Summary: Endogenous insurance and informal relationships

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0.1 Key Notation and Terms

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<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$G_i$</td>
<td>The $i^{th}$ group of agents</td>
</tr>
<tr>
<td>$Z$</td>
<td>The number of agents in a group. The cardinality of a set of agents.</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Arrow-Pratt degree of absolute risk aversion of agent $i$. See The Model section for full explanation</td>
</tr>
<tr>
<td>$u_i(x)$</td>
<td>The amount of utility agent $i$ receives from consuming $x$ units of output</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Agent $i$’s degree of risk tolerance, $\frac{1}{r_i}$</td>
</tr>
<tr>
<td>$p$</td>
<td>A project that can be undertaken by a pair</td>
</tr>
<tr>
<td>$Y_p$</td>
<td>The return of project $p$, and random variable</td>
</tr>
<tr>
<td>$y_p$</td>
<td>The realized output of project $p$</td>
</tr>
<tr>
<td>$s(.)$</td>
<td>Sharing rule of realized output among pair</td>
</tr>
<tr>
<td>$\mu(.)$</td>
<td>Match function that maps each agent in group 1 to a single agent in group 2</td>
</tr>
<tr>
<td>$v_i$</td>
<td>The expected utility of a project for agent $i$, the surplus $\psi(r_i, \mu(r_i), v_i)$</td>
</tr>
<tr>
<td>$\psi(r_1, \mu(r_1), v_1)$</td>
<td>Positive Assortative Matching (PAM) The $i^{th}$ least risk-averse person in $G_1$ is matched with the $i^{th}$ least risk averse person in $G_2$</td>
</tr>
<tr>
<td>$\psi(r_2, \mu(r_2), v_2)$</td>
<td>Negative Assortative Matching (NAM) The $i^{th}$ least risk-averse person in $G_1$ is matched with the $i^{th}$ most risk averse person in $G_2$</td>
</tr>
<tr>
<td>$(r_1, r_2)$</td>
<td>A matched pair</td>
</tr>
<tr>
<td>$p^*(r_1, r_2)$</td>
<td>A matched pair’s chosen project</td>
</tr>
<tr>
<td>$CE(r_1, r_2, p^*(r_1, r_2))$</td>
<td>The certainty-equivalent of a matched pair’s project in equilibrium. A project with a certain return that would provide the same utility as risky project. Since all agents in this model are risk averse, the return of the certainty equivalent is always lower than the expected return of the risky project.</td>
</tr>
<tr>
<td>$V(p)$</td>
<td>The variance cost of a project with mean return $p$</td>
</tr>
<tr>
<td>$M(p)$</td>
<td>The marginal variance cost of a project with mean return $p$</td>
</tr>
<tr>
<td>$CV(p)$</td>
<td>The coefficient of variation of a project $p$. A unitless, and therefore convenient tool for comparing portfolios</td>
</tr>
<tr>
<td>$R$</td>
<td>$R_1 + R_2$ The representative risk tolerance of a matched pair $(r_1, r_2)$</td>
</tr>
<tr>
<td>$p^*(r_i, \mu(r_i))$</td>
<td>The mean return of the project chosen by a matched pair $(r_i, \mu(r_i))$</td>
</tr>
<tr>
<td>$[p_L, p_H]$</td>
<td>Government imposed price bands on a crop’s price</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>The yield of crop $i$</td>
</tr>
<tr>
<td>$P_i$</td>
<td>The net price of crop $i$</td>
</tr>
<tr>
<td>$\pi(i)$</td>
<td>The profit of crop $i$</td>
</tr>
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1 Introduction

The risk-averse poor often lack access to formal risk-management tools. In order to address this, they create informal relationships with each other that function as insurance. This paper advances the literature by endogenizing the structure of informal insurance (the size and properties of the groups reaching these agreements) rather than assuming a fixed, isolated group of individuals. It seeks to shed light on how people in these relationships respond to policy...
interventions and implications regarding income inequality, entrepreneurship, and the structure
of informal firms.

Since risk is a well-documented burden on the poor, Wang draws from a wide literature. This includes Rosenzweig and Stark (1989) on how farmers marry off their daughters to distant farms as a diversification strategy as well Stiglitz (1974), and Ackerberg and Botticini (2002) on informal arrangements that help sharecropping. The point is that the poor devise creative solutions to problems created by a lack of formal insurance and credit institutions.

1.0.1 Key Elements of the Model

1. Risk-averse individuals with exponential utility work together to be productive (e.g. A land-owner, and a landless farmer)

2. Matching is assumed to be pairwise in the benchmark model, but this assumption is relaxed to allow group size to be endogenous.

3. Two types of heterogeneity: in preferences and in technology.

4. Individuals vary in their degree of constant absolute risk aversion.

5. A matched group chooses a joint income distribution from a set of options (projects) that differ in risk. Note: This is equivalent to individuals choosing distributions and sharing pooled realizations.

6. Projects with higher expected return have a higher variance in returns.

7. Members of a group share the return of a project according to a rule determined before the realization of the return.

8. The model allows for a large class of symmetric and skewed return distributions. Distributions may have infinitely many higher order cumulants. The intuition behind a cumulant is that it is equivalent to a moment, and measures deviation from a normal distribution. Therefore, the distributions in this model can greatly differ from a normal distribution.

1.0.2 Summary of Results

1. A policy which reduces aggregate risk is a strict Pareto improvement if informal insurance is assumed to stay fixed.

2. But, with an endogenous network response, there may be unintended consequences that harm the most risk-averse (the people whom the policy is most intended to help). The least risk-averse are incentivized to no longer insure the most risk-averse – instead partnering with other less risk-averse agents. The most risk-averse are forced to match with each other instead, which is harmful in two ways. One is that both partners want to consumption smooth, but neither is willing to take on volatility to allow this. The other, resulting from the first one, is that the whole group must rely on income-smoothing to manage risk. In order to do this, they select low-variance/low-expected return projects, so the more risk-averse agents can’t take advantage of the high-expected return projects (i.e. entrepreneurial projects) whose variance of risk is reduced by the policy. This holds despite the ability of agents to commit ex ante to a return-contingent sharing rule.

3. This paper breaks from models with exogenous informal insurance, which find that informal and formal insurance serve complements, by showing the theoretical possibility that they can be substitutes under certain circumstances.
1.0.3 Summary of Approach

1. Find a transferable utility representation of the model. Expected utility, represented by the certainty-equivalent, is transferable. Utility is not, due to the fact that agents have heterogeneous risk attitudes, hence the utility generated by a unit of output differs between agents.

2. Find conditions in which the total certainty-equivalent of a matched group exhibits supermodularity and submodularity in risk attitudes. In this context, that means the conditions for which the risk attitudes of the matched pairs are complementary.

3. Key: The total certainty equivalent of a matched group is the product of the cumulant-generating function of the return distribution, and the group’s representative risk tolerance.

4. Despite the fact that the model allows for infinitely many higher order cumulants (see Key Elements of the Model #8), unique assortative matching in risk attitude is determined by the mean and variance alone (the first two cumulants), and does not depend on the distribution of risk attitudes throughout the population.

5. Define the coefficient of variation of return across all projects as the ratio of the standard deviation and the mean of these returns. If projects with higher expected return have a lower coefficient of variation (the coefficient is decreasing in the mean) then unique positive assortative matching results in the pairwise equilibrium. By contrast, if projects with lower expected return have a lower coefficient of variation, unique negative assortative matching results in pairwise equilibrium. Note: the preference structure is not driving this result.

6. Think of the variance of project return for a project that has a return $p$ as the “cost” of obtaining an expected return of $p$.

7. It follows from #6 that the coefficient of variation is decreasing (increasing) in mean return if and only if the marginal cost function is concave (convex) in the mean $p$. **Intuition:** There is a trade-off between preference for a similar partner when choosing risk before the return is realized, and preference for a differing partner when sharing risk after the realization of the returns. This trade-off drives the matching equilibrium, and is reflected in the marginal cost function.

8. When the ability to share risk *ex post* is eliminated (e.g. by government mandate) positive assortative matching is always the unique equilibrium. Conversely, when the ability to choose risk *ex ante* is removed, negative assortative mating is the unique equilibrium.

9. Wang allows group size to be endogenous. She shows that whole-group matching (maximal connectedness) is the unique equilibrium under unique positive assortative matching in the pairwise case. Under unique negative assortative matching in the pairwise case, pairwise matching (minimal connectedness) is the unique equilibrium. Network shapes are tied to within-group composition when achieving the largest number of matches.

10. In other models, the marginal benefit of an additional group member eventually becomes negative. In this model, it is possible for the marginal benefit to be positive even as the group size grows infinitely large. This is because of an increased ability to share risk *ex post* allows the group to take on additional risk when adding a new member – meaning that it can take up a higher mean, higher variance project. The countervailing effect to
this is that the surplus gets divided among more people. It is shown that the curvature of the marginal variance cost function determines which effect is stronger.

2 The Model

Assumptions in benchmark model, later to be relaxed: Matching is pairwise, group size is not endogenous.

2.1 Setup

2.1.1 The population of agents:

1. The economy consists of two groups of agents, \( G_1 \) and \( G_2 \). The amount of agents in each group is equal, and is an integer \( > 1 \). In mathematical notation: \( |G_1| = |G_2| = Z \), and \( Z \in \{2, 3, 4, \ldots\} \). The case \( |G_1| \neq |G_2| \) doesn’t significantly change analysis, just leaving the most risk-averse in larger group unmatched.

2. Note: The Arrow-Pratt degree of absolute risk aversion is a measure of risk aversion such that, if \( c \) is consumption: \( r = A(c) = -\frac{u(c)}{u(c)} \). A higher ratio of the second and first derivatives implies more concavity and therefore more risk aversion.

3. Agents differ in \( r \), where an agent \( i \) of type \( r_i \) receives utility \( u_i(x) = -e^{-r_ix} \) from consuming \( x \) units of output. Let \( r_i > 0 \) \( \forall i \) so all agents are risk-averse. Let \( R_i = \frac{1}{r_i} \) be agent \( r_i \)'s degree of risk tolerance. An example of CARA can be seen in Figure 1.

4. No assumptions are imposed on the distributions of risk preferences with or across groups.

2.1.2 The risky environment:

1. There is a spectrum of available projects, with return distributions parameterized by \( p \in \Pi \subseteq \mathbb{R}_+^+ \).

2. A project \( p \) returns \( Y_p \), a random variable described by: \( Y_p = p + V(p)Y \). \( Y \) is a random variable with a well-defined cdf: \( F_Y : \mathbb{R} \to [0, 1] \), and \( E(Y) = 0, V(Y) = 1 \).
Note: As alluded to earlier, this allows for a large class of possible distributions for returns, both symmetric or skewed, including Normal, Laplace, and generalized extreme value (e.g. Gumbel) distributions.

3. It follows that \( E(Y_p) = p \) and \( V(Y_p) = V(p) \). The function \( V : \Pi \rightarrow \mathbb{R}_0^+ \) is the variance of a project with expected return \( p \). **Assumption:** \( \Pi = \mathbb{R}_0^+ \), so that there exists a portfolio which is not strictly dominated by achieving expected return \( p \) for each \( p \geq 0 \).

Let \( V(. \) be thrice-differentiable and have the following three properties.

i. \( V(0) = 0 \), and \( V(p) = 0 \) for \( p > 0 \). This ensures the variance is non-negative and that an action with a certain return of 0 exists (e.g. do nothing).

ii. \( V'(0) = 0 \), and \( V'(p) = 0 \) for \( p > 0 \). This ensures that projects with higher expected return also have a higher variance of return.

iii. \( V''(p) > 0 \). This ensures an interior solution for project choice for any agent \( r \).

4. A subset of the risky projects available might be represented like this:

![Figure 2: Choosing Among Projects Which Vary in Risk Structure: Gumbel, V(p)\(=\sigma^4\)](image)

2.1.3 Production:

1. Any project \( p \) requires a partnership of one agent from \( G1 \) and one agent from \( G2 \).

2. All matched pairs face the same spectrum of projects, each agent can be involved in at most one project, and a pair’s project choice does not affect availability or returns of other pairs’ projects (no "project externalities").

3. There is no moral hazard in the model, as the focus is on equilibrium matching of the trade-off between \( ex \ ante \) and \( ex \ post \) risk management across partnerships of different risk compositions.

2.1.4 Information and commitment:

1. Perfect information is assumed. All agents know each other’s risk types and the risk environment.
2. A given matched pair, \((r_1, r_2)\) undertaking project \(p_{12}\) realizes output \(y_{p12}\) from their partnership, and is able to commit *ex ante* to a feasible return-contingent sharing rule \(s: \mathbb{R} \rightarrow \mathbb{R}\) (there is no limited liability). \(r_2\)'s share of realized output is \(s(y_{p12})\). Feasibility implies that the income \(r_1\) receives must be less than or equal to \(y_{p12} - s(y_{p12})\). By the monotonicity of the agents’ preferences, \(r_1\)'s share of realized output will be equal to \(y_{p12} - s(y_{p12})\).

2.1.5 The equilibrium

An equilibrium is:

1. **The matching pattern**: a match function \(\mu: \mathbb{R} \rightarrow \mathbb{R}\), matches each agent in group 1 to an agent in group 2. \(r_1\)'s partner is denoted by \(\mu(r_1)\), and \(\mu(.)\) assigns members of group 1 to distinct members in group 2. The matching pattern must be stable. It must be that no agent is able to propose a feasible project and sharing rule to an agent not matched to her under \(\mu\) such that both agents are happier than they are with the partners assigned by \(\mu\).

2. **The risky projects**: for each matched pair, a project is chosen so that no pair can achieve an outcome that is weakly better for both partners and strictly better for at least one.

3. **Individual payoffs and sharing rules**: each matched pair has a sharing rule, determining the amount each partner receives given a potential return. The allocation of returns must be feasible, i.e. their sum can’t exceed total return. The sharing rule is chosen so that no pair can achieve an outcome that is weakly better for both partners and strictly better for at least one.

Individual payoffs will not be unique in the equilibrium of this model. The stability conditions will determine a set of equilibrium surplus divisions. Let \(v_i\) denote the expected utility of \(\mu(r_i)\) for a matched pair \((r_i, \mu(r_i), v_i)\). Let \(\phi(r_i, \mu(r_i), v_i)\) denote \(r_i\)'s maximal expected utility given that \(\mu(r_i)'s\) expected utility is \(v_i\). Then a vector of individual payoffs described by \((v_1, ..., v_N)\) can be supported in equilibrium if and only if for each \(r_i\):

\[
\phi(r_i, \mu(r_i), v_i) \geq \phi(r_j, \mu(r_j), v_j) \quad \forall j \neq i
\]

The intuition: no two individuals who are unmatched are able to match with each other instead, and split the surplus in a way that is weakly better for both partners and strictly better for at least one.

2.1.6 Matching patterns:

Let \(G_j = \{r^1_j, r^2_j, ..., r^Z_j\}, j \in \{1, 2\}\), ordered from least to most risk-averse.

1. **Positive assortative matching (PAM)**: the \(i^{th}\) least risk-averse person in \(G1\) is matched with the \(i^{th}\) least risk-averse person in \(G2\). \(\mu(r^i_1) = r^i_2, i \in \{1, ..., Z\}\).

2. **Negative assortative matching (NAM)**: the \(i^{th}\) least risk-averse person in \(G1\) is matched with the \(i^{th}\) most risk-averse person in \(G2\). \(\mu(r^i_1) = r^{Z-i+1}_2, i \in \{1, ..., Z\}\).

The unique equilibrium matching pattern is PAM (NAM), if and only if, the only \(\mu\) which is stable under optimal within-pair sharing rules and projects is the match function which assigns agents to each other positive (negative) assortatively in risk attitudes.
3 Results

The intuition of this section is as follows in Professor Wang’s own words:

A less risk-averse person enjoys the premium a more risk-averse partner is willing to pay her to smooth his consumption, but acting as the informal insurer and bearing her partner’s risk forces the pair to choose a safer project with lower expected return. If she instead matches with a less risk-averse partner, she forgoes the premium from providing insurance, but she and her partner are able to undertake a riskier project with higher expected return. Whether a less risk-averse individual prefers to be an informal insurer or an entrepreneur, and thus whether negative or positive assortative matching results, depends on whether partnerships generate the most value through insurance or production. When the ratio of standard deviation to expected return is higher for projects with higher expected return, the less risk-averse will prefer to be informal insurers; when the ratio is lower, the less risk-averse will prefer to be entrepreneurs and choose high risk, high return projects.

Recall that $r_i \neq r_j \in \{G_1, G_2\}$, $\forall i \neq j$ and that for all agents, $u(x) = -\exp(-r_ix)$ so that $u(r_1) \neq u(r_2)$ even when both consume the same $x$. For transferable utility, we need homogenous utility for the agents but here we clearly do not have that; hence the nontransferable utility setting. Additionally, agents choose potential partners not only about the dimension of potential output but also in how that output is shared in a nontransferable utility setting. However, to establish a matching algorithm, the potential partners must be able to see their utility in a transferable utility representation. For this, she uses certainty-equivalents, the amount the pair would be willing to receive in risk-free money to be indifferent about a particular gamble with potentially higher payoff. This relationship between certainty-equivalents and the type of matching algorithm is established in Lemma 1:

$$\frac{\delta}{\delta r_1 r_2} CE(r_1, r_2, p^*(r_1, r_2)) > 0 \iff \text{unique PAM}$$

$$\frac{\delta}{\delta r_1 r_2} CE(r_1, r_2, p^*(r_1, r_2)) < 0 \iff \text{unique NAM}$$

where $CE$ designates a twice-differentiable certainty-equivalent function of a matched pair in equilibrium.

The intuition is as follows. Suppose $r_1$ and $r_2$ paired up and chose an optimal project $p$. Although Professor Wang invokes the surplus $v \in \mathbb{R}$ from the project the cost of the project is never explicitly characterized and so it seems to be implicitly equivalent to the income from the project $y_p$. Nevertheless, we’ll use it for notational consistency, in brief though, the total the agents can divide up cannot exceed the income $y_p$. Before embarking on the project, they decide what proportion $s$ of income goes to agent $r_1$ and what proportion goes to $r_2$; hence $s(y_p) + (1-s)y_p \leq y_p$. However, because the utilities are strictly increasing monotone functions (Figure 1), both agents will clearly maximize their share of the optimal and not accept anything less than the remainder from the $sy_p$. Mathematically, the each, say $r_1$, maximizes over all possible project outputs:

$$\max_{s(y_p)} \int_{-\infty}^{\infty} -\exp(-r_1[y_p - s(y_p)])f(y_p|p)dy_p$$

$$\text{s.t.} \int_{-\infty}^{\infty} -\exp(-r_2s(y_2))f(y_p|p)dy_p \geq -\exp(-v)$$

Where the constrain is just the same problem for $r_2$ because $r_1$ knows that $r_2$ will not accept anything less than the remainder of $y_p$. When $r_2$ is more risk averse than $r_1$ solving the system
gives $s = \frac{r_2^2}{r_1 + r_2} y_p + c(v) < \frac{1}{2} y_p + c(v)$ where $c(v)$ is a constant that is a function of only the surplus $v$. This enables her to write the certainty-equivalent function for each member of the pair as

$$CE_{r_1} = \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \log \int_{-\infty}^{\infty} -\exp\left(\frac{r_1}{r_1 + r_2} y_p\right) f(y_p|p^*(r_1, r_2)) dy_p - \frac{1}{r_2} v$$

(1)

$$CE_{r_2} = \frac{1}{r_2} v$$

Since the above certainty equivalence function is a monotonic transformation of expected utility it also depicts the transferability of utility. Moreover, it shows that by characterizing the change in risk, we can determine whether the agents will pair as PAM or NAM. Recall that $Var(p)$ is the variance of the project output $y_p$ so then $Var'(p)$ is the marginal variance cost of a project and $Var''(p)$ shows whether marginal variance is concave or convex. By Proposition 1

a. PAM is the unique equilibrium match if the marginal variance cost is concave or $M''(p) < 0$ for $p > 0$.

b. NAM is the unique equilibrium match if the marginal variance cost is convex or $M''(p) > 0$ for $p > 0$.

c. Any match is stable if the marginal variance cost is linear or $M''(p) = 0$ for $p > 0$.

If we define $R = \frac{1}{r_1} + \frac{1}{r_2}$ then $R(r_1, r_2)$ and we can rewrite Equation 1 as

$$CE(r_1, r_2) = CE(R(r_2, r_2)) = R \log \int_{-\infty}^{\infty} \exp\left(\frac{-1}{R} y_p f(y_p|p^*(R))\right) dy_p$$

The above can be re-expressed as a series expansion, by property of cumulant-generating functions

$$CE(p, R) = p - \sum_{n=2}^{n=2} \frac{(-1)^n}{n! R^{n-1}} V(p)^{n/2} \kappa_n(y)$$

which can be interpreted as the mean project return minus a penalty that is a function of risk aversion and variance. Interestingly, when the agents are less risk-averse then the penalty is smaller than it would be for the same project for a more risk-averse pair. Consequently, the less risk-averse would be more willing to undertake a riskier project for the same certainty-equivalence as a more risk-averse pair.

As mentioned in the beginning, the pair weighs both which project $p$ to take on and how to share the risk $s$. The two channels balance to either provide a PAM or NAM matching. Namely, by Proposition 2, if the pair can only choose $s$ then the matching will be NAM. Intuitively, all agents face the same income distribution, irrespective of their own risk preferences, and so the only way they can adjust to their risk preferences is to determine $s$ once they are allocated a project. On the other hand, Proposition 3, if one shuts down the avenue for determining the sharing of the project income $s y_p$, by say a government rule that requires a 50-50 share, then the only way the agents can mitigate risk is to pick the preferred projects. For example, under normal circumstances, a more risk averse would be more willing to give away a higher $s$ to the less risk averse for more risky projects but here, we cannot adjust $s$ and so the less risk averse have no incentive to bear the risk. This leads to PAM.

---

1 She mentions that the reformulation exhibits supermodularity (in convex $CE(R)$) or submodularity (in concave $CE(R)$) in $r_1$ and $r_2$. Supermodularity arises in game theory and is defined as follows $f(\max\{x, y\}) + f(\min\{x, y\}) \geq f(x) + f(y)$ and submodularity occurs if the above is true for $-f$

2 log of a moment generating function
4 Policy

In the policy section Professor Wang discusses the potential implications of risk reduction in risky environments that do not have access to formal insurance. Since group formation in the paper is endogenous to the risk environment and groups form as an informal insurance policy, changes in the risk environment may alter the status quo groups in a way that leaves the most risk-averse (typically the poorest) worse off. She provides an example of this in regard to price stabilization policy in the context of farmers who lack access to formal insurance. The price stabilization policy binds all project returns to a band of $[p_L, p_H]$ so that the variance $V(p) \forall p$ is reduced. The policy may, thus, alter the Marginal Variance Cost $M(p)$ or $\nabla V(p)$ from convex to concave; thereby shifting the groupings from NAM to PAM. In this case, the groups reform: the less risk averse pair with other less risk averse agents to pursue riskier projects while the most risk averse pair with other risk averse agents to pursue lower variance, lower return projects. This leaves the poorest farmers to become more poor and the wealthier or less risk-averse ones to become less poor thereby leading to an exacerbation of inequality.

Mathematically, the above is formalized as follows: "...let $Q_i$ be the yield of crop $i$, and $P_i$ the net price of crop $i$. Assume that an individual farmer’s production does not affect the world price, so that $P_i$ is independent of $Q_i$. Then, for a given production level $Q_i$..." assuming that $Q_i$ is non-stochastic, the expected profit $\pi_i$ is given by

$$E[\pi_i] = E[P_iQ_i] = Q_iE[P_i]$$

and the variance of the profit is given by

$$V(\pi_i) = V(P_iQ_i) = Q_i^2V(P_i)$$

Now, assume that the price $P_i$ for each crop follows a uniform distribution $Unif[a_i, b_i]$ where $0 \geq a_i \geq b_i$. The expected value of a uniform distribution is $\frac{a_i + b_i}{2}$ and the variance is given by $\frac{(b-a)^2}{12}$, plugging these facts in we get that

$$E[\pi_i] = Q_i\frac{a_i + b_i}{2}$$

and

$$Var(\pi_i) = Q_i^2\frac{(b-a)^2}{12}$$

Recall a key assumption of the paper: higher return projects have higher volatility. Does the assumption of a uniform distribution of crop prices satisfy that? Order projects from lowest to highest expected return, so that $i < j \implies E[\pi_i] < E[\pi_j]$ then it is sufficient to check that the derivative of $E[\pi_i]$ and $Var(\pi_i)$ is positive with respect to crop $i$ and indeed that is the case when $a_i + b_i > 0$ and $b_i > a_i$ for mean and variance respectively. Combining the two conditions, we get $b_i > a_i \geq 0$ so under these assumptions, higher return projects do have higher volatility -let this assumption hold from hereon. Now we must check whether it is credible for the Marginal Variance Cost to shift enough so that we go from NAM to PAM.

Recall Professor Wang’s Corollary 1 that says:

**Theorem 1** Let $CV(p) = \frac{Var(p)^{\frac{1}{2}}}{p}$ represent the coefficient of variation of a project $p$. Then:

a) $CV'(p) < 0 \forall p > 0$ iff $M''(p) < 0 \forall p > 0 \implies (PAM)$

b) $CV'(p) > 0 \forall p > 0$ iff $M''(p) > 0 \forall p > 0 \implies (NAM)$
Plugging in the distributional assumptions gives us:

\[
CV_i = \frac{\text{Var}(\pi_i)}{E[\pi_i]} = \frac{b_i - a_i}{\sqrt{3(a_i + b_i)}}
\]

\[
\frac{\delta}{\delta_i} CV(i) > 0 \iff a_jb_j < a_ib_i
\]

\[
\frac{\delta}{\delta_i} CV(i) < 0 \iff a_jb_j > a_ib_i
\]

When will the above \(a_jb_j\) and \(a_ib_i\) relationship hold? Suppose that before the implementation of the policy, prices for crop \(i\) followed the ensuing conditions:

\[
P_{\text{pre}}^i \sim [1, \bar{P}_i] \quad \text{s.t.} \quad \bar{P}_i \geq 1, \bar{P}'_i > 0.
\]

So all it says is that the price for the crop falls between 1 and some upper bound \(\bar{P}_i\) with the upper bound being greater than one and increasing in \(i\).

Now "the government imposes a price floor and a price ceiling, to reduce the price volatility, so that post policy:

\[
P_{\text{post}}^i \sim \left[ \frac{1}{2}(\bar{P}_i + 1) - c, \frac{1}{2}(\bar{P}_c + 1) \right], \; c \in [0, \frac{1}{4}(\bar{P}_c - 1)]
\]

so that for all crops, the expected income is now \(\frac{1}{2}(\bar{P}_i + 1)\) so that \(E[P_{\text{pre}}^i] = E[P_{\text{post}}^i]\) but now the variance for each project is reduced and whereas before the policy the \(CV\) for projects with smaller expected returns is smaller but now the \(CV\) is smaller for projects with higher expected returns. This results in a switch from NAM to PAM matching. Professor Wang illustrates the shift by returning to the initial formulation of project spectrum so that pre-policy the spectrum is characterized by:

\[
Y_i = Q_i \frac{\bar{P}_i + 1}{2} + Q_i \frac{\bar{P}_c - 1}{\sqrt{12}} Y_0
\]

where \(Y_0 \sim \text{Unif}[\frac{-\sqrt{12}}{2}, \frac{\sqrt{12}}{2}]\) and post-policy the spectrum is:

\[
Y_i = Q_i \frac{\bar{P}_i + 1}{2} + Q_i \frac{c}{\sqrt{3}} Y_0
\]

For analyzing the change in welfare as a result of the shift from NAM to PAM assume that pre-policy profits for project \(p\) are distributed normally

\[
\pi_{p}^{\text{pre}} \sim N(p, p_{1}^{N})
\]

where \(N1 > 2\) and post policy project \(p\) returns are distributed about

\[
\pi_{p}^{\text{post}} \sim N(p, p_{2}^{N})
\]

where \(N2 \in (1, 2)\). The the marginal variance cost is the derivative with respect to \(p\) of the variance, differentiating the respective variances gives us:

pre: \(M_{\text{pre}}(p) = N_1 p^{N_1 - 1} \implies \text{convex}\)

post: \(M_{\text{post}}(p) = N_2 p^{N_2 - 1} \implies \text{concave}\)

At \(N = 2\) the curvature is linear and so all pairs are stable in equilibrium. Since policy is typically only able to reduce volatility by a small amount, consider \(N_1 = 2 + \epsilon\) and \(N_2 = 2 - \epsilon\).
According to Lemma 2, if the volatility of every project is reduced without changes in pairing then the policy is Pareto Improving for everyone. For example, suppose you make on average $100 a day but on certain days you can make as much as -$400 and on others $500. Now that variance is reduced to $80 at the lower bound and $120 at the upper bound then certainly, you are better off. If the pair switches projects then they are strictly better off under the new project than the old project with decreased variance.

If the groups do switch though, the risk reduction is no longer Pareto improving due to the shift in groups. Namely, the most risk averse agents lose the less risk averse agents as their insurers and so cannot smooth their consumption. Additionally, since they are not paired with other highly risk-averse agents, the pair must opt for projects that have lower mean returns and lower variances since neither can smooth the others’ consumption.

On the other hand, the least risk-averse agents pair with each other and undertake riskier ventures with higher returns by, say, adopting a new technology so these agents are better off from the policy. This can be graphically seen in 3, the lower the rank, the less risk averse are the individuals. So the two least averse pairs gain substantially from the policy while the two more risk averse pairs lose from the policy in their expected payoff.

Figure 3: Equilibrium Project Choice Pre- and Post-Policy

![Figure 3](image1.png)

Figure 4:

![Figure 4](image2.png)
5 Endogenous Group Size

Until now the unique equilibrium under the condition when group size is constrained to be of size two.

The following equation defines the representative risk tolerance of a group of N matched people:

\[ R = \sum_{i=1}^{N} \frac{1}{r_i} \]

The author makes some adjustments to the benchmark model, in order to adapt to an endogenous group size.

In the benchmark model individuals are matching across two unique groups. However, in an endogenous group size model, all individuals belong to one group. They will be matched within this group G.

It requires at least two collaborators. Every group of matched agents will have a risky project \( p \), where \( p \geq 0 \) and the project’s return can be described in \( Y_p = p + V(p)Y \).

Where \( Y \) is a random variable with cdf and \( E(Y) = 0 \), \( V(Y) = 1.20 \).

Individuals in the group commit to a sharing rule \( s_2(y_p), s_3(y_p), \ldots s_N(y_p) \). It describes a formation where each possible output there is a share realized output.

**Proposition 5** Each project \( p \) has a marginal variance cost \( M(p) \equiv V'(p) \)

a. If \( M''(p) < 0 \), the unique equilibrium is the maximal-connectedness, which means every individuals in the group G is matched.

b. If \( M''(p) > 0 \), the unique equilibrium is the minimal-connectedness, which means the \( i^{th} \) least risk averse person is matched with the \( i^{th} \) most risk averse one.

**Proof:** Through the proof of proposition 1 we have

\[ M''(p) > 0 \Leftrightarrow CE(R_1 + R_2) < CE(R_1) + CE(R_2) \forall R_1, R_2 \in \mathbb{R}_0^+ \]

\[ M''(p) < 0 \Leftrightarrow CE(R_1 + R_2) > CE(R_1) + CE(R_2) \forall R_1, R_2 \in \mathbb{R}_0^+ \]

In addition, the R of a matched group is sum of R of individuals. Therefore, if \( M'(p) \leq 0 \) and \( CE(.) \) is concave in R, maximizing the sum will require minimal matching and vice versa.

This proposition proves that when \( V'(p) \) is concave, the unique equilibrium, or the optimum, is the matching when group size equals two which means pairwise matching. However, when \( V'(p) \) is convex in expected return, all individuals in the group is maximally-connected. Therefore, the \( V'(p) \) has effect on both the extremal match composition and the extremal structure of how the risk sharing network is matched.

In a real world case, a maximal-connected network economy will have a large firm that chooses a risky project and a minimal connected network economy will have many small and heterogeneous firms, each choosing less risky projects with lower mean and lower variance in turns of project return.
However, this conclusion contrasts with Genicot and Ray (2003), which shows that the group cannot reach a sustainable equilibrium when individuals are homogeneously risk-averse.

6 Falsifiability

In the section the author provides a variety of approaches to test the theory through empirical methods. The author suggested the researcher can take an approach of calculating the $V(p)$ and $CV(p)$, and verify the theory through:

1. relationship of curvature in $V'(p)$
2. monotonicity of coefficient of variation in $E(p)$

The author suggested that $V(p)$ and $CV(p)$ might not be always be reliably constructed. So taking the proposition 4 is another approach.

6.1 Example

Suppose $G_1 = \{r_{A1}^A, r_{A2}^A, r_{A3}^A\}$ and $G_2 = \{r_{B1}^B, r_{B2}^B, r_{B3}^B\}$, where $r_{A1}^{AB} < r_{A2}^{AB} < r_{A3}^{AB}$. Suppose $M''(p) < 0$. This follows the approach of proposition 4, using observed matching to calculate risk tolerance $R$ of each pair. Then check the convexity or concavity of mean returns in risk tolerance:

\[ p_i = \beta_1 + \beta_2 R_i + \beta_3 R_i^2 + \epsilon_i \]

if $\beta_3 > 0$ than it suggests $p_i$ is convex in $R_i$ which supports the theory.

6.2 Experiment

The author uses the dataset from Attanasiao et al (2012) to seek support or falsification of the theory. Attanasiao’s experiment includes 70 Colombian communities for a laboratory test among risk-sharing groups. Players can choose their preferred gamble where higher mean return have higher variance. The sharing rule is fixed which means that the group member can choose what risk but not how the risk is shared. The information of kinship and friendship is also included in the experiment because people will likely have more information about the risk attitudes of people they have close relationships with. In addition, the close relationship has a significant influence on risk sharing activity.

Because the sharing rule is fixed in this experiment, the data cannot be used for a full test for the model. Rather, we can use Proposition 4 for a partial test. It describes the relationship between the curvature of mean return and risk tolerance of matched groups and $V'(p)$. The theory will be proven if the mean income is concave in risk tolerance.

The author regresses the mean returns of chosen gambles with following equation

\[ p_i = \beta_1 + \beta_2 \left( \frac{1}{r_i} \right) + \beta_3 \left( \frac{1}{r_i} \right)^2 + \epsilon_i \]

The result is in the following table, and shows mean return is concave in risk tolerance and players with higher risk tolerance will choose riskier gamble:
7 Conclusion

The main contribution of this paper is that it develops and explores the implications a theory of endogenous relationship formation between heterogeneously risk-averse people without access to formal insurance and credit markets. Particularly, how they choose what risk to face and how to share that risk. The implications of this framework are not limited to insurance – because informal risk management influences entrepreneurship, entrepreneurship’s interaction with income inequality, and the optimal structure of informal firms.

Breaking away from the previous literature, Wang makes the structure of the groups that engage in these risk-management relationships endogenous. In doing so, she studies the emergence of a network shape that depends on the possibilities of other structures the network could have assumed. And she identifies connections between the equilibrium network and the risk environment. A remarkable feature of her model is that it allows for a large class of distributions, yet she finds that equilibrium matching is found with only the first two cumulants: the mean and the variance. The mean-variance trade-off is neatly captured by the coefficient of variation.

A main take-away is the importance of heterogeneity in developing economies. A natural constraint on informal risk-management is the fact that everyone is risk-averse to some degree. Yet since people do differ in risk-aversion, there is still opportunity for informal insurance, and the degree of heterogeneity determines the strength of the insurance. In practice, circumstances are a key factor in determining which informal roles individuals assume.

The endogeneity of insurance has significant policy implications that include those discussed in this paper but reach farther, and leave plenty of directions to take future research.

8 Critique & Extensions

Overall, this paper is highly robust, mostly because potential caveats and critiques are addressed by Professor Wang as they come up. Moreover, the assumptions made in the model are also all addressed in a manner that keeps the paper incredibly general. The model is highly sensitive to the assumptions one makes on the variance parameters of returns, certainty equivalence, or any other function of variance that may shift the marginal cost of variance from convex to concave. Namely, recall from the Policy section the, albeit arbitrary and changeable, assumption that returns are normal $N(p, p^2 N^2)$. As the paper mentions, the variance on returns must be on the brink of $p^2$ so that in case of a policy shift we have a change in curvature. Although the model is robust to different types of distributions of project returns, it nevertheless seems dependent on the distribution parameter values. Of course, this may be seen as a flexibility that gives more leeway in the approach of policy evaluation and model testing. This model reminds us of

<table>
<thead>
<tr>
<th>Gamble</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Coeff. of Variation</th>
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<tr>
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<td>0</td>
<td>0</td>
</tr>
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</tr>
<tr>
<td>5</td>
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<td>5000</td>
<td>0.83</td>
</tr>
</tbody>
</table>
labour models that are frictionless, i.e., no moving costs, no monopsony, etc. In Professor Wang’s original, and arguably current rendition, this paper pertains to the pairing between employer and employee where once paired, the employee works on the land of the employer and the project they pursue together is the yield of employer’s land. One particular concern is the cost of moving if there is indeed a shift in matching paradigm. The cost may include not only the pecuniary costs of transporting one’s family, belongings but also the time and effort of moving, as well as, the cost of acquiring information on potential pairings. Consequently, although in a frictionless environment a policy may imply a shift from NAM to PAM, the moving costs may in fact disincentivize the repairing. Adding an additional layer would involve the distribution of moving costs among the employees. For example, the less risk averse typically tend to be higher income farmers who may be more inclined to bear the immediate cost of moving for a potentially much higher income later on. On the other hand, the poorest tend to be the most risk-averse so they would not only lack the means to moving but may also be unwilling to incur the cost of moving. As a result, we have a force whose effect on the outcome is unclear but one that may be critical in considering actual policy impact. One way of addressing this would be perhaps to characterize people not only according to their risk attitudes $r_i$ but also according to their moving disutility $c_i$ so that consists of tuples, namely, $G^2 = \{(r_i, c_j)\}_{i \in R, j \in C}$. Incorporating this into the model would involve concocting another transferable utility specification, perhaps by adding the heterogeneous moving costs to the pair-wise CE equations. At this point, delving any further into it would require us to write a new paper as this may also influence the ex-ante sharing of the income and may also necessitate the use of time-inconsistent preferences to capture the perceived cost of moving now but reaping benefits later.

8.1 Additional Extensions

In her conclusion, Wang suggests potential uses for the model. She briefly explains: "...understanding the equilibrium matching between individuals with different risk attitudes and the risky health behaviors they choose may help us design more effective approaches to encouraging vaccination, boosting health investments and sanitation practices, and preventing and treating HIV.”

She has gone on to extend it in her future work. In Wang (2013),”Risk, Incentives, and Contracting Relationships,” she uses ideas from this paper to analyze relationships between employers and employees. In this model, there is a moral hazard, because the returns for the employer depend on unobservable employee effort, and the employee’s contract does not depend on her effort. She finds that better insurance comes at the cost of weaker incentives. In Wang (2015),”Policymaking and the Adaptability of Informal Institutions,” she expands on the implications of this framework on entrepreneurship and inequality.

Like many models, it is easy to imagine an application of this one to marriage. People have a preference for a certain level of risk that is revealed in their career choices. It may be beneficial to marry someone who has a career that is more or less risky. For instance, a NAM-like pairing could be between an accountant and an aspiring writer. A career as an accountant won’t make you the wealthiest person in the world, but it would ensure a low-risk, stable income. On the other hand, writing is a field with a superstar effect – you could become J.K. Rowling, but you will likely spend most of your time behind the counter at Starbucks. It is an extremely risky career to go into, but it has a much higher potential return than an accountant. A marriage between an accountant and an aspiring writer could provide exposure to the high potential gains of an acting career for the former, and a form of insurance for the latter. A PAM matching would similar to a marriage between two lawyers, which is actually what we see more of in practice. Policy-makers use the social insurance system, the tax system, and other forms of
social policy to incentivize certain marriage arrangements. A model where group structure is endogenous suggests that these policy-makers should be cognizant of the potential effects on informal relationships that their preferred policies may have.

Another potential application is college student buddy-system. A student who likes to party and consume mind-altering substances could be thought of as being less risk-averse, but could also (by some people’s evaluations) have more fun (higher returns) than less ”active” students. In a NAM-like matching, it is conceivable that the partier would gain from having a straight-edge friend to look out for them (i.e. insurance). The straight-edge friend benefit from being exposed to more potential fun. One can imagine several policies that a college could implement that would have implications in a model where group structure is endogeneous. For instance, if the college provides more alcohol-free activities that the straight-edge students consider fun, then they have less incentive to go out with their partying friends. Or the college could provide weekend van services or have more forgiving policies for students who become too intoxicated. Then partiers would have less incentive to bring their straight-edge friends with them.

This model may also be applicable to the happenings of Europe, where before the crisis we can consider the international bond buying as being in a risky environment without the existence of a formal insurer. Germany is notoriously risk-averse whereas Greece can be considered as less risk-averse (though is that the same thing disregarding risk entirely?). Before the crisis, there was a PAM-like pairing where less-averse lenders lent to other less risk-averse countries. After the crisis; however, in order to keep the union together (maximum transferable utility), there was an endogenous shift in pairing to NAM, where risk-averse Germany now lends to dilly dallying Greece.

9 Appendix

Professor Wang’s paper is surprisingly very readable and the Appendix is no exception. Consequently, we felt no need to include all 30 pages of the Appendix and an elaborate explanation of the proofs therein. Instead, we include proofs we thought were of particular importance.

Corollary 1 is particularly important for many reasons. One, it incorporates the Sharpe Ratio that Professor Becker asks about in the questions. Second, it is used as an empirically testable hypothesis through the availability of this information and it illustrates, again, the effect on the matching algorithm of a change in the curvature of the marginal cost of variance.

Theorem 2 Let \( CV(p) = \frac{\text{Var}(p)^{1/2}}{p} \) represent the coefficient of variation of a project \( p \). Then:

\[ CV'(p) < 0 \forall p > 0 \iff M''(p) < 0 \forall p > 0 \implies \text{(PAM)} \]

\[ CV'(p) > 0 \forall p > 0 \iff M''(p) > 0 \forall p > 0 \implies \text{(NAM)} \]

Proof: According to the Wikipedia page on the Sharpe Ratio, is \( \frac{E[R_r - R_b]}{\sigma} \) where \( R_r \) is the expected return on a risky venture, \( R_b \) is a benchmark project, such as a risk-free bond, and \( \sigma \) is the standard deviation of the excess return, namely \( \sqrt{\text{Var}(R_r - R_b)} \). The Sharpe Ratio is used as a gauge for the superiority of the risky venture over a non-risky venture. The higher the ratio, the better. In the context of the paper, the expected return on the risky venture is \( p \) and the return on non-risky venture is 0. Hence the Sharpe Ratio takes the form:

\[ SR(p) = \frac{p}{\sqrt{V(p)^{1/2}}} = \frac{1}{CV(p)} \]
where $SD(p) = V(p)^{1/2}$ and coefficient of variance as defined in the paper. Differentiating the above:

$$\frac{\delta}{\delta p} SR(p) = \frac{\delta}{\delta p} pV(p)^{-1/2} = 1[V(p)^{-1/2}] + p[\frac{\delta}{\delta p} V(p)^{-1/2}] = V(p)^{-1/2} - p[\frac{1}{2} V(p)^{-3/2} V'(p)]$$

where we used the product rule and chain rule. Now, put the two terms under the same denominator $V(p)$ to get:

$$SR(p) = \frac{V(p)^{1/2} - \frac{1}{2} V(p)^{-1/2} V'(p)}{V(p)}$$

Since $V(p) > 0 \forall p$ by assumption that $Var(p) = 0 \iff p = 0$, and property of variance always being non-negative, the above is strictly positive if $V(p)^{1/2} > \frac{1}{2} V(p)^{-1/2} V'(p)$ and negative if $\frac{1}{2} V(p)^{-1/2} V'(p) > V(p)^{1/2}$. To establish the condition under which each of the above is true, differentiate $V(p)$ and $\frac{1}{2} p V'(p)$ with respect to $p$ to get $V'(p)$ and $pV''(p)$ and then differentiate again to get $V''(p)$ and $V'''(p) + pV'''(p)$ so now the terms are comparable. Moreover, as established in the paper before, $V'''(p) = M''(p)$ which determines whether we have PAM or NAM matching. Consequently, by tracing the inequalities back: $SR'(p) > 0$ if $0 > pV'''(p)$ and $SR'(p) < 0$ if $pV'''(p) > 0$ which implies that $SR'(p) < 0 \implies NAM$ and $SR'(p) > 0 \implies PAM$. 