.8932
Technology	1.0326	1.1339	1.2696	1.3264
Media & Telecom	0.9271	0.9154	0.7691	0.8603
Stores & Services	0.9803	0.9933	0.9079	0.9728
Healthcare	0.8873	0.8746	0.9015	0.9751
Utilities	0.7946	0.6860	0.5575	0.5380
Others	1.1046	1.0871	1.0422	1.0037

5.3.2 Interesting Example

From Figure 4, Technology industry illustrated a significant rally during the dot-com boom era between 1999 and 2002. Intuitively, such conclusion makes sense. During the tech-bubble, market indices were mostly driven by Technology firms, and thus betas for Technology industry peaked while betas for others industries (represented by Non-Durables sector in the figure) dunked. Similarly, Telecom & Media industry betas show similar trend in this era and therefore the same conclusion can be drawn (refer to Appendix D).

Figure 4: Technology & Non-Durables Industry Beta Estimation (1999 - 2003)



5.3.3 Model Evaluation

DM-test result changes are minimal for contraction periods or expansionary periods. GARCH still do appear to have an advantage in beta estimation in the U.S. Equity Markets. This is true especially in the case where GARCH is unlikely to over-estimate or over-predict spikes or nosedives given its mean-reversion property.

Table 7: DM Test: Static vs. GARCH Model Precision (Contractions & Expansions)

U.S. Industry	CAPM		Fama-French	
	Contraction	Expansion	Contraction	Expansion
Non-Durables	-1.893**	-0.447	1.146	1.890*
Durables	2.885***	2.331**	1.568**	1.967**
Manufactoring	-2.070***	0.891	-0.526	1.106
Energy	0.085	1.776*	2.124**	3.145***
Technology	-1.579	1.629	2.089**	1.475
Media & Telecom	0.866	3.228***	3.660***	2.249**
Stores & Services	1.181	-1.658	2.939***	1.776*
Healthcare	1.476	3.282***	2.7158***	3.6747***
Utilities	-1.579	-1.879*	1.824*	1.440
Others	-0.774	4.595***	1.769*	1.839*

where *** $<$ 1% significance level; ** $<$ 5% significance level; * $<$ 1% significance level

6 Conclusion

Our objective in this paper is to estimate the daily betas for industry portfolios from a range of variance and covariance models in an asset-pricing context using sector equity data from the U.S. Equity Markets. We first derive beta estimates for individual industry and the use out-of-sample prediction for returns. We then compare our predicted returns to actual returns to measure the effectiveness of the models in the application. Lastly, we observe the trends and movements of the betas during U.S. economic booms and recessions.

In most cases, average beta estimates are consistent under the four methods with some exceptions in Fama-French unconditional beta modeling. The discrepancy in model performance could be due to the estimation mechanisms (i.e. SMB and HML factors) as well as different equity behaviors for valued and growth stocks. On a daily basis, Static and GARCH demonstrates similar model performance under the CAPM framework; GARCH estimates result in more accurate estimation compared to Static model using Fama-French asset pricing model. From our findings, Fama-French model do perform better than CAPM as widely perceived. We also find trends of average betas change by sectors in economic recessions or booms. The trends are almost consistent for the GARCH modeling while varies quite significantly for the Static method.

In our analysis, we carry out daily estimations of 10 industry portfolios for sample period of 52 years. One possible direction for future research is to extend time period back to potentially 1930s. This may help distinguish the underlying beta trends given more business cycles. The thesis can be further directed to compare the industry betas among large-cap, mid-cap, and small-cap stocks within each sector. Also, since U.S. Equity Markets is representative of a financial system that is well developed, it would be also of interest to perform same exercise comparing beta estimates to Emerging Markets by sectors and compare how results differ.

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A Industry Portfolio Descriptions and SIC Codes

Industry Portfolio	Abbrev.	Description & SIC Codes
Consumer Non-Durables	NoDur	Food, Tobacco, Textiles, Apparel, Leather, Toys {0100-0999, 2000-2399, 2700-2749, 2770-2799, 3100-3199, 3940-3989}
Consumer Durables	Durbl	Cars, TV's, Furniture, Household Appliances {2500-2519, 2590-2599, 3630-3659, 3710-3711, 3714-3714, 3716-3716, 3750-3751, 3900-3939, 3990-3999}
Manufacturing	Manuf	Machinery, Trucks, Planes, Chemicals, Office Furniture, Paper, Online Printing {2520-2589, 2600-2699, 2750-2769, 2800-2829, 2840-2899, 3000-3099, 3200-3569, 3580-3621, 3623-3629, 3700-3709, 3712-3713, 3715-3715, 3717-3749, 3752-3791, 3793-3799, 3860-3899}
Energy	Enrgy	Oil, Gas, and Coal Extraction and Products {1200-1399, 2900-2999}
Technology	HiTec	Business Equipment, Computers, Software, Electronic Equipment {3570-3579, 3622-3622, 3660-3692, 3694-3699, 3810-3839, 7370-7372, 7373-7373, 7374-7374, 7375-7375, 7376-7376, 7377-7377, 7378-7378, 7379-7379, 7391-7391, 8730-8734}
Media & Telecom	Telcm	Telephone and Television Transmission {4800-4899}
Stores & Services	Shops	Wholesale, Retail, Some Services (Laundries, Repair Shops) {5000-5999, 7200-7299, 7600-7699}
Healthcare	Hlth	Healthcare, Medical Equipment, and Drugs {2830-2839, 3693-3693, 3840-3859, 8000-8099}
Utilities	Utils	{4900-4949}
Others	Other	Mines, Construction, Building Materials, Transportation, Hotels, Bus Service, Entertainment, Finance

B NBER Business Cycle Dates

Table 8: Business Cycle Reference Dates (1960 -2012)

Peak	Trough	Duration in Months	
		Contraction	Expansion
April 1960	February 1961	10	24
December 1969	November 1970	11	106
November 1973	March 1975	16	36
January 1980	July 1980	6	58
July 1981	November 1982	16	12
July 1990	March 1991	8	92
March 2001	November 2001	8	120
December 2007	June 2009	18	73

C ARMA, GARCH, and DCC Parameters

Table 9: U.S. Industry Sector GARCH Parameters

	ζ	ψ	θ
Non-Durables	(0.0007)	0.8687	(0.7633)
Durables	(0.0001)	0.8454	(0.7632)
Manufacturing	0.0001	0.8672	(0.7845)
Energy	(0.0001)	0.8384	(0.7523)
Technology	0.0003	0.9045	(0.8028)
Media & Telecom	0.0001	0.8213	(0.7267)
Stores & Services	(0.0001)	0.8548	(0.7731)
Healthcare	0.0000	0.8923	(0.7963)
Utilities	0.0002	0.7559	(0.6238)
Others	(0.0001)	0.8322	(0.7138)

Table 10: U.S. Industry Sector GARCH Parameters

	ω	α	β
Non-Durables	0.0000	0.1839	0.7662
Durables	0.0000	0.1382	0.8046
Manufacturing	0.0001	0.1448	0.8533
Energy	0.0001	0.1276	0.8463
Technology	0.0000	0.2238	0.8217
Media & Telecom	0.0000	0.1983	0.8086
Stores & Services	0.0001	0.1533	0.7681
Healthcare	0.0000	0.0561	0.9265
Utilities	0.0000	0.0953	0.8730
Others	0.0001	0.1237	0.8659

Table 11: Industry Sector GARCH Parameters

	Durbl	Manuf	Enrgy	HiTec	Telcm	Shops	Hlth	Utils	Other
Non-Durables	$\eta = 0.0643$	0.0633	0.0443	0.0440	0.0498	0.0761	0.1314	0.0262	0.0762
	$\phi = 0.8828$	0.9046	0.9822	0.9333	0.9413	0.8833	0.4736	0.9758	0.8997
Durables		0.0735	0.0434	0.0731	0.0697	0.0885	0.0253	0.0357	0.0773
		0.8299	0.9312	0.8716	0.8774	0.8467	0.9387	0.9380	0.8820
Manufacturing			0.0396	0.0373	0.0411	0.1891	0.1376	0.0453	0.1078
			0.9350	0.9403	0.9078	0.7092	0.1651	0.9241	0.7688
Energy				0.0356	0.0420	0.0493	0.0472	0.0751	0.0311
				0.9155	0.9456	0.9282	0.8821	0.8764	0.9245
Technology					0.1049	0.0649	0.0394	0.0256	0.0579
					0.4561	0.9110	0.8345	0.9113	0.9073
Media & Telecom						0.0612	0.0425	0.0231	0.0567
						0.9121	0.8170	0.8979	0.9121
Stores & Services							0.1218	0.0177	0.0798
							0.4966	0.9541	0.8142
Healthcare								0.0417	0.0399
								0.9612	0.9771
Utilities									0.0263
									0.9476

D Beta Estimates by Sector

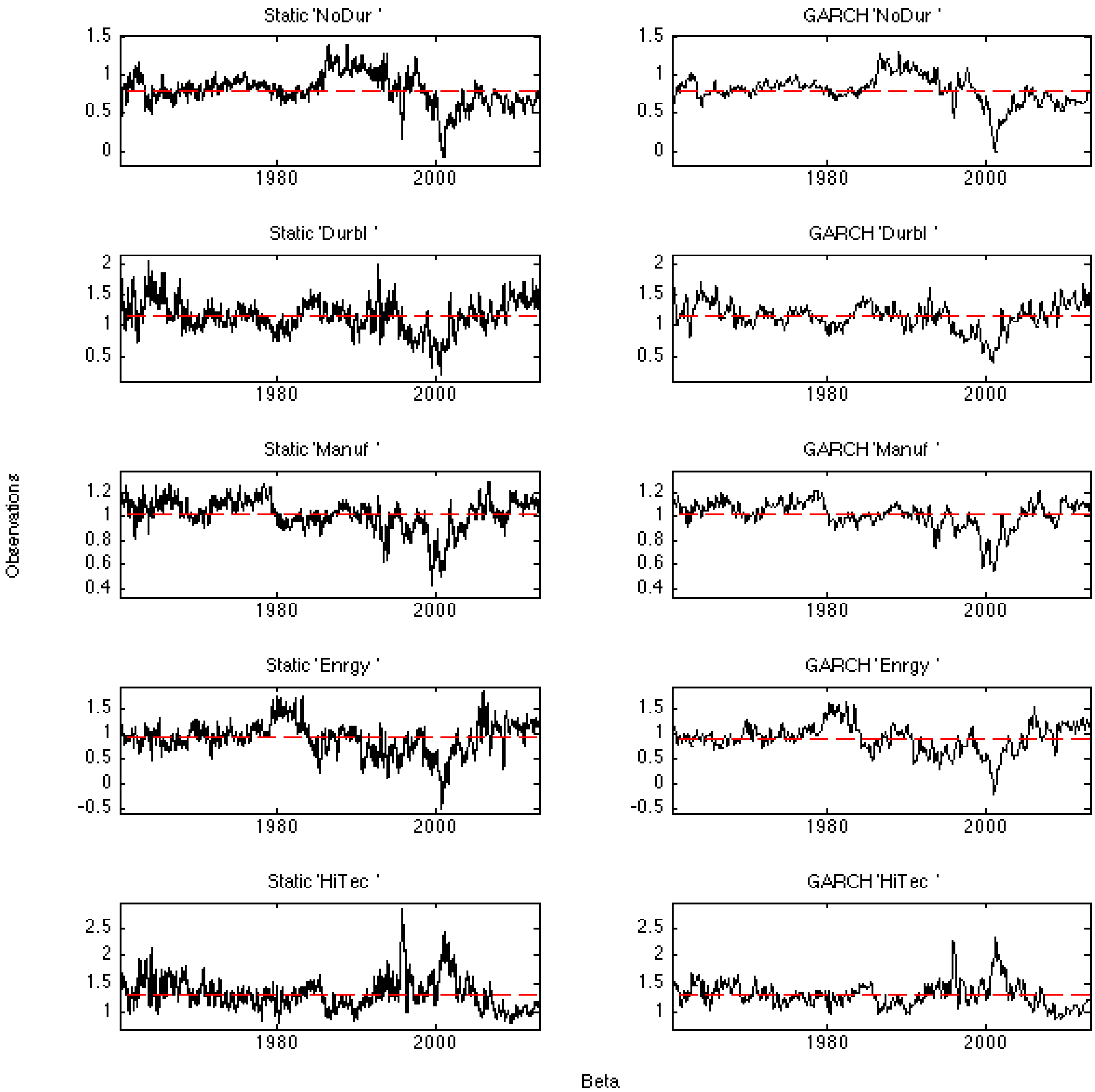

Figure 5: CAPM Betas by U.S. Sector (Part 1)

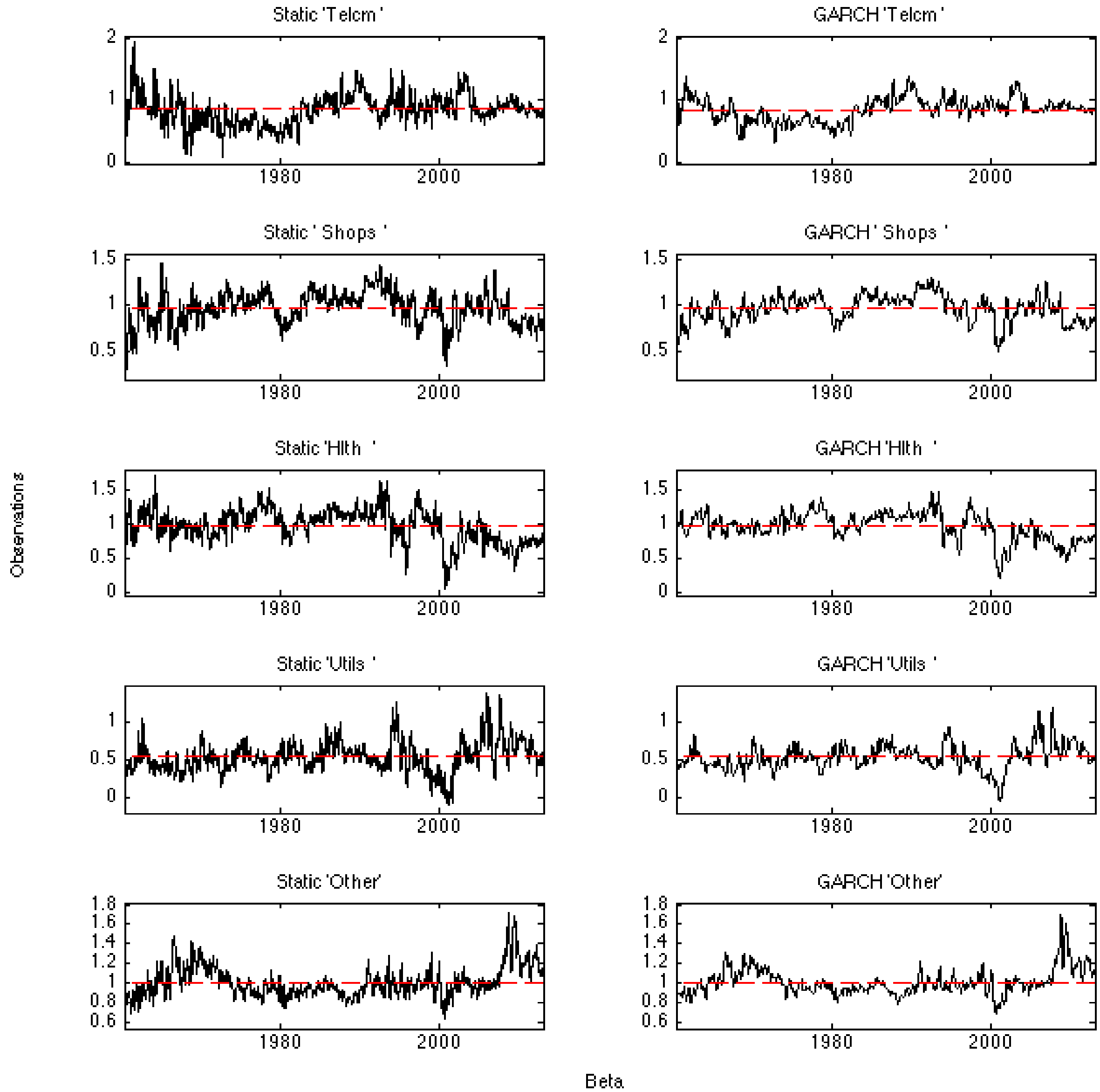

Figure 6: CAPM Betas by U.S. Sector (Part 2)

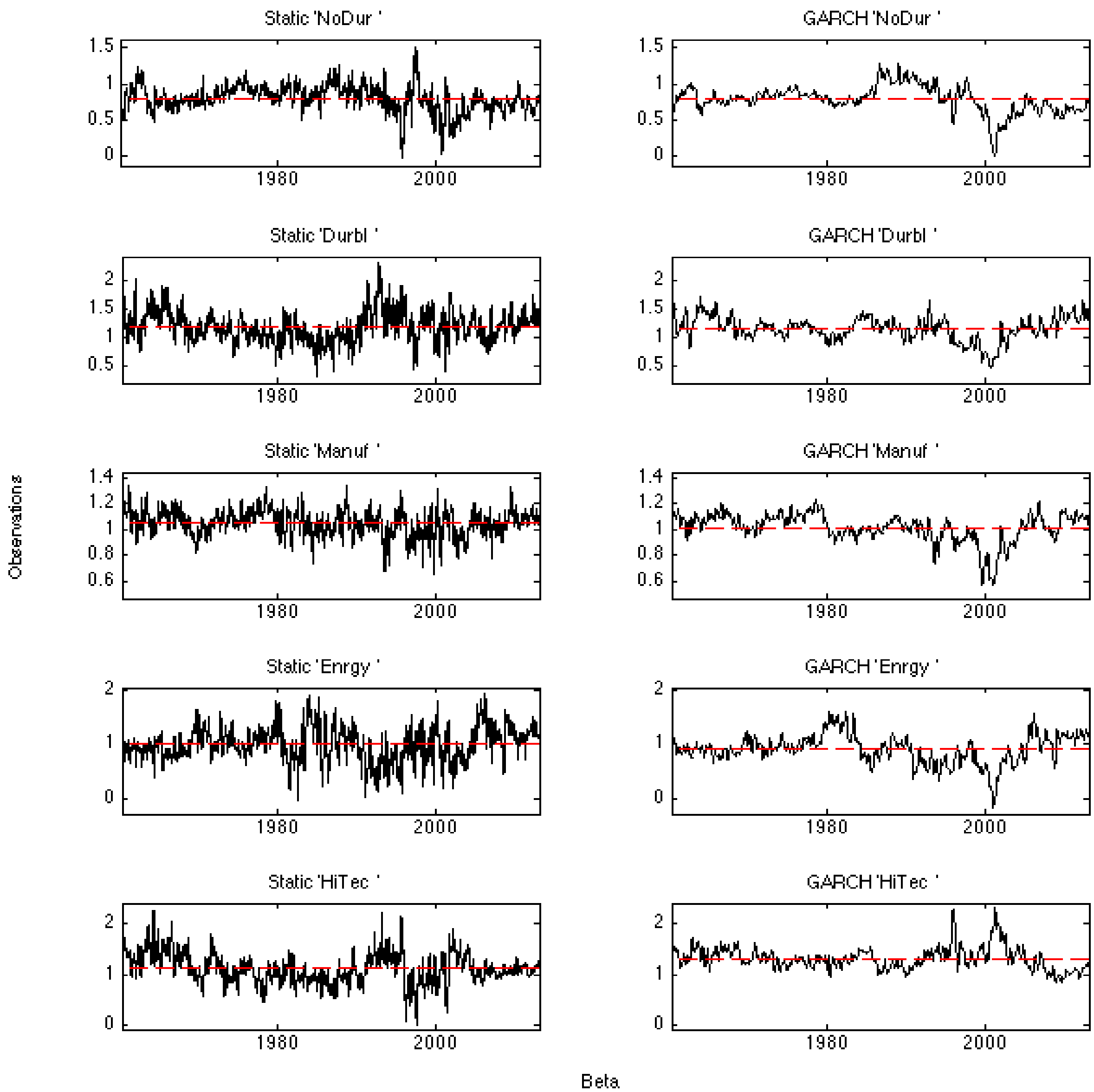

Figure 7: Fama French Betas by U.S. Sector (Part 1)

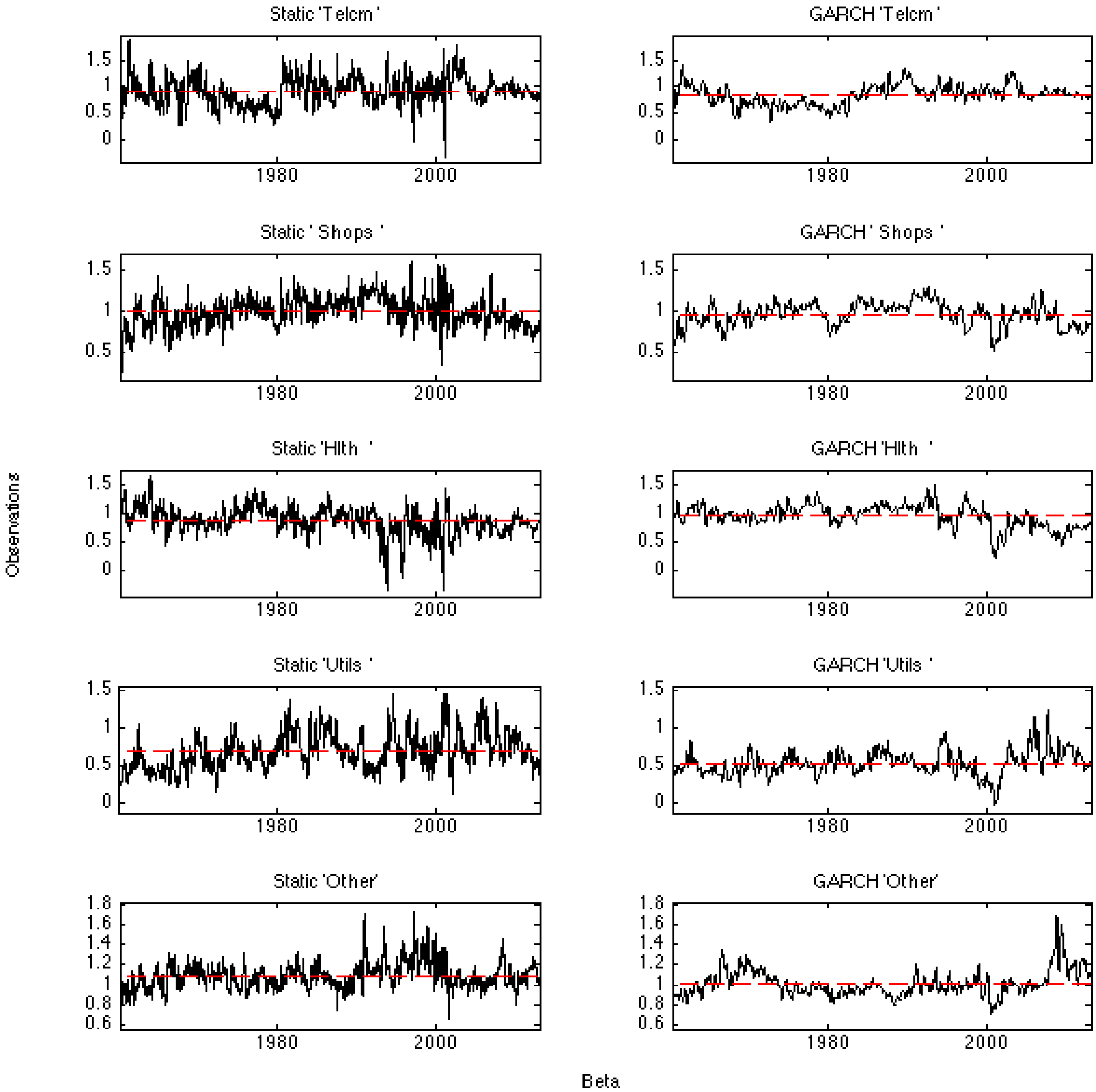

Figure 8: Fama French Betas by U.S. Sector (Part 2)

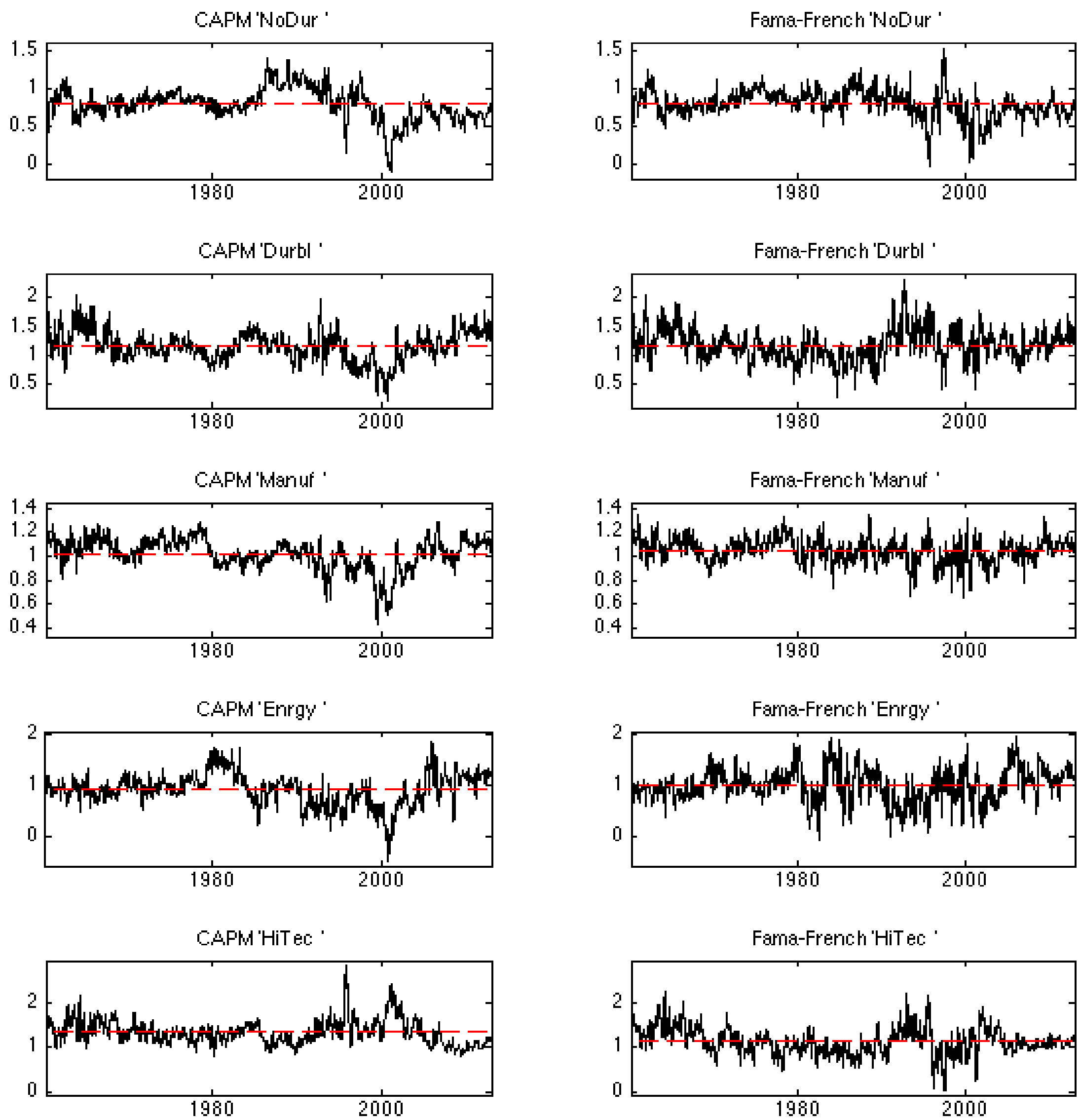

Figure 9: Static (Rolling Window) Model Betas by U.S. Sector (Part 1)

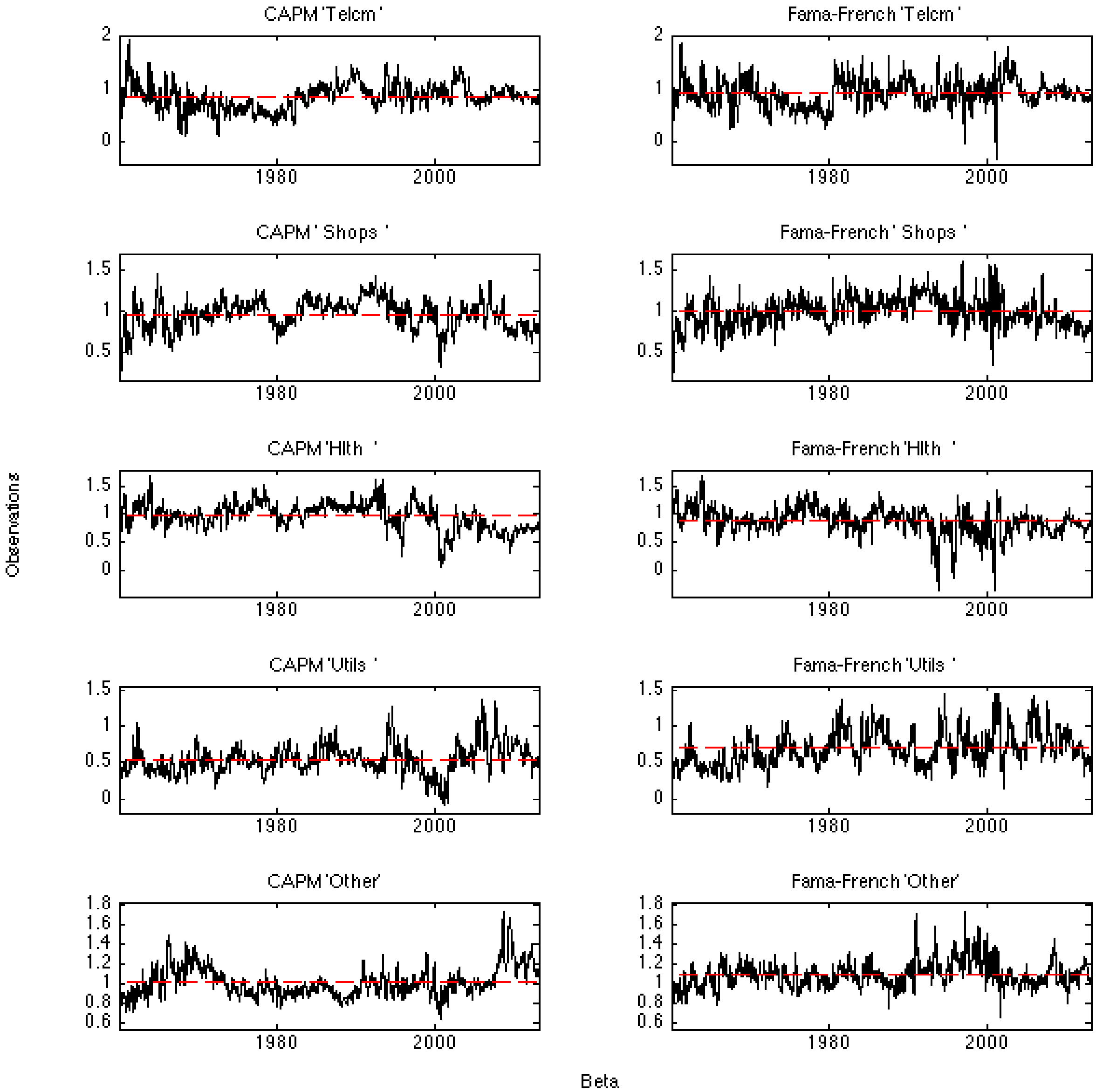

Figure 10: Static (Rolling Window) Model Betas by U.S. Sector (Part 2)

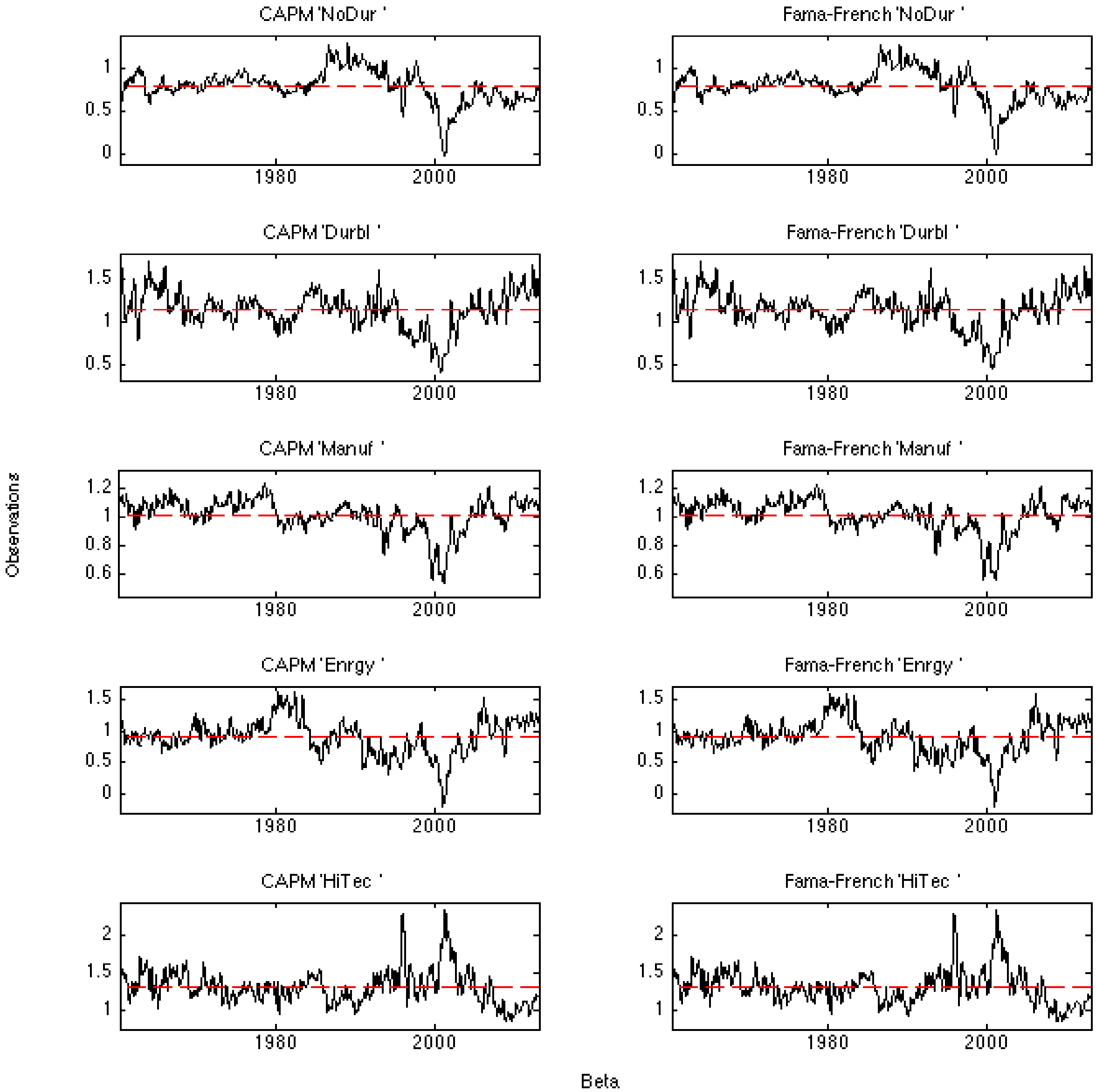

Figure 11: DCC-GARCH Model Betas by U.S. Sector (Part 1)

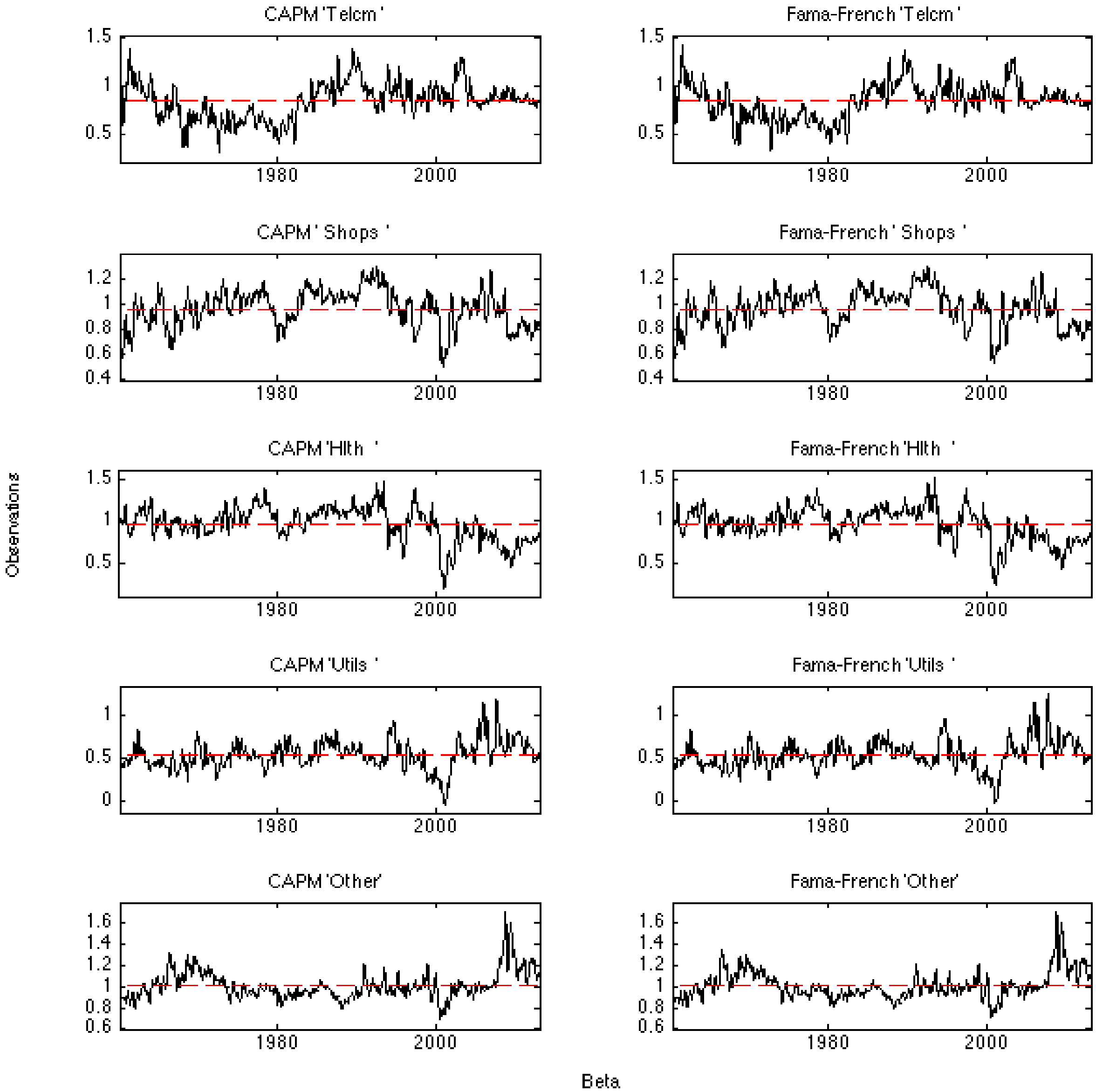

Figure 12: DCC-GARCH Betas by U.S. Sector (Part 2)