Game Theory and The World Marathon Majors

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Department of Economics
Duke University

Advisors:
Professor Curtis Taylor
Professor Michelle Connolly

Ben Jones

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Abstract

The World Marathon Majors (WMM) Series Prize was enacted in 2006 as a million dollar prize handed out annually to the top man and woman competing at five of the most important marathons. This paper considers the motivations behind setting up this prize, as well as the theoretical rationale for its existence and whether the empirical data supports these results. We find that the game theory model supports the ideas that the World Marathon Majors organizers state as their goals in creating the prize, but at the same time, there is not much empirical support as of yet to support any quantifiable changes within marathoning in the past few years. The regressions do not produce statistically significant data for finishing times decreasing even though the world record has been broken three times in these races since the implementation of the WMM. This may be due to the small number of observations and the fact that the series is so new. However, there are other areas of interest, such as an increase in World Record-breaking times or an increase in overall publicity, that may justify such a lucrative prize for these races. These topics are not included within the regressions and could be an area for further study.

Introduction

Along with long bike races like the Tour de France, the marathon is one of the most grueling physical tests of endurance and is the longest race (26 miles, 385 yards) currently in the Olympics. Because of high prize values and sponsorship opportunities, marathons are the longest races that have a strong cadre of professional athletes who exclusively run for their income. In order to increase exposure for marathon running in general and especially for the participating races, a group of five of the most elite international marathons united to create the World Marathon Majors (WMM). The WMM is a series of world-class marathons that was developed as a parallel to the Grand Slam Tournaments in tennis and Major Tournaments in golf. The races involved were all already in existence, but they were grouped into a series with the implementation of the WMM in 2006. It seems at first that these races could maintain their exclusivity and prestige without having to dole out $1 million every year. However, the race organizers decided to implement the series in a fashion where the top performing man and woman over a two-year period earn an extra $500,000 each on top of their other earnings from the individual races. If the WMM organizers' goals were to be accomplished by creating a series of elite competitions similar to golf or tennis, there is no formal requirement that a monetary prize be attached to the overall series, as neither golf nor tennis have annual awards given to the top performers across the spectrum of their respective "major" events. The WMM can be thought of as a cartel for elite marathons, as the series includes five of the biggest marathons in the world (Boston, New York City, Chicago, Berlin, and London) and the two most significant inter-country competitive marathons (the IAAF World Championships and the Summer Olympics). The World Championships happen once every two years (in the summer of every odd year), and the Summer Olympics are in the summer of years evenly divisible by four. All of the other races are annual, with the sequence each year being Boston and London (both late April), Berlin (late September), Chicago (mid-October), and New York City (early November). The series takes place over two years but the prize is awarded every year, meaning that each year is the first year in a series as well as the second for the previous series. For example, the first series awarded the prize for the races in the 2006-2007 calendars (January 2006 – December 2007), but the second award was for 2007-2008. Additionally, only the points from each runner's top four races are counted, so if a runner gets third in four races and fourth in a fifth all in the two-year span, they will only accumulate points based on the third place finishes. In each of the 7 races involved, points are awarded in the following pattern: 1^{st} – 25, 2^{nd} – 15, 3^{rd} – 10, 4^{th} – 5, 5^{th} – 1. In this paper, we would like to explore and analyze the effects of this relatively new series of races and determine whether or not the series is achieving its goals. These possible effects include changes in race entrants, in winning times at races, in average times of the top few finishers, in changes of the "closeness" of races, and in publicity for races. It will be important to view these changes temporally (comparing the WMM races pre- and post-2006) and also across other large marathons not included within the WMM. These questions are significant for all of these races because the WMM races have contributed quite a significant amount of money in order to implement the series prize, and the other races have possibly lost business and exposure as a result of the others' predicted increase. The central question surrounding this paper is: What have the impacts of this annual million-dollar prize been?

The Game Theory aspect within marathon racing is particularly interesting because of the relative simplicity of competitive marathon running compared to other sports. Because of the physical rigor involved in running a world-class marathon, elite marathoners typically only run one or two races each year, indicating the importance for most runners of the need to maximize their financial gains from racing (O'Toole, 2009). This fact was likely considered when the WMM was implemented with the four race limit as a mandated precaution so that runners would not feel pressure to race too many times in a year in order to boost their chances of winning the overall prize. Furthermore, in order further to disincentivize runners from competing in too many races in a year, a runner can only count three races in any individual year. In addition, the data on competitive marathoners is concrete and compared to other sports, lends itself more readily to analysis, due to the relative objectivity of results. Using the finishing times as a proxy for overall effort exerted by marathoners in a race is a better metric than something like minutes played or points scored in basketball, due to the relatively simplistic nature of running as a sport, in that there are few important statistics, and there is little argument over who are the best runners. There are certain qualities of marathons, such as differences in course difficulty, differences in weather, and complicated prize structures that prevent the data from being perfect, but it is evident that it is still much easier to estimate the overall effort of athletes.

Research Questions

The main question we want to address is what added value the race organizers of the WMM races expect to extract from their new series. This question involves many other questions because we must take many factors into account, such as athletes' probability of choosing or winning different races (within and outside of the WMM), publicity and sponsorship opportunities, effects on the races themselves, etc. We have modeled different race scenarios with heterogeneous runners and different prize values which show under what circumstances certain runners may self-select into certain races depending on their expected utility. They will attempt to maximize their overall utility, which is a function of both their expected prize winnings and the costs of entering the race. Within this question, it is important to consider the differences between certain athletes and reasons why they may deviate from what is ideal gameplay for others. Elite marathoners get paid in a variety of different capacities, which will all obviously affect the ways in which they respond to monetary incentives in their running careers. This overall concept is indicative of the idea that players have different payoff functions illustrated by Robert Gibbons in *Game Theory for Applied Economists* (Gibbons, 1992). One key deviation from his idea is that we will assume that most players have complete information about the other runners' payoff functions, as sponsorship and prize values are relatively well-known figures. For most non-elite marathoners (our low-ability players), entrants must pay an entry fee for participating in a race. However, elite marathoners are often actually paid an "appearance fee" for showing up to a race, even a race with a large purse (the purse is the total amount of money given away to top finishers). In addition, they may make speeches at the races or promote products in commercials and at race expositions for which they get paid. Essentially, all of these payments can be lumped together into the larger category of endorsements, which will have an impact on players' decisions in selecting which races to run. When we consider these areas of alternate pay, we have to look at certain athletes who may decide that the WMM prize is not worth the effort involved. For example, the former marathon world record holder, Haile Gebrselassie is unusual in several capacities. He has broken 27 world records in various distances, including having two of the official fastest marathon times ever.[1] His illustrious career has provided him sponsorship opportunities that would be unthinkable for other runners, and he owns numerous businesses in his native Ethiopia, including a hotel resort, a fitness center, and a car dealership (Meier, 2011). Taking these considerations into account, Haile would be much

[1] Geoffrey Mutai's 2011 Boston time is an unofficial world's fastest time due to the unusual nature of Boston's course, the IAAF states, "Due to the elevation drop and point-to-point measurements of the Boston course, performances are not eligible for world record consideration"

less likely to be affected by the WMM series prize than another elite marathoner who depends more heavily on race winnings for his or her consumption needs.

The WMM website suggests that there are a number of expected economic benefits from the incorporation of the WMM (WMM Website, 2011), but it seems hard to verify some of these numbers independent of other factors, like the general rise of marathon interest. It is also important to look at the change in race times from before and after the implementation of the WMM in order to try to isolate the effects of the WMM. The previously mentioned financial boons to these races would also result from increased publicity because of a "better" overall race. We can measure how "good" a race is through looking at how low the overall finish times are, as well as analyzing the time lag between top finishers, with less time lag indicating a more exciting, or "better" race experience for fans. Because of the magnitude of the WMM series prize, we hypothesize that races will have significantly lower times, and less time lag, especially for races towards the end of the series schedule, where the winner of the series may depend on who wins one specific race.

The WMM does not mention any effects on other races not involved in the WMM, but we hypothesize that there will be relatively strong effects on these other races as a result of the increased prize money for a specific seven races. If an elite runner thinks s/he can win the WMM, each race involved essentially doubles its prize money ($500,000 split up between the four races s/he would participate in), which should significantly impact the races that did not have this drastic increase in prize money. Additionally, for sub-elite marathoners who assume their probability of winning the series prize is 0, we expect their results to be consistent with the pre-series times across all these races. On the other hand, it is not unreasonable to postulate that some trickle-down effects could exist from an increased pace by the elite marathoners within the series races. In addition, it is possible that elite marathoners will self-select more into the WMM races, which may increase the possibilities of winning for sub-elites in various other marathons. This general idea is similar to the tournament theory concept of firms hiring graduates of the best schools and the best graduates of other schools (Gibbons, 1992). In other words, the top races award money to a larger number of runners, and the next tier of races award money just to the very best few runners. We expect these phenomena to be more apparent within women's marathons because of the greater heterogeneity in women's marathon running, resulting in top women having greater control over their races (with less randomness due to errors of judgment).

Literature Review

Most of the relevant literature will come from the field of tournament theory, which is a sub-field of economics and the related field of sports economics. Lazear and Rosen, with their 1981 paper "Rank-Order Tournaments as Optimal Labor Contracts" were some of the initial few theorists involved in looking at aspects of basing compensation around relative rank instead of absolute output (Lazear & Rosen, 1981). This structure is important because it is often much harder to measure absolute output than it is to measure relative rank, and overall output-contingent contracts are hard to enforce and often imperfect (whether in sports or in corporations) (Gibbons, 1992). Two aspects of tournament theory that are very important to my paper and to marathon prize structures in general are that prizes are fixed in advance and that compensation spreads are large enough to induce those at lower levels to put forth more effort. The first point is quite obvious, but the second point is somewhat more subtle. The compensation spreads between finishers in a race should be large enough to make the opportunity cost of dropping one spot high enough that a competitor who is not winning maximally exerts their energy in an attempt to move up one place. In other words, the prize money needs to be increasing at an increasing rate in order to offset the increasing costs and uncertainty that two similar runners face in competing for a prize, or to "buy off" their risk aversion (Rosen, 1986). This effect should, in turn, not negatively impact the exertion of the top performer in a race (by decreasing overall absolute output due to the need only to win only at the margin) because of the relative homogeneity of elite marathon runners. Marathons typically attempt to offset this effect by initiating additional incentive prizes for course and/or world records. This impact from tournament theory also increases the likelihood of an "exciting" race in terms of time lags because the runners will be more closely spaced because in a race that strictly adheres to tournament theory, as there is no added incentive for winning by a large amount of time (one minute) instead of a short amount of time (five seconds).

In their model of compensation schemes, Lazear and Rosen (1981) showed quantitatively why effort depends on the spread of compensation. $C(u)$ is the cost of their investment of effort, the function $g(0)$ is a probability density function measuring the distribution of the error term (accounting for random factors in the tournament), and each W is a monetary prize: $C'(u_i) = (W_1 - W_2)g(0)$ (Lazear & Rosen, 1981). It can be seen that as the difference between W_1 and W_2 grows larger, the investment of effort also increases. Lazear and Rosen point out that this factor can work contrary to established goals for those establishing prize structures because a spread that is too large may induce excessive investment that has negative impacts in other areas. This idea is clearly possible within marathon running because of the high risk of injury during each race and an added prize of $500,000 which vastly skews the compensation spread for that race. Keeping this in mind, it is important for prize structures to maintain an optimum compensation spread, something that may be impossible considering the effects of the final race in each WMM series, with the top two runners of the series having a compensation spread that is multiple times greater than that which is normally encountered. Along these same lines, this idea of compensation spread can help explain why certain companies pay their presidents so much more than they pay their vice presidents (Lazear & Rosen, 1981). If the working environment is thought of as a tournament game in which the salary of the president is the prize, a large compensation differential will induce high amounts of effort among the vice presidents. Ehrenberg and Bognanno found conclusive evidence that the structure of prizes in PGA golf tournaments has a positive impact on player performance, in addition to players being more likely to have better scores in rounds that have a clear impact on their overall winnings (Ehrenberg & Bognanno, 1990).

Theoretical Framework

Most of the theoretical background within this paper comes from tournament theory, where many of the relevant concepts point strongly to the idea that the WMM should have a positive effect on the races involved, both in terms of audience satisfaction and in terms of the athletes' performances (lower times).

Prior to the existence of the WMM, there were still multiple different ways of earning money through marathon racing. Many of the marquee races give out prize money for the top finishers (total number varies from race to race). There are also sometimes bonuses paid out for running under a certain time or for breaking the course or world record. Furthermore, many elite marathoners earn sponsorships from companies like Nike, Asics, and Adidas. There is definitely a trade-off in terms of these methods of earning money because tournament theory would suggest that the best runner will only run fast enough to beat the second-best runner but no faster. If this were the case, bonuses would only be earned if multiple runners were fast enough and willing to break the threshold. They may decide not to maximize their performance because of the possibility of injuries, the lengthened training and recovery time from a more strenuous race, or because the bonus is not a sufficient incentive compared to the overall race prize. For example, the best runner in the race may be more likely to win if s/he chooses a safer strategy as opposed to a riskier strategy that may result in a more impressive (lower) total time, increasing the chances of winning the comparatively larger prize for placing first. On the other hand, better overall times, especially course and world records, are more likely to attract sponsorships from shoe and apparel companies. These conflicting incentives present an interesting scenario where runners receive different pressures and must make decisions in order to maximize their utility within marathon racing.

In order to show what one would expect with respect to the WMM Prize from a rational standpoint, we use a simple model, similar to those used by Lazear and Rosen in their seminal 1981 paper. However, instead of applying tournament theory to a company's compensation structure, we instead apply it to the WMM series prize and analyze the relevant effects on elite marathon races and to the overall efforts expended by runners. We later expand upon this simple model to allow for a change in prize value and see how the prize value affects the runners' utilities with multiple possible races.

<u>2 Homogeneous Runners & 1 Race</u>

We first start out with a simplistic model, which we expand upon later in the paper:

- 2 equal ability (cost) runners & 1 race
- P = Prize value (including sponsorship and prestige value)
- e = effort
- α = Ability of runners (lower alpha corresponds to higher-cost runners)
- Utility = $P(e_1)/(e_1 + e_2) - (\alpha_1)(e_1)$

After taking the first order conditions of the original effort equations where we are trying to maximize the utility function (full methodology in Appendices 1 & 2):

Equation 1: $u_1 = P(e_1)/(e_1 + e_2) - (\alpha_1)(e_1)$

The Nash Equilibrium we arrive at is the following:

Equation 2: $e_1 = (\alpha_2)(P)/(\alpha_1 + \alpha_2)^2$

The effort for player 2 is exactly the same, except that all of the subscripts are reversed. This is essentially a cost-benefit analysis gauging whether a runner would want to enter the given race, with the first multiplicative term being the probability of winning times the prize money (estimating financial returns) minus the costs of racing, which is the runners' effort indexed by their ability[2]. The indirect utility function for player 1 within this model is the following[3]:

Equation 3: $u_1 = P(\alpha_2)^2/(\alpha_2 + \alpha_1)^2$

Comparative Statics (full methodology in Appendix 3) yield

4. $\partial e_1/\partial P = \alpha_2/(\alpha_1 + \alpha_2)^2$ Change in Effort as Prize value changes
5. $\partial e_1/\partial \alpha_2 = P(\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2)^3$ Change in Effort as Opponent's ability changes
6. $\partial e_1/\partial \alpha_1 = P(-2\alpha_2)/(\alpha_1 + \alpha_2)^3$ Change in Effort as Own ability changes

When we look at player one's effort dependent on the opponent's ability (the derivative of e_1 with respect to α_2), we get in the numerator: $P(\alpha_1^2 - \alpha_2^2)$. All else equal, this means that effort is maximized when the two have the same ability, $\alpha_1 = \alpha_2$. When player 1 has lower ability (higher

[2] We at first tried to use a model with quadratic costs, but the Nash Equilibria did not make intuitive sense, and when we used this model, the numbers are much clearer and easier to understand. Although it seems strange to say that effort costs are perfectly linear, they are definitely increasing over time and the model maintains all of the meaningful relationships involved.

[3] In all cases of maximizing utility, the Second Order Conditions all were checked in order to assure the extrema is a maximum.

costs), $\alpha_1 > \alpha_2$, Player 1's effort is higher, and if $\alpha_1 < \alpha_2$, his/her effort would be lower, indicating that a higher ability runner would exert less energy in racing.

The Best Response Function (full methodology in Appendix 4) for Player 1 is

Equation 7: $$e_1 = e_2 + \sqrt{(Pe_2/\alpha_1)}$$

We got this solution by using the quadratic formula to solve for the Nash equilibrium, so there is obviously also a negative square root. The positive square root is the one that makes more sense as a response function because it is increasing in P instead of decreasing, which means that as the prize money increases, Player 1's effort will increase as well. However, the function including the negative square root $(e_1 = e_2 - \sqrt{(Pe_2/\alpha_1)})$ may make more sense, as it is decreasing in own cost, while the other function increases. This relationship is less clear in a real-life scenario because low-ability (high-cost) runners may decide to increase effort in order to increase their overall chances of winning the prize money. Because there are legitimate arguments possible for both increasing and decreasing own costs, but the function should definitely be increasing in prize value, we chose the function with the positive square root. In addition, the function we chose has a more noticeable impact of a change in e_2 than the other function because the negative mitigates the effect of a change in e_2.

Case of 4 Runners (2 Low-ability, 2 High-ability) & 2 Races

There are many different ways that the 4 runners can divide themselves among the two races, several of which (but not all) are of interest to us in this paper, with the less relevant cases included in Appendix 5. The distinction between races containing two high-ability runners or one high-ability and one low-ability is most crucial, so they are the two comparisons we make in the main body of the paper. All else equal, it should be the case within the game theory model that a race with one each of low-ability and high-ability runners would be a stable equilibrium, with the high-ability runner obtaining higher utility in this race than in a race of two high-ability runners. This basic premise is the reason that the WMM organizers introduced such a high prize, to make it so the high-ability runners would have sufficient reason to deviate from the stable equilibrium of heterogeneous races; namely, that the series prize would push the two high-ability runners' expected utility to the point where it superseded the expected utility in the heterogeneous game with a sufficiently lower total prize value. It is also important to note, as Lazear and Rosen (1981) did, that tournament structures with participants of heterogeneous abilities require signaling and credentials of some kind, which marathon organizers usually accept as previous marathon times or other similar metrics that are highly correlated with marathon times (half-marathon, 10k, etc.). The following scenarios are created in much the same way as the standard example above, but there are a few key differences. The low-ability runners have subscript L for their parameters (effort, e and ability α), while the high-ability runners have subscript H for their parameters. This point is reflected in the idea that α_L is strictly greater than α_H (because for lower ability runners, effort comes at a higher cost, so low-ability runners have a higher value for their cost parameter, which is α_L multiplied by effort). On the other hand, effort levels for the runners are not related to each other by definition like the ability/cost parameters are; instead, the subscripts are used for clarity and representative purposes.

<u>One High-ability and one Low-ability in both Races:</u>

We start by maximizing utility functions $u_H = P(e_H/(e_H + e_L)) - e_H\alpha_H$ and $u_L = P(e_L/(e_H + e_L)) - e_L\alpha_L$. The first order conditions, taking the derivative with respect to each runner's effort, are the following: $0 = \partial u_H/\partial e_H = P(e_L/(e_H + e_L)^2) - \alpha_H$ and $0 = \partial u_L/\partial e_L = P(e_H/(e_H + e_L)^2) - \alpha_L$. Upon solving for the equilibrium efforts, the Nash Equilibria of efforts are $e_H = P(\alpha_L)/(\alpha_H + \alpha_L)^2$ and $e_L = P(\alpha_H)/(\alpha_H + \alpha_L)^2$, which gives indirect utility functions of $u_H = P(\alpha_L^2)/ (\alpha_H + \alpha_L)^2$ and u_L

$= P(\alpha_H^{2})/ (\alpha_H + \alpha_L)^2$. Each race would have its own race-specific prize value (so the other situations have a P_1 value and a P_2 value), but because of the symmetric races, it can be avoided in this case.

Comparative Statics (methodology in Appendix 3) yield

8. $\partial e_H/\partial P = (\alpha_L)/(\alpha_H + \alpha_L)^2$ Change in Effort as Prize value changes

9. $\partial e_L/\partial P = (\alpha_H)/(\alpha_H + \alpha_L)^2$`

10. $\partial e_H/\partial \alpha_L = P(\alpha_H - \alpha_L)/(\alpha_H + \alpha_L)^3$ Change in Effort as Opponent's ability changes

11. $\partial e_L/\partial \alpha_H = P(\alpha_L - \alpha_H)/(\alpha_H + \alpha_L)^3$

12. $\partial e_H/\partial \alpha_H = P(-2\alpha_L)/(\alpha_H + \alpha_L)^3$ Change in Effort as Own ability changes

13. $\partial e_L/\partial \alpha_L = P(-2\alpha_H)/(\alpha_H + \alpha_L)^3$

In this scenario, we see that the high-ability runner will in equilibrium exert a higher amount of effort (with both effort functions having the same denominator but $P(\alpha_L)$ in the e_H numerator being larger than $P(\alpha_H)$ in the e_L numerator) in order to have higher utility gained from a greater chance of winning the race. It is easy to see this being the case in actuality because a more highly-skilled (lower-cost) runner may demoralize the lower-ability runner into not working as hard because he sees his probability of winning as very low and working less hard will minimize the costs associated with racing. In these heterogeneous races, total equilibrium efforts are lower than in homogeneous races because of this distinct advantage that some runners have over others.

<u>Two High-ability in one Race and two Low-ability in one Race (methodology shown in Appendices 1 & 2):</u>

This maximization problem is the exact same set-up as in the homogeneous case with runners 1 & 2 (the first example given), so the indirect utilities are just changed from 1 and 2 to H_1 and H_2 and L_1 and L_2, respectively. The equilibrium efforts are calculated in the exact fashion as in the first example, with the two high-ability runners exerting $e_H = P_1/(4\alpha_H)$ and the two low-ability runners exerting $e_L = P_2/(4\alpha_L)$. Because the high- and low-ability runners are homogeneous within their respective races, these games give indirect utilities of $u_H = P_1(\alpha_H)^2/(\alpha_H + \alpha_H)^2$, which simplifies to $u_H = P_1/4$, and the same logic holds true for the low-ability runners, with indirect utility being $u_L = P_2/4$ because of the same mathematical simplification.

Comparative Statics (methodology in Appendix 3) yield

14. $\partial e_H/\partial P_1 = 1/(4\alpha_H)$ Change in Effort as Prize value changes

15. $\partial e_L/\partial P_2 = 1/(4\alpha_L)$

16. $\partial e_H/\partial \alpha_L$ = N/A because runners are in separate races

17. $\partial e_L/\partial \alpha_H$ = N/A because runners are in separate races

18. $\partial e_H/\partial \alpha_H = -P_1/4(\alpha_H)^2$ Change in Effort as Own ability changes

19. $\partial e_L/\partial \alpha_L = -P_2/4(\alpha_L)^2$

We see that efforts for both runners are increasing with respect to prize value for all values in our domain (14 & 15). The runner's effort is not related to the opposite-ability runner's ability level because they are in separate races. We also see that effort is negatively related to own-cost, with less effort exerted by runners with lower ability levels (18 & 19).

Discussion of Theory

This section is devoted to discussing the differences within the above scenarios and what we would expect in equilibrium under certain triggers, such as a change in prize value of one of the races within the scenario. There are several different scenarios that we will compare in order to figure out which race set-ups are equilibria when the prize values are the same and when players move simultaneously. Because this set-up is not realistic, the simultaneous move games are discussed in Appendix 6. We will now introduce high-ability players being able to move first to reflect the fact that the most elite marathoners often do have higher priority in entering races.

First-Mover Advantage by High-Ability Runners

If we make a reasonable assertion that high-ability runners have a first-mover advantage, this eliminates the two-person homogeneous races as an equilibrium. This is actually more realistic than allowing simultaneous entry by all runners because in reality, most races give preferences to the best runners in choosing to run their races. As previously mentioned, many races actually pay the best runners an appearance fee. Even though many sub-elites (low-ability runners in our model) will get preferential treatment from races compared to casual runners, elites will usually get preferential treatment compared to people just a few minutes slower than they are. This set-up of sequential entry and equal prize values across races leads to a four-player game where the only stable equilibrium is one with each race having one high- and one low-ability runner.

Implementation of the WMM Prize

The driving force behind this theoretical framework is the idea that the organizers of the WMM can provide incentives to the best marathoners in the world to run in their races. The fact that our model with identical prizes produced an equilibrium with heterogeneous-ability runners under the most realistic conditions (allowing better runners to choose first) is troublesome for a race that desires the best runners (all high-ability in the same race). This general idea is likely a key driver behind the choice to increase the prize level. At this point, the key question is whether a prize increase will actually convince high-ability runners to race against each other instead of exerting less effort and individually winning each of the prizes from the races they enter. A critical follow-up to that question is how costly this prize increase must be in order to induce an equilibrium with both high-ability runners in the same race (for our simplistic model, we can think of one race as the WMM series and the other race as all the rest of the possible races a runner may choose to participate in). It should be easy to see that the WMM has no problem with more competitors in their race; it is just focused on getting the best runners. In other words, the WMM race is best off in the scenario with all four runners selecting into it, but the overall strategy of increasing prize value can be attributed to an attempt at having either HH, HHL, or HHLL as the participants in the WMM race. The WMM is better off with more runners because it increases the effort of the high-ability runners, as well as likely increasing in publicity and other benefits that accompany larger races. These races often have bigger sponsors and are better economically for their cities due to the number of entrants eating in restaurants, staying in hotels, and visiting the city in general[4].

Even though the WMM race would like to have as many participants as feasible, it is obviously easiest to induce an equilibrium of HH and LL due to the utility functions we have previously derived, as well as the intuition that accompanies this logic. It should be easier to get two runners to compete against each other for one prize than it is three or four, no matter what the ability levels are. At a minimum, the WMM should want to implement the prize value so that high-ability runners are marginally better off choosing to run against each other than splitting into the two different races (with the WMM race being race two). The WMM race would have prize value equal to P_2, where

Equation 20: $P_1(\alpha_L^2)/(\alpha_H + \alpha_L)^2 \quad < \quad P_2/4$

indirect utility for high-ability runners in HL < indirect utility for high-ability runners in HH

[4] Marathons will often have a required confirmation form where runners have to fill out information about hotels, rental cars, flights, etc. in order to track these factors that contribute to the city's economy due to the marathon.

which is equivalent to

$$P_2/P_1 > 4(\alpha_L^{2})/(\alpha_H + \alpha_L)^2$$

The ratio of WMM prize value to the other race's prize must get increasingly large as the difference in ability levels between the elites (high-ability runners) and sub-elites (low-ability runners) increases. Under a scenario where both runner types are of identical abilities ($\alpha_H = \alpha_L$), they will already essentially be split up into the ideal two-runner scenario, but as the ability gap increases between the runner types, it makes intuitive sense that the race organizers will have to offer more lucrative prizes in order to maintain the same race structure in an attempt to offset the runners' added cost of having to exert more effort to race against someone very similar (or identical) to one's own ability level.

If the WMM organizers truly wanted to have the ideal race in this scenario, they would have to make it so both low-ability runners would want to switch into their race by creating a prize that made the following inequality hold (Appendix 6, Equation 41): $P_1 < P_2(\alpha_L - 2\alpha_H)^2/(2\alpha_H + 2\alpha_L)^2$. This is certainly possible, and a prize worth \$500,000 in cash alone is definitely not something to ignore. The main problem with this race set-up is that the difference in ability will likely make entering this race prohibitive due to the almost impossibility of a sub-elite (low-ability) runner actually winning the overall prize series. For the actual series, the winner each year has been a runner who would certainly be considered high-ability in our model, with many Olympic medals among their collective accomplishments (WMM Website, 2011). The male winner of the past two series was Sammy Wanjiru who was noted for his 1st place finish at the Beijing Olympics, as well as two wins at the Chicago Marathon and one in London.

The most important question once we have a new prize value involved is whether the new equilibria are in fact stable. We know that if that ratio of prize values from Equation 41 holds then the two high-ability runners self-select into the same race, but if it is possible that a low-ability runner selects into it, then the indirect utility function for the high-ability runners will change as a result of its being a different race set-up. It should be noted that this equilibrium being unstable is only a problem because the WMM race organizers should want to invest as little money as possible in order to have both high-ability runners in the same race. If the prize value is high enough at the margin to induce a low-ability runner to deviate, then the indirect utility for the high-ability runners will no longer be sufficient to have both of them in the same race (because if the prize ratio were just at the margin, it would definitely no longer be a strong enough incentive once a third runner joined the race). For the minimum condition in the WMM, the prize ratio they need to implement is Equation 20: $P_2/P_1 > 4(\alpha_L^2)/(\alpha_H + \alpha_L)^2$ or $P_2 > 4P_1(\alpha_L^2)/(\alpha_H + \alpha_L)^2$. To determine whether this is a stable equilibrium with the new prize values, we compare the indirect utilities for the low-ability runner in the HHL race with the prize value for P_2 and for the low-ability runner in the LL race with the prize value for P_1. As long as his indirect utility for race LL is higher than the indirect utility in the other race, we have a stable equilibrium. As mentioned above, the indirect utility for low-ability runners in race LL is $P_1/4$ and the indirect utility for low-ability runners in race HHL is $P_2(2\alpha_H - \alpha_L)^2/(2\alpha_H + \alpha_L)^2$. After substituting in P_2 in the inequality of indirect utilities, we get the following inequality:

Equation 21: $$P_1/4 > (4P_1(\alpha_L^2)(2\alpha_H - \alpha_L)^2)/((\alpha_H + \alpha_L)^2(2\alpha_H + \alpha_L)^2)$$

which simplifies to

$$1 > (64\alpha_L^2\alpha_H^2 - 64\alpha_L^3\alpha_H + 16\alpha_L^4)/(4\alpha_H^4 + 12\alpha_H^3\alpha_L + 13\alpha_L^2\alpha_H^2 + 6\,\alpha_L^3\alpha_H + \alpha_L^4).$$

If we index the high-ability runner's ability to be equal to 1 and only analyze the changes in the low-ability runner's ability level relative to the high-ability runner, we get the following inequality:

$$1 > (64\alpha_L^2 - 64\alpha_L^3 + 16\alpha_L^4)/(4 + 12\alpha_L + 13\alpha_L^2 + 6\,\alpha_L^3 + \alpha_L^4)$$

which holds when $\alpha_H = 1$ with our domain of $\alpha_L = (1,2)$, which we restrict because the costs of running will not be multiple times greater for sub-elite marathoners (low-ability) than for elite marathoners (high-ability). It should be obvious that because HHL will not be an equilibrium, HHLL will also not be an equilibrium because the prize value increase would need to be even larger to induce both runners to switch, rather than just one.

Now that we know that we have established certain values of P_2 that create a stable equilibrium with the minimum WMM condition met (both high-ability runners entering the same race), it is important to see whether certain prize values will result in low-ability runners also entering. The WMM prize value will have to be higher to prevent the low-ability runners from deviating from these races because of their lower chance of winning, so the trade-off for the low-ability runners is the constraint that we must consider for the prize value required. For the race with two high-ability and one low-ability, we compare the indirect utilities from the HHL race and the LL race because those would be the potential races the low-ability runner would be choosing between. The ratio of P_2/P_1 for this to be true must be greater than $(\alpha_L + 2\alpha_H)^2/(4(2\alpha_H - \alpha_L)^2)$. In order for the runners to select into the HHLL race, we compare the indirect utilities for the low-ability runner in the HHLL race and the L race. For the HHLL race to be a stable equilibrium, it must be the case that the ratio of P_2/P_1 be greater than $(2\alpha_L + 2\alpha_H)^2/(\alpha_L - 2\alpha_H)^2$. These ratios are shown with certain values in Table 4, with Table 3 as the same basic idea except for when the high-ability runners will be satisfied in their current race. Specific analysis of these ratios is below each table for ease of comparison with hard numbers.

TABLE 1: Indirect Utilities for Each Runner in Each Race Scenario	**1 High-Ability, 1 Low-Ability in each race (stable equilibrium with equal prize values)**		**2 High-Ability, 1 Low-Ability in one race**		**2 Low-Ability, 1 High-Ability in one race**		**2 High-Ability in one race, 2 Low-Ability in one race (may be stable depending on parameter values)**		**2 High-Ability, 2 Low-Ability in one race (Will not observe when P is equal)**	
Race Set-up	**HL**	**HL**	**HHL**	**L**	**HLL**	**H**	**HH**	**LL**	**HHLL**	**0**
Low-ability Indirect Utility	$P(\alpha_H^2)/(\alpha_H + \alpha_L)^2$	$P(\alpha_H^2)/(\alpha_H + \alpha_L)^2$	$P(2\alpha_H - \alpha_L)^2/(2\alpha_H + \alpha_L)^2$	P	$P(\alpha_H)^2/(\alpha_H + 2\alpha_L)^2$	N/A	N/A	P/4	$P(\alpha_L - 2\alpha_H)^2/(2\alpha_H + 2\alpha_L)^2$	N/A
High-ability Indirect Utility	$P(\alpha_L^2)/(\alpha_H + \alpha_L)^2$	$P(\alpha_L^2)/(\alpha_H + \alpha_L)^2$	$P(\alpha_L)^2/(2\alpha_H + \alpha_L)^2$	N/A	$P(2\alpha_L - \alpha_H)^2/(\alpha_H + 2\alpha_L)^2$	P	P/4	N/A	$P(\alpha_H - 2\alpha_L)^2/(2\alpha_H + 2\alpha_L)^2$	N/A

TABLE 2: Efforts for Each Runner in Each Race Scenario	**1 High-Ability, 1 Low-Ability in each race (stable equilibrium with equal prize values)**		**2 High-Ability, 1 Low-Ability in one race**		**2 Low-Ability, 1 High-Ability in one race**		**2 High-Ability in one race, 2 Low-Ability in one race (may be stable depending on parameter values)**		**2 High-Ability, 2 Low-Ability in one race (will not observe when P is equal)**	
Race Set-up	**HL**	**HL**	**HHL**	**L**	**HLL**	**H**	**HH**	**LL**	**HHLL**	**0**
Low-ability Effort	$P(\alpha_H)/(\alpha_H+\alpha_L)^2$	$P(\alpha_H)/(\alpha_H+\alpha_L)^2$	$P_1(4\alpha_H - 2\alpha_L)/(2\alpha_H+\alpha_L)^2$	0	$P_1(2\alpha_H)/(\alpha_H+2\alpha_L)^2$	N/A	N/A	$P_2/4$	$P_1(6\alpha_H - 3\alpha_L)/(2\alpha_H+2\alpha_L)^2$	N/A
High-ability Effort	$P(\alpha_L)/(\alpha_H+\alpha_L)^2$	$P(\alpha_L)/(\alpha_H+\alpha_L)^2$	$P_1(2\alpha_L)/(2\alpha_H+\alpha_L)^2$	N/A	$P_1(4\alpha_L - 2\alpha_H)/(\alpha_H+2\alpha_L)^2$	0	$P_1/4$	N/A	$P_1(6\alpha_L - 3\alpha_H)/(2\alpha_H+2\alpha_L)^2$	N/A

Table 3: Ratio of Prize Values (P_2/P_1) For High-Ability Runners Staying in their Race

alpha (L)	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**
HH vs HL	1.0	1.1	1.2	1.3	1.4	1.4	1.5	1.6	1.7	1.7
HHL vs. HL	2.3	2.2	2.1	2.1	2.0	2.0	1.9	1.9	1.8	1.8
HHLL vs. H	16.0	12.3	9.9	8.3	7.1	6.3	5.6	5.1	4.6	4.3

The above table shows the ratios of prize values (P_2/P_1) that will result in the high-ability runners staying in each race for the various scenarios (these prize values may not be large enough to provide an incentive for low-ability runners to stay in the race). In the second row, we see the steadily increasing prize ratio of $P_2/P_1 > 4\alpha_L^2/(\alpha_L + \alpha_H)^2$. This prize value continually increases with respect to the divergence in ability levels because the high-ability runner is more and more likely to deviate when he is more likely to win the other race and must be compensated through the prize value in order not to do so. In the second and third rows, we see a decreasing pattern of prize value as the ability levels diverge, which may at first seem unlikely. However, this is due to the fact that low-ability runners are selecting into the high prize value race, and as the ability levels diverge, those runners no longer are as significant a hurdle in collecting the prize value. When the runners are virtually identical, the prize value must be significantly higher, as we would expect, but as they get worse, the high-ability runners care less about them being in their race, and in our model, this effect dominates the need to be compensated in order not to deviate. The ratios for the second and third rows are $P_2/P_1 > (\alpha_H + 2\alpha_L)^2/(\alpha_L + \alpha_H)^2$ and $P_2/P_1 > (2\alpha_L + 2\alpha_H)^2/(\alpha_H - 2\alpha_L)^2$, respectively.

Table 4: Ratio of Prize Values (P_2/P_1) For Low-Ability Runners Staying in their Race

alpha (L)	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**1.6**	**1.7**	**1.8**	**1.9**
HH vs HL	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
HHL vs. LL	2.3	3.0	4.0	5.6	8.0	12.3	20.3	38.0	90.3	380.3
HHLL vs. L	16.0	21.8	30.3	43.2	64.0	100.0	169.0	324.0	784.0	3364.0

The ratios in this table are the prize value ratios (P_2/P_1) that represent the values where the low-ability runners would be indifferent between the races listed in the far left column. As is expected, the low-ability runners require enormous prizes that increase quickly as the ability levels diverge. As the values approach the upper end of our domain (α_L: (1,2)), the low-ability runners require the ratio to be infinite. Although this prize ratio is impossible, it indicates that a runner whose costs are twice as high as another's would be unlikely to enter the same race provided that he desired to win the prize value.

Data

We have collected data on the finishing times and ages for the top 20 finishers of both genders for all of the races involved in the WMM series, as well as for select other races that also have significant amounts of prize money and are similar in structure to the WMM races. The list of non-WMM races we have considered are the following: Houston, Honolulu, Los Angeles, Rock 'n Roll San Diego, and Grandma's (Duluth, MN) marathons, which are all races with large first place prize values and elite competitors. For each race, we consider data from between 2000 and 2011 (with slight exceptions like not including certain races for 2011 because they occur late in the fall). Data for the Olympics and the World Championships are only present in years that these races occur. Our data include the prize structures for races, overall finish times for races, age of competitors, humidity at races, and temperatures at races.

There are unfortunately a few drawbacks within the data. First, the age data is not complete, so certain races and individuals had to be dropped, though there was nothing strikingly different about these races or individuals that would indicate some specific reason for its non-existence. If we drop the age variable to include more observations, the regressions do not change in meaningful ways. The number of observations (especially for the time lag regressions) is unfortunately not as high as desired, but nothing can be done about this, as the prize has only been in existence for five years.

Empirical Specification

There are many different regressions that will need to be run to measure the variety of changes that could have taken place over the past five years with the implementation of the WMM. These include changes in World Records or World's Fastest Times, Overall Faster Race Times for the top finishers, Time Lags for the top finishers in Races, and Changes in the Times of the 6th-20th finishers in WMM races. The last two categories are included in Appendix 7. One point that requires mentioning about the regressions is that they do not incorporate the dynamic aspect of the WMM Series, so there is a section about these issues after discussion of the regressions. It is certainly the case that some of the top runners have lower incentives during some races because they came into the Series with such large leads from the previous year, for which the static regressions do not account.

World Records/World's Fastest Times

From the theoretical model, we predict that one of the quantifiable benefits of the WMM Prize is that it will increase effort in the high-ability runners and increase the number of high-ability runners in these races, so that world records are more likely to occur in these races. This effect should be seen in two different ways. We would expect there to be more world records being produced because the model predicts an increase in overall effort in the homogeneous-ability case (with multiple high-ability runners in the same race), and we also anticipate that if there are more world records being set, they should occur in the races with the WMM prize. The runners will have strong incentives to win these races, due not only to the high prize attached to the WMM Prize, but also because they get 25 points for a first place WMM finish instead of just 15 for second place. This makes a minor increase in effort to get first in one race a significantly more attractive strategy based on how much effort they would have had to exert in order to be in second place. Below are charts detailing the overall trend in world record marathon times for men:

Table 5: World Record Times in Men's Marathoning

Time	Name	Date	Race
2:09:36	Derek Clayton	December 3, 1967	Fukuoka Marathon
2:09:28	Ron Hill	July 23, 1970	Edinburgh, Scotland
2:09:12	Ian Thompson	January 31, 1974	Christchurch Marathon
2:09:06	Shigeru So	February 5, 1978	Beppu-Ōita Marathon
2:09:01	Gerard Nijboer	April 26, 1980	Amsterdam Marathon
2:08:18	Robert De Castella	December 6, 1981	Fukuoka Marathon
2:08:05	Steve Jones	October 21, 1984	Chicago Marathon
2:07:12	Carlos Lopes	April 20, 1985	Rotterdam Marathon
2:06:50	Belayneh Dinsamo	April 17, 1988	Rotterdam Marathon
2:06:05	Ronaldo da Costa	September 20, 1998	Berlin Marathon
2:05:42	Khalid Khannouchi	October 24, 1999	Chicago Marathon
2:05:38	Khalid Khannouchi	April 14, 2002	London Marathon
2:04:55	Paul Tergat	September 28, 2003	Berlin Marathon
	WMM Starts		
2:04:26	Haile Gebrselassie	September 30, 2007	Berlin Marathon
2:03:59	Haile Gebrselassie	September 28, 2008	Berlin Marathon
2:03:02*	Geoffrey Mutai	April 18, 2011	Boston Marathon
2:03:38	Patrick Makau	September 25, 2011	Berlin Marathon

*Not an official world record (known as world's fastest marathon time)

Ever since the marathon world record dropped below 2:10:00 (roughly where our low-ability runners anticipate finishing) in 1967, a new world record has been set 16 times. Of those 16, 13 happened prior to the WMM and 3 have happened in the 5 years since the implementation of the WMM. The important issue is whether or not world records are happening more frequently in the WMM races. Although this is not statistically significant due to the low sample size, only 5 of the 13 world records prior to 2006 occurred in the future WMM races and all 3 in the post-WMM world have taken place at those races, without including a new world's fastest time in the 2011 Boston Marathon. Apart from acknowledging that there has been no change, which itself is significant, we do not consider the Marathon World Records for women. The actual time has not changed since 2003, so there has clearly been no discernible impact from the implementation of the WMM on this aspect of the women's half of the sport.

Table 6: Chart of World Records Since 1900

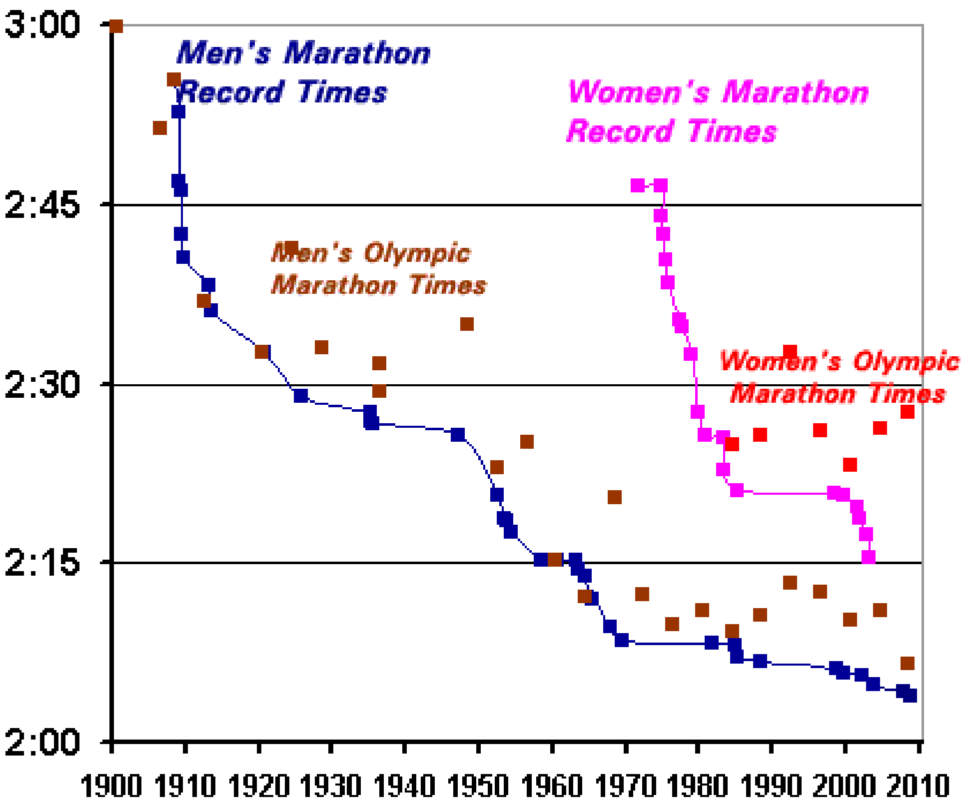

Figure 1 - (WMM Website, 2011)

After looking at this graph, it is clear that world records do not decrease linearly, as one person often drops a significant amount of time from the previous world record and afterwards there may be a prolonged time when no one improves upon that time. This is especially the case with the women's marathon, perhaps because women's marathoning is much more heterogeneous than men's marathoning. For example, the three fastest women's times have all been performed by one woman, Paula Radcliffe. It is also the case that the women's world record times have been under controversy lately because of disagreements over whether times completed in races that involve men should be allowed (Leicester, 2011). The strange thing about this new rule is that it attempts to prevent men pacing women for new record times, but men's marathons have used "rabbits" (runners paid not to win but to establish a good pace and lead the race) for a long time, and times run in these races are eligible as world records. In fact, this disagreement resulted in the former world record being demoted to "world's best" time and the new world record was established as, coincidentally, Paula Radcliffe's third best time of 2:17:42 (her best two races both involved men in the race). It is a combination of the relative dominance of Paula Radcliffe in the early 2000s and the relatively young sport of women's marathoning in general that result in such a different graph of world record times. For these reasons (especially the strength of Radcliffe's world's best time of 2:15:25, which no woman has come within three minutes of), it is not surprising that there have not been new world records established in women's marathoning after the implementation of the WMM Prize. We chose to look at the world record times starting in 1970 because this is when marathon times were much less volatile and the sport was much more similar to how it exists today compared to earlier in the 1900s.

In terms of viewing increased effort across all the races, in 2011 alone, new course records were set in all five WMM races, as well as Houston and Los Angeles. Although this shows nothing conclusive or statistically about the WMM prize, it does indicate that times are dropping and that runners may be exerting more effort overall.

Faster Overall Race

If the results in the theoretical model hold true, we should see an increase in effort in both WMM and the Non-WMM races, as the homogeneous races encourage runners to exert more effort because they are racing against others with more similar ability levels. It is the case that with a large prize value, we expect to see the homogeneous set-up become the equilibrium given our theoretical model. We are most interested in seeing if the WMM races have a decrease in average times for the top five finishers in the years after 2006, which would suggest that the runners are dividing themselves among the two races, as well as potentially exerting more effort within those races.

We use a regression similar to that used by Ehrenberg and Bognanno (1990): ours uTo their approach, we add course and runner fixed effects to measure the effects on the overall race time of the top five finishers (for individuals *i* in race *j* at time *t*).

Equation 22:

$$t_{ijt} = a_0 + a_1 2006Dummy_{jt} + a_2 Year_{jt} + a_3 TempDev_{jt} + a_4 Humidity_{jt} + a_5 Age_{ijt} + a_6 Age^2_{ijt} + v_{ijt}$$

In the regression, t_{ijt} is the individual's final finishing time (the lower the better), a_0 is a constant, $2006Dummy_{jt}$ is a dummy for whether the WMM series was in effect at the time of the race, $Year_{jt}$ is the year that each race took place in, $TempDev_{jt}$ is the absolute value of the temperature deviation in degrees Fahrenheit from 57.7 degrees for the maximum temperature on the day of each race[5], $Humidity_{jt}$ is the maximum percent humidity on the day of each race, Age_{ijt} is the individual runner's age on race day, Age^2_{ijt} is the individual's age squared on race day, and v_{ijt} is an error term. There is no accepted best temperature for a marathon, but these races are designed to have fast times run at them, with many of their financials and related

[5] 57.7 is the temperature used for the benchmark of TempDev because it is the average of the temperatures in each WMM race's city during the time of the race.

incentive prizes stating so. Because of this, their average temperature is likely a good proxy for an optimal marathon temperature. The $Year_{jt}$ variable is used as a way of controlling for the fact that marathon times tend to decrease over time, simply because people keep getting faster. The course fixed effects control for individual course difficulty levels, including such things as prize values (including sponsorships, non-monetary prizes, and monetary prizes), terrain, topography, and sharp turns. Furthermore, the individual runner fixed effects combined with the age and age^2 variables control for the ability levels of runners over time, with the natural rise and fall of a marathoner's running career, not to mention appearance fees that certain runners receive from races. If the tournament theory predictions hold for this set of data, the coefficients for $2006Dummy_{jt}$ should be negative, indicating that the runners' times decrease if the race occurs after the creation of the WMM.

It must be noted that the inter-country competitions, such as the Olympics, fuel nationalistic pride, so they have higher prestige and sponsorship opportunities for the runners, but these factors should be included in the course fixed effects. However, the governments of certain countries provide ample support to the athletes who win medals in these races, including lifetime leases on apartments, college tuition, and sometimes comparable cash prizes to other major marathons (*New York Times*, 2004). These lucrative contracts are controlled for through the fixed effects for race name, provided that the benefits are constant across time for each race, which is likely the case between 2000-2011. Furthermore, the fixed effects do not account for changes in the course over time, but any changes to these courses are likely insignificant as the course records are important historical metrics and would be invalidated with a significant change to the course.

The results of the regressions are in the tables below, and it is again the case that the variable we are most concerned with is the 2006 dummy:

Table 7: Individual Top 5 Time Regressions with WMM Races Separated

	WMM Women top 5	**Non-WMM Women top 5**	**WMM Men top 5**	**Non-WMM Men top 5**
2006 Dummy	.0006016	.0010551	-.0002454	.0004777
Year	.0001067	-.0003316	-.0002323	-.0004003
TempDev	.0001005***	-.0000304	.0000918***	.00000301
Humidity	-.00000563	.0000244	-.00000506	.0000314*
Age	-.0003621	-.0017005**	-.0007843*	-.0005082
Age^2	.00000849	.0000276**	.0000173**	.0000132
Constant	-.1111415	.8058374*	.5602714*	.9045184
Observations	264	226	315	228
Adj. R^2	.526	.792	.255	.865
Significance level:		**.1*	*** .05*	**** .01*

The variables are all either insignificant or significant in the ways that we would expect. None of the variables concerning the effects of time are significant, so we lack conclusive evidence supporting the idea that the WMM could be inducing faster overall times. $TempDev_{jt}$ is positive and highly significant for the WMM races and not significant for the Non-WMM races, indicating that as temperatures deviate from 57.7 degrees, race times get worse. This effect is intuitively comfortable because sub-optimal temperatures have large impacts on fatigue and overall ability to run a fast race. The Age^2 variable is represented as we would expect, with age being correlated with times in a quadratic relationship indicating that there is an optimal age for each runner and as they deviate in either direction, the times get worse.

It is understandable that there would be no statistically significant decline in overall times for a number of reasons relating to the theoretical foundation. First of all, the theoretical model only includes four runners splitting between two races, and the reality is much more complicated. It is reasonable to think that prior to the WMM prize, most, if not all, of the marathoners were already maximally exerting effort, especially considering that there were already hundreds of thousands of dollars in prize value already given out. Additionally, many of the runners competing must acknowledge that they have no chance of winning the overall WMM prize, so their results would not necessarily be affected by the WMM prize's implementation. It is for these reasons that the World Record setting times and Time Lag regressions are likely more closely impacted by the actual implementation of the WMM prize (because the WMM prize is only awarded to one runner each year). The theoretical model also shows the impacts of the WMM prize on the top two runners in each race more clearly.

Faster Overall Race WMM and Non-WMM Combined Regression

In order to control for the difference between marathoners getting faster overall and the impact of the WMM dummy, we added an interaction dummy "WMM Dummy Interact" into the previous regression and combined the two data sets, in order to compare directly the difference between the WMM and Non-WMM races. This dummy is a 0 if the race was either a non-WMM race or if it took place prior to 2006 and is a 1 if it was both a WMM race and took place after the implementation of the WMM. The coefficient of this variable will indicate if the effect of the WMM is what we should theoretically expect.

Table 8: Individual Top 5 Time Regressions with WMM and Non-WMM Races Combined

	Women top 5 Combined	Men top 5 Combined
2006 Dummy	-.0009635	-.0000467
WMM Dummy Interact	.00219***	.0003915
Year	.000128	-.0003256***
TempDev	.0000613***	.0000755***
Humidity	.0000113	.00000779
Age	-.0008968*	-.0006408
Age^2	.0000147**	.0000148**
Constant	-.1286439	.7438773***
Observations	490	542
Adj. R^2	.839	.8785
*Significance level: *.1*	***.05*	****.01*

The 2006 Dummy indicates that times are decreasing (though lacking statistical significance), but the Interaction Dummy for men is not significant and for women is actually positive, indicating that the WMM implementation is having a worsening impact on women's racing times. This statistic shows the exact opposite relationship that we would expect due to an increase in prize value. As we discussed previously, it seems that marathon runners were already exerting themselves to their fullest, and increased prize values may have no quantifiable effect on marathon times in general. Furthermore, the Year variable is highly significant and negative for men, which indicates that the overall decrease in men's marathon times is more closely tied to men getting faster overall, rather than because of an increased prize value. In sum, this regression indicates that there is not solid evidence to say that marathon times are getting better as a result of the WMM prize, and any improvement in the overall times seems more tied to the general increase in ability and performance that we see over time.

The regressions regarding Time-Lag and 6th-20th place finishers are included in Appendix 7 because they are less related to the main points of this discussion paper but are included for the sake of completeness. One aspect important to the WMM that is not taken into account in the regressions is the question of whether the timing of each race matters in terms of who becomes the overall champion. In other words, at what point during the two-year series is the champion essentially determined? If two runners had accumulated a similar amount of points, we would expect the races later in the series (Berlin, Chicago, and New York) to be extremely competitive between these two runners because the $500,000 is a more salient award that may change incentives from what the runners would have previously done. This is relatively simple to view just by examining the race results from each of the series in the past. We will now examine each of the series prizes to see whether this could have had a strong impact. We assume that runners will only be able to do one Fall marathon (unless they choose to do the risky Berlin/New York City combination) and one Spring marathon in each year, as more than this will likely lead to underperformance or injury (No one is likely to do both the Chicago and New York City marathons in order to win the overall prize – in actuality, it is unlikely that anyone is capable of winning two world-class marathons within such a short time period). The below table summarizes the results from each series, with the point differential going into New York listed alongside the winner's name. In each case, the winner of the overall series was winning going into the last race, except the 2011 men's series, so the cell representing that point differential has a negative number:

Table 11: WMM Point Lead for Series Winner after Penultimate Race

	Female Winner	Point Differential Before Last Race	Male Winner	Point Differential Before Last Race
2007	Gete Wami	10	Robert Kipkoech Cheruiyot	30
2008	Gete Wami/Irina Mikitenko	0	Martin Lel	21
2009	Irina Mikitenko	50	Sammy Wanjiru	40
2010	Irina Mikitenko	44	Sammy Wanjiru	10
2011	Liliya Shobukhova	30	Emmanuel Mutai	(-5)

2007 Series – Winners: Gete Wami and Robert Kipkoech Cheruiyot

For the women, Wami had 40 points heading into the Fall races, trailing Jelena Prokopcuka by 15. In fact, once Wami had won Berlin in September, she was leading Prokopcuka by 10, and they were both registered to run New York City in just over a month. Wami miraculously edged out Prokopcuka by one place to solidify her crown and $500,000. While she did not make a new World Record, her time of 2:23 is spectacular considering that she had just won a world-class marathon 6 weeks prior. Without the WMM prize, it is hard to conceive that Wami would have voluntarily run these marathons (April, September, November) in such quick succession. Independent of the regression results and WMM prize, it is hard to argue that this was ideal racing strategy that a coach would recommend for an elite runner without a lucrative prize purse causing a change in strategies.

For the men, it was a more certain outcome: there was no challenge to Cheruiyot in the Fall of 2007, as he maintained a 25-point cushion over then 2nd place Haile Gebrselassie heading into the Fall races. Cheruiyot completed the series with a 5th place finish in Chicago, totaling 80 points, 15 ahead of 2nd place Martin Lel who, with a 1st place finish in New York City, passed Gebrselassie, who did not race in the Fall (as we have previously discussed, Gebrselassie's financial well-being is much higher than most of his competitors, and he is likely not concerned with the WMM prize).

2008 Series – Winners: Gete Wami/Irina Mikitenko and Martin Lel

If possible, this series was even more exciting than the 2007 one for the women. Wami had a 25 point cushion heading into the Fall races, so it seemed that only if Irina Mikitenko won one of the Fall races, would there be a tie for the WMM prize. As it often happens in sports, Mikitenko ended up winning Chicago in 2008, and the two women tied for the overall series prize, splitting the money. It is interesting to note that Wami competed in the last race in the 2008 series, but missed out on earning the entire WMM prize by placing 6th in the race. It is possible that the previous Fall's races had taken their toll on her body, or maybe she just misjudged how difficult it would be to score points in the 2008 New York City race. The 5th place finisher in this race beat out Wami by 11 seconds, basically an instant in marathons (about .1% of the racing time – in percentage terms, it would be similar to losing in a 100 meter dash by .01 seconds, a photo finish). This close race supports the tenets from tournament theory: Wami was racing for a prize over $250,000, which gave her great incentive to come in 5th place or better, leading to the close finish.

For the men, the series was basically determined prior to the 2008 Fall marathons once Robert Kipkoech Cheruiyot decided not to run in any of them. This effectively eliminated him from contention and Martin Lel ended up the WMM series champion, as he had a comfortable 36 point lead over Abderrahim Goumri.

<u>2009 Series – Winners: Irina Mikitenko and Sammy Wanjiru</u>

For the women, this series was never close. Mikitenko accumulated 75 points heading into the Fall season, with the next closest woman (Dire Tune) only having 40. As it were, Mikitenko added to her lead with a 3rd place finish in Chicago to win by 50 points with a total of 90.

For the men, we have a similar story. Sammy Wanjiru had accumulated 65 points prior to the Fall marathons, and his only real competition was Haile Gebrselassie (with 50 points, all coming from 2 Berlin wins), who is more concerned with world records than with the WMM, so Wanjiru's only competition came from Robert Kipkoech Cheruiyot who only had 26 points at the time. Wanjiru ended up winning the WMM with 90 points (compared to Gebrselassie's 50 and Cheruiyot's 41).

<u>2010 Series – Winners: Irina Mikitenko and Sammy Wanjiru</u>

For the women, the series was not very close, as Shobukhova had a 20-point lead going into the Fall 2010 season. She won Chicago in 2010 and won the overall prize by an astonishing 44 points. The big difference in this series was that Irina Mikitenko placed 5th in Chicago while Shobukhova won, adding a 24-point cushion to her lead.

For the men, there were four men in contention entering the Fall 2010 season, with two having 50 points (Sammy Wanjiru and Tsegaye Kebede) and two having 35 points (Emmanuel Mutai and Deriba Merga). The 2010 Chicago Marathon basically determined the winner, as both Wanjiru and Kebede ran in it, giving whoever won an insurmountable lead (maintaining the assumption that runners choosing to race in Chicago could not come back in only 3 weeks to place in New York City). Wanjiru beat Kebede by 19 seconds over the 26.2 miles and earned the overall WMM prize with a total of 75 points.

<u>2011 Series – Winners: Liliya Shobukhova and Emmanuel Mutai</u>

For the women, the series had already ended by the time of the New York City Marathon. Shobukhova had a comfortable 30-point lead after the Spring marathons, but Edna Kiplagat won the World Championships over the Summer, which narrowed the gap. However, Shobukhova won in Chicago, which sealed the WMM prize for her. It is the case that her time in Chicago was the fourth fastest ever done (2:18:20 and the fastest done by someone other than Paula Radcliffe), so it is likely that knowing the prize was on the line pushed her to run much faster than she may have otherwise.

For the men, four of the top six point holders signed up to race in New York City, but the overall leader, Patrick Makau, did not. He is the current world record holder, and as we have discussed, certain runners have lucrative sponsorships and are less affected by the WMM prize (world record holders are likely in this group). Emmanuel Mutai ended up winning the WMM prize after getting 2nd place in New York, just behind Geoffrey Mutai, who was the runner up in the WMM series. The top 4 finishers in New York City all finished in the top 5 of the WMM series, showing how important the last race was to this year's series.

Conclusion

There are always going to be imperfections in data being collected over twelve years, with this sample being no exception. There are inherent differences within the data: weather at different races could impact times in ways we have not have accounted for; there is significant difficulty in gauging health, injury, and prestige factors. In terms of the weather, the error term in the game theory model included the idea of weather affecting the individual runner, but in terms of comparing races in general, the weather can have large impacts, like increasing (rain, wind, snow) or decreasing (tailwinds) the overall times of every runner that year by a few minutes. The negative impacts of weather are obviously a bigger concern, as tailwinds are much less common than all of the rest of the possible weather patterns (though they did play a significant role in the 2011 Boston Marathon). This is also a significant issue because certain runners will obviously respond differently to different weather conditions; those more accustomed to certain conditions like heat or humidity may perform better than others in different circumstances. The course difficulty levels definitely do vary, but most race directors try to advertise their courses as flat and fast, so it is reasonable to assume that the courses are relatively homogeneous, especially over such a long distance and considering their relatively similar course record times, though this is controlled for through fixed effects. The relative difficulty of courses is also something that is relatively well established within the marathoning community. The difficulty of gauging health and prestige factors is unavoidable and we must admit that the numbers we have come up with through analysis are estimates.

In addition, another interesting aspect of the races to analyze empirically would be the possible increase in publicity. Another potential goal for the organizers of the WMM may be to increase publicity from more exciting races. We could use Google searches for the marathons as a proxy for publicity, using data from one month leading up to the race and one month after the race to see if there are increases. If there are statistically significant differences, controlling for race size, and if there is a significant impact of lag times, race times, record times, or prize money on the overall results, then the prize may be adding value through publicity. Each of these impacts would have significantly different readings, with the impact of race times, lag times, and record times more as a measure of the "excitement" of the race, and prize money more being a measure of straight publicity from the large prize money available. If there is a statistically significant change in searches, but it is not accounted for by any of those factors, it may be the case that the incorporation of the series itself and any publicity involved in its naming was the driver behind the difference.

The theoretical framework behind this analysis provides support in many different ways for this increase in prize value, but the empirical results suggest that there may not be quantifiable effects. Whether through increased effort or different race set-ups, there is a legitimate theoretical rationale that underpins why times should be decreasing as a result of the WMM prize's implementation. Unfortunately, the regressions and analysis within this paper would seem to suggest that the WMM prize is not producing significant tangible results for its member races, though there has undoubtedly been an increase in the number of world records being broken within men's marathoning. It is important for the organizers of the WMM to ask themselves if their goals are being achieved through this $1,000,000 annual cash prize because there may be more efficient uses for that money.

Appendix 1

This appendix contains a brief overview of the methodology used in the theoretical framework. The overall process is the same in the different set-ups for each game theory scenario (with Appendix 2 showing slight differences in the more complicated scenarios):

We first took the utility functions:

$$u_H = P(e_H/(e_H + e_L)) – e_H\alpha_H$$

$$u_L = P(e_L/(e_H + e_L)) – e_L\alpha_L$$

In order to maximize these, we took the derivative of utility for each of the runners (u_H) with respect to that runner's effort (e_H), using the quotient rule:

$$\partial u_H/\partial e_H = P(((e_H + e_L)(1) – (e_H)(1))/(e_H + e_L)^2) – \alpha_H = 0$$

$$\partial u_L/\partial e_L = P(((e_L + e_H)(1) – (e_L)(1))/(e_H + e_L)^2) – \alpha_L = 0$$

Which simplify to the following:

$$\alpha_H = P(e_L/(e_H + e_L)^2)$$

$$\alpha_L = P(e_H/(e_H + e_L)^2)$$

After dividing the top function by the bottom, the Prize values (P) and $(e_H + e_L)^2$ terms cancel, leaving the following:

$$\alpha_H/\alpha_L = e_L/e_H$$

Which gives equilibrium effort for each runner as the following:

$$e_H = (e_L\alpha_L)/\alpha_H$$

$$e_L = (e_H\alpha_H)/\alpha_L$$

Plugging each equilibrium effort back into the first order conditions gives us the effort level with maximum utility for each of the runners:

$$\alpha_H = P((e_H\alpha_H)/\alpha_L)/(e_H + (e_H\alpha_H)/\alpha_L)^2$$

$$\alpha_L = P((e_L\alpha_L)/\alpha_H)/(e_L + (e_L\alpha_L)/\alpha_H)^2$$

These can be rewritten as the following by getting common denominators:

$$\alpha_H = P((e_H\alpha_H)/\alpha_L)/((e_H\alpha_L + e_H\alpha_H)/\alpha_L)^2$$

$$\alpha_L = P((e_L\alpha_L)/\alpha_H)/((e_L\alpha_H + e_L\alpha_L)/\alpha_H)^2$$

Which, by cancelling out a α_L from the denominators and a e_H from the numerators of the α_H function, can be rewritten as

$$\alpha_H = P(\alpha_H)/(e_H(\alpha_L + \alpha_H)^2/\alpha_L)$$

$$\alpha_L = P(\alpha_L)/(e_L(\alpha_H + \alpha_L)^2/\alpha_H)$$

Cancelling the left-hand side with the numerators and moving the effort variable to the other side yields the equilibrium effort for each runner

$$e_H = P(\alpha_L)/(\alpha_L + \alpha_H)^2$$

$$e_L = P(\alpha_H)/(\alpha_H + \alpha_L)^2$$

Taking this point and plugging it back into the original utility function gives us the indirect utility functions for each runner, which allows us to compare the different expected utilities for each runner under the different scenarios:

$$u_H = P((P(\alpha_L)/(\alpha_L + \alpha_H)^2)/((P(\alpha_L)/(\alpha_L + \alpha_H)^2) + (P(\alpha_H)/(\alpha_H + \alpha_L)^2)) – (P(\alpha_L)/(\alpha_L + \alpha_H)^2)\alpha_H$$

$$u_L = P((P(\alpha_H)/(\alpha_H + \alpha_L)^2)/((P(\alpha_L)/(\alpha_L + \alpha_H)^2) + (P(\alpha_H)/(\alpha_H + \alpha_L)^2)) – (P(\alpha_H)/(\alpha_H + \alpha_L)^2)\alpha_L$$

All of the $(\alpha_H + \alpha_L)^2$ terms in the first expression cancel out, as well as a P from each expression, leaving

$$u_H = P(\alpha_L/(\alpha_H + \alpha_L)) – (P(\alpha_L)/(\alpha_L + \alpha_H)^2)\alpha_H$$

$$u_L = P(\alpha_H/(\alpha_L + \alpha_H)) – (P(\alpha_H)/(\alpha_H + \alpha_L)^2)\alpha_L$$

Getting a common denominator of $(\alpha_H + \alpha_L)^2$ and combining like terms gives

$$u_H = P((\alpha_L)(\alpha_L) + (\alpha_L)(\alpha_H) – (\alpha_L)(\alpha_H))/(\alpha_L + \alpha_H)^2)$$

$$u_L = P((\alpha_H)(\alpha_H) + (\alpha_H)(\alpha_L) – (\alpha_H)(\alpha_L))/(\alpha_H + \alpha_L)^2)$$

Combining like terms gives the final indirect utility functions for each runner:

$$u_H = P(\alpha_L^2)/(\alpha_H + \alpha_L)^2$$

$$u_L = P(\alpha_H^2)/(\alpha_H + \alpha_L)^2$$

Appendix 2

This appendix contains a brief overview of the methodology used in the theoretical framework for the initial set-up of the game theory for races involving combinations of homogeneous and heterogeneous runners.

We first took the utility functions:

$$u_e = P_1(e/(e + e_H + e_L)) – e\alpha$$

$$u_L = P_1(e_L/(2e_H + e_L)) – e_L\alpha_L$$

In order to maximize these, we took the derivative of utility for each of the runners (u_H) with respect to that runner's effort (e_H), using the quotient rule:

$$\partial u_H/\partial e = P_1(((e + e_H + e_L)(1) – (e)(1))/(e + e_H + e_L)^2) – \alpha_H = 0$$

$$\partial u_L/\partial e_L = P_1(((e_L + 2e_H)(1) – (e_L)(1))/(2e_H + e_L)^2) – \alpha_L = 0$$

After differentiating, we then substituted e_H back into the first order conditions for e in order to carry out the rest of the process outlined in Appendix 1 to find the equilibrium efforts and indirect utility functions:

$$\partial u_H/\partial e_H = P_1(((e_H + e_H + e_L)(1) – (e_H)(1))/(e_H + e_H + e_L)^2) – \alpha_H = 0$$

$$\partial u_L/\partial e_L = P_1(((e_L + 2e_H)(1) – (e_L)(1))/(2e_H + e_L)^2) – \alpha_L = 0$$

So we get these functions that can be solved in the same way that we solved the functions in Appendix 1:

$$\partial u_H/\partial e_H = P_1((e_H + e_L)/(2e_H + e_L)^2) – \alpha_H = 0$$

$$\partial u_L/\partial e_L = P_1((2e_H)/(2e_H + e_L)^2) – \alpha_L = 0$$

Appendix 3

Comparative Statics:

From $E_1 = P\alpha_2/(\alpha_1+\alpha_2)^2$

- It is clear that the partial derivative of E_1 with respect to P is $\partial E_1/\partial P = \alpha_2/(\alpha_1 + \alpha_2)^2$.
- The partial derivative of $\partial E_1/\partial \alpha_2$ is derived in the following way (first using the quotient rule):
    - $\partial E_1/\partial \alpha_2 = P((\alpha_1 + \alpha_2)^2 - 2\alpha_2(\alpha_1 + \alpha_2))/(\alpha_1 + \alpha_2)^4$
        - Expanding terms
    - $\partial E_1/\partial \alpha_2 = P((\alpha_1)^2 + 2\alpha_1\alpha_2 + (\alpha_2)^2 - 2\alpha_1\alpha_2 - 2(\alpha_2)^2)/(\alpha_1 + \alpha_2)^4$
        - Cancelling out like terms
    - $\partial E_1/\partial \alpha_2 = P((\alpha_1)^2 - (\alpha_2)^2)/(\alpha_1 + \alpha_2)^4$
        - Cancelling out a $(\alpha_1 + \alpha_2)$ from the numerator and denominator
    - $\partial E_1/\partial \alpha_2 = P(\alpha_1 - \alpha_2)/(\alpha_1 + \alpha_2)^3$
- The partial derivative of $\partial E_1/\partial \alpha_1$ is derived in the following way (again first using the quotient rule)
    - $\partial E_1/\partial \alpha_1 = P((\alpha_1 + \alpha_2)^2(0) - 2\alpha_2(\alpha_1 + \alpha_2))/(\alpha_1 + \alpha_2)^4$
        - Cancelling out like terms
    - $\partial E_1/\partial \alpha_1 = (-2\alpha_2 P)/(\alpha_1 + \alpha_2)^3$

Appendix 4

To find the Best Response Function, we solve the first order condition of player one's utility function with respect to player one's effort in order to find where player one's utility is maximized with respect to what player two's effort level is, namely player one's best response to player two's choices:

$$\partial u_1/\partial e_1 = P(((e_1 + e_2)(1) – (e_1)(1))/(e_1 + e_2)^2) – \alpha_1 = 0$$

Simplifying this equation yields the following:

$$\partial u_1/\partial e_1 = P(e_2)/(e_1 + e_2)^2) – \alpha_1 = 0$$

Which simplifies to the following:

$$\partial u_1/\partial e_1 = P(e_2) = \alpha_1(e_1 + e_2)^2$$

Subtracting *$P(e_2)$* from both sides, expanding the squared term and dividing by α_1 gives the following:

$$\partial u_1/\partial e_1 = 0 = e_1^2 + 2e_1e_2 + e_2^2 + P(e_2)/\alpha_1$$

Setting up a quadratic equation to solve for e_1 gives the following:

$$e_1 = (2e_2 \pm \sqrt{}(4e_2^2 – 4e_2^2 + (4P(e_2)/\alpha_1))/2$$

Which, upon cancelling out like terms and cancelling out the twos in both numerator (including the 4 within the square root) and denominator yields the following:

$$e_1 = e_2 \pm \sqrt{}(P(e_2)/\alpha_1)$$

For reasons explained in the body of the paper, the positive root is used, and the results are symmetric for e_1 and e_2, giving the following Best Response Functions:

$$e_1 = e_2 + \sqrt{}(P(e_2)/\alpha_1)$$

$$e_2 = e_1 + \sqrt{}(P(e_1)/\alpha_2)$$

Appendix 5

Two High-ability, one Low-ability in one Race and only one Low-ability in the other Race (methodology shown in Appendices 1 & 2):

Maximizing utility functions $u_L = P_1(e_L/(2e_H + e_L)) – e_L\alpha_L$ for the low-ability runners and $u_e = P_1(e/(e + e_H + e_L)) – e\alpha$ for the high-ability runners (to get the type-specific Nash Equilibrium, we substitute e_H in for e after differentiating in order to isolate the derivative with respect to only one of the two runners in question). The first order conditions, taking the derivative with respect to each runner's effort, are the following: $0 = \partial u_L/\partial e_L = P_1(2e_H)/(2e_H + e_L)^2 – \alpha_L$ and $0 = \partial u_H/\partial e_H = P_1(e_H + e_L)/(2e_H + e_L)^2 – \alpha_H$. Upon solving for the equilibrium efforts, the Nash Equilibria for efforts are $e_L = P_1(4\alpha_H – 2\alpha_L)/(2\alpha_H + \alpha_L)^2$ and $e_H = P_1(2\alpha_L)/(2\alpha_H + \alpha_L)^2$, which gives indirect utility functions of $u_L = P_1(2\alpha_H – \alpha_L)^2/(2\alpha_H + \alpha_L)^2$ and $u_H = P_1(\alpha_L)^2/(2\alpha_H + \alpha_L)^2$. For the second race, the utility is simply the value of the prize (P_2), with effort being essentially 0 because there is only one entrant (who happens to be low-ability) in the race.

Comparative Statics (methodology in Appendix 3) yield

23. $\partial e_H/\partial P_1 = (2\alpha_L)/(2\alpha_H + \alpha_L)^2$ Change in Effort as Prize value changes

24. $\partial e_L/\partial P_1 = (4\alpha_H – 2\alpha_L)/(2\alpha_H + \alpha_L)^2$

25. $\partial e_H/\partial \alpha_L = P_1(4\alpha_H – 2\alpha_L)/(2\alpha_H + \alpha_L)^3$ Change in Effort as Opponent's ability changes

26. $\partial e_L/\partial \alpha_H = 4P_1(3\alpha_L – 2\alpha_H)/(2\alpha_H + \alpha_L)^3$

27. $\partial e_H/\partial \alpha_H = P_1(–8\alpha_L)/(2\alpha_H + \alpha_L)^3$ Change in Effort as Own ability changes

28. $\partial e_L/\partial \alpha_L = 2P_1(\alpha_L – 6\alpha_H)/(2\alpha_H + \alpha_L)^3$

We see that in 23 & 24, efforts for both runners are increasing with respect to prize value for all values in our domain. Efforts are also increasing compared to other runners' costs of running, so as the opposing runner gets worse, the other runner exerts more effort in order to earn the prize (25 & 26). In 27 & 28, we see that the opposite is true in own-cost of effort, with less effort exerted by runners with lower ability levels.

<u>One High-ability, two Low-ability in one Race and one High-ability in one Race (methodology shown in Appendices 1 & 2):</u>

Maximizing utility functions $u_e = P_1(e/(e + e_H + e_L)) – e\alpha$ for the low-ability runners and $u_H = P_1(e_H/(e_H + 2e_L)) – e_H\alpha_H$[6]. The first order conditions, taking the derivative with respect to each runner's effort, are the following: $0 = \partial u_L/\partial e_L = P_1(e_H + e_L)/(e_H + 2e_L)^2 – \alpha_L$ and $0 = \partial u_H/\partial e_H = P_1(2e_L)/(e_H + 2e_L)^2 – \alpha_H$. Upon solving for the equilibrium efforts, the Nash Equilibria for efforts are $e_H = P_1(4\alpha_L – 2\alpha_H)/(\alpha_H + 2\alpha_L)^2$ and $e_L = P_1(2\alpha_H)/(\alpha_H + 2\alpha_L)^2$, which gives indirect utility functions of $u_H = P_1(2\alpha_L – \alpha_H)^2/(\alpha_H + 2\alpha_L)^2$ and $u_L = P_1(\alpha_H)^2/(\alpha_H + 2\alpha_L)^2$. For the second race, the utility is simply the value of the prize (P_2), with effort being essentially 0 because there is only one entrant (who happens to be high-ability) in the race.

Comparative Statics (methodology in Appendix 3) yield

29. $\partial e_H/\partial P_1 = (4\alpha_L – 2\alpha_H)/(\alpha_H + 2\alpha_L)^2$ Change in Effort as Prize value changes

30. $\partial e_L/\partial P_1 = (2\alpha_H)/(\alpha_H + 2\alpha_L)^2$

31. $\partial e_H/\partial \alpha_L = 4P_1(3\alpha_H – 2\alpha_L)/(\alpha_H + 2\alpha_L)^3$ Change in Effort as Opponent's ability changes

32. $\partial e_L/\partial \alpha_H = P_1(3\alpha_L – 2\alpha_H)(4\alpha_L – 2\alpha_H)/(\alpha_H + 2\alpha_L)^3$

33. $\partial e_H/\partial \alpha_H = 2P_1(\alpha_H – 6\alpha_L)/(\alpha_H + 2\alpha_L)^3$ Change in Effort as Own ability changes

34. $\partial e_L/\partial \alpha_L = P_1 (–8\alpha_H)/(\alpha_H + 2\alpha_L)^3$

We see that efforts for both runners are increasing with respect to prize value for all values in our domain (29 & 30). Efforts are also increasing compared to the other runners' costs of running, so as the opposing runner gets worse, the other runner exerts more effort in order to earn the prize

[6] To get the type-specific Nash Equilibrium, we substitute e_H in for e after differentiating in order to isolate the derivative with respect to only one of the two runners in question.

(31 & 32). We see that the opposite is true in own-cost of effort, with less effort exerted by runners with lower ability levels (33 & 34).

Two High-ability, two Low-ability in one Race and No one in one Race (methodology shown in Appendices 1 & 2):

This race situation is highly unlikely, but we included it for the sake of argument in such a circumstance where one race had an extremely high prize relative to the other, so it would be an equilibrium for all four runners to enter the same race. For example, if the ability levels were not significantly different, it is easy to see that a race with a five-million dollar prize would attract all the athletes if the competing race only offered a prize of five hundred dollars. Each athlete's expected winnings would be drastically higher by entering the first race even though they could exert no effort and earn five hundred dollars.

Maximizing utility functions $u_e = P_1(e/(e + e_H + 2e_L)) - e\alpha$ for the high-ability runners[7] and $u_e = P_1(e/(e + 2e_H + e_L)) - e\alpha$[8]. The first order conditions, taking the derivative with respect to each runner's effort, are the following: $0 = \partial u_H/\partial e_H = P_1(e_H + 2e_L)/(2e_H + 2e_L)^2 - \alpha_H$ and $0 = \partial u_L/\partial e_L = P_1(2e_H + e_L)/(2e_H + 2e_L)^2 - \alpha_L$. Upon solving for the equilibrium efforts, the Nash Equilibria for efforts are $e_H = P_1(6\alpha_L - 3\alpha_H)/(2\alpha_H + 2\alpha_L)^2$ and $e_L = P_1(6\alpha_H - 3\alpha_L)/(2\alpha_H + 2\alpha_L)^2$, which gives indirect utility functions of $u_H = P_1(\alpha_H - 2\alpha_L)^2/(2\alpha_H + 2\alpha_L)^2$ and $u_L = P_1(\alpha_L - 2\alpha_H)^2/(2\alpha_H + 2\alpha_L)^2$. The utility from entering the second race would be its prize value (P_2), but there is no prize money awarded because there is no one winning (or even participating).

Comparative Statics (methodology in Appendix 3) yield

35. $\partial e_H/\partial P_1 = (6\alpha_L - 3\alpha_H)/(2\alpha_H + 2\alpha_L)^2$ Change in Effort as Prize value changes

[7] To get the type-specific Nash Equilibrium, we substitute e_H in for e after differentiating in order to isolate the derivative with respect to only one of the two runners in question.
[8] Done for the same reasons as the high-ability runners.

36. $\partial e_L/\partial P_1 = (6\alpha_H - 3\alpha_L)/(2\alpha_H + 2\alpha_L)^2$

37. $\partial e_H/\partial \alpha_L = 12P_1(2\alpha_H - \alpha_L)/(2\alpha_H + 2\alpha_L)^3$ Change in Effort as Opponent's ability changes

38. $\partial e_L/\partial \alpha_H = 12P_1(2\alpha_L - \alpha_H)/(2\alpha_H + 2\alpha_L)^3$

39. $\partial e_H/\partial \alpha_H = 6P_1(\alpha_L - 5\alpha_H)/(2\alpha_H + 2\alpha_L)^3$ Change in Effort as Own ability changes

40. $\partial e_L/\partial \alpha_L = 6P_1(\alpha_H - 5\alpha_L)/(2\alpha_H + 2\alpha_L)^3$

We see that efforts for both runners will be increasing with respect to prize value for all values in our domain. Efforts are also increasing compared to the other runners' costs of running, so as the opposing runner gets worse, the other runner exerts more effort in order to earn the prize. We see that the opposite is true in own-cost of effort, with less effort exerted by runners with lower ability levels.

Appendix 6

Comparing indirect utilities and finding equilibria race-entering scenarios with Simultaneous Entry

Under the assumption that the two prize values are the same, the indirect utilities from these possible races are presented in Table 1. After comparing these indirect utilities, it is clear that many race scenarios are not likely to exist with identical prizes, rational players, and complete information. For example, anyone who deviates from the HHLL race will immediately get the same prize value that they were previously competing with three other players for. In other words, we can effectively eliminate the two columns on the far right as races that will never exist in equilibrium (in fact, they result in lower utility compared to all the rest of the strategies for both ability types). With equal prizes (and sponsorship opportunities and prestige are assumed to be incorporated into the value of *P*), we will never witness the far right strategy being played in equilibrium.

We now have to analyze whether any of the three-person races are possible under equilibrium. As it turns out, neither three-person race will exist in equilibrium because there is always an incentive to deviate. In fact, there are circumstances when both ability types want to deviate away from the three-person race in order to reap the benefits that are gained from being involved in a two-person race. First, we can compare the race with two low-ability and one high-ability runners with one of the race scenarios with only two runners (with the actual set-up depending on who deviates). For each runner in the three-person race, the indirect utilities are the following: $u_H = P(2\alpha_L - \alpha_H)^2/(2\alpha_L + \alpha_H)^2$ and $u_L = P(\alpha_H)^2/(2\alpha_L + \alpha_H)^2$. If one of the low-ability runners were to deviate, he could get indirect utility of $u_L = P(\alpha_H)^2/(\alpha_H + \alpha_L)^2$ which is the function derived in the race with one high-ability, one low-ability. This clearly shows that the low-ability runner would prefer to deviate because his indirect utility in both cases has the same numerator but in the three-runner scenario has a larger denominator, indicating lower overall utility. This, on its own, is enough to eliminate the HLL race as a possible scenario, but the high-ability runner actually will often also have reason to deviate.

Depending on parameter values (for α_L and α_H), even the high-ability runner may be tempted to deviate from the three-person race because of his indirect utility described by $u_H = P(2\alpha_L - \alpha_H)^2/(2\alpha_L + \alpha_H)^2$. The closer the values of α_L and α_H (the closer the ability levels) the more likely he will be to deviate, which makes intuitive sense. If the runners are very similar, it makes little sense for there to be three of them in the same race. Only when the parameter values are sufficiently different will the high-ability runner choose to stay (presuming that a low-ability runner had not already deviated). Under the circumstances that $4(2\alpha_L - \alpha_H)^2/(2\alpha_L + \alpha_H)^2 - 1 < 0$, which results in HH in one race and LL in one race becomes an equilibrium.

Because of the symmetry of the race set-up and the stated utility functions, the other three-person race (HHL) will also not be an equilibrium. In fact, it is intuitively much easier to see why this race would not support a stable Nash equilibrium. Why would two higher-ability runners choose to race against each other when one of them could choose to run against a lower-ability runner, exert less effort and earn the same overall prize value? The indirect utilities for the high-ability runner in each case are $u_H = P(\alpha_L)^2/(\alpha_L + 2\alpha_H)^2$ for the three-person race and $u_H = P(\alpha_L)^2/(\alpha_H + \alpha_L)^2$ for the two-person race to which they could deviate. For the same reason that a low-ability runner would switch from the HLL race, a high-ability runner will switch from the HHL race (same numerator for utility but smaller denominator). The previous analysis for the high-ability runner in the HLL race now holds true for the low-ability runner in this (HHL) race. He will deviate from this scenario whenever $P(2\alpha_H - \alpha_L)^2/(\alpha_L + 2\alpha_H)^2 < P/4$, which for obvious reasons (smaller increase in numerator and smaller decrease in denominator) will be true for a narrower range of parameter values than in the previous scenarios.

Independent of what parameter values are for ability, the result is both ability types wanting to deviate from the three-person race. It is clear that these races are not stable equilibria since there is always at least one person who will deviate. This leaves only two different race scenarios as possible equilibria. All the rest provide lower utility for at least one type of runner compared to at least the heterogeneous strategy with two runners in each race. When we compare the two race set-ups where each race has two runners, both scenarios possibly support stable equilibria depending on parameter values.

Theoretical Findings

The heterogeneous race set-up (HL and HL) is stable independent of parameter values (except for prize value) because neither ability type will deviate under any circumstance where the prize values are identical. On the other hand, in the homogeneous game (HH and LL), the high-ability types are tempted to deviate under the circumstance that the following equation holds:

Equation 41: $P/4 < P(2\alpha_L - \alpha_H)^2/(2\alpha_L + \alpha_H)^2$

What this inequality actually represents is that they are tempted to deviate under the same circumstances that they were tempted to stay within the HLL race in the previous analysis; namely, when their ability levels are more heterogeneous and they receive much higher likelihood of winning the prize relative to the other two (lower-ability) runners.

Appendix 7

Time Lag

The time lag of the finishers of a race is important to measure because it measures the "closeness" of a race, and closer races are typically considered more exciting. Time lag is the amount of time separating finishers of races, and the theoretical model indicates that time lag should decrease as the efforts and prize values increase in tandem. If we think of these races as tournaments with runners of identical abilities, we would expect time lag to be relatively low. To win the prize value for first place, each runner only needs to beat the other runners by one second and has little incentive to put forth effort in order to attain some absolute low time. It is obviously the case that the runners are not of identical ability, and this is possibly where the difference between homogeneous men's marathoning and more heterogeneous women's marathoning could be seen quantitatively. With a large prize value, the theoretical model would predict that the effort exerted would be more varied for heterogeneous races, so the impact of the prize should be larger for women in terms of decreasing the time lag. In sum, it is unclear which of these two effects would dominate or even which one the race organizers would prefer. We run a regression to analyze the Time Lag for the top 2 finishers in races and to see if there is a statistically significant difference between WMM races before 2006 and those after. The regression run was the following:

Equation 42:

$$lag_{jt} = a_0 + a_1 2006Dummy_{jt} + a_2 PrizeValue_{jt} + a_3 PrizeDiff_{jt} + a_4 WinningTime_{jt} + v_{ijt}$$

In the regression, lag_{jt} is the time lag between the first and second place finishers, a_0 is a constant, $2006Dummy_{jt}$ is a dummy for whether the WMM series was in effect at the time of the race, $Year_{jt}$ is the year in which each race was run, $PrizeValue_{jt}$ is the amount of prize money awarded for first place in each year (apart from the WMM), $PrizeDiff_{jt}$ is the difference in prize values for the first- and second-place finishers, Winning $Time_{ij}$ is the winning time in each race, and v_{ijt} is a random error term.

Table 9: Time-Lag Regressions

	WMM Women Timelag	**WMM Men Timelag**
2006 Dummy	-.0002599	-.0001125
PrizeValue	-.00000217	.000000862
PrizeDiff	-.00000512	.00000201
Winning Time	-.1082827**	-.0458806
Constant	.0124532***	.0042613
Observations	54	63
Adj. R^2	.256	-.028
*Significance level: *.1*	**.05	*** .01

The variable we are most concerned with is the $2006Dummy_j$. The 2006 Dummy is not statistically significant in either case, which fails to give evidence that the WMM prize is resulting in closer races (at least between the top 2 finishers). The women have a significant negative relationship between Winning Time and Time Lag, which could indicate that as winning times decrease, time lags increase because second-place finishers just cannot keep up. However, it seems that this regression is not that strong of a predictor of Time Lag, especially with the men's regression having a negative Adjusted R^2. The heterogeneity difference between men's and women's marathoning is supported by this regression, as we witness that women's winning times are much more significant than men's, which shows that the winners of women's marathons are creating a much wider gap in times than in their male counterparts' races.

6^{th} – 20^{th} Place Finishers in the WMM

Another possible effect of the WMM prize implementation is an indirect effect. This effect is with the runners who have no chance of winning the WMM prize but in the years before the WMM, still chose to enter the race. The theoretical model suggests that the lower-ability runners will switch out of the high prize value race because they can maximize their chances of winning prizes because of the high-ability runners' earlier decisions. It is possible that they will self-select out of the WMM races because others are self-selecting in and leaving smaller prize values in other races up for grabs. This follows exactly along the lines of the lower-ability runners in the theoretical model deviating and choosing the lower prize races. To test this effect, we set up a regression identical to the faster time regressions but with the sixth-place through twentieth-place runners:

Equation 43:

$$t_{ijt} = a_0 + a_1 2006Dummy_{jt} + a_2 Year_{jt} + a_3 TempDev_{jt} + a_4 Humidity_{jt} + a_5 Age + a_6 Age^2 + v_{ijt}$$

Table 10: Individual 6^{th}-20^{th} Places Time Regressions

	WMM Women 6-20	**WMM Men 6-20**
2006 Dummy	.0006522	.0006524*
Year	.0009128	-.0000791
Tempdev	.0000769**	.0001156***
Humidity	.0000319	-.0000339***
Age	-.0017735**	-.0015805***
Age^2	.0000136	.0000243***
Constant	-1.671248	.2763319
Observations	762	862
Adj. R^2	.799	.727
*Significance level: *.1*	*** .05*	**** .01*

The variable we are most concerned with is the 2006 Dummy, but all the other variables (except Humidity) are either represented in the ways we would expect or are not statistically significant. Although the 2006 Dummy is positive, which is what we would expect if good runners were leaving these races in order to have a better chance at winning smaller prizes in other races, it is not highly statistically significant in either regression, and is only significant at the 10% level for men, which fails to provide us with significant evidence to support our hypothesis of this change.

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