HARRIS-TODARO MODEL OF URBAN UNEMPLOYMENT

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1. INTRODUCTION

Since the wage in cities is higher than one in village people migrate into the cities hoping to get urban job. The probability to get a job depends on the size of unemployment pool in relation to the number employed in industries. Therefore, in many mostly less-developed countries urban unemployment is a big issue. W. Max Corden in his book *Trade Policy and Economic Welfare* claims that the possible reason for urban unemployment is the wage differential. This coexists with usually high minimum wage in industries and with a marginal product of labor in agriculture less than the urban minimum wage\(^1\).

The model presented is derived from *Migration, Unemployment and Development: A Two Sector Analysis* original article by John R. Harris and Michael P. Todaro (1970)\(^2\) and W.M. Corden’s book mentioned above. In our approach we will assume as Harris and Todaro did that the expected urban wage is equal to the average wage of both urban employed and unemployed. Authors’ main claim is that the best policy to improve employment is to protect agricultural sector rather that manufacturing sector of the country. In this paper we will build a Harris-Todaro model of urban unemployment, discuss two cases: 1) subsidizing manufacturing, and 2) subsidizing agriculture, and test Harris and Todaro’s claim. For that purpose, we will run simulations for both cases in MS Excel, and try to analyze outcomes and suggest possible policies. The approach of the project will be comparative static.
2. MODEL

2.1 GENERAL ASSUMPTIONS

- Two sectors: urban (manufacture) and rural (agriculture)
- Rural-urban migration condition: when urban real wage exceeds real agricultural product
- No migration cost
- Perfect competition
- Cobb-Douglas production function
- Static approach
- Low risk aversion

2.2 VARIABLES AND PARAMETERS

**Exogenous variables**

\( \bar{L} \) – total labor force (workers)

\( W_M \) – minimum wage rate in manufacturing (dollars)

**Endogenous variables**

\( L_M \) - urban labor in manufacturing (workers)

\( L_U \) – unemployed labor force (workers)

\( L_A \) - rural labor force in agriculture (workers)

\( W_A \) – wage rate in agriculture (dollars)

\( EW_M \) – expected wage rate in manufacturing (dollars)

**Parameters**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_M )</td>
<td>elasticity of urban labor demand</td>
<td>-1</td>
</tr>
<tr>
<td>( \eta_A )</td>
<td>elasticity of rural labor demand</td>
<td>-1 / -0.1 / -5000</td>
</tr>
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<td>( \theta_M )</td>
<td>share of employed in manufacturing in urban labor force</td>
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</tr>
<tr>
<td>( \theta_U )</td>
<td>share of unemployed in total urban force</td>
<td>0.5</td>
</tr>
<tr>
<td>( \varphi_U )</td>
<td>share of unemployed in total labor force</td>
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</tr>
<tr>
<td>( \varphi_M )</td>
<td>share of employed in manufacturing in total labor force</td>
<td>0.25</td>
</tr>
<tr>
<td>( \varphi_A )</td>
<td>share of employed in agriculture in total labor force</td>
<td>0.5</td>
</tr>
</tbody>
</table>
2.3 BASIC EQUATIONS

1) Expected wage in manufacturing sector:
\[ EW_M = \frac{L_M \cdot W_M}{L_M + L_U} \]

2) Wage in agricultural sector:
\[ W_A = W_A(L_A) \]

3) Labor force in manufacturing sector:
\[ L_M = L_M(W_M) \]

4) Total labor force in the country:
\[ \bar{L} = L_M + L_A + L_U \]

5) Equilibrium condition:
\[ W_A = EW_M \]

Note: in our simulations in Excel we will use the differential form of these five equations.

2.4 THE EXPECTED URBAN WAGE AND THE UNEMPLOYMENT POOL

In Figure 1 there are two sectors: agriculture and manufacturing. Each sector has a specific factor (agriculture – land, manufacturing – capital) and labor which is mobile between these two sectors. In this model we assume that prices of agricultural and manufacturing goods are constant.
The horizontal axis shows total labor force. The marginal product curves are LL’ for agriculture and MM’ for manufacturing. O*W is the fixed minimum wage in manufacturing, and corresponding employment is given at NO*. According to Harris and Todaro’s approach in equilibrium the **expected** urban wage must be equal to the agriculture wage. We draw a rectangular hyperbola through J and let it intersect LL’ at R. This gives agricultural wage OV and rural employment OG. The remained part GN will be an unemployment pool. Manufacturing wage-bill NJWO* rectangular area is spread over the whole urban labor force, and we get the expected urban wage GR, which is the average of the minimum wage O*W received by the employed and the zero wage received by the unemployed. Since two shaded areas are equal the expected urban wage is equal to the rural wage.
3. SIMULATIONS AND INTERPRETATION OF THE RESULTS

3.1 POLICY 1: SUBSIDIZING EMPLOYMENT IN MANUFACTURING

When we subsidize manufacturing as it can be seen from Figure 2 by QJ’ per man we expand manufacturing output by N’N. The shaded area N’QJN is the value of extra output in manufacturing. Then we draw a new rectangular hyperbola R’J’, and get the new equilibrium allocation. Labor in agriculture declines by G’G, and the output also declines by the area of the shaded area G’R’RG. So we need to compare the two shaded areas in order to measure effect on total output. This depends on the size of the unemployment pool. The flatter the slope of LL’ and steeper the slope of MM’, the bigger number of the unemployment people and lower real output.

Can we restore full employment subsidizing manufacturing? Yes. More and more workers will leave agriculture increasing marginal product in agriculture till it reaches the fixed minimum wage in industry. Here both wages are equalized, so there will be no unemployment. In Figure 2, agricultural output declines by OG”. Because marginal product of labor in manufacturing will be below that in agriculture, this would not be first-best solution, and not even second-best solution. However, a very low wage subsidy may maximize real output.
If the marginal product of labor in agriculture stayed unchanged (horizontal line LL’) when labor leaves agriculture a wage subsidy lowers real output as it is demonstrated in Figure 3.

![Figure 3](image)

By the properties of rectangular hyperbolae the two shaded areas (increases output in industry and decreased output in agriculture) must be equal. However, there is the output fall in manufacture QJ’J which caused by the wage subsidy.

We put this model in Excel and did computer simulations of two policies: subsidizing manufacturing and subsidizing agriculture. Also, we had three cases in each policy: inelastic, unity elasticity, and elastic urban labor demand.

**Results of simulation:**

**Case 1: Unity elasticity of labor demand**

<table>
<thead>
<tr>
<th>Lbar^</th>
<th>W_M^</th>
<th>S_M^</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>5</td>
</tr>
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Case 2: Inelastic labor demand

<table>
<thead>
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<th>W_M^</th>
<th>S_M^</th>
</tr>
</thead>
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</tr>
<tr>
<td>0</td>
<td>-1</td>
<td>10</td>
</tr>
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<td>0.181818182</td>
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<td></td>
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<tr>
<td>1.818181818</td>
<td>9.090909091</td>
<td></td>
</tr>
<tr>
<td>1.818181818</td>
<td>9.090909091</td>
<td></td>
</tr>
</tbody>
</table>

Case 3: Elastic labor demand

<table>
<thead>
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<th>S_M^</th>
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<td>1.999600080</td>
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<td>-0.000399920</td>
<td>0.0019996</td>
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</tr>
<tr>
<td>-0.000399920</td>
<td>0.0019996</td>
<td></td>
</tr>
</tbody>
</table>

Economic explanation:

Lbar^:

An increase in the population causes an increase in the level of unemployment for all three cases. The largest effect is observed for the inelastic case, and the smallest effect occurs for the elastic case. When population increases, wage decreases because more people are willing to work. If labor demand is elastic, the sector demands more workers and is able to hire the available ones. If labor demand is inelastic, production is already set and not as many workers are necessary. So, more people are out of jobs, and employment increases by a large amount.

An increase in the population has no effect on the labor employed in manufacturing because the demand for labor in manufacturing is unit elastic.

An increase in the population causes an increase in the level of employed in agriculture, which is larger for more elastic demands in agriculture. The elastic sector is more sensitive to wages (which fall as labor supply increases) and can absorb more of the workers.
An increase in the population causes a fall in the wage in the agricultural sector, which is larger for inelastic conditions. This comes from equation 2: for a given change in La, wage must fall by a larger amount to balance the equation that has a smaller elasticity. In equilibrium, expected manufacturing wage must be equal to the agricultural wage.

\[ W_M^\wedge: \]

An increase in the urban minimum wage impacts only the urban sector, decreasing labor and increasing unemployment proportionately. There are no changes in the agricultural sector because we move along the manufacturing labor demand curve – there is no shift.

\[ S_M^\wedge: \]

An 10% subsidy of wage in manufacturing decreases unemployment for the inelastic case, but increases it for the elastic case! This is the Todaro Paradox. This peculiar result occurs because the rural to urban migration induced by the subsidy outweighs the number of jobs created.

A 10% subsidy of wage in manufacturing causes a proportional increase in the number of laborers in manufacturing. This comes from the equation for urban labor, which depends directly on the wage and the subsidy. This increase is the same for all of our cases because the manufacturing sector is unit elastic.

A 10% subsidy of wage in manufacturing causes a decrease in the number of laborers in agriculture. The decrease is larger for the elastic case than for the inelastic case. If the agricultural sector is inelastic, there is a smaller difference between the rectangular hyperbolas due to the steep slope of the labor demand in agriculture. Hence, the effect is smaller.

A 10% subsidy of wage in manufacturing causes a rise in the wage in the agricultural sector, which is larger for inelastic conditions. This comes from equation 2: for a given decrease in La, wage must rise by a larger amount to balance the equation that has a smaller elasticity. In equilibrium, expected manufacturing wage must be equal to the agricultural wage.

3.2 POLICY 2: SUBSIDIZING AGRICULTURAL EMPLOYMENT

Subsidizing agriculture rather than manufacturing would reduce the wage differential which will employ some of the urban unemployed, reducing thus unemployment pool. In Figure
4 TJ is the wage subsidy in agriculture per man. Employment in manufacturing stays unchanged at NO*, but employment in agriculture rises from OG to ON, absorbing all the unemployed.

The shaded area GRTN is the extra agricultural output and pure gain. Thus, the author claims that agriculture rather than manufacturing must be subsidized. Even though there is a gain, this policy is still not the first-best because if there is no unemployment it will lead to excessive movement of labor into agriculture compared to the first-best solution.

**Results of simulation:**

**Case 1: Unity elasticity of labor demand**

\[
\begin{array}{|c|c|c|}
\hline
L_{bar}^\land & W_{M}^\land & S_{A}^\land \\
\hline
2 & 1 & -10 \\
0 & -1 & 0 \\
1 & 0 & 5 \\
-1 & 0 & 5 \\
-1 & 0 & 5 \\
\hline
\end{array}
\]

equals

**Case 2: Inelastic labor demand**
Case 1: Elastic labor demand

<table>
<thead>
<tr>
<th>$L_U^\wedge$</th>
<th>$W_M^\wedge$</th>
<th>$S_A^\wedge$</th>
</tr>
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<tbody>
<tr>
<td>3.6363636366</td>
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<td>1.818181818</td>
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</tr>
</tbody>
</table>

$\begin{align*}
L_U^\wedge & = 0.00079984 \\
W_M^\wedge & = 19.9960008 \\
S_A^\wedge & = 9.9980004
\end{align*}$

**Economic explanation:**

The results of changes in $Lbar^\wedge$ and $W_M^\wedge$ are the same as before: our simulation shows that none of the numbers change.

**$S_A^\wedge$:**

An 10% subsidy on the wage in agriculture decreases the unemployment level in every case, though the effect is more pronounced for the elastic case than the inelastic case. If the demand for labor in agriculture is elastic, the sector can absorb more workers. In the case of an agricultural subsidy, there is no Todaro Paradox regardless of elasticity.

A 10% subsidy on the wage in agriculture has no effect on the labor in manufacturing because the demand for labor in manufacturing is unit elastic.

A 10% subsidy on the wage in agriculture increases the labor in agriculture, and the effect is more pronounced for an elastic labor demand than for an inelastic one because the elastic sector can absorb more workers.

A 10% subsidy on the wage in agriculture increases the wage in the agriculture sector as a positive beneficial effect of the subsidy. In equilibrium, expected manufacturing wage must be equal to the agricultural wage.
3.3 FIRST-BEST POLICY

According to the first-best solution labor should be allocated such that:

a) marginal products in agriculture and manufacturing must be equal

b) no unemployment

This solution is represented by the point Z in Figure 5.

As Harris and Todaro stated the first-best solution is to subsidize manufacturing (at the rate ZZ' per man) and restrict migration out of agriculture. Or, subsidize both sectors equally by ZZ' per man. To finance this subsidy we need to find some sectors that are taxable either directly, or indirectly through trade policy, so that it will not affect supplies of the taxed factors as a result.
4. SUBSIDY FINANCING PROBLEM

So, who pays taxes?

The author implies that a subsidy to labor in manufacturing cannot be financed by a tax on labor in agriculture because this tax will increase the wage-differential, assuming that potential migrants compare their after-tax wage with expected urban wage and move to cities. This will lead to unemployment increase.

If the subsidy is financed by taxing labor in manufacturing, the after-tax real wage in manufacturing will fall. With tax illusion the real disposable wage falls, which will lead to decrease of wage-differential and, as a result decreases unemployment. This might solve problem of unemployment. However, in absence of tax illusion, taxing manufacturing will lead to a rise of pre-tax wage which will reduce employment. Therefore, it will worsen the situation at the labor market.

It implies that in absence of tax illusion a wage subsidy must be financed by taxes on manufacturing profits, on agricultural rents, or both. The financing problem is illustrated in Figure 6.

![Figure 6](image)

The rectangular T’W’WT is the total cost of the subsidy. The wage-subsidy will increase agricultural rents by the shaded area T’VRZ and will increase manufacturing profits by the
shaded area ZJWT. If increased profits and rents can be recovered through taxes, the remained three areas will be financed:

1) VW’R’R is the part of the subsidy which has increased the wages in agriculture which prevents rural labor to migrate to the cities.

2) RR’Z’Z is the part of the subsidy will allow unemployed people to be employed in agriculture, and is the excess of the wages they receive over the value of their output.

3) ZZ’J refers to previously unemployed and now employed in manufacturing, and is the excess of the wages they receive over the value of their output.

If there is not a fixed revenue constraint, we should take into account disbursement costs, collection costs, and any by-product distortion costs. Then making a full correction with a uniform wage subsidy will no longer be first-best policy. A more limited subsidy, mostly for labor in agriculture and improvement of public infrastructure in order to keep people in the villages (and not improving mobility into the cities) may be first-best policy.

Corden makes the following assumptions here:

1) Low risk aversion and labor turnover in manufacturing high, so there is no incentive to be un- or under-employed in the cities because of wage-differential.

2) The urban after-tax real wage cannot be reduced.

3) A rise in the agricultural wage caused by subsidy to the rural labor will not make the urban wage to rise.

4) Harris-Todaro type of unemployment only.

5) Labor in agriculture is paid by its marginal product.

6) Investment cost of migration does not affect movement to the cities.

7) The wage-differential cause the rise to unemployment with zero productivity.
5. CONCLUSION

Harris Todaro model explains some issues of rural-urban migration. This migration happens in case when expected rural income is higher than rural wages. In this case economy may have high rates of unemployment. The equilibrium condition of this model is when expected rural wage is equal to rural wage.

When government subsidize manufacturing sector Harris Todaro paradox may happen. According to the authors job creation instead of dealing with unemployment problem actually may cause increase of unemployment. This happens when urban-rural wage differential is high enough, so rural workers move to the cities hoping to find a job with high wage. Obviously, not all these workers succeed in finding jobs which leads to unemployment.

Another issue is that inducing minimum wages creates labor market distortions. Therefore, policy makers should not set the minimum wage rates.

In addition, simulations showed that different policies’ outcomes depend on elasticity of labor demand in different sectors and on marginal product of labor.

As Harris and Todaro suggested the first-best policy would be subsidizing manufacturing along with restrictions of rural migration.
6. REFERENCES
