Financial Intermediation, Loanable Funds, and the Real Sector

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1 Background and Summary

The authors of this paper are Bengt Holmstrom and Jean Tirole. Holmstrom is a Professor in the Economics department at MIT, while Tirole, a recent Nobel Laureate, teaches at the IDEI. These two authors have written many papers together, including the one we will discuss today regarding financial intermediation. The paper was published in the QJE in 1997.

In this paper, Holmstrom and Tirole explore an incentive model of financial intermediation between firms and intermediaries, a topic that had already been studied. However, in this paper, the authors view a situation in which both firms and intermediaries are capital constrained, the latter of which had not been explored as yet. They analyze how the distribution of wealth across three parties (firms, intermediaries, and investors) affect three different fronts—levels of investment, interest rates, and intensity of monitoring (done by the intermediary). The paper attempts to construct a simple equilibrium model of credit in order to replicate some of the stylized facts on lending patterns observed in recent (note the year of publication, 1997) financial crises.

Perhaps the most important takeaway from the paper is that in times of capital tightening (of which we will explore three types), poorly capitalized firms are hit the hardest. Further, firms with substantial capital can rely on cheaper and less information sensitive forms of financing. Another finding is that interest rate effects and the intensity of monitoring will depend heavily on the relative changes in the various components of capital. That is to say that there are scenarios we will encounter with certain ambiguities in the direction of interest rates.

The authors discuss how these findings are consistent with various situations around the world, most notably in Scandinavia—a topic we will later explore more thoroughly.
2 Key Variables

2.1 Key Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Each firm’s capital</td>
</tr>
<tr>
<td>$G(A)$</td>
<td>Cumulative distribution function of assets across firms</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment costs</td>
</tr>
<tr>
<td>$R$</td>
<td>Total return on investment</td>
</tr>
<tr>
<td>$B$ (or $b$)</td>
<td>Private benefit for entrepreneurs</td>
</tr>
<tr>
<td>$c$</td>
<td>The cost of monitoring faced by an intermediary</td>
</tr>
<tr>
<td>$K$</td>
<td>The aggregate amount of capital</td>
</tr>
<tr>
<td>$\beta$</td>
<td>The equilibrium rate of return on intermediary capital</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The market rate of return on investor capital</td>
</tr>
<tr>
<td>$S(\gamma)$</td>
<td>The supply function which determines the aggregate amount invested uniformed capital</td>
</tr>
<tr>
<td>$m, u, f$</td>
<td>These subscripts correspond to intermediary, investors, and firm, respectively</td>
</tr>
</tbody>
</table>
3 The Basic Model

The model includes the following types of agents:

1. Firms
2. Intermediaries
3. Investors

(as defined earlier)

Assumptions

1. All agents are risk neutral.
2. They are protected by limited liability so that no one can end up with a negative cash position.

There are also two periods in our model. In the first period, financial contracts are signed, and investment decisions are made. During the second period, investment returns are realized, and claims are settled.

3.1 The Real Sector

There are a continuum of firms. They have access to the same technology, the only difference among them is that they begin with different amounts of capital, denoted as $A$. We assume that initial capital is cash, or any other type of asset that can be pledged as collateral with the same value as $A$. The distribution of assets across firms is described by the cumulative distribution function $G(A)$, which indicates the fraction of firms with assets less than $A$

The aggregate amount of firm capital is given by the following equation,

$$K_f = \int A dG(A).$$

Basic Version of the Model

Each firm has one economically viable project or idea. It costs $I > 0$, in period 1 to undertake a project. If $A < I$, then a firm needs at least $I - A$ in external funds to be able to invest. In period 2, the investment generates a verifiable, financial return equaling either 0 (failure) or $R$ (success).

The Firm’s Moral Hazard Problem
Firms are run by entrepreneurs, who may deliberately reduce the probability of an investment’s success to secure a private benefit. The author formalizes this moral hazard problem by describing three discrete versions of the firm’s project. First, we will use the following equation to describe the change in probability of an investment’s success caused by the entrepreneur’s self-interest.

\[ \Delta p = p_H - p_L > 0 \]

\( p_H \) denotes the probability of the high success project. \( p_L \) denotes the probability of success for the projects that have a lower rate of success, but generate a private benefit for the entrepreneur. From these probabilities, we can create the following table, which describes the firm’s 3 scenarios.

<table>
<thead>
<tr>
<th>Project</th>
<th>Good</th>
<th>Bad (low private benefit)</th>
<th>Bad (high private benefit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Benefit</td>
<td>0</td>
<td>b</td>
<td>B</td>
</tr>
<tr>
<td>Probability of Success</td>
<td>( p_H )</td>
<td>( p_L )</td>
<td>( p_L )</td>
</tr>
</tbody>
</table>

Private benefits are ordered \( B > b > 0 \). Also note, that both bad projects have the same probability of success. This is analytically convenient, because we can assume that the entrepreneur will prefer the high private benefit project to the low private benefit project (i.e. \( B > b > 0 \)) irrespective of the financial contract. Eventually, we will need two types of bad investments to sufficiently model monitoring.

The following decision tree, from a handout created by the University of Essex, outlines the firm’s choices.

![Decision Tree](image)

**Private benefit**

- \( p_H \) if monitored
- \( 1 - p_H \) if not monitored

- \( p_L \) if monitored
- \( 1 - p_L \) if not monitored

**Figure 1: Firms’ investment opportunities**

Interpretation: Each firm chooses \( H \) or \( L \) (and \( B \) or \( b \) if \( L \) is chosen); then ‘Nature’ chooses success or failure with the stated probabilities. Everyone is assumed to know the probabilities \( p_H \) and \( p_L \) that Nature will act in a particular way.
Furthermore, we assume that in the relevant range of rate of return on investor capital, denoted by $\gamma$, only the good project is economically viable. That is,

$$p_H R - \gamma I > 0 > p_L R - \gamma I + B$$  \hspace{1cm} (1)

Here we should make a couple notes about the preceding equation. First, the probability of success $P_L$ is not affected by the level of private benefit. Therefore, the entrepreneur will always prefer the high benefit project ($B$) over the low private benefit project ($b$). Furthermore, we can easily alternatively interpret the private benefits as opportunity costs from managing the project diligently.

### 3.2 The Financial Sector

The financial sector consists of many intermediaries. Their function is to monitor firms’ investments and mediate the previously mentioned moral hazard problem. In practice, this means the intermediary will inspect the firm’s cash flows, its balance sheet position, its management, etc. The intent of monitoring is to ultimately lower the opportunity cost of being diligent. Therefore, we assume that can prevent the undertaking of the $B$ project. This reduces the firm’s opportunity cost from $B$ to $b$.

We assume that intermediaries will have to pay some amount $c > 0$ in order to eliminate the $B$ project. Thus, we have another moral hazard problem.

#### The Intermediary’s Moral Hazard Problem

While we assume that each intermediary has the capacity to monitor an arbitrary number of firms, the moral hazard problem puts a limit on the actual amount of monitoring that will take place. The solution to this problem is actually quite simple, which can be seen in reality.

The moral hazard forces intermediaries to inject some of their own capital into the firms they monitor. This makes the aggregate amount of intermediary or "informed” capital $K_m$. This will be one of the important constraints on aggregate investment. The equilibrium rate of return on intermediary capital, $\beta$ will be discussed later.

One important point to mention here is that all projects financed by an intermediary are \emph{perfectly correlated}, and that the capital of each intermediary is sufficiently large relative to the scale of a project.
3.3 Investors

Individual investors are small. We will refer to them as "uniformed" investors, because, unlike intermediaries, they will not directly monitor the firms they invest in. Uninformed investors demand an expected rate of return $\gamma$.

The author’s assumptions on $\gamma$ will occasionally change but will be noted. Sometimes, we assume that $\gamma$ is exogenously given. Other times, the aggregate amount of uninformed capital invested in firms is determined by a standard, increasing supply function $S(\gamma)$.

We assume that firms cannot monitor other firms. This leads to the assumption that firms with excess capital will have to invest their surplus cash in the open market, earning $\gamma$ (i.e. the uninformed rate of return).

In other words, an investment project that an uninformed investor takes part in, must return at the very least $\gamma$, as evidenced by Equation (1).

Now, that we have described the three agents, we can move on to questions of investment.

4 Fixed Investment Scale

In this section, we assume that the investment scale $I$ is fixed.

4.1 Direct Finance

Direct finance is defined as financing a project without intermediation.

Note: This brings about an important distinction. That is in the case of Direct Finance, only two parties are necessary. Firms and Investors (uniformed Investors specifically).

Consider a firm that only borrows from uniformed investors, treated here as a single party. A contract dictates how much each side should invest and how much each side is paid as a function of outcome. It is clear that one optimal contract will have the following structure:

1. The firm invests all of its funds $A$, while the uninformed investors put up the difference between $I - A$.

2. If the investment fails, both parties receive no compensation
3. If the investment is successful, the firm is paid $R_f > 0$ and the investors are paid $R_u > 0$.

From this it is clear that we can derive the following equation from point (3),

$$R = R_f + R_u$$

From equation (1), we realize that a necessary condition for direct finance is that the firm prefers to be diligent,

$$P_H R_f \geq P_L R_f + B$$

Direct finance requires that the firm get paid at least the following,

$$P_H R_f \geq P_L R_f + B
\quad (\Delta P + P_L)R_f \geq (\Delta P + P_H)R_f + B
\quad P_L R_f + P_H R_f \geq B
\quad (\Delta P)R_f \geq B
\quad R_f \geq \frac{B}{\Delta P}$$

We refer to this last line as the incentive constraint of the firm ($IC_f$). The last equation leaves at most $R_u = R - \frac{B}{\Delta P}$ to compensate investors. Therefore, the maximum expected income (or pledgeable expected income) is given by

$$P_H [R - \frac{B}{\Delta P}]$$

It should also be noted that the pledgeable expected income cannot be less than $\gamma[I - A]$, the market value of funds supplied by the uninformed investors.

This brings us to a necessary and sufficient condition for the firm to have access to direct finance. That is,

$$\gamma[I - A] \leq P_H [R - \frac{B}{\Delta P}]$$

$$\quad (I - A) = \frac{P_H}{\gamma} [R - \frac{B}{\Delta P}]$$

$$A(\gamma) = I - \frac{P_H}{\gamma} [R - \frac{B}{\Delta P}]$$

(2)

We define Equation (2) as $A(\gamma)$. We conclude that only firms with $A \geq A(\gamma)$ can use direct finance. That is, firms with sufficient capital.

Note that $A(\gamma)$ could be negative, in which case firms could invest without their own capital. However, the authors rule out this case as it is uninteresting.
Explanation of the assumption that $A(\gamma) > 0$:

If $A(\gamma) < 0$, the firm does not need to invest anything and can totally rely on the uninformed capital. To eliminate this undesired case, we assume,

$$p_H R - \gamma I < \frac{-p_H B}{\Delta p}$$

so that the benchmark for firm’s initial capital endowment is positive, $A(\gamma) > 0$. The intuition is that: if all the investment comes from uninformed capital and the firm itself invests nothing, it has to pay at least $\gamma I$ to the uniformed investors. Under this circumstance, the total surplus from the good project is not sufficient to pay the minimum return for firms to perform diligently. So the firm has to choose to invest its own capital to reduce the $I$ in the inequality so that the interests saved on the uninformed capital will be directed to the return to firms until the total surplus from the good project is sufficient to pay the firms to perform diligently.

Like other models with liquidity constraints, efficiency is not defined by total surplus maximization. It is true aggregate surplus could be increased by reallocating funds from uninformed investors to firms that are capital constrained, such transfers are NOT Pareto improving. There are no externalities in this model that the firm and the investor cannot internalize just as effectively as a social planner facing the same informational constraints.

### 4.2 Indirect Finance

The structure for Indirect Finance follows the same basic framework as Direct Finance. However, in the case of Indirect Finance we must have three parties to the financial contract. These are the two from the Direct Finance case, Firms and Uniformed Investors, but now also an Intermediary. Intermediaries can be thought of as a commercial bank. The purpose of these banks is to offer external capital to capital constrained firms, as well as monitoring the firm. By doing so, the bank reduces the firm’s opportunity cost of being diligent, and thus eliminates the high benefit $B$-project.

The payoffs of this three party case takes the same form as before where if the project fails, all parties are left with zero. If the project succeeds, the payoff $R$ is split three ways, such that

$$R = R_f + R_u + R_m$$

Here $R_m$ denotes the share given to the intermediary, as $R_f$ and $R_u$ denote the firm and uninformed investor share as they did prior. If the project is successful, $R_f, R_u, R_m > 0$
If the project fails, $R_f, R_u, R_m = 0$

Suppose the intermediary monitors. Because monitoring eliminates the high benefit project (the B-project), the firm is left between deciding between the good project (with probability of success $p_H$), and the low benefit project (the b-project, with probability of success $p_L$).

The firm’s incentive constraint, denoted as $(IC_f)$, now becomes

$$R_f \geq \frac{b}{\Delta P}$$

This is derived the same way as before in the case of the high-benefit project B, but replacing the high benefit $B$ with the low benefit, $b$.

In the case of Indirect Finance, we can safely assume that $R_f < \frac{B}{\Delta P}$, else otherwise, the firm would behave without monitoring.

In order to properly incentivize the intermediary to monitor, we must have $R_m \geq \frac{c}{\Delta P}$. This is the intermediary’s incentive constraint denoted as $(IC_m)$.

Note: $(IC_f)$ and $(IC_m)$ imply minimum returns for the firm and the intermediary, respectively.

Here, the pledgeable expected income (maximum expected income that can be promised to uninformed investors without destroying firm or intermediary investors) is given by:

$$P_H[R - \frac{b + c}{\Delta P}]$$

Take a second to note that $(IC_m)$ implies that

$$P_H R_m - c > 0$$

Which ensures that monitoring earns a positive net return in period 2. Also, competition among intermediaries reduces this surplus by forcing intermediaries to investment in the firm’s project during period 1. Furthermore, intermediary capital is entirely invested in the monitoring of firm’s projects.

For now, assume that monitoring capital is scarce - this ensures that intermediaries make a strictly positive profit.

Let $I_m$ be the amount of capital that an intermediary invests in a firm that it monitors. The rate of return on intermediary capital is then:

$$\beta = \frac{p_H R_m}{I_m}$$
Monitoring is a costly procedure, and thus $\beta$ must exceed $\gamma$. In words, this states that informed capital rate of return is strictly greater than that of uninformed capital rate of return. Consequently, firms prefer (whenever possible) to be supplied with uninformed capital rather than informed capital.

However, since the intermediary incentive constraint ($IC_m$) requires the intermediary to be paid at least $R_m = \frac{c}{\Delta p}$, the intermediary will contribute at least:

$$I_m(\beta) = p_H \frac{R_m}{\beta}$$

$$I_m(\beta) = \frac{p_H c}{\Delta p \beta}$$

This yields a very important assumption. That is that all firms that are monitored will demand exactly this minimum level of informed capital. Demanding any amount higher would be excessively costly, and any amount less would negate existing incentives.

The intermediary incentive is being drive by the return $R_m$, and the required investment of $I_m(\beta)$ regulates the intermediary capital so that the market for informed capital clears. In fact, we could take either $I_m$ or $\beta$ as the equilibrating variable, as the relationship between the two is monotone.

Now, the uninformed investors will supply the remaining investment ($I_u$), given by:

$$I_u = I - A - I_m(\beta)$$

Therefore, a necessary and sufficient condition for a firm to be financed is then

$$\gamma [I - A - I_m(\beta)] \leq p_H [R - \frac{(b + c)}{\Delta p}]$$

which states that for the uninformed investors, the expected return on the project must exceed their risk free rate.

We rewrite the above condition as:

$$A \geq A(\gamma, \beta) = I - I_m(\beta) - \frac{p_H}{\gamma} [R - \frac{(b + c)}{\Delta p}]$$ (5)

A firm with less than $A(\gamma, \beta)$ in initial capital assets will not be able to convince uninformed investors to supply enough capital for the project.
If there were a combination of interest rates such as $\beta$ and $\gamma > A(\gamma)$, it would indicate that the price of monitoring is too high, and there will be no demand for monitoring. To offset this, the value of $\beta$ would have to come down. Note however, that $\beta$ must still be high enough to make the intermediary prefer monitoring in the firms’ project rather than investing its own capital in the open market, earning the rate of return $\gamma$. It follows that the minimum acceptable rate of return $\beta$ is given by the following condition.

$$\frac{p_{HC}}{\Delta p} - c = \gamma I_m(\beta) = \gamma \frac{p_{HC}}{\Delta p}$$

which translates into

$$\beta = \frac{p_H \gamma}{p_L} > \gamma$$

If $A(\gamma, \beta) > A(\gamma)$, then there is no demand for informed capital, even at the lowest acceptable rate of return to the monitor. It would mean that the monitoring technology is too costly to be socially valuable. In order for monitoring to be socially valuable, it must hold that $c \Delta p < p_H [B - b]$. This condition is met for a small enough $c$, following from the fact that $B > b$.

### 4.3 Summary of 4.1 and 4.2

Here is a summary of the preceding discussion.

We assume that $A(\gamma, \beta) > A(\gamma, \beta)$.

$$A(\gamma, \beta) = I - I_m(\beta) - \frac{p_H}{\gamma} \left[ R - \frac{(b+c)}{\Delta p} \right] \Rightarrow I - A(\gamma, \beta) = \frac{p_H}{\gamma} \left[ R - \frac{(B)}{\Delta p} \right]$$

is the maximum external capital (only informed capital) available to the firm under direct finance.

**Important Explanation of the Assumption that $(b + c) < B$:**

We also assume that $(b + c) < B$. Otherwise, there might be cases in which the intermediary will monitor without investing anything in the firm. For instance, if a firm starts with an initial capital endowment in the range below, the firm is eligible to use indirect finance. However, the maximum uninformed capital will be greater than the gap $(I - A)$; this implies that the intermediary don’t need to invest anything but still monitor.
**Case 1:** If $A < A(\gamma, \beta)$. In this case, the maximum total external capital available to the firm is this length, and given the initial capital endowment, there is always an unfilled gap. So if a firm starts with an initial capital endowment less than $A(\gamma, \beta)$, there is no way for the firm to have sufficient external capital to launch the project.
Case 2: If $A(\gamma, \beta) < A < A(\gamma, \beta)$ and $A + I_m(\beta) < I$, the firm is eligible to use indirect finance. In this case, the total external capital needed is $(I - A)$. The maximum total external capital available to the firm is more than sufficient to fill the gap. In addition, the optimal informed capital alone is not sufficient to fill the gap, so the firm needs to use uninformed capital as well.

In summary, the firm itself will invest $A$, the intermediary will invest $I_m(\beta)$, and the uninformed investor will invest the rest $(I - A - I_m(\beta))$. 

\[ (I_m + I)_{\text{max}} = I_m(\beta) + \frac{\psi(R - \frac{\theta + \epsilon}{\Delta R})}{\gamma} \]
Case 3: If $A(\gamma, \beta) < A < A(\gamma, \beta)$ and $A + I_m(\beta) > I$, the only difference between this case and the previous case is that, the informed capital alone will be sufficient to cover the total external capital needed ($I - A$). In summary, the firm itself will invest $A$, the intermediary will invest $I_m(\beta)$, and there is no need for uninformed capital.

Case 4: If $A > A(\gamma, \beta)$ the firm is eligible to use direct finance. In summary, the firm itself will invest $A$ and the uninformed investors will invest the rest ($I - A$).
4.4 Certification versus Intermediation

It follows from the previous section that firms fall into three categories, dependent on their demand for informed (intermediary) capital. At one extreme, there are firms with sufficient capital to invest in their projects without any assistance from the intermediaries. At the other extreme are the poorly capitalized firms that cannot invest at all without external capital. The typical firm, however, lies somewhere in between. For completeness, the firm falls in one of three possible cases:

1. Well-Capitalized Firms
   \[ A(\gamma, \beta) > A(\gamma) \]
   These firms can invest directly and demand no informed capital.

2. Poorly-Capitalized Firms
   \[ A(\gamma, \beta) < A(\gamma) \]
   These firms cannot invest at all (at least not without external capital).

3. The Typical Firm - Somewhere in between
   \[ A(\gamma, \beta) < A < A(\gamma) \]
   These firms can invest, but only with the help of monitoring.

The typical firm falls into this third category, and finances its investment with a mix of informed and uninformed capital. We must now clarify when it is mixed financing takes place, but more importantly, in what form. The first of which is Certification.
Certification As previously discussed, uninformed investors are independent investors that invest directly into the firm. However, they only do so once the monitor has taken a large enough financial interest in the firm so that the investor can be assured the firm will behave diligently. Recall that investors are uninformed in the sense that they are not able to distinguish if a firm is behaving diligently. However, seeing that an intermediary has invested a sufficient amount into the firm certifies to the investor that the investment project is worthwhile. In this interpretation, the monitor represents a venture capitalist, a lead investment bank, or any other form of sophisticated investor. Their stake in the firm certifies, or validates that the firm is sound. This opens up the possibility for the firm to raise capital from uninformed investors.

Intermediation An alternative interpretation is to view the intermediary as a commercial bank. Investors will then invest via the bank rather than directly into the firm. The bank will invest the uninformed capital along with its own funds into the firms that they are monitoring.

The amount of uninformed capital that an intermediary can attract will depend on two factors. The first is how much equity the intermediary possesses. The second is the value of $\beta$ and $\gamma$, or the rates of return on the markets for informed and uninformed capital respectively.

In the intermediary case, the investors will demand that intermediaries meet certain solvency conditions that put a lower bound on the ratio of their equity to total capital. These solvency ratio conditions will be explored in the variable investment model (Section 5 of Handout).
4.5 Equilibrium in the Credit Market

Recall that each firm demands the minimum amount of informed capital, namely $I_m(\beta)$, derived earlier. The aggregate demand for informed capital is:

$$D(\gamma, \beta) = [G(A(\gamma)) - G(A(\gamma, \beta))]I_m(\beta)$$

Assuming that there is no excess supply of informed capital at the minimum acceptable rate of return $\beta$, an equilibrium in the monitoring market is obtained when $\beta$ satisfies the following:

$$K_m = D_m(\gamma, \beta) = [G(A(\gamma)) - G(A(\gamma, \beta))]I_m(\beta)$$ (6)

Note: Equation (6) describes the equilibrium rate of return demanded by uninformed capital, but only if this $\gamma$ is determined exogenously.

$D_m$ is the demand for informed capital, and it is decreasing in $\beta$. This is because $I_m(\beta)$ is decreasing and $A(\gamma, \beta)$ is increasing in $\beta$. Hence, for each $\gamma$, there is a unique value of $\beta$ that clears the market for informed capital.

However, the effect of $\gamma$ on $D_m$ is ambiguous. This is because an increase in $\gamma$ causes increases in both $A(\gamma)$ as well as $A(\gamma, \beta)$. The effect then depends upon the distribution function $G(A)$, and the aggregate demand can either increase or decrease with changes in $\gamma$. 

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Now what if $\gamma$ is endogenous? That is, if the supply of uninformed capital $S(\gamma)$ is imperfectly elastic. We now need to add an equilibrium condition for uninformed capital.

Let $D_u$ denote the demand for uninformed capital.

$$D_u = (\gamma, \beta) = \int_{A(\gamma, \beta)}^{A(\gamma)} [I - A - I_m(\beta dG(A))] + \int [I - A]dG(A) \quad (7)$$

The Demand, $D_u$, is decreasing in $\gamma$. On the one hand, firms with assets at a level just above $A(\gamma, \beta)$ are squeezed out by an increase in $\gamma$. Meanwhile, on the other hand, there are firms with assets at a level just above $A(\gamma)$ now move from direct financing to indirect financing. This uses less uninformed capital as $I_m(\beta) > 0$.

However, there are ambiguities in the effect that an increase in $\beta$ has on $D_u$. This is because there are two opposing effects going on. The first is that firms with assets just above $A(\gamma, \beta)$ are dropping out, thereby reducing the demand for uniformed capital. The second is that there are firms relying on intermediation, who now demand more uninformed capital. This is because intermediaries must now invest less per each firm—following from $I_m(\beta)$ decreasing with $\beta$.

Next, the market for uninformed capital will clear under the following condition.

$$D_u = (\gamma, \beta) = S(\gamma) \quad (8)$$

Note that for each $\beta$ there is a unique $\gamma$ that solves Equation (8).

We can obtain these specific values of $\beta$ and $\gamma$ by substituting Equation (6) into Equation (8).

$$\int_{A(\gamma, \beta)}^{\infty} [(I - A)dG(A)] = S(\gamma) + K_m \quad (9)$$

The left hand side $\rightarrow \int_{A(\gamma, \beta)}^{\infty} [(I - A)dG(A)]$ $\rightarrow$ is the firms’ aggregate demand for capital.

The right hand side $\rightarrow S(\gamma) + K_m$ $\rightarrow$ is the total supply of external capital.

Thus, Equation (9) equates the firms’ aggregate demand for capital with the total supply of external capital.

### 4.6 Changes in the Supply of Capital

The authors main interest is with the effects that changes in asset values and capital supply have on the equilibrium outcome. First, we are a bit limited
in what we can say about the behavior of interest rates in this section, which stems from neither \( D_u \) nor \( D_m \) being monotone. We are also limited in our analysis here because of the fixed investment scenario. Because the size of the investment is fixed, it causes discontinuities in individual firm demands. This makes the distribution function \( G(A) \) play a critical role in the analysis. Rather than introducing new assumptions on the distribution function, \( G(A) \), we will instead restrict our analysis here on the behavior of investment in response to changes in the supply of capital.

In order to perform this analysis, the authors consider three (3) forms of capital tightening, which correspond to three forms of capital in the model.

1. Credit Crunch
   \[ \rightarrow \] The supply of intermediary capital \( K_m \) is reduced.

2. Collateral Squeeze
   \[ \rightarrow \] Aggregate firm capital \( K_f = \int A dG(A) \) is reduced.
   \[ \rightarrow \] Also, these reduction affects firms in proportion to their assets.

3. Savings Squeeze
   \[ \rightarrow \] The savings function \( S(\gamma) \) shifts inward.

Any of these capital shocks leads to a drop investment. This is the basis for Proposition 1.

**Proposition 1**

In either type of capital squeeze, aggregate investment will go down, and \( A(\gamma, \beta) \) will increase. Consequently, poorly capitalized firms will be the first to lose their financing in a capital squeeze.

**Proof**

1. Suppose \( A(\gamma, \beta) \) goes down as a result of any of the above three mentioned capital squeezes.

2. A reduction in \( A \) causes an increase in aggregate investment.
   This would be financed by consumer loans. Specifically by more uninformed capital, so \( S(\gamma) \) goes up.

3. An increase in the supply of capital, \( S(\gamma) \) causes the interest rate \( \gamma \) to rise as well. As uninformed capital becomes more expensive, fewer firms have access to direct finance. With higher interests rates, the threshold of \( A(\gamma) \) to become higher.

4. A lower \( A(\gamma, \beta) \) and higher \( A(\gamma) \) implies that more firms are being financed by banks (via indirect finance). This means that intermediation spans a strictly large set of firms. Because the supply of capital is fixed, this implies that each firm receives less informed capital. In other words, each bank loan is smaller. Also note that direct financing is now more expensive.
5. Therefore, it follows that $I_m$ decreases, and from this that $\beta$ is pushed up. The interpretation of a higher $\beta$ is the intermediary rate of return is increased.

6. As both informed and uninformed capital have become more expensive, it follows that $A(\gamma, \beta)$ cannot go down. Thus, we have reached a contradiction.

Q.E.D.

Proposition 1 implies that at least one of the interest rates, $\gamma$ or $\beta$, must go up when there is a capital squeeze. If both had in fact gone down, it would imply that $A(\gamma, \beta)$ goes down as well.

Corollary to Proposition 1

The equilibrium in the fixed investment model must be unique. If there were in fact two different equilibria, it follows from Proposition 1 that $A(\gamma, \beta)$ would have to be the same in both. It would also follow that $A(\gamma)$ must also be the same. If they weren’t, then $\gamma$ and $\beta$ would be lower in one of the equilibria. However, as we have noted, this is not possible.

It is worth mentioning that since all forms of capital tightening result in the same outcome (that capital poor firms lose their financing), this effect is strongest when occurring on all three fronts mentioned above. This conclusion reassures the empirical evidence that small firms are more highly leveraged and bear the brunt of a capital squeeze.

Consider a recession. Naturally, one can imagine the the values of $R$ and $p_H$ decrease during such an economic state. Following the logic of Proposition 1, it implies that either of these changes will cause the threshold of $A(\gamma, \beta)$ to increase, which again simply states that capital for firms are the first to be neglected of financing, and pushed out of the market.

Stepping away from the theoretical model and imagining the real world for a moment, one may argue that small firms are the first to push out due to economies of scale, especially in terms of monitoring. In times of a credit crush, it would not be worth it to a bank to cover the fixed cost in order to serve (monitor) a small firm. While the model does not explicitly discuss such economies of scale, it implicitly does so in this fixed cost argument. That is, large firms that are monitored pay the same absolute amount for monitoring as a small firm. Hence, at least in terms of per unit of net worth, monitoring costs do in fact decrease with size.
5 Variable Investment Scale

For the remainder of the paper, the model is concerned with a variable level of investment. This section really doesn’t offer many new insights. The authors seem to include it to justify their earlier discussion of the market equilibrium.

We assume that investments can be undertaken at any scale $I$. Benefits and costs are proportional to $I$. That is,

1. The private benefits are $B(I) = BI$ (respectively, $b(I) = bI$).
2. The cost of monitoring is $c(I) = cI$.
3. The return from a successful investment is $R(I) = RI$.
4. The probability of success remain as before ($p_H$ and $p_L$) and depend on the firm’s actions.

5.1 The Firm’s Program

Take the rates of return $\beta$ and $\gamma$ to be given. A firm holds initial assets $A_0$. The firm will choose its overall level of $I$, its own capital contribution $A$, and the variables $R_f, R_m, R_u, I_m, I_u$ to solve program $A_0$ (Recall, that $f, m, u$ refer to the firm, investors, and intermediary, respectively):

$$\max U(A_0) = p_H R_I - p_H R_M - p_H R_u + \gamma (A_0 - A)$$

subject to,

1. $A \leq A_o$
2. $A + I_m + I_u \geq I$
3. $p_H R_m \geq \beta I_m$
4. $p_H R_u \geq \gamma I_u$
5. $R_m \geq cI/\delta p$
6. $R_f \geq bI/\delta p$
7. $R_f + R_m + R_u \leq RI$

From their results in Section III, the author’s assume that it is desirable to employ an intermediary and that informed capital is scarce. We will return to these assumptions later.

Now, we divide each of the equations in Program $A_0$ by $A_0$. This creates a program in which all choice variables are scaled by $A_0$, and all the parameters are independent of $A_0$. This results in an optimal solution taking the form
of $R_f = \tilde{R}_f A_0$. Simply put, firms with different levels of assets use the same optimal policy scaled by their assets. This is a useful conclusion for aggregate analysis.

In equilibrium all constraints will bind. So we plug constraints 1, 3, 4, 5, 6, and 7 into constraint 2.

$$
A + I_m + I_u \geq I \\
\Rightarrow A_0 + \frac{p_H R_m}{\beta_m} + \frac{p_H R_u}{\gamma} \geq I \\
\Rightarrow A_0 + \frac{I p_H c}{\beta_m A p} + \frac{I p_H}{\gamma} (R - \frac{b + c}{A p}) \geq I
$$

A simple mathematical manipulation shows that the highest sustainable level of investment is

$$
I(A_0) = \frac{A_0}{A_1(\gamma, \beta)}
$$

The denominator $A_1(\gamma, \beta)$ is

$$
A_1(\gamma, \beta) = 1 - \frac{p_H c}{\beta_m A p} - \frac{I p_H}{\gamma} (R - \frac{b + c}{A p})
$$

This represents the amount of firm capital needed to undertake an investment of $I = 1$. Clearly, $A_1(\gamma, \beta) < 1$. This reflects the fact that the firm can lever its own capital. The lower $A_1(\gamma, \beta)$ is the higher the leverage. In equilibrium, rates of return must also be such that $A_1(\gamma, \beta) > 0$, or else the firm would want to invest without limit.

Substituting the constraints into the objective function gives the firm’s maximum payoff. That is,

$$
U(A_0) = \frac{p_H b I(A_0)}{A p}
$$

The net value of leverage to the firm is

$$
\left[ \frac{P h b}{(A p A_1(\gamma, \beta))} - \gamma \right] A_0
$$

Assuming that monitoring is valuable, the term in the brackets is positive. It is the difference between the internal and external rate of return on firm capital.
5.2 Equilibrium in the Capital Markets

Because firms choose the same optimal policy per unit of own capital, an equilibrium is easily found by aggregating across firms.

Let \( K_f \) be the aggregate of firm capital, \( K_m \) the aggregate amount of informed capital, and \( K_u \) the aggregate supply of uninformed capital. The first two are fixed, while the third is determined so that the demand for uniformed capital (the sum of the pledge able expected returns of individual firms, discounted by \( \gamma \)) equals the supply \( S(\gamma) \).

Let \( \gamma = \gamma(K_u) \) be the inverse supply function. The equilibrium in the market for uninformed capital obtains when

\[
p_h(K_f + K_m + K_u)[R - \frac{(b + c)}{\Delta}] = \gamma(K_u)K_u
\]

Let \( K = K_f + K_m + K_u \), where \( K \) is the total amount of capital invested. Then, the equilibrium rates of return in the two capital markets are

\[
\gamma = p_hK\frac{R - \frac{(b + c)}{\Delta}}{K_u}
\]

\[
\beta = \frac{p_H eK}{(\Delta p)K_m}
\]

Figure IV provides a graph of how \( K_u \) is determined.
As can be seen from the preceding figure, in order for investment to be finite, the equilibrium value of \( \gamma \) must be such that it exceed the pledgeable expected income \( p_H\left[R - \frac{(b+c)}{\Delta p}\right] \) (per unit of investment).

Equations (16) and (17) show that the equilibrium rates of return on firm and intermediary capital depend on the relative scarcity of the two forms of capital. However, (15) shows that the aggregate level of investment only depends on the sum of firm and intermediary capital. This is a consequence of the model’s assumptions. Only uninformed capital responds to changes in the rate of return.

If firms had more than one type of investment opportunity, the optimal choice would generally depend on the relative costs of capital.

5.3 Changes in the Supply of Capital

In addition to analyzing the effect that changes in the supply of capital have on interest rates and investment, the authors also consider the effect these changes have on the solvency ratio of firms and intermediaries.

Each firm’s solvency ratio equals the aggregate solvency ratio. This defined by \( r_f = \frac{K_f}{K} \). The intermediary’s solvency ratio is defined by \( r_m = \frac{K_m}{K_m + K_u} \).

**Proposition 2**

1. A decrease in \( K_m \) (credit crunch)
   → decreases \( \gamma \) → increases \( \beta \)
→ decreases $r_m \to$ increases $r_f$

2. A decrease in $K_f$ (collateral squeeze)
→ decreases $\gamma \to$ decreases $\beta$
→ increases $r_m \to$ decreases $r_f$

3. An inward shift in $S(\gamma)$ (savings squeeze)
→ increase $\gamma \to$ decreases $\beta$
→ increases $r_m \to$ increase $r_f$

In all cases, investment and the supply of uninformed capital decline.

These results follow directly from equations (15) - (17). To prove this consider equation (15) and a credit crunch (i.e. when intermediary capital contracts).

When intermediary capital contracts, less uninformed capital can be attracted. This lowers $K_u$ and $\gamma$. Now, if we divide equation (15) through by $K_u$, we see that $K_m/K_u$ must decrease since $K_f/K_u$ increases and $\gamma$ goes down.

The contraction in uninformed capital is less than proportional to the contraction in $K_m$. Informed capital will be relatively scarcer than before, which will increase $\beta$ and lowers $r_m$. Since both informed and uninformed capital contracts, the solvency ratio $r_f$ will increase.

5.3.1 The Scandinavian Recession

The author’s now go on a brief aside to compare the model to historical data, specifically the Scandinavian Recession. Note that this model is a bit difficult to compare to an actual recession, because a recession is going to affect many of the model’s variables. In other words, it could be difficult to parse out the affects. On the other hand, if reality looked very different from the model’s simple prediction, then the model wouldn’t be very useful.

The Scandinavian recession started as a credit crunch (1. in Proposition 2). Banks were overextended and had to rein in on lending. Lending and deposit rates widened at this stage. That matches our prediction that $\beta$ would increase and $\gamma$ would decrease. Overall, investment dropped by more than the reduction in bank lending. Banks were forced to consolidate their balance sheets. Again, this matches our prediction in Proposition 2.

The solvency of banks dropped romantically and recovered only with government support and monetary easing. According to our model, $r_m$ should go down according to Proposition 2. This result cannot be directly matched, since
regulatory rules clearly govern the behavior of banks.

This model may have some bearing on the policy debate surrounding the regulation of capital ratios. That is, should these ratios vary with the business cycle and, if so, how? Ultimately, the author’s shy away from making any strong policy statements. They do mention that their model does seem to suggest that adequacy ratios be procyclical. However, they are quick to mention that their are several other issues to consider before discussing the regulation of solvency.

5.4 Endogenous Monitoring

So far, the level of monitoring intensity has been fixed. However, the logic of the model suggests than monitoring intensity should actually vary in response to changes in capital.////

Modeling varying monitoring intensity is actually simple. Let the opportunity cost $b$ be a continuous rather than discrete variable. This effectively implies that a firm has a continuum of bad projects, which are distinguished by differing levels of private benefit $b$. Now, monitoring at level $c$ eliminates all bad projects with a private benefit higher than $b(c)$. Again, $c$ represents the cost of monitoring, and $b(c)$ is the functional relationship between monitoring intensity and the firm’s opportunity cost of being diligent.

5.4.1 Fixed Investment

Let’s revisit the fixed investment model.

In this case, every firm that was monitored demanded the same amount of informed capital because the monitor had to be paid a minimum return. However, it is clear that all firms, except the most poorly capitalized, would choose to reduce the intensity of monitoring. Any firm for which $A > A(\gamma, \beta)$ can reduce its cost of capital by letting $b$ increase.

That is, a higher $b$ implies a lower $c$. This relaxes the intermediary’s incentive compatibility constraint ($IC_m$). Thus, this lowers the amount the intermediary has to be paid, $R_m$, and the intermediary has to invest, $I_m$.

The model implies that the intensity of monitoring is positively related to the amount of capital that the intermediary has to put up. This seems consistent with reality. Consider commercial banks, which do not monitor very intensely, which partly explains why they can leverage their capital so extensively. By contrast venture capitalists have a much higher stake in the projects they finance, and they monitor their projects much more intensely.
5.4.2 Variable Investment

Now, we can focus on the variable investment section of our model.

In the variable investment model with endogenous monitoring, all firms would be monitored at the same intensity. This is because the choice of $b$ in Program $A_0$ is independent of $A_0$. However, this level would vary with the relative amounts of intermediary and firm capital. Using (11) - (13), we see that a firm would choose $b$ to minimize $A_1(\gamma, \beta)/b$, i.e. the amount of own assets per unit of private benefit.

It is clear that $b$ increases in response to an increase in $\beta$ (keeping $\gamma$ exogenous). So, when informed capital becomes scarcer, the response is to shift towards less intense monitoring. Conversely, when informed capital becomes more abundant relative to firm capital requires that it be employed for more intensive monitoring.

The author’s now go on a brief aside to consider a variation of the model in which investment will depend on the relative amounts of firm and intermediary capital.

In this case, investment is continuous but subject to decreasing returns to scale. Let $R(I)$ denote a firm’s gross profit in case of success, with $R' > 0$, $R'' > 0$, $R'(0) = \infty$, and $R'(\infty) = 0$.

For given rates of return $\beta$ and $\gamma$, a firm’s utility function is similar to before. That is,

$$U(I) = p_H R(I) - \gamma I - (\beta - \frac{\gamma}{\beta})(\frac{p_H c}{\Delta p})I$$

$U(I)$ is maximized at some investment $I^*$. A firm’s utility depends on its asset level only through its borrowing capacity. The latter is obtained by replacing "$R_I$" by "$R(I)$" in the derivation of equation (11). The incentive compatibility for the firm requires that $I \leq I(A_0)$, where $I(A_0)$ is given by

$$\Delta p[R(I) - \frac{c I}{\Delta p} - \frac{\gamma}{p_h} (I - \frac{p_H c I}{\beta \Delta p} - A_0)] = b I$$

In this version, firms with more assets will have a higher solvency ratio $r_f$. For this reason, it is evident that the distribution of capital across firms and intermediaries influences aggregate investment.

We arrive with two conflicting effects: lower leverage makes large firms less sensitive, while lower marginal returns make them more sensitive to a rise in $\beta$. 


6 Conclusion and Commentary

This model sought to provide a first step towards understanding the role played by the distribution of capital across differently informed sources of capital. This model does accomplish that goal, however, like all good theoreticians, Holmstrom and Tirole are quick to note the limitations of their model. In their words, "We have been careful not to get ahead of ourselves on policy matters; the models are too primitive for that." They are also quick to note their use of "unpalatable assumptions". They spend a significant amount of time dealing with some of their more reductive assumptions in their conclusions.

In fairness to Holmstrom and Tirole, their model does reproduce some of the facts associated with capital crunches, and it does seem to provide at least an initial step towards more complex and accurate models. Their final comments request a broader reacher agenda associated with scarce loanable funds.