1 Perfect Competition

This should be revision, but it is good stuff to be reminded of...

1.1 Structure of IO
1.2 Competition in agriculture

- In poorest nations of the world, 50 to 80 percent of population live on farms
- Less than 2 percent in the US, less than 10 percent in Europe
- Highly skewed distribution: largest 2.6 percent of wheat farms raise roughly half of all wheat
- However, still very large numbers of firms: the 2 percent of farms producing half the grain comprise 27000 farms
- Can any one farm have any effect on price of corn?
- How difficult is it to enter farming?

1.2.1 Cost structure: textbook depiction

![Cost Structure Diagram](image)

- Fixed costs
  1. Mortgage payments
  2. Taxes
  3. Depreciation of equipment (why?)

- Variable costs
  1. Preharvest costs ($31 per acre of corn, 1971 prices)
    - Labor: about 4 hours
– Seed
– Fertilizer
– Fuel
– Insecticides/herbicides

2. Harvesting costs ($10 per acre of corn, 1971 prices)
– Labor: about 2 hours
– Fuel
– Trucking

Is this an accurate depiction of what we think the farmer looks like?

What do you think the average and marginal costs of the farm look like?

1.2.2 Profit maximization decision of the corn farmer: Revision

• \( \pi(q) = qP(q) - TC(q) \)

• What’s \( \frac{dP(q)}{dq} \)? Why?

• \( P = MC \)

• But is this enough?
  – MC at current production = 41 cents
— What happens if price is 40 cents per bushel?
— If price is 50 cents per bushel?

1.2.3 Market Equilibrium: Revision

![Graph showing market demand and supply with shaded areas for consumer surplus and producer surplus]

1.2.4 Welfare and Efficiency: Revision

- Social welfare = Consumer Surplus + Producer Surplus
- Consumer surplus: area under the market demand curve minus price paid by consumers
- Why is this a measure of consumer welfare?

2 Monopoly

Overview

Definition: A firm is a monopoly if it is the only supplier of a product in a market. A monopolist’s demand curve slopes down because firm demand equals industry demand.

Five cases:

1. Base Case (One price, perishable good, non-IRS Costs).
2. Natural Monopoly
3. Price Discrimination
4. Bundling
5. Durable Goods

2.1 Base case

2.1.1 Revenue

Demand Curve Facing Monopolist (MC = 0). Decreasing price by $\Delta p$ reduces revenue on the infra-marginal unit, but increases revenue on the extra marginal unit.

Which revenue effect is larger?

Revenue = $P \times Q$

Is the % decrease in $P$ greater or less than the % increase in $Q$?

It depends on the price elasticity of demand:

$$\varepsilon_d = \frac{P \frac{dQ}{dP}}{Q \frac{dP}{dP}}$$

Moving toward the point where $\varepsilon_d = -1$ increases total revenue.
Given a price drop, revenue increases if
\( \Delta q \times p > -\Delta p \times q \)
\( p\Delta q > -q\Delta p \)
Which implies:
\[ \frac{(p\Delta q)}{(q\Delta p)} < -1 \]
\( \varepsilon < -1 \)

2.1.2 Linear Demand:

Marginal revenue is less than price

2.1.3 Profit Maximization:

Monopolist’s Profit Maximization Problem:

\[ \max_Q \pi = P(Q)Q - C(Q) \]

(Choosing \( P \) or \( Q \) makes no difference because we are selecting a single point on the demand curve.)
This will not be true when we consider oligopoly problems.)

F.O.C. are:
\[
\frac{d\pi}{dQ} = P(Q) + Q \frac{dP}{dQ} - \frac{dC}{dQ} = 0
\]
\[
\Rightarrow P(Q) + Q \frac{dP}{dQ} = \frac{dC}{dQ}
\]
\[
\Rightarrow MR = MC
\]

\((P^*, Q^*)\) is profit-maximizing choice.

Note that Marginal Revenue may be written:
\[
MR = \frac{dR}{dQ} = \frac{d[P(Q)Q]}{dQ} = P + \frac{dP}{dQ} Q
\]
\[
P + \frac{dP}{dQ} Q = P \left(1 + \frac{1}{\varepsilon_d}\right)
\]

2.1.4 Inverse Elasticity Rule for Monopolist:

Price Cost Margin, Markup, or Lerner Index is:
\[
L = \frac{P - MC}{P}
\]
The monopolist chooses output such that the markup equals the inverse of the elasticity of demand:

\[
\frac{P(Q) - \frac{dC(Q)}{dQ}}{P(Q)} = -Q \frac{dP(Q)}{dQ} = \frac{-Q}{P} \frac{dP(Q)}{dQ} = 1 - \varepsilon_d > 0
\]

### 2.1.5 Welfare and Efficiency

- What is the welfare impact of monopoly?
  - Graphically
  - Algebra

- What is the reason that there is DWL?

**Graphically:**

![Graph showing welfare and efficiency](image-url)
2.1.6 Summary

To Reiterate: The Pricing Rule of A Monopolist Is:

\[ MR = P \left( 1 + \frac{1}{-|\varepsilon_d|} \right) = MC \]

- or equivalently:
  \[ \frac{P - MC}{P} = \frac{1}{-\varepsilon_d} \]

Flatter demand implies higher \( \varepsilon_d \) holding \( P \) and \( Q \) fixed, a lower monopoly markup and lower DWL.

Monopolists induce inefficient rent-seeking behavior and monopoly profit is a transfer from consumers.

But: there are benefits to monopoly: Incentives to innovate (new products, more efficient production).

2.2 Natural Monopoly

- A firm is a natural monopoly if it can produce the market quantity \( Q \), at lower cost than two or more firms.
• To argue for a natural monopoly, have to establish subadditivity of cost function: \( C(Q) < C(q_1) + C(q_2) + ... + C(q_K) \) where \( Q = \sum_{i=1}^{K} q_i \).

• Claim: if \( AC \) is declining everywhere, subadditivity is satisfied.

• However, the reverse is not always true, i.e. economies of scale is sufficient but not necessary for natural monopoly.

• It is often argued that electrical, gas, telephone and other utilities are natural monopolies. Why?

\[ \text{Price} \]

\[ \begin{align*}
P_M & \quad Q_M \\
P_S & \quad Q_S \\
MC & \quad D = AR \\
\end{align*} \]

Monopoly will maximize profits at \((P_M, Q_M)\).

\((P_s; Q_s)\) are perfectly competitive and, hence, welfare maximizing (not counting fixed costs), but a firm would make a loss in this case.

How to regulate a natural monopoly?

1. Pay a subsidy to the firm to cover losses,

2. or have firms bid based on price: ”franchise bidding”

3. or, even better, have firms bid based on both a fee to operate, but also the price they will charge consumers.
2.3 Price Discrimination

We distinguish between three types of price discrimination:

1. First degree ("personalized" prices to extract full surplus)
2. Second degree (have customers choose between a menu of quantities/prices - "self-selection")
3. Third degree ("market segment" based pricing)

Price discrimination always increases firm profits, but its effects on consumer surplus are ambiguous.

2.3.1 First-Degree Price Discrimination

Monopolist knows the WTP of each customer, and charges a different price from each.

The key idea is all the consumer surplus is extracted.

The best way to think of this is that it is done with a lump sum fee and quantity restriction, or an access fee and $P = MC$
Claim: This type of price discrimination leads to an efficient outcome.

What’s the MR curve of a monopolist who can charge the area under the demand curve?

Implementation problem: how does the monopolist know its demand curve?
2.3.2 Third Degree

This is "market segmentation".

Two segments:

- \( \max_{q_1,q_2} \pi(q_1, q_2) = TR_1(q_1) + TR_2(q_2) - TC(q_1 + q_2) \)
- \( MR_1(q_1^m) = MC(q_1^m + q_2^m) \)
- \( MR_2(q_2^m) = MC(q_1^m + q_2^m) \)
- Equate marginal revenues across two markets!
- Marginal revenues might be the same, but not the resulting prices!

- Claim: \( p_1^m (1 + \frac{1}{|\varepsilon_1|}) = p_2^m (1 + \frac{1}{|\varepsilon_2|}) \)

- Price in market with less elastic demand is higher (Ramsey pricing principle)
- Examples: Cinema pricing; Book Pricing
- In general, welfare consequences are ambiguous.
2.3.3 Second-Degree Price Discrimination

This is the case where you can’t effectively segment the market. Instead you get people to choose between different pricing plans ("tariffs").

- Cell phone plans are a good example.
- Quantity discounts (ever wonder how Starbucks sets prices across sizes?)

In some cases, a much harder problem to analyze. Two type example:

- Consumer surplus from drinking $q$ ounces of coffee and paying $t_i$ is given by:
- $\theta_i q - t$ where $i = 1; 2$;
- with $(\lambda, 1 - \lambda)$ percent of each type in population
- $\theta_1 < \theta_2$, i.e. $\theta_2$ people get higher marginal utility from each ounce of coffee ("H" types).
- Costs are given by $C(q_i)$

- Monopolist offers two sizes of coffee: $(q_1, t_1)$, $(q_2, t_2)$ and corresponding pricing.

- Maximization problem:
  $$\max_{\{q_1, t_1, q_2, t_2\}} \lambda(t_1 - C(q_1)) + (1 - \lambda)(t_2 - C(q_2))$$
  such that people (a) buy and (b) buy the bundle they are supposed to
  $$\theta_1 q_1 - t_1 \geq 0 \quad (\text{IR-L})$$
  $$\theta_2 q_2 - t_2 \geq 0 \quad (\text{IR-H})$$
  $$\theta_1 q_1 - t_1 \geq \theta_1 q_2 - t_2 \quad (\text{IC-L})$$
  $$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 \quad (\text{IC-H})$$

- Let’s get rid of some of these constraints:

- **Claim**: IR-L and IC-H automatically imply IR-H.

  **Proof**:
  $$\theta_2 q_2 - t_2 \geq \theta_2 q_1 - t_1 \quad (\text{IC-H})$$
  $$> \theta_1 q_1 - t_1 \quad \text{(since } \theta_2 > \theta_1)$$
  $$> 0 \quad (\text{IR-L})$$
• **Claim**: IR-L binds at an optimum.

**Proof**: Suppose not. If monopolist increases \( t_1 \) and \( t_2 \) by \( \varepsilon \), he still satisfies IR-L. IC-L and IC-H are unchanged (\( \varepsilon \) cancels from both sides), and IR-H continues to hold. But then monopolist increases profit (price increased by \( \varepsilon \), so we can’t be at an optimum. Contradiction.

• **Claim**: IC-H binds.

**Proof**: Suppose not. Increase \( t_2 \) by \( \varepsilon \) such that IC-H still holds. IR-L not affected, so IR-H still holds. IC-L not affected. So once again monopolist increased profits! Contradiction.

• **Claim**: IC-L is satisfied automatically given other constraints. Assume for now and verify at optimum.

"Simplified" Maximization problem:

\[
\max_{\{(q_1,t_1),(q_2,t_2)\}} \{ \lambda(t_1 - C(q_1)) + (1 - \lambda)(t_2 - C(q_2)) \}
\]

\[
s.t. \quad t_1 = \theta_1 q_1 \\
\quad t_2 - t_1 = \theta_2 q_2 - \theta_2 q_1
\]

So we can write the problem as:

\[
\max_{(q_1,q_2)} \lambda(\theta_1 q_1 - C(q_1)) \\
\quad + (1 - \lambda)(\theta_2 q_2 + (\theta_1 - \theta_2)q_1 - C(q_2))
\]

Giving FOC’S:

\[
\theta_1 - (1 - \lambda)\theta_2 = \lambda C''(q_1) \\
\theta_2 = C'(q_2)
\]

For H-types, "large" coffee size is socially optimal; for L-types, suboptimal:

\[
\theta_1 - (1 - \lambda)\theta_2 < \theta_1 - (1 - \lambda)\theta_1 = \lambda \theta_1, \text{ so } \theta_1 > C'(q_1).
\]

That is, small coffee size is too small. Moreover, L-types enjoy zero surplus: \( \theta_1 q_1 - t_1 = 0 \), and are just indifferent between buying coffee and not. Last step:
• Verify IC-L: since IR-L binds, $\theta_1 q_2 \leq t_2$. Subtract IR-L (which binds) from both sides: $\theta_1 q_2 - \theta_1 q_1 \leq t_2 - t_1$.

• H-types enjoy surplus: $\theta_2 q_2 - t_2 \geq 0$. Proof: Use IC-H and IR-L.

More than 2 types of consumers:

If there are $K$ types, $\theta_1 < \theta_2 < \theta_3 < ... < \theta_K$

1. Optimality at the top. Marginal cost of largest coffee cup is equal to marginal benefit of "$H" types.
2. Lowest type gets no surplus
3. All other types enjoy some surplus
4. Only downward IC constraints matter, i.e.

$$t_i - t_{i-1} = \theta_i q_i - \theta_{i-1} q_{i-1}$$

5. $q_i > q_{i-1}$

**Second-Degree Price Discrimination with Competition**

What happens to the results above when there are many competing firms, with each competitor offering a menu of prices and quantities? This is probably a situation that you are more likely to encounter in everyday life. Intuition:

1. Individual rationality constraint becomes important. No longer clear that participation constraint will bind for lowest type consumer; this person can be "lured" by the competitor.
2. You’ll end up leaving some surplus to the marginal consumer.
3. As $n$ grows larger, quantity distortion decreases

Largely an empirical question right now. Borenstein and Rose (1994): Competition and Price Dispersion in Airline Industry

• Considerable price dispersion within airline (across customers). Average dispersion between two tickets on same carrier and same route is 36 percent of ticket price

• Much less price dispersion across airlines serving the same route (97 per cent of total price dispersion is within airline)
• Variation in dispersion across markets: 3.6 percent of ticket price (Eastern Boston-LaGuardia shuttle) to 86 percent of ticket price (TWA, Phoenix-Las Vegas flight)

• Price dispersion increases with number of competing airlines

2.4 Durable Goods

2.4.1 Coase Conjecture

Consider the example of a monopolist who owns all the land in the world and wants to sell it at the largest discounted profit.

• In year 1, the monopolist sets a monopoly price and sells half the land. (Think of a linear demand curve with marginal cost at zero.)

• In year 2, the monopolist will want to do the same with the remaining land, but unless the population is growing very quickly, demand for land will be lower. Thus, the monopoly land price in year 2 will be lower.

Coase conjecture: if consumers do not discount time too heavily and if consumers expect price to fall in future periods, current demand facing the monopolist will fall, implying that the monopoly will charge a lower price (compared to a perishable good).

(Price is driven to marginal cost “in the blink of an eye”)

Crucial assumptions:

1. durable good

2. demand does not grow quickly over time

3. consumers anticipate price cuts

As we will see, the assumption of a downward sloping demand (with a continuum of consumers) is also a crucial assumption.

Model I: Continuum of consumers, downward sloping demand.

Assumptions:

• Consumers live for 2 periods
• Product lasts for 2 periods

The aggregate one-period demand for the services of the good is:

\[ p = 100 - q. \]

That is, if the good were only sold in period 2, this is what demand in period 2 would look like. If a consumer buys in period 1, that consumer gets twice the consumer surplus that they would get if that had bought at the same price in period 2. That is, they get to consume for an extra period...

Define a game:

1. Set of players: seller and buyers

2. Set of actions/strategies for each player:
   - seller chooses price in period 1
   - buyers choose whether to buy in period 1
   - seller chooses price in period 2
   - buyers choose whether to buy in period 2

3. Pay-off function (producer and consumer surplus)

Use backward induction to solve:

In period 2, demand is

\[ p_2 = 100 - \bar{q}_1 - q_2 \]

because people who bought in period 1 do not need to buy again.

Set \( MR = MC \) (assume) = 0, just like always. Then

\[ MR = 100 - \bar{q}_1 - 2q_2 = 0 \]

Implying

\[ q_2 = 50 - \frac{\bar{q}_1}{2} \]

(turns out \( p = q \) in this example)

and

\[ \pi_2 = p_2q_2 = \left(50 - \frac{\bar{q}_1}{2}\right)^2 \]
If the monopolist sells to the $q_1$ buyers with the highest valuations in period 1, then the marginal buyer must be indifferent between purchasing the good in period 1, or waiting until period 2. Thus, the marginal buyer’s utility is equivalently given by:

\[
\text{value in period 1} - \text{price in period 1} = \text{value in period 2} - \text{price in period 2}
\]

\[2(100 - \bar{q}_1) - p_1 = (100 - \bar{q}_1) - \left(50 - \frac{\bar{q}_1}{2}\right)
\]

Now it is straightforward to solve for $p_1$ as a function of $\bar{q}_1$:

\[p_1 = 150 - \frac{3\bar{q}_1}{2}.
\]

Now we can write the firm’s profit maximization problem as a function of $q_1$ only:

\[
\max_{q_1}(\pi_1 + \pi_2) = \left(150 - \frac{3q_1}{2}\right) q_1 + \left(50 - \frac{q_1}{2}\right)^2
\]

Take F.O.C. and solve. . .

We find $q_1 = 40; q_2 = 30; p_2 = 30; p_1 = 90$. This implies that total profits for both periods (no discounting) are 4,500.

**An alternative strategy set: RENT**

Define selling vs. renting. (See Shy: "Selling means charging a single price for an indefinite period. . . Renting means charging a price for using the product for a limited time period.")

How does the firm maximize discounted profits if it rents out the good? Recall that the aggregate one-period demand for the services of the good is $p = 100 - q$. Therefore, solve the static model twice (a repeated game). F.O.C. (MR = MC) imply

\[p_t = 50 \Rightarrow \pi_t = 2,500 \Rightarrow \pi_1 + \pi_2 = 5,000.
\]

The firm does better by renting out the product. That is, by renting (and turning a durable good into a non-durable) they firm avoids the coase conjecture problem of having to charge lower prices.

Two outstanding issues:
1. How does the analysis work when demand is discrete?

2. Does the firm have any other choice variables? . . .

The relative level of “durability.” The usual example is lightbulbs. This highlights the role of the firm’s cost function, which we have been assuming is zero. Choosing a lower relative level of durability is one way of solving the problem of consumers’ expectations. Other ways include:

1. Renting (we just did that)
2. Planned obsolescence (new car models, new fashions... as long as costs are not too high)
3. Capacity constraints (numbered prints)
4. Buy-back provisions (not useful if consumers can damage good, or easily resell)
5. Announcements/advertising future prices

Discrete demand (Shy’s example in his textbook with two types of consumers):

- Two types of consumers
- Consumers are ”different enough” in their willingness to pay for the good
- Two periods for consumption

Shy shows: selling may be more profitable than renting.

This reverses the coase conjecture intuition - instead, the extreme heterogeneity in consumer types means that the durability enables a form of type 2 price discrimination to work...

3 Oligopoly Theory

We will cover the following topics:

- Cournot
- Cournot with Sequential Moves
- Bertrand
• Cournot versus Bertrand

After these basic static models we will examine:

• Dynamic oligopoly and Self-enforcing Collusion

3.1 Cournot

Cournot wrote in 1838 - well before John Nash! He proposed an oligopoly-analysis method that is (under many conditions) in fact the Nash equilibrium w.r.t. quantities.

Firms choose . . . production levels.

3.1.1 Version 1: Two-seller game

Cost: \( TC_i(q_i) = c_i q_i \)

Demand: \( p(Q) = a - bQ \) where \( Q = q_1 + q_2 \)

Define a game:

• Players: firms.
• Action/Strategy set: production levels/quantities.
• Payoff function: profits, defined:

\[
\pi_1(q_1, q_2) = p(q_1 + q_2)q_1 - TC_1(q_1)
\]

\[
\pi_2(q_2, q_1) = p(q_2 + q_1)q_2 - TC_2(q_2)
\]

• Now we need an equilibrium concept.

\( \{ p^c, q_1^c, q_2^c \} \) is a Cournot equilibrium ("Nash-in-quantities" equilibrium) if:

1. a) given \( q_2 = q_2^c \), \( q_1^c \) solves \( \max_{q_1} \pi_1(q_1, q_2^c) \)
   
b) given \( q_1 = q_1^c \), \( q_2^c \) solves \( \max_{q_2} \pi_2(q_2, q_1^c) \)
2. \( p^c = a - b(q_1^c + q_2^c) \), for \( p^c, q_1^c, q_2^c \geq 0 \)

"No firm could increase its profit by changing its output level, given what other firms produced."

F.O.C. (for firm 1):

\[
\frac{\partial \pi_1(q_1, q_2)}{\partial q_1} = a - 2bq_1 - bq_2 - c_1 = 0
\]

As always, set marginal revenue \((a - 2bq_1 - bq_2)\) equal to marginal cost \((c_1)\).

How do we know this is a maximum and not a minimum?

\[
\frac{\partial^2 \pi_1(q_1, q_2)}{\partial (q_1)^2} = -2b < 0 \ \forall q_1, q_2
\]

Solve the F.O.C. for \( q_1 \) ...

Note: \( q_1 \) is a function of \( q_2 \). It is the best-response function, a.k.a. reaction function of firm 1. Call this \( R_1(q_2) \).

\[
q_1 = R_1(q_2) = \frac{a - c_1}{2b} - \frac{1}{2}q_2
\]
Note both best-response functions are downward sloping. For each firm: if the rival’s output increases, I lower my output level. (i.e., an increase in a rival’s output shifts the residual demand facing my firm inward, leading me to produce less.) This feature of "strategic substitutes”.

Now we can compute Cournot equilibrium output levels.

\[ q_1^* = \frac{a - 2c_1 + c_2}{3b} \]
\[ q_2^* = \frac{a - 2c_2 + c_1}{3b} \]

Equilibrium quantity supplied on the market is

\[ Q^* = q_1^* + q_2^* = \frac{2a - c_1 - c_2}{3b} \]

We can also find equilibrium price.

\[ p^* = a - bQ^* = \frac{a + c_1 + c_2}{3} \]

What are the pay-off functions in equilibrium?

\[ \pi_i^* = (p^* - c_i)(q_i^*) = b(q_i^*)^2 \]

### 3.1.2 Version 2: Extending this to N firms.

It’s harder to see the reaction functions, but the story is exactly the same. Now each firm maximizes profits according to:

\[ \pi_i(q_1, q_2, ... q_N) = p(Q)q_i - TC_i(q_i) \]

We would derive the best response function for all N firms. For firm 1,

\[ q_1 = R_1(q_2, ... q_N) = \frac{a - c_1}{2b} - \frac{1}{2} \left( \sum_{j=2}^{N} q_j \right) \]

We need N of these equations. However, if we assume that firms’ marginal costs are the same \( (TC_i(q_i) = cq_i \forall i) \), it’s a lot easier. Each firm has the same reaction function, which is

\[ q_i = R_i(q_{-i}) = \frac{a - c}{2b} - \frac{1}{2} \left( \sum_{j \neq i} q_j \right) \]

Look at the 2-firm example. When costs (c_1 and c_2) are the same, output levels of the two firms are the same. So, we guess in this case that all the output levels \( (q_1, ..., q_N) \) are going to be the same too. If so, denote \( q_i = q \forall i \).
WATCH OUT! We cannot substitute in \( q_i = q \) before we’ve derived the best response function. This is a common mistake. Why is it a mistake?

Assuming that \( q_i = q \) before we take first order conditions implies that firm \( i \) has control over all firms’ output decisions (the \( q_{-i} \)’s). Here, we substitute \( q_i = q \) into the already-derived best-response functions. We do this to make the process of solving \( N \) equations for \( N \) unknowns easier. And, hopefully because we believe the assumption that firms’ cost functions are truly the same.

Back to the reaction function.

\[
q_i = R_i(q_{-i}) = \frac{a - c}{2b} - \frac{1}{2} \left( \sum_{j \neq i} q_j \right)
\]

\[
q = \frac{a - c}{2b} - \frac{1}{2} (N - 1)q
\]

Thus,

\[
q^c = \frac{(a - c)}{(N + 1)b} \\
Q^c = \frac{N(a - c)}{(N + 1)b}
\]

Now it is straightforward to solve for \( p^c \) (the market price) and \( \pi^c_i \) (profits for each firm).

Sanity checks. . .

Do we get the monopoly result for \( N = 1 \)?

Do we get the duopoly result for \( N = 2 \)?

What is the Cournot solution for \( N = \infty \)?

Take the limit as \( N \to \infty \) for \( q^c \) and \( Q^c \) and \( p^c \). They are:

\[
\lim_{N \to \infty} q^c = 0 \\
\lim_{N \to \infty} Q^c = \frac{a - c}{b} \\
\lim_{N \to \infty} p^c = c
\]

Under the assumption of Cournot competition, market supply approaches the competitive supply as \( N \to \infty \).

Note that market supply depends on the slope and intercept of demand, and the (common) marginal cost. Individual firms’ output levels approach zero as \( N \to \infty \).
Convince yourself that you can find the equilibrium quantity supplied and price in the market under the assumption of perfect competition.

Simple way: let number of firms = 1, but assume that 
\[ \frac{\partial p}{\partial Q} = 0 \]

and take F.O.C. to find the Q that maximizes profits for the firm under assumptions of perfect competition. This is what we did in the first week.

We have obtained producer surplus under the assumption that N firms in a market set outputs levels simultaneously according to a "Nash-in-quantities" equilibrium. What about consumer surplus?

We can compute consumer surplus as the area under the demand curve up to the output level that is supplied on the market.

\[ CS^c(N) = \frac{N^2(a-c)^2}{2b(N+1)^2} \]

Does consumer surplus go up or down as the number of firms increases in this model?

As for social welfare, we want to compare the sum of firm profits and consumer surplus as the number of firms increases. (Firms are owned by consumers in the end...)

We can compute this as a function of the number of firms in the market.

The bottom line: firm profits go down as more firms enter, but consumer surplus goes up, and goes up by more.

\[ W^c(N) = CS^c(N) + N\pi^c(N) = \left( \frac{(a-c)^2}{2b} \right) \left( \frac{N^2 + 2N}{N^2 + 2N + 1} \right) \]

The important point:
\[ \frac{\partial W^c(N)}{\partial N} > 0. \]

3.1.3 Version 3: Different Marginal Costs

We have been assuming that all firms are identical, which implies symmetric equilibrium outcomes. Take a look at the results when firms have different levels of marginal cost.
Let marginal cost of firm $i$ be $c_i$. The F.O.C. is only a slight modification from before.

$$\frac{\partial \pi_i}{\partial q_i} = a - 2bq_i^* - b \sum_{i \neq j} q_j^* - c_i = 0$$

Rewrite the F.O.C. as

$$a - bQ^* - bq_i^* = c_i$$

Substitute in for price:

$$p - c_i = bq_i^* \frac{Q^*}{Q^*}$$

Rearrange...

$$\frac{p - c_i}{p} = \frac{bq_i^*}{Q^*} \frac{Q^*}{p} = \frac{-\partial p}{\partial Q} \frac{q_i^*}{Q^*}$$

The term $q_i^*/Q^*$ the market share of firm $i$. Denote this simply as $s_i$. We now have the inverse elasticity rule of Cournot oligopoly:

$$\frac{p - c_i}{p} = \frac{s_i}{-\varepsilon_d}$$

3.1.4 Cournot - Stackelberg

Cournot with Sequential Moves:

We could also think about this in a game where firm 1 moves first, firm 2 moves second, etc. We call this a leader-follower market structure, or a Stackelberg game.

Keep everything as in the 2-firm cournot model, but assume firm 1 sets quantity ($q_1$) first, then firm 2 sees this quantity and then chooses its quantity, $q_2$.

- Work backwards: Suppose firm 1 sets output level to $q_1$. What would firm 2 do?
- $R_2(q_1) = \frac{a - c}{2p} - \frac{1}{2} q_1$
- Firm 1 can figure this out. What will Firm 1 do in response?
- $\max_{q_1} \left[p(q_1 + R_2(q_1))q_1 - cq_1\right]$
- Firm 1 can figure this out. What will Firm 1 do in response?
max_{q_1} [p(q_1 + R_2(q_1))q_1 - cq_1]

What’s different between this profit function and the Cournot profit function?

Turns out leader output level is: $q_1^S = \frac{a-c}{2b} = \frac{3}{2}q^c$.

Follower output level is: $q_2^S = \frac{a-c}{4b} = \frac{3}{4}q^c$.

Equilibrium price is lower than Cournot, output is larger than Cournot – hence more consumer surplus.

First-mover advantage: “leader” makes more profit than “follower”. “Leader” is better off than in Cournot.

3.2 Bertrand

When do you think price setting makes more sense than setting quantity? In general, economists may believe that different assumptions hold for different settings. Then we have to argue about which one is more consistent with the data. Bertrand reviewed Cournot’s work 45 years later.

Go through a two-firm example again. Now firms set prices. We need two assumptions.

1. Consumers always purchase from the cheapest seller (recall definition of homogeneous goods).
2. If two sellers charge the same price, consumers are split 50/50.

The second assumption is that demand takes the form:

$$q_i = \begin{cases} 
0 & \text{if } p_i > p_j \text{ or } p_i > a \\
\frac{a-p}{2b} & \text{if } p_i = p_j = p < a \\
\frac{a-p}{b} & \text{if } p_i < \min(a, p_j)
\end{cases}$$

Equilibrium:

$\{p_1^b, p_2^b, q_1^b, q_2^b\}$ is a Bertrand equilibrium (“Nash-in-prices” equilibrium) if:

1. a) given $p_2 = p_2^b$, $p_1^b$ solves $\max_{p_1} \pi_1(p_1, p_2^b) = (p_1 - c_1)q_1$

   b) given $p_1 = p_1^b$, $p_2^b$ solves $\max_{p_2} \pi_2(p_1^b, p_2) = (p_2 - c_2)q_2$

2. $q_1$ and $q_2$ are determined as above.
That is:

“No firm could increase its profit by changing its price, given the price set by the other firm.”

1. If costs are the same, a Bertrand equilibrium is price = marginal cost, with quantity supplied on the market equal to the perfectly competitive outcome (equally split between the two firms).

2. If costs differ, (say firm 1 has cost = $c_1$ where $c_1 < c_2$), then the firm with the lower cost charges $p_1 = c_2 - \varepsilon$, firm 2 sells zero quantity, and firm 1 sells quantity given by $q_1^b = \frac{(a - c_2 + \varepsilon)}{b}$.

Intuition:

If costs are the same, undercutting reduces price to marginal cost. If costs differ, undercutting reduces price to ”just below” the cost of the high-cost firm.

3.2.1 Bertrand w capacity constraints

Keep the assumption that costs are the same. Previous results (with no capacity constraints):

1. price competition reduces prices to unit costs

2. firms earn zero profits.
Do we believe these results? Maybe, maybe not. We might worry about the fact that the number of firms makes no difference. Undercutting reduces price to marginal cost with only 2 firms. (Consider the policy implications of this for the anti-trust authorities.)

Edgeworth had the idea that in the short run, firms may be constrained by capacity that limits their production levels.

Bertrand with capacity constraints:

- Consider the following example. There are two hair salons, both charging an initial price of $35.
- Under the basic Bertrand model (no capacity constraints), Salon-2 could take all customers if it set its price at $34.
- Suppose initial business is 12 haircuts per day per firm, and that the maximum capacity for either firm is 15 haircuts in a day.

Now the basic Bertrand result of marginal cost pricing won’t arise.
Assume the marginal cost of a haircut is simply \( c \). The regular Bertrand solution is of course

\[
p_1 = p_2 = c
\]
\[
\pi_1 = \pi_2 = 0
\]

And the total market output is the same as the competitive level of output

\[
q_1 + q_2 = Q^{\text{Competn}} = a - bp = a - bc
\]

But now add in the capacity constraint \((k_i)\) with the following features:

\[
k_i < Q^{\text{Competn}}
\]
\[
k_i > \frac{1}{2}(Q^{\text{Competn}})
\]

If \( p_2 > p_1 = c \), firm 2 loses some customers, but not all of them, because firm 1 cannot serve the entire residual demand.

\[\Rightarrow \pi_2 > 0 \text{ for } p_2 = p_1 + \varepsilon\]

In other words, there exists a unilateral profitable deviation from \( p_1 = p_2 = c \).

\( p_1 = p_2 = c \) is not a Nash equilibrium of the Bertrand game with capacity constraints (of the type described above).

In order the determine the Nash equilibrium of the price-setting game with capacity constraints, we can consider a two-stage game:

1. Firms invest in (choose) capacities; then
2. Firms choose prices

Example:

- Let \( c = 0 \), \( p = 1 - Q \), for \( Q = q_1 + q_2 \).
- Capacity of each firm: \( k_1, k_2 \)
- Capacity has been acquired at unit cost \( c_0 \in \left[\frac{3}{4}, 1\right] \)
Claim 1: Firms won’t invest in capacities above \( \frac{1}{3} \).

Proof 1: Monopoly profit is \( \frac{1}{4} \). Net profit is negative for \( k_i > \frac{1}{3} \).

Claim 2: If \( k_i \leq \frac{1}{3} \), both firms charging \( p^* = 1 - (k_1 + k_2) \) is an equilibrium, and this equilibrium is unique.

Proof: Suppose firm 2 is charging \( p^* \) and has capacity \( k_2 \). What should firm 1 do?
- Can firm 1 lower price? No, because at \( p^* \), both firms are at capacity, so there’s no use in lowering price.

- Can firm 1 raise price? Assume she charges \( p_1 \geq p^* \). Let \( q_1 \) be the quantity that firm 1 sells at this high price. Then profit is \( (1 - q_1 - k_2)q_1 \). Derivative is \( 1 - 2q_1 - k_2 \). This is greater than 0 when \( q_1 = k_1 \) since \( k_i \leq \frac{1}{3} \), i.e. lowering quantity, or equivalently, raising price is not profitable.

Claim 3: Given Claim 2, it is optimal for firms to set capacities equal to Cournot levels in the first stage.

Proof 3, step 1: In second stage, \( p^* = 1 - (k_1 + k_2) \).

Proof 3, step 2: Hence, firm 1’s profit function is: \( (1 - k_1 - k_2)k_1 - c_0k_1 \) with derivative (or best-response function), \( 1 - 2k_1 - k_2 - c_0 \). Hence, \( k_1 = k_2 = \frac{1 - c_0}{3} \).

CRUCIAL ASSUMPTION: setting capacities above \( \frac{1}{3} \) is not profitable. This was due to the high investment costs.

We made this rather problematic assumption since it turns out that if firms build capacity above the Cournot level, then there is no pure strategy equilibrium in the second stage. Mixed strategies are pretty tough to derive; you can look at Kreps and Scheinkman (1983) for a full derivation.

Main message of the above model, however, is that capacity constraints can soften Bertrand competition.

Sometimes regulation/government can help firms "coordinate" on capacity limits. Example: zoning laws in communities prevent large stores from being set up.
3.3 Overview

Comparison of Equilibrium in Models Covered So Far:

Monopoly (M), Cournot (C), Stackelberg

\[ CS^M < CS^C < CS^S < CS^{PC} \]

\[ PS^M > PS^C > PS^S > PS^{PC} = 0 \]

\[ DWL^M > DWL^C > DWL^S > DWL^{PC} = 0 \]