Overlapping Generation (OLG) Modeling in GAMS

Econ 567 Computer Modeling: Project II

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Objectives

- How to optimize consumption to maximize utility?
- How things will be affected by a saving tax? (tax on capital)
- How can tax-given-back benefit taxpayers, and exhibit good externality?
- Sensitivity Analysis on: (a) optimal saving tax: max utility; (b) inter-temporary consumption preference.
- Interpret this OLG Model in different dimensions=)

Basic Assumptions

- People live for two periods, and only work in period one.
- Cobb-Douglas production function; two factors: K and L.
- 100 % depreciation rate. No capital accumulation.
- Market is clear. Products are consumed/as capital input.
- An ad valorem saving tax is imposed on capital.
- CRS Cobb-Douglas utility function in base model; extends to IRS utility function when tax is given back.
- In the model "with saving tax", the tax is used only by the king (not increase any utility of taxpayer); in "tax given back" model, the tax is used to increase people's utility.

Symbol System

Endogenous Variables		Exogenous Variables		
		note	meaning	values
U	*utility level*	**rho**	*share parameter of current consumption in utility*	0.5
C1	*consumption in period 1*	**beta**	*share parameter of tax in utility*	0.5
C2	*consumption in period 2*	**alpha**	*share parameter of labor in production function*	0.5
K	*capital level*	**A**	*technology factor*	2
Q	*output level*	**L**	*labor supply*	10
IR	*interest rate*	**t**	*tax rate*	0.01
W	*wages*			
Tax	*Total tax*			

Equation System

$$Utility\ \ function: \qquad U_{base} = C_1^{\rho} * C_2^{(1-\rho)};$$

$$U_{tax-given-back} = C_1^{\rho} * C_2^{(1-\rho)} + (Tax)^{\beta};$$

$$Production\ \ function: \qquad Q = A * K^{(1-\alpha)} * L^{\alpha};$$

$$Wage\ \ rate\ \ function: \qquad W = A * \alpha * (K / L)^{(1-\alpha)};$$

$$Interest\ \ rate\ \ function: \qquad 1 + IR = A * (1-\alpha) * (L / K)^{\alpha};$$

$$1 + IR_{taxed} = A * (1-\alpha) * (L / K)^{\alpha} - t;$$

$$Consumption\ \ allocation: \quad C_1 + C_2 / (1 + IR) = W * L;$$

$$Utility\ \ maximization\ \ : \qquad C_2 / C_1 = (1 + IR) * (1-\rho) / \rho$$

$$Market\ \ clear\ \ condition: \quad Q = C_1 + C_2 + K + Tax;$$

$$Tax\ \ quation: \qquad Tax = t * K$$

Optimized Solutions from GAMS

Table.2 CGE optimized output for the three models under initial setting

Exogenous Variables		Base Value	with Saving Tax	Tax Given Back
U	*utility level*	5	4.950495	5.263592
C1	*consumption in period one*	5	4.950495	4.950495
C2	*consumption in period two*	5	4.950495	4.950495
K	*capital level*	10	9.802960	9.802960
Q	*output level*	20	19.801980	19.801980
IR	*interest rate*	0	0	0
W	*wages*	1	0.990099	0.990099
Tax	*saving tax*	NA	0.098030	0.098030

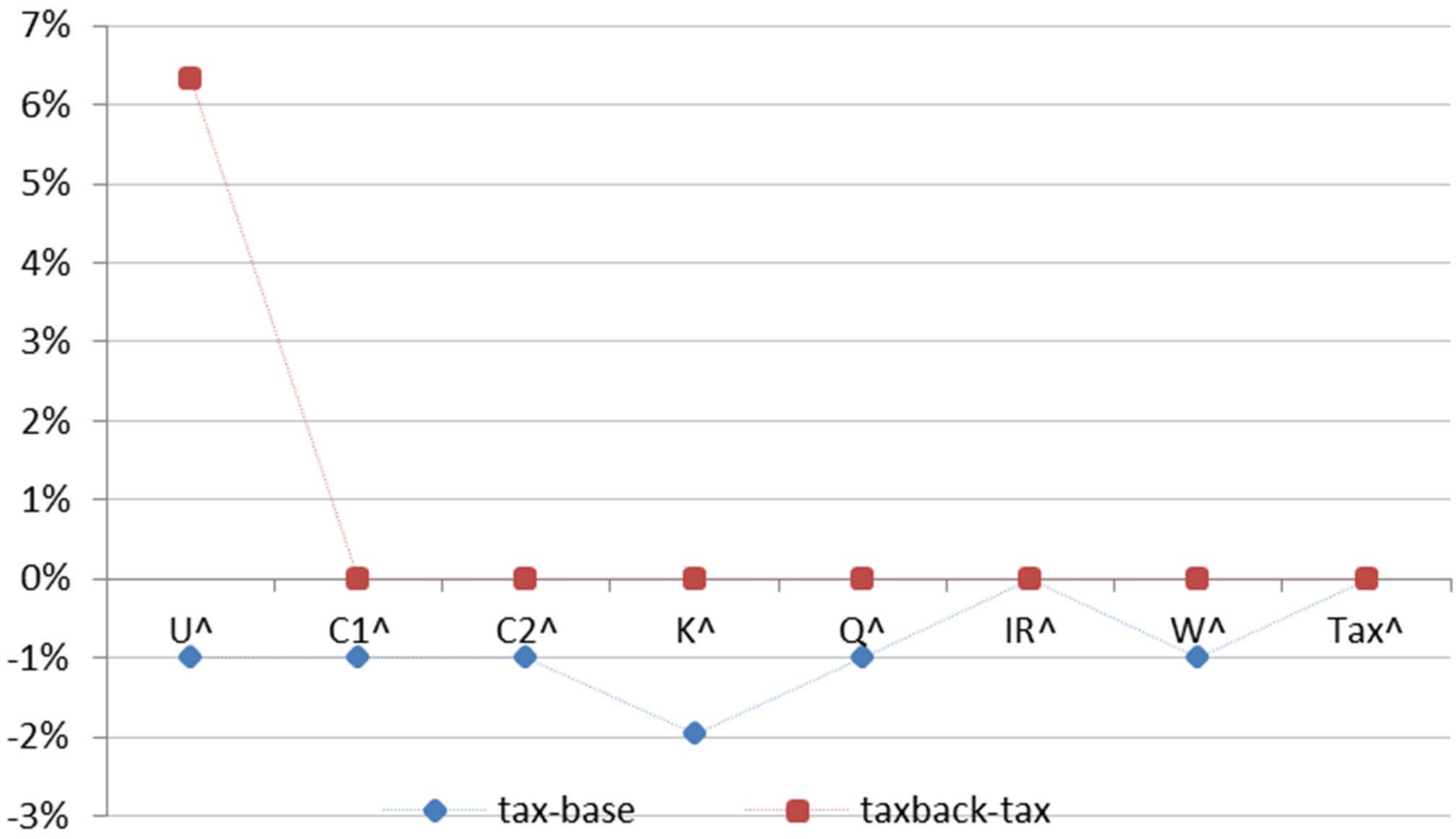

Figure.2 Percentage change in basic models

Extension: Sensitivity Analysis

(1) Change t: find out the optimal tax that maximize utility. how can a rise of tax affect the optimal solution?

(2) Change rho: does inter-temporary consumption preference matters?

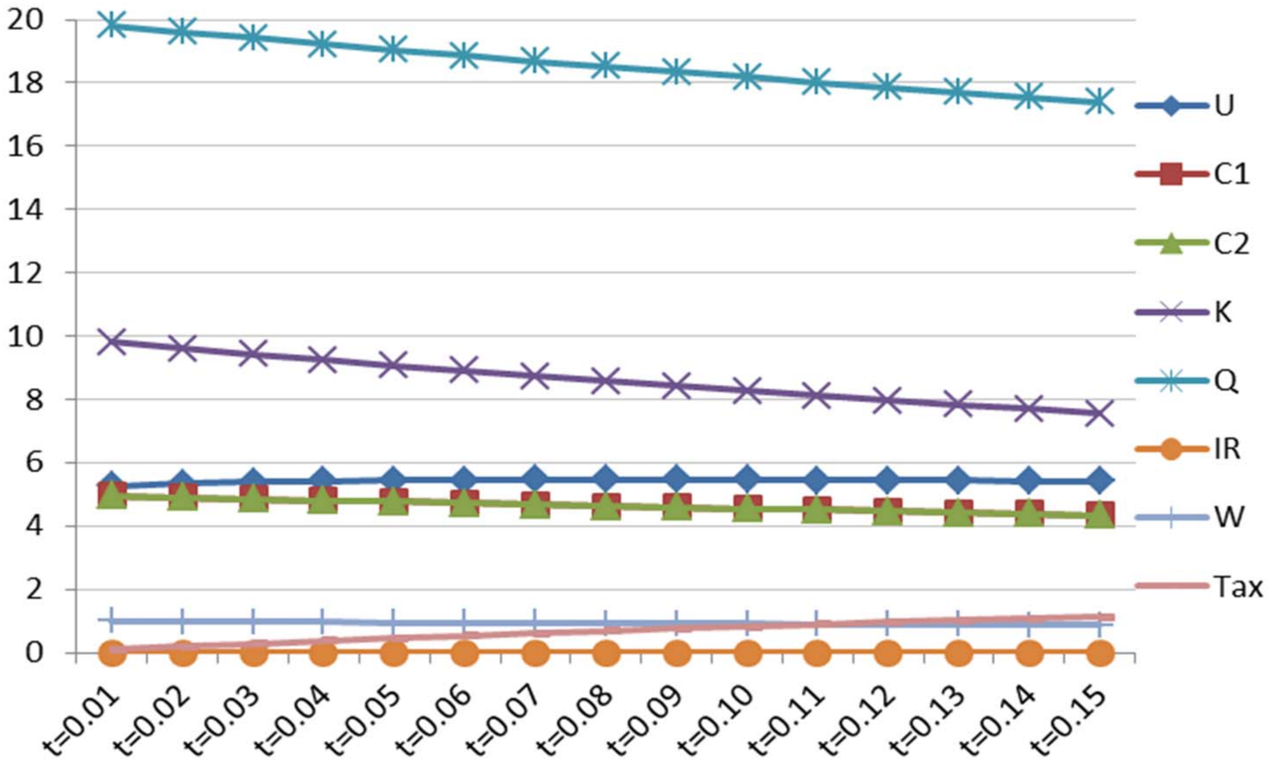

Figure.3 sensitivity analysis for tax rate on tax back model

	t=0.01	t=0.02	t=0.03	t=0.04	t=0.05	t=0.06	t=0.07
U	5.264	5.34	5.386	5.416	5.435	5.448	5.455
C1	4.95	4.902	4.854	4.808	4.762	4.717	4.673
C2	4.95	4.902	4.854	4.808	4.762	4.717	4.673
K	9.803	9.612	9.426	9.246	9.07	8.9	8.734
Q	19.802	19.608	19.417	19.231	19.048	18.868	18.692
IR	0	0	0	0	0	0	0
W	0.99	0.98	0.971	0.962	0.952	0.943	0.935
Tax	0.098	0.192	0.283	0.37	0.454	0.534	0.611
marginal utility analysis							
U(Tax)=tax^0.5	0.313	0.438	0.532	0.608	0.674	0.731	0.782
U(C)=(C1*C2)^0.5	4.950	4.902	4.854	4.808	4.762	4.717	4.673
dU/U	NA	0.014	0.009	0.006	0.004	0.002	0.001
dU(T)/d(T)	NA	1.331	1.031	0.877	0.780	0.712	0.661
dU(C)/d(C)	NA	0.500	0.500	0.500	0.500	0.500	0.500

	t=0.08	t=0.09	t=0.10	t=0.11	t=0.12	t=0.13	t=0.14	t=0.15
	5.458	5.458	5.455	5.449	5.442	5.434	5.424	5.413
	4.63	4.587	4.545	4.505	4.464	4.425	4.386	4.348
	4.63	4.587	4.545	4.505	4.464	4.425	4.386	4.348
	8.573	8.417	8.264	8.116	7.972	7.831	7.695	7.561
	18.519	18.349	18.182	18.018	17.857	17.699	17.544	17.391
	0	0	0	0	0	0	0	0
	0.926	0.917	0.909	0.901	0.893	0.885	0.877	0.87
	0.686	0.758	0.826	0.893	0.957	1.018	1.077	1.134
marginal utility analysis								
	0.8282512	0.87063195	0.90884542	0.94498677	0.97826377	1.00895986	1.03778611	1.06489436
	4.63	4.587	4.545	4.505	4.464	4.425	4.386	4.348
	0.00055	0	-0.0005497	-0.0010999	-0.0012846	-0.00147	-0.0018403	-0.002028
	0.6211504	0.58862201	0.56196275	0.53942315	0.51995306	0.50321463	0.48858042	0.47558344
	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

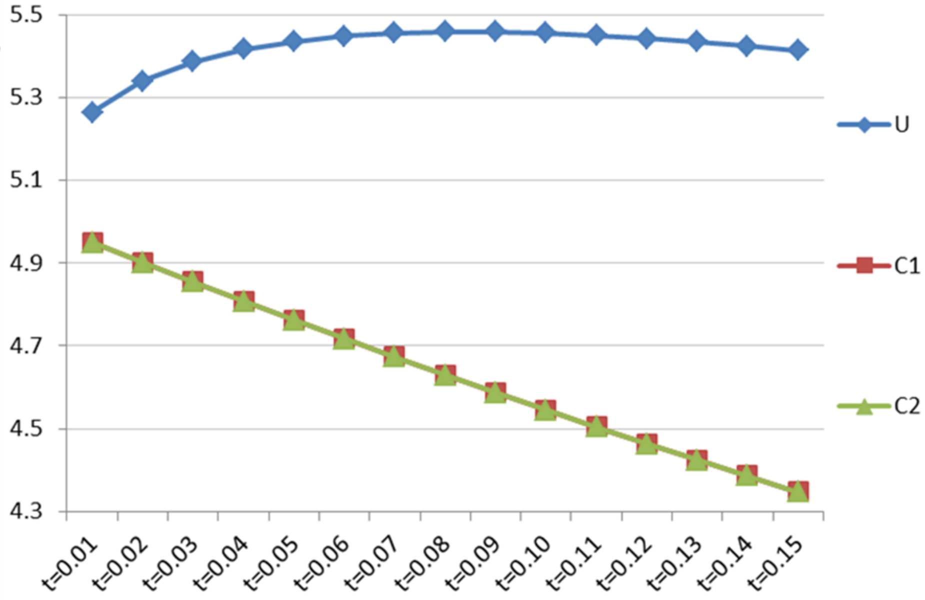

Figure.4 simplified version of Figure.3: focus on utility and consumptions

In figure.4

(1) utility reaches the highest when t=0.09 (the optimal saving tax);

(2) The consumption in both periods decreases at the same rate in our settings

In figure.5

(1) Marginal utility decreases (too small to detect, due to scale)

(2) Marginal consumption utility decreases as t increases.

(3) Marginal tax utility decreases as t increases.

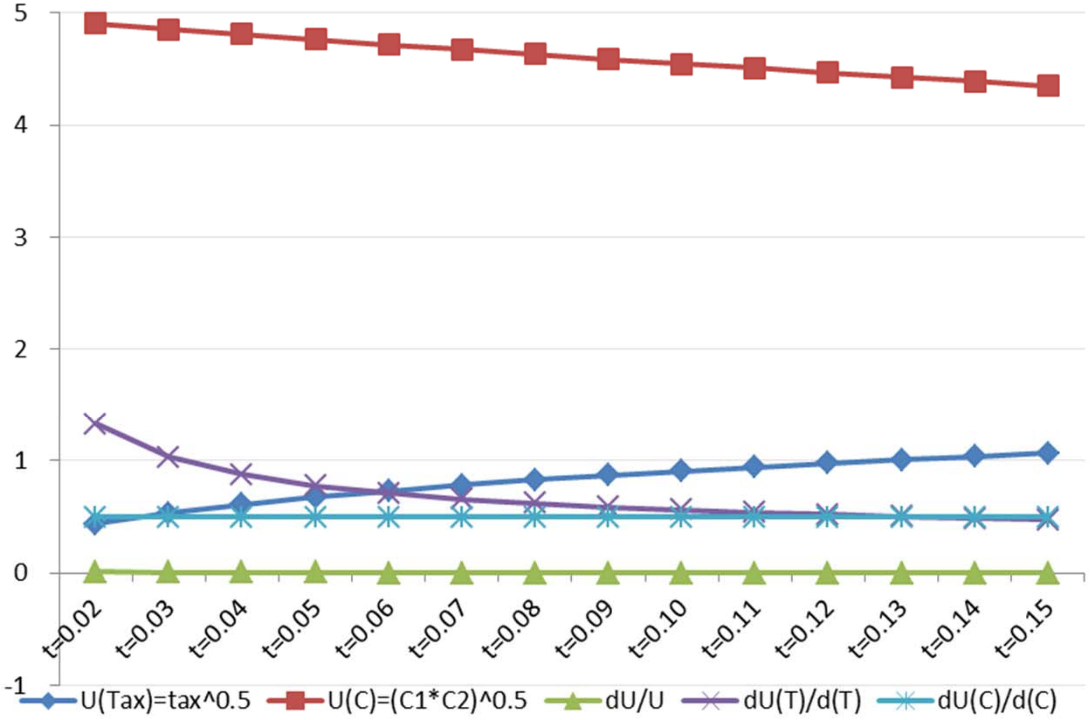

Figure.5 sensitivity analysis of marginal effects in tax-given-back model with varied

(2) Inter-temporary consumption preference

Table.3 Change rho based on tax-given-back model

	rho=0.3	rho=0.4	rho=0.5	rho=0.6	rho=0.7
U	5.688	5.364	5.264	5.364	5.688
C1	2.97	3.96	4.95	5.941	6.931
C2	6.931	5.941	4.95	3.96	2.97
K	9.803	9.803	9.803	9.803	9.803
Q	19.802	19.802	19.802	19.802	19.802
IR	0	0	0	0	0
W	0.99	0.99	0.99	0.99	0.99
Tax	0.098	0.098	0.098	0.098	0.098

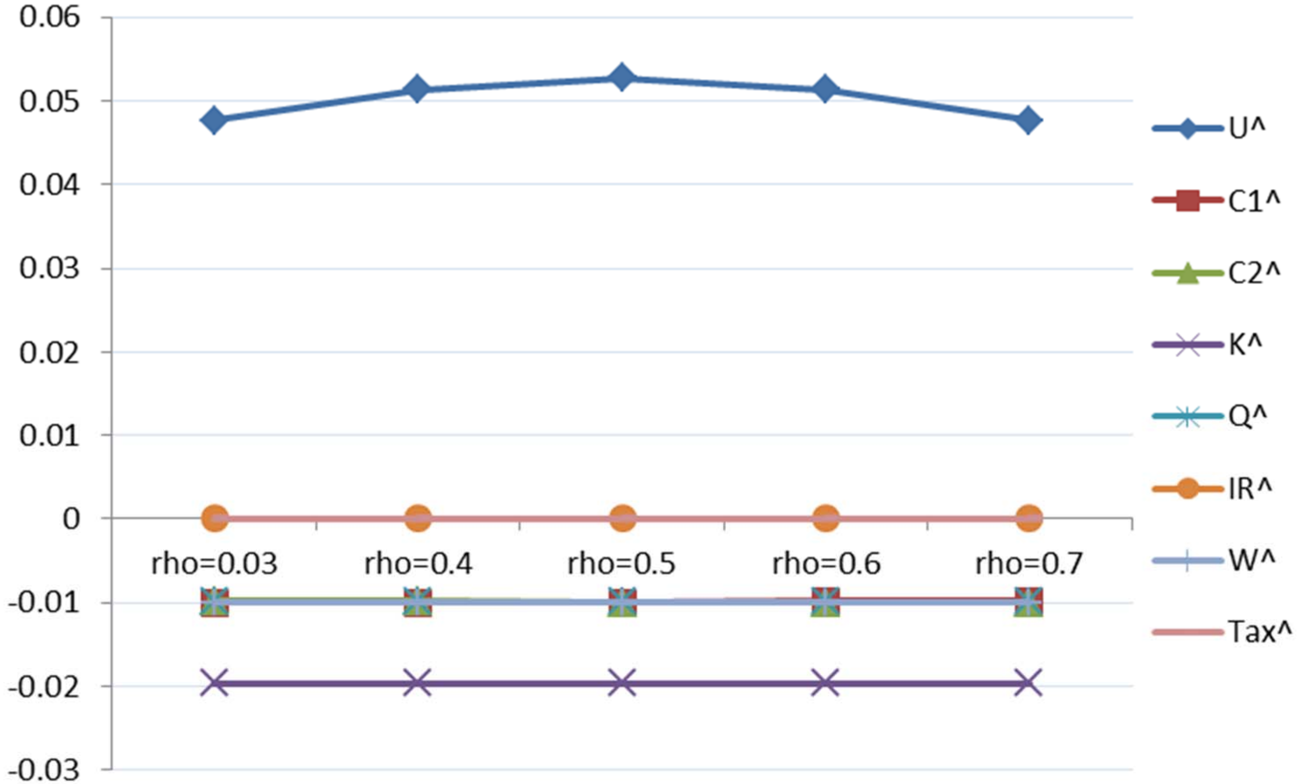

Figure.7 percentage change from model1 to 3 as rho changes

Conclusion and Recommendations

- The three ***Basic Models*** show: saving tax will decrease the utility, and will be partly compensated by a tax-given-back;
- The ***optimal saving tax*** : tax has decreasing marginal utility while consumption has constant marginal utility. Utility first increases and then decreases as tax rate rises.
- The ***inter-temporary consumption preferences*** shows: tax policy works best where people have equal consumption preference (rho=0.5) in our model settings. Policy makers should take people's consumption behavior into account.

Some interesting tricks in GAMS …

- GAMS Code:

```
t=0.01;
MODEL
ca3/Utility2,Production,Wage,InterestRate2,Decision,Consumption,MarketClear,TaxTotal/;
SOLVE ca3 using nlp maximizing U;
```

```
*REPORT
K_R('SAVTAXBACK')=K.L;
Q_R('SAVTAXBACK')=Q.L;
IR_R('SAVTAXBACK')=IR.L;
U_R('SAVTAXBACK')=U.L;
C1_R('SAVTAXBACK')=C1.L;
C2_R('SAVTAXBACK')=C2.L;
W_R('SAVTAXBACK')=W.L;
TAX_R('SAVTAXBACK')=TAX.L;
```

```
*change tax rate in the Tax-back Model
t=0.03;
SOLVE ca3 using nlp maximizing U;
t=0.05;
SOLVE ca3 using nlp maximizing U;
t=0.07;
SOLVE ca3 using nlp maximizing U;
```

- **Thank you!!!**