DUKE UNIVERSITY
ECONOMICS MA

Overlapping Generation (OLG) Modeling in GAMS

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An OLG model is a type of economic model in which agents live a finite length of time long enough to overlap with at least one period of another agent's life. We first developed a base model without tax, then add a tax, followed with tax-benefit, and extended with sensitivity analysis of free parameters.

Abstract:

An overlapping generations model, abbreviated as OLG model, is a type of economic model in which agents live a finite length of time long enough to overlap with at least one period of another agent's life. In this report, we first developed a base model with no tax but only saving and capital accumulation across different generations. Then we added a saving tax into the model. Finally we added tax into the utility function to explore the externality of the tax. This OLG model is a standard CGE application with GAMS.

Keywords:

Computable General Equilibrium (CGE), Two-Period Overlapping Generation (OLG) Model, Sensitivity Analyses

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1. Introduction

An overlapping generations model (OLG) is a type of economic model in which agents live a finite length of time long enough to overlap with at least one period of another agent's life. Notable improvements have been made by Maurice Allais in 1947, Paul Samuelson in 1958, and Peter Diamond in 1965. The most well-known models are Samuelson's model and Diamonds' model.

Samuelson's OLG model has the following characteristics.

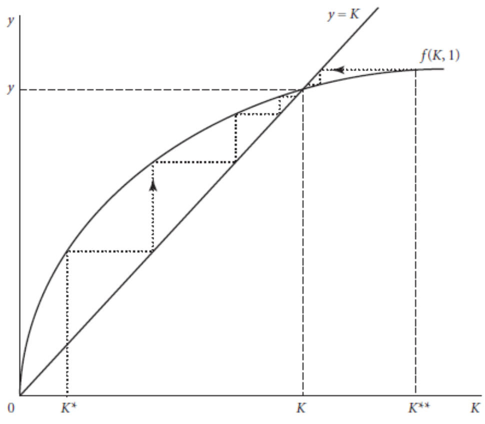
First, individuals live for two periods, young and old. Some individuals are born in period t, denoted by L^t_t. L^{t-1}_t denotes number of old people in period t. Since the economy begins in period 1, there is a group of people who are already old. They are referred to as the initial old and denoted as L_0 .People born in period t will not die until the end of period t+1, thus $L^t_t = L^t_{t+1}$.Population grows at a constant rate n: $L^t_t=(1+n)^tL_0$.

Second, there is only one good in this economy, and it cannot endure for more than one period. But the good could be exchanged for money, which endures more than one period.

Third, each individual receives a fixed endowment of this good at birth. This endowment is denoted as k. Individuals work with this endowment and creates a real income equal to the value of good k produced. Under this framework, individuals only work during the young phase of their life.

Fourth, utility of an individual comes from consumptions in both periods. The individual could consume all his products in the first period and has nothing to consume in the next period. But this is not a Pareto optimal allocation since the young generation could give half of product to the old to make everyone better off without making one worse off, given that the time span is infinite and the utility function is symmetric and strictly concave. Each individual could consume half of the good and sell the rest to the old generation in exchange for money. They could save this money to buy good when they become old. In the context of modern societies, it can be seen as people exchange good with government in exchange for debt obligation, which would be paid back with interest in the form of pension. In the Samuelson's OLG model, policies affect only the distribution of welfare between the young and the old; the higher is the interest rate, the higher is the consumption of the old relative to that of the young.

Diamond added production into this model. He assumes that labor and capital markets are perfectly competitive and the aggregate production technology is CRS, Y = F(K,L). Individuals are endowed with k_t, with which they work and create a real production of y and real income w_t. They consume part of the income and save the rest. The aggregate savings serve as the initial endowment for the next generation. The generation would consume their savings in the second phase. In Diamond's model, individuals tend to save more than is socially optimal, leading to dynamic inefficiency.



2. Objectives of Our OLG modeling

Our OLG model mainly bases on Diamond's model.
We aim to find out the optimal solutions in the following several cases:

(1) How can people optimize their consumption in different life periods, in order to maximize utility?
(2) What will happen if government imposes a saving tax? (capital is taxed)
(3) How and to what extent could a tax-given-back benefit taxpayer, and exhibit good externality?
(4) All of the analyses above need to fix some free parameters first. But these free parameters contain many interesting information and can affect the results a lot. By taking a second look at these free parameters, we did three sensitivity analyses to explore the in-depth logical and practical impacts of: (a) tax level and corresponding effects; (b) inter-temporary consumption preference.
(5) Provide not only analytical analysis of this standard framework, but also numerical and visualized explanation of the computable general equilibrium solution for this overlapping generation issue.

3. Basic Assumptions and Simple Explanations

(1) People live for two generations, young and old. They only work in young generation.
(2) Only one good in the society, seed, which can be invested, produced, consumed and saved.
(3) Production function has the Cobb-Douglas form and there are two factors of production K and L.
(4) People maximize the utility from today's consumption and tomorrow's consumption within budget constraint.
(5) Market is clear. All goods produced are either consumed or reinvested.
(6) Capital is fully depreciated.
(7) In the second and third models, a saving tax is imposed. Capital tax is an ad valorem tax.
(8) In the third model, taxes are used for the production of public good, which brings utility to the people. In this case, the tax given back increases people's utility, so the utility function is no longer a CRS Cobb-Douglas function as in Model 1 and 2. Now it is IRS (increasing Return to Scale) utility.
(9) In the extension, all the previous assumptions are still held, but we change the free parameters to a reasonable interval, to see a better refined whole picture. Suppose all other assumptions work well in this extension.

4. Computable General Equilibrium (CGE) Model development:

4.1 Symbol System

Table.1 Endogenous and Exogenous Variables used in our Model

Endogenous Variables		**Exogenous Variables**		
U	*utility level*	note	Meaning	values
C1	*consumption in period one*	**rho**	*share parameter of current consumption in utility*	0.5
C2	*consumption in period two*	**beta**	*share parameter of tax in utility*	0.5
K	*capital level*	**alpha**	*share parameter of labor in production function*	0.5
Q	*output level*	**A**	*technology factor*	2
IR	*interest rate*	**L**	*labor supply*	10
W	*wages*	**t**	*tax rate*	0.01
Tax	*Total tax*			

* Here, assume equal inter-temporary consumption preference, capital-labor ratio = 1, and certain tax utility.

4.2 Equation System

$$Utility\ \ function: \quad U_{base} = C_1^{\rho} * C_2^{(1-\rho)};$$

$$U_{tax-given-back} = C_1^{\rho} * C_2^{(1-\rho)} + (Tax)^{\beta};$$

$$Production\ \ function: \quad Q = A * K^{(1-\alpha)} * L^{\alpha};$$

$$Wage\ \ rate\ \ function: \quad W = A * \alpha * (K / L)^{(1-\alpha)};$$

$$Interest\ \ rate\ \ function: \quad 1 + IR = A * (1-\alpha) * (L / K)^{\alpha};$$

$$1 + IR_{taxed} = A * (1-\alpha) * (L / K)^{\alpha} - t;$$

$$Consumption\ \ allocation: \quad C_1 + C_2 / (1 + IR) = W * L;$$

$$Utility\ \ maximization\ \ : \quad C_2 / C_1 = (1 + IR) * (1-\rho) / \rho$$

$$Market\ \ clear\ \ condition: \quad Q = C_1 + C_2 + K + Tax;$$

$$Taxe\ \ quation: \quad Tax = t * K$$

$$-\ \ in\ \ base\ \ model: \quad t = 0\ \ no\ \ saving\ \ tax$$

$$-\ \ in\ \ saving\ \ tax\ \ model: \quad t \neq 0\ \ tax\ \ not\ \ given\ \ back(no\ \ utility)$$

$$-\ \ in\ \ tax\ \ back\ \ model: \quad t \neq 0\ \ tax\ \ given\ \ back\ \ to\ \ tax\ \ payers,$$

$$positively\ \ affect\ \ utility\ \ function$$

$$-sensitivity\ \ analysis: simulations\ \ are\ \ conducted\ \ by\ \ resetting\ \ the$$

$$values\ \ of\ \ share\ \ parameters\ \ to\ \ a\ \ possible$$

$$interval\ \ to\ \ characterize\ \ different\ \ cases$$

4.3 Optimized Solutions from GAMS

Beta=0.7 indicated the utility-rising-up by the given back of tax. The impact of a 30% saving tax[1] can now be shown as:

Table.2 CGE optimized output for the three models under initial setting

Exogenous Variables		Base Value	with Saving Tax	Tax Given Back
U	*utility level*	5	4.950495	5.263592
C1	*consumption in period one*	5	4.950495	4.950495
C2	*consumption in period two*	5	4.950495	4.950495
K	*capital level*	10	9.802960	9.802960
Q	*output level*	20	19.801980	19.801980
IR	*interest rate*	0	0	0
W	*wages*	1	0.990099	0.990099
Tax	*saving tax*	NA	0.098030	0.098030

From both Table.2 and Figure.1 (next page), we can compare the three group of result horizontally.

[1] Solution came out in this order: K_R, Q_R, IR_R, U_R, C1_R, C2_R, W_R, TAX_R, as GAMS calculated. We re-organized the output to put them together for a comparison. You can see some interesting following simulation by changing the values for free parameters in later part.

The utility level decreases a little as a small saving tax (on capital) implemented, and utility level increases a lot (bigger than the previous decrease) when the tax used for people and add a separate term to the power of beta to the utility function. The total utility level is now above the initial utility level in this setting. It seems pretty good: tax should be collected to benefit taxpayers☺.

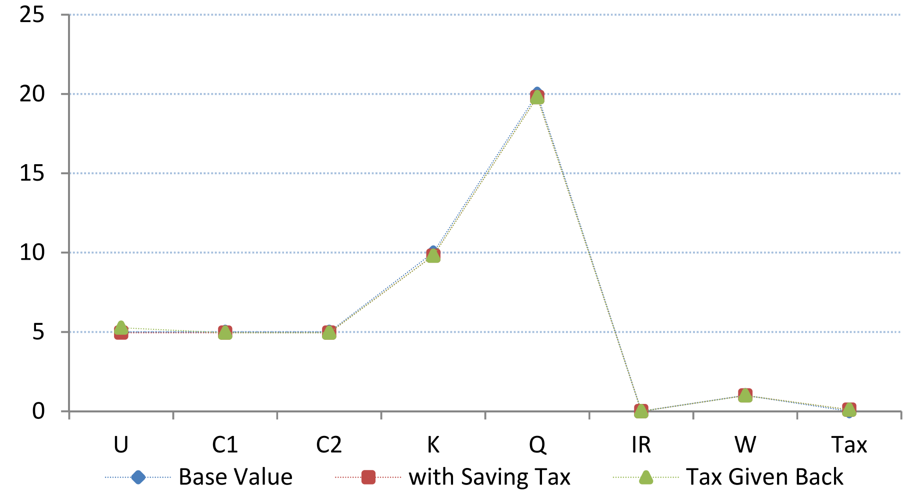


Figure.1 Graphical comparison of three CGE output under initial settings

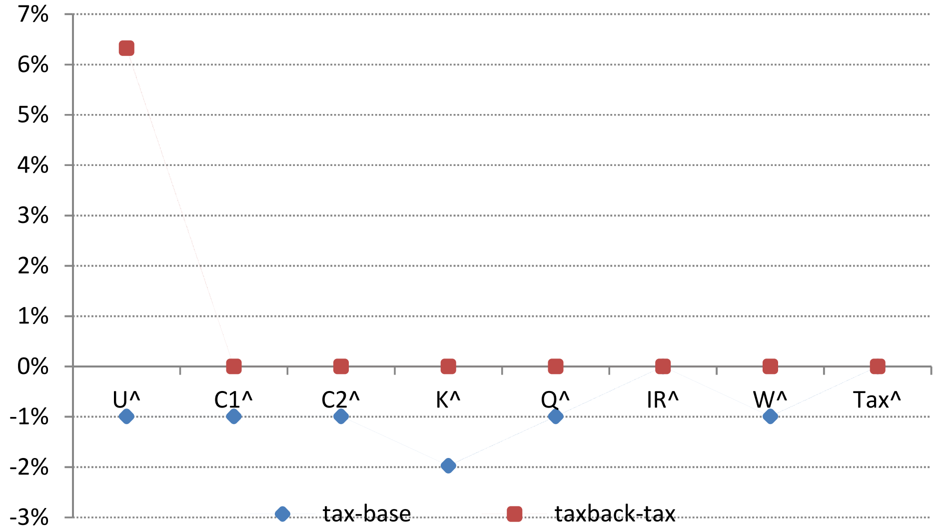


Figure.2 Percentage change in basic models

4.4 Simple interpretations:

In this part, we focus on the causal relationship inside each independent model.

(1) Basic Story:

People live for two generations, young and old. They only work in young generation. Everyone is endowed with K at birth, with which they produce Q= A*K^(1-alpha)*L^alpha. People receive the products paid to labor as real income. Part of their income is consumed now and the rest are saved for old time (Consumption). Whatever they saved today would allow them to consume (1+IR)*savings tomorrow. They allocate consumptions in two periods in a way that maximize their utility within the budget constraint (Decision). The initial capital is fully depreciated. Products paid to capital would cover the original capital and a certain rent, but the rent could be negative (Interest). All products produced in a certain period are either consumed by the young generation, the old generation or reinvested (Market Clear).

(2) With a Saving Tax:
Saving tax reduces rent paid to savings. The real rent to capital is the original rent minus tax rate. Part of the payment to capital is taken away from government and not returned, thus less capital can be reinvested. Q decreases subsequently. Meanwhile, wage rate declines since workers become less productive due to the decline of capital per capita. The decrease of real income force one to consume less in the first period (C1 declines) and the second period C2. As both C1 and C2 decrease, utility decreases.

(3) Tax Externality (Saving tax used for people's benefit):
Unlike in the second case, taxes are used for production of public good which brings utility to people later. This increases people's utility without changing the formation of any other terms. Since the effect of tax is different from consumption, we set it as an additive term, to the power of beta. Here

4.5 Extension: Sensitivity Analysis via reset free parameters

What will happen if we change the values of some free parameters? ...
Logic structure: we most interested in analyzing two free parameters: t (tax rate) and rho (share parameter of inter-temporary consumption parameter).

(1) Change tax rate: how can a rise of tax affect the optimal solution?

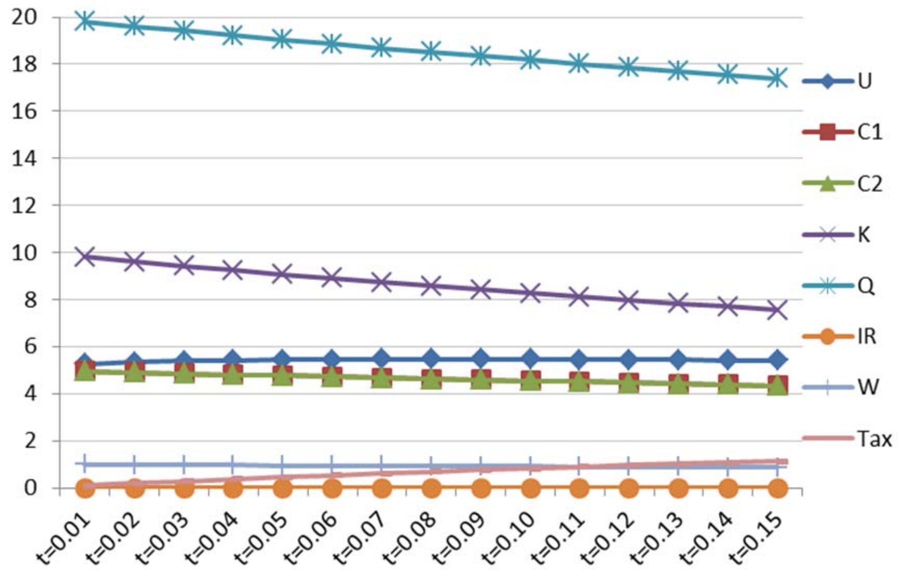


Figure.3 sensitivity analysis for tax rate on tax back model

The choice of an optimal tariff is always something of interest in policy analysis. So, we want to explore the optimal saving tax in our initial settings, under a tax-given-back model. This is very applicable and reliable. We set t into an interval from 0.01 to 0.15, with each step length equal to 0.01. We define the "optimal saving rate" as the rate that maximizes our utility. A simple glance of the overall graph above confirms our choice of the interval: the line of utility first increases, and decreases slightly after certain values.

Table.3 Change tax rate based on tax-given-back model

	t=0.01	t=0.02	t=0.03	t=0.04	t=0.05	t=0.06	t=0.07
U	5.264	5.34	5.386	5.416	5.435	5.448	5.455
C1	4.95	4.902	4.854	4.808	4.762	4.717	4.673
C2	4.95	4.902	4.854	4.808	4.762	4.717	4.673
K	9.803	9.612	9.426	9.246	9.07	8.9	8.734
Q	19.802	19.608	19.417	19.231	19.048	18.868	18.692
IR	0	0	0	0	0	0	0
W	0.99	0.98	0.971	0.962	0.952	0.943	0.935
Tax	0.098	0.192	0.283	0.37	0.454	0.534	0.611
marginal utility analysis							
U(Tax)=tax^0.5	0.313	0.438	0.532	0.608	0.674	0.731	0.782
U(C)=(C1*C2)^0.5	4.950	4.902	4.854	4.808	4.762	4.717	4.673
dU/U	NA	0.014	0.009	0.006	0.004	0.002	0.001
dU(T)/d(T)	NA	1.331	1.031	0.877	0.780	0.712	0.661
dU(C)/d(C)	NA	0.500	0.500	0.500	0.500	0.500	0.500

t=0.08	t=0.09	t=0.10	t=0.11	t=0.12	t=0.13	t=0.14	t=0.15
5.458	5.458	5.455	5.449	5.442	5.434	5.424	5.413
4.63	4.587	4.545	4.505	4.464	4.425	4.386	4.348
4.63	4.587	4.545	4.505	4.464	4.425	4.386	4.348
8.573	8.417	8.264	8.116	7.972	7.831	7.695	7.561
18.519	18.349	18.182	18.018	17.857	17.699	17.544	17.391
0	0	0	0	0	0	0	0
0.926	0.917	0.909	0.901	0.893	0.885	0.877	0.87
0.686	0.758	0.826	0.893	0.957	1.018	1.077	1.134
marginal utility analysis							
0.8282512	0.87063195	0.90884542	0.94498677	0.97826377	1.00895986	1.03778611	1.06489436
4.63	4.587	4.545	4.505	4.464	4.425	4.386	4.348
0.00055	0	-0.0005497	-0.0010999	-0.0012846	-0.00147	-0.0018403	-0.002028
0.6211504	0.58862201	0.56196275	0.53942315	0.51995306	0.50321463	0.48858042	0.47558344
0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

1. U=C1^0.5*C2^0.5+Tax^0.5: Utility first increase because of the externality of tax. However, the marginal utility of tax is decreasing while the marginal utility of consumption is constant.

An important finding!!! The marginal utility reaches 0 at t=0.09, i.e., a saving tax equal to 9% can give people highest utility in this setting. When t<0.09, the marginal utility is positive with decreasing absolute values, which mean the utility increases at decreasing speed. After the threshold of t=0.09, when t>0.09, utility decreases.

2. C1 and C2 remains the same as tax increase since C2/C1= (1+IR)*(1-rho)/rho, IR=0 and rho=0.5.

3. The optimal interest rate is zero in all cases because in the stable-state, capital stabilize at a certain level, rent paid to capital is only sufficient for reproduction of the original capital.

Since (1+IR)=A*(1-alpha)*(L/K)*alpha, IR remains zero in all stable states.

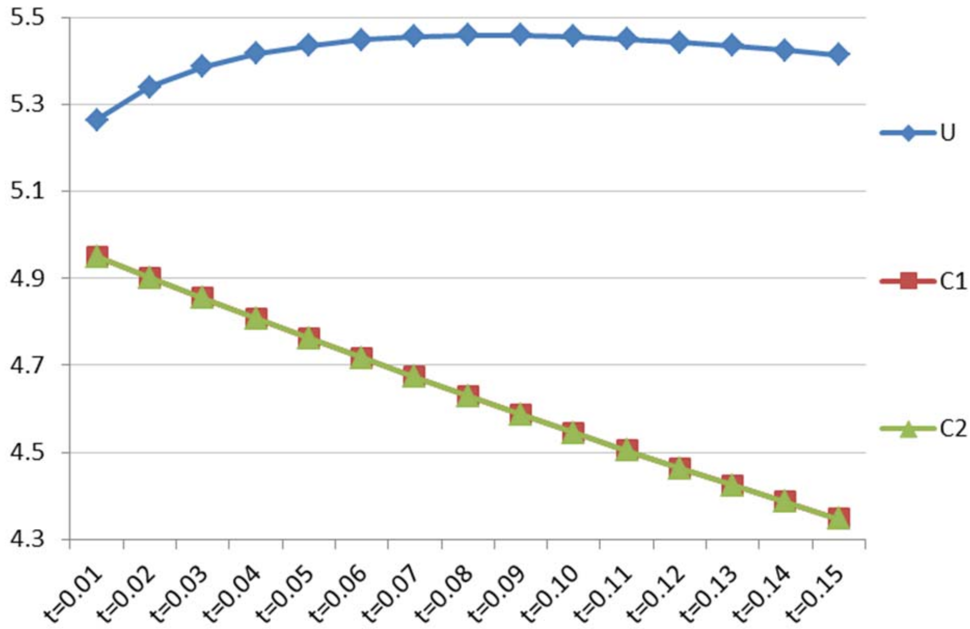


Figure.4 simplified version of Figure.3: focus on utility and consumptions

"Figure.4" is in line with our conclusions above. Now we only focus on the change of utility and consumptions in two periods. The utility does increase at decreasing rate at reaches the highest point when t=0.09. The consumptions in two periods (C1 and C2) are always equal and they change together in the same direction with the same magnitude.

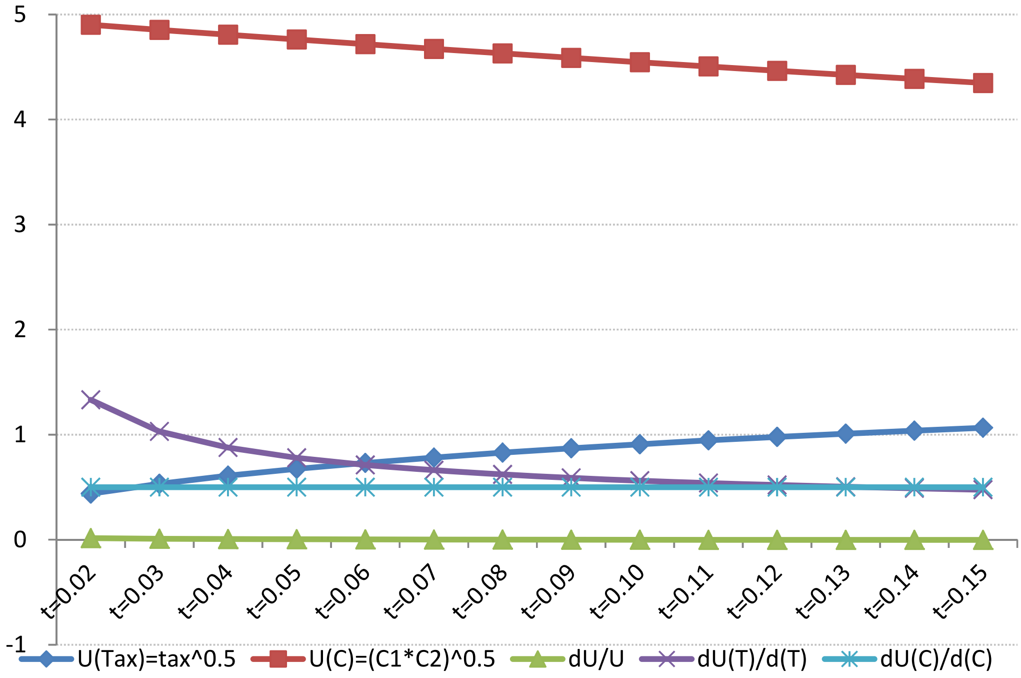


Figure.5 sensitivity analysis of marginal effects in tax-given-back model with varied

"Figure.5" is a visualization of different marginal utilities change with varied tax. Here "marginal utility" only indicate the format, they are actually marginal effect of different elements. It's interesting that: as the tax rate increases, marginal utility from consumption is monotonic decreasing while marginal utility of tax is monotonic decreasing. This is due to the change in relative proportion of consumption and tax in the utility function. Marginal change in the total utility is too small to be notice in this graph. Marginal change of Tax and Consumption both reach the highest when t=0.02.

(2) Change rho: does inter-temporary consumption preference matters?

Table.3 Change rho based on tax-given-back model

	rho=0.3	rho=0.4	rho=0.5	rho=0.6	rho=0.7
U	5.688	5.364	5.264	5.364	5.688
C1	2.97	3.96	4.95	5.941	6.931
C2	6.931	5.941	4.95	3.96	2.97
K	9.803	9.803	9.803	9.803	9.803
Q	19.802	19.802	19.802	19.802	19.802
IR	0	0	0	0	0
W	0.99	0.99	0.99	0.99	0.99
Tax	0.098	0.098	0.098	0.098	0.098

Simple observations and conclusions:

1. The ratio between C1 and C2 equals to rho/(1-rho) since C2/C1=(1+IR)*(1-rho)/rho, IR=0. This numerical result matches the economics theory behind it. We can get the same conclusion by using Lagrangian Multiplier and solve for the optimized solutions.

2. All other terms except for U, are not changed as preference parameter rho changes. This can be straightforwardly seen in the Figure.6 below. These terms won't change with rho since their final formula expressions don't have rho. We can call them "preference-robust" variables.

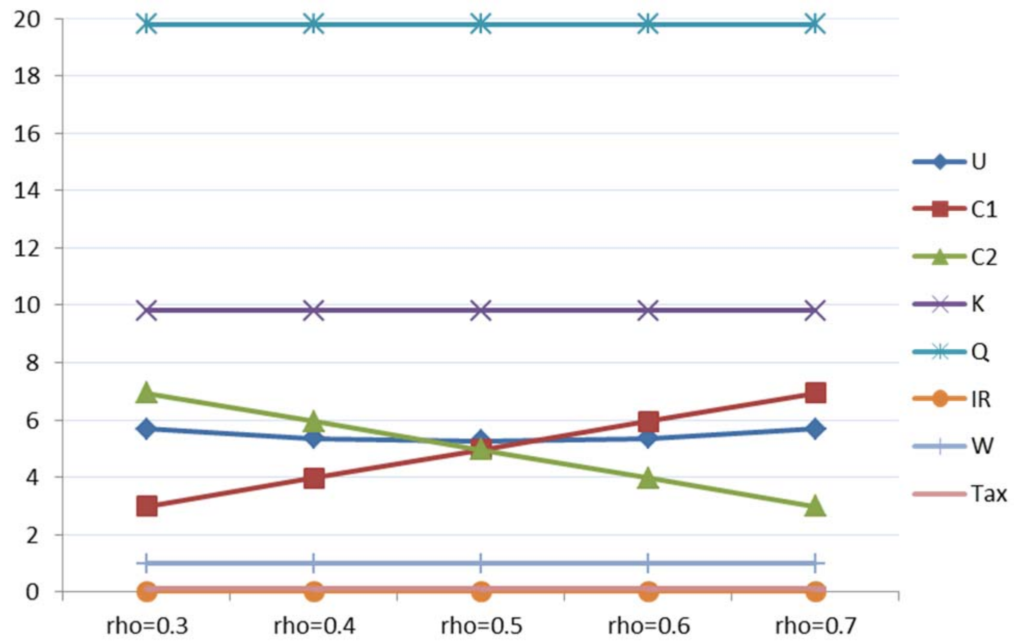


Figure.6 sensitivity analysis for inter-temporary consumption on tax back model

**figure6 is not very scientific since we should not make inter-personal comparison. Figure7 is better.*

As we can see above in figure.6, the utility changes as rho changes. Holding all others constant, utility reaches the lowest when rho=0.5, and get higher as rho decreases or increases. It seems that: people can get higher utility by allocating different consumption in young period and old period, thus, "Carpe Diem" or keep patient are both good ways to increase your total utility (keep all others constant).

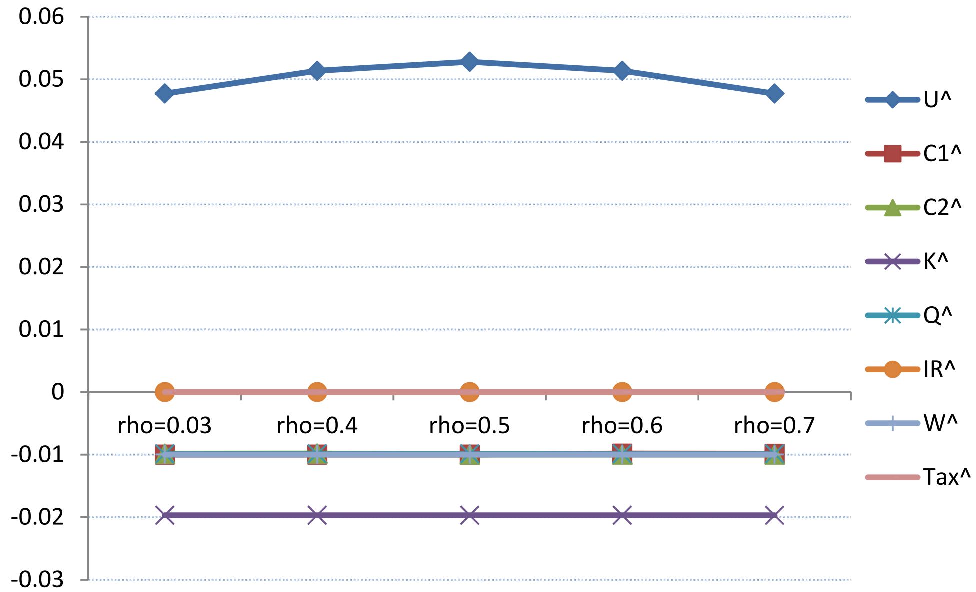


Figure.7 percentage change from model1 to 3 as rho changes

5. Conclusions and Recommendations

(1) This two-period overlapping generation model assumes that in the stable-state, people are endowed with a certain amount of capital for production and would save the same amount of capital for the next generation. Thus interest rate (IR) is always zero at optimum. Imposition of a saving tax in the second model implies that a certain share of saving K*t is taken away from the people and would not bring utility. Therefore, the stable-state capital level (K) is lower compared with the case without tax. As a result, the stable-state output (Q), consumption in both periods (C1 and C2) and wage rate (W) are higher without tax. People's utility is lower if tax brings no externality. If tax brings good externality, people's utility could be higher at certain tax rates than the case without tax.

(2) **Optimal Tax.** Tax decreases private consumption but also brings good externality in the tax-base model. In our setting, tax has decreasing marginal utility while consumption has constant marginal utility, U(tax)= $tax^{0.5}$. Therefore, utility first increases and then decreases as tax rate rises. Utility reaches optimum when t=0.9. The policy implication is government should set the tax rate at the point where marginal utility of tax collected equals zero, assuming decreasing MU of tax.

(3) **Rho** decides the inter-temporal consumption preference. A rho close to one indicates the tendency to consume today instead of tomorrow. A country with higher rho has higher C1 and lower C2 than country with lower rho. (even though it's not very scientific to make this comparison) The percentage variable changes are almost the same from model1 to model3, except the change in utility is maximized at rho=0.5.

6. Acknowledgment

Xiaolu and Ying thank Prof. Edward Tower for leading us into this beautiful Computable General

Equilibrium (CGE) modeling world, and for his help in developing and refining the OLG model for our project. And thanks Tevy Chawwa for her great help in the whole semester and resourceful ideas in GAMS operation. At last, thanks all our dear classmates in this Computer Modeling class, we learn a lot in your wonderful projects and presentations!!!

7. Appendix

```
********************************** GAMS Code **************************************
*** Try project 2_Xiaolu & Ying ***
* A simplified OLG Model (Overlapping Generation) *
* 2 period in life: first work, second retired *
OPTIONS
limcol=0, limrow=0, solprint=on, decimals=6;

PARAMETER
rho share parameter of current consumption in utility
beta share parameter of tax in utility
alpha share parameter of labour in production function
A technology factor
L labour supply
t tax rate
tO initial saving tax rate
UO initial utility level
C1O initial consumption in period one
C2O initial consumption in period two
KO initial capital supply
QO initial output level
IRO initial interest rate
WO initial wage rate
TAXO initial tax;

* Assign values to the parameters
rho=0.5;
beta=0.5;
alpha=0.5;
A=2;
L=10;
tO=0;
t=tO;
UO=20;
C1O=5;
C2O=5;
KO=10;
QO=20;
IRO=1;
WO=1;
TAXO=0;
```

VARIABLES

```
U utility level
C1 consumption in period one
C2 consumption in period two
K capital level
Q output level
IR interest rate
W wages
TAX tax;
```

*** Assign initial values to variables, and set lower bounds**

```
K.L=KO;
Q.L=QO;
IR.L=IRO;
W.L=WO;
C2.L=C2O;
C1.L=QO-KO-C2O;
U.L=C1O**rho*C2O**(1-rho);
TAX.L=KO*tO;

U.LO=0;
C1.LO=0;
C2.LO=0;
K.LO=0;
Q.LO=0;
W.LO=0;
TAX.LO=0;
```

EQUATIONS

```
Utility utility function
Utility2 utility function including tax
Production Production function
Wage Wage rate function
InterestRate Interest rate function
InterestRate2 Interest rate function with tax
Decision decision between current consumption and future consumption
Consumption Consumption allocation
MarketClear Market clear condition
TaxTotal tax equation;

Utility..U=e=C1**rho*C2**(1-rho);
Utility2..U=e=C1**rho*C2**(1-rho)+TAX**beta;
Production..Q=e=A*K**(1-alpha)*L**alpha;
Wage..W=e=A*alpha*(K/L)**(1-alpha);
InterestRate..1+IR=e=A*(1-alpha)*(L/K)**alpha;
InterestRate2..IR=e=A*(1-alpha)*(L/K)**alpha-1-t;
```

```
Decision.. C2/C1=e=(1+IR)*(1-rho)/rho;
Consumption..C1+C2/(1+IR)=e=W*L;
MarketClear..Q=e=C1+C2+K+TAX;
TaxTotal..TAX=e=t*K;

* This is model without tax saving
MODEL
ca1 /Utility,Production,Wage,InterestRate,Decision, Consumption,MarketClear,TaxTotal/;
SOLVE ca1 using nlp maximizing U;

*REPORT of Result----------------------------------------------------------------------
SET
CASE cases
/BASE base case
  SAVTAX with saving tax
  SAVTAXBACK saving tax brings utility/;

parameters
K_R(CASE),Q_R(CASE),IR_R(CASE),U_R(CASE),C1_R(CASE),C2_R(CASE),W_R(CASE),TAX_R(CASE);

*REPORT
K_R('BASE')=K.L;
Q_R('BASE')=Q.L;
IR_R('BASE')=IR.L;
U_R('BASE')=U.L;
C1_R('BASE')=C1.L;
C2_R('BASE')=C2.L;
W_R('BASE')=W.L;
TAX_R('BASE')=TAX.L;

*change rho in the Base Model
rho=0.30;
SOLVE ca1 using nlp maximizing U;

rho=0.40;
SOLVE ca1 using nlp maximizing U;

rho=0.50;
SOLVE ca1 using nlp maximizing U;

rho=0.60;
SOLVE ca1 using nlp maximizing U;

rho=0.70;
SOLVE ca1 using nlp maximizing U;
```

```
* MODEL with saving tax
rho=0.50;
t=0.01;
MODEL
ca2 /Utility,Production,Wage,InterestRate2,Decision,Consumption,MarketClear,TaxTotal/;
SOLVE ca2 using nlp maximizing U;

*REPORT
K_R('SAVTAX')=K.L;
Q_R('SAVTAX')=Q.L;
IR_R('SAVTAX')=IR.L;
U_R('SAVTAX')=U.L;
C1_R('SAVTAX')=C1.L;
C2_R('SAVTAX')=C2.L;
W_R('SAVTAX')=W.L;
TAX_R('SAVTAX')=TAX.L;

*This is the model when tax brings utility
t=0.01;
MODEL
ca3 /Utility2,Production,Wage,InterestRate2,Decision,Consumption,MarketClear,TaxTotal/;
SOLVE ca3 using nlp maximizing U;

*REPORT
K_R('SAVTAXBACK')=K.L;
Q_R('SAVTAXBACK')=Q.L;
IR_R('SAVTAXBACK')=IR.L;
U_R('SAVTAXBACK')=U.L;
C1_R('SAVTAXBACK')=C1.L;
C2_R('SAVTAXBACK')=C2.L;
W_R('SAVTAXBACK')=W.L;
TAX_R('SAVTAXBACK')=TAX.L;

display
K_R,Q_R,IR_R,U_R,C1_R,C2_R,W_R,TAX_R;

*change tax rate in the Tax-back Model
t=0.02;
SOLVE ca3 using nlp maximizing U;

t=0.03;
SOLVE ca3 using nlp maximizing U;

t=0.04;
SOLVE ca3 using nlp maximizing U;
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t=0.05;
SOLVE ca3 using nlp maximizing U;

t=0.06;
SOLVE ca3 using nlp maximizing U;

t=0.07;
SOLVE ca3 using nlp maximizing U;

t=0.08;
SOLVE ca3 using nlp maximizing U;

t=0.09;
SOLVE ca3 using nlp maximizing U;

t=0.10;
SOLVE ca3 using nlp maximizing U;

t=0.11;
SOLVE ca3 using nlp maximizing U;

t=0.12;
SOLVE ca3 using nlp maximizing U;

t=0.13;
SOLVE ca3 using nlp maximizing U;

t=0.14;
SOLVE ca3 using nlp maximizing U;

t=0.15;
SOLVE ca3 using nlp maximizing U;

*change rho in the Tax-back Model
t=0.01;
rho=0.30;
SOLVE ca3 using nlp maximizing U;
rho=0.40;
SOLVE ca3 using nlp maximizing U;
rho=0.50;
SOLVE ca3 using nlp maximizing U;
rho=0.60;
SOLVE ca3 using nlp maximizing U;
rho=0.70;
SOLVE ca3 using nlp maximizing U;
***   END   ***
```