Notes on:
Prospect Theory: An Analysis of Decision Under Risk
(Kahneman and Tversky 1979)

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1 Introduction

The expected utility principle was formulated in the 18th century by Daniel Bernoulli (1738), then axiomatized by Von Neumann and Morgenstern (1944), and further developed by Savaga (1954) who integrated the notion of subjective probability into expected utility theory. Expected utility theory aims to help make decisions among various possible prospects and has been used in economics as a descriptive theory to explain various phenomenon such as the purchase of insurance and gambling. However, the experimental tests of utility theory imply that the expected utility theory tends to be betrayed by people's choices. In this paper, Kahneman and Tversky develop an alternative model, which is called prospect theory.

2 Critique

2.1 Expected Utility Theory

Let \( X = \{x_1, x_2, \ldots, x_n\} \) be the set of outcomes. Then \( G \), the set of prospects, is given by

\[
G = \left\{ (p_1 \circ x_1, \ldots, p_n \circ x_n) \left| \sum_{i=1}^{n} p_i = 1 \right. \right\}
\]

The three properties of the Expected Utility Theorem that are applied to choices between prospects are:

(i.) Expectation: \( U(x_1, p_1; \ldots; x_n, p_n) = p_1 u(x_1) + \ldots + p_n u(x_n) \).

(ii.) Asset Integration: \( (x_1, p_1; \ldots; x_n, p_n) \) is acceptable at asset position \( w \) iff

\[
U(w + x_1, p_1; \ldots; w + x_n, p_n) > u(w)
\]

(iii.) Risk Aversion: \( u \) is concave(\( u'' < 0 \))

Property (ii) argues that the utility function relies on final states, rather than on gains or losses. Property (iii) implies that people are risk averse and that their utility function is concave. In the following sections the author demonstrates several arguments that violate these properties. All arguments are based on experimental tests to hypothetical choice problems, which are taken among college students and university faculty. Note that these kinds of studies can only provide qualitative predictions, for probabilities and utilities cannot be adequately measured in such contexts. Yet, these counterexamples still cast doubt on the reliability of expected utility theory.
2.2 Allais’ Paradox: Certainty, Probability, and Possibility

In expected utility theory, people use probability to evaluate the utility of outcomes. Kahneman and Tversky argue that people overweight outcomes they consider to be certain relative to uncertain outcomes, this phenomenon is called the certainty effect. The authors made a survey based on a modified version of Allais’ Paradox proposed by Maurice Allais in 1953. In the survey, N denotes the number of respondents who answered each problem, and the percentage who choose each option is given in brackets.

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<td>2400 with certainty</td>
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<td>2400 with probability 0.66</td>
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PROBLEM 2: Choose between

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<td>2500 with probability 0.33</td>
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<td>0 with probability 0.67</td>
<td>0 with probability 0.66</td>
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Suppose \( u(0) = 0 \), then we get the following expression,

\[
\begin{align*}
   u(2400) &> 0.33u(2500) + 0.66u(2400) & (1) \\
   0.34u(2400) &< 0.33u(2500) & (2)
\end{align*}
\]

Equation (1) represents the preference of problem 1, while equation (2) represents the preference of problem 2. It is easy to see that these two equations contradict each other, which implies expected utility theory is not consistent in people’s preferences. One decent explanation is people overweight the significance of certainty. As Kahneman and Tversky put it, “A greater reduction in desirability can be observed when the prospect is changed from a sure gain to a probable one, than when both the original and final prospects are uncertain.”

Further, the certainty effect implies that the substitution axiom of utility theory is not supported empirically. This is demonstrated by Problems 3 and 4.

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PROBLEM 4: Choose between

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<td>(4000 , 0.20)</td>
<td>(3000 , 0.25)</td>
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If we were to replace \( C = (4000, 0.2) \) by \( A, 0.25 \) and \( D = (3000, 0.25) \) by \( B, 0.25 \), then problem 4 implies \( A > B \). According to the substitution axiom, any \( (A, p) \) should be strictly preferred to \( (B, p) \) i.e \( pu(A) > pu(B) \). So when \( p = 1 \), we get \( u(A) > u(B) \). Since \( 0.25u(A) = 0.2u(C) \), we can multiply the expression by 4 to get: \( 4(0.25u(A) = 0.2u(C)) \), such that \( u(A) = 0.8u(C) \). Since \( B \) here is actually equal to \$3000, the substitution axiom argues that \( A = (4000, 0.8) \) should be strictly preferred to \( B = (3000) \), which contradicts the actual choice made. Another violation of the substitution axiom is illustrated in problems 7 and 8.
PROBLEM 7: Choose between

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<td>(6000, 0.45)</td>
<td>(3000, 0.9)</td>
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PROBLEM 8: Choose between

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<tr>
<td>(6000, 0.001)</td>
<td>(3000, 0.002)</td>
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From the result we find that when the probabilities of winning are substantial (0.45 and 0.90) most people would choose the low return prospect with high probability of winning. However, when the probabilities become minuscule (0.001 and 0.002), people will change their strategy and choose the riskier prospect with higher returns. Kahneman and Tversky believe if \((y, pq)\) is equivalent to \((x, p)\), then \((y, pqr)\) is preferred to \((x, pr)\), \(0 < p, q, r < 1\).

2.3 The Reflection Effect

It is normal to wonder if expected utility theory is also applicable to negative prospects (i.e. losses). The left-hand column of Table I displays four of the choice problems that were discussed in the previous section, and the only difference between the left-hand side and the right-hand side is the sign of the outcomes.

<table>
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<tr>
<th>Positive Prospects</th>
<th>Negative Prospects</th>
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<tr>
<td>Problem 3 (N=95)( \begin{array}{c} 4000, 0.80 \ 3000 \end{array} ) &amp; Problem 3' (N=95)( \begin{array}{c} -4000, 0.80 \ -3000 \end{array} )</td>
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<td>Problem 4 (N=95)( \begin{array}{c} 4000, 0.30 \ 3000, 0.25 \end{array} ) &amp; Problem 4' (N=95)( \begin{array}{c} -4000, 0.20 \ -3000, 0.25 \end{array} )</td>
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<tr>
<td>Problem 7 (N=66)( \begin{array}{c} 3000, 0.90 \ 6000, 0.45 \end{array} ) &amp; Problem 7' (N=66)( \begin{array}{c} -3000, 0.90 \ -6000, 0.45 \end{array} )</td>
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<tr>
<td>Problem 8 (N=66)( \begin{array}{c} 3000, 0.002 \ 6000, 0.001 \end{array} ) &amp; Problem 8' (N=66)( \begin{array}{c} -3000, 0.002 \ -6000, 0.001 \end{array} )</td>
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The results in Table I suggest that changing the sign of the outcomes will lead to the reversion of preference. Kahneman and Tversky refer to this pattern as the reflection effect. Several characteristics can be observed from the reflection effect. First, risk aversion in the domain is replaced by risk seeking in the negative domain. It is worth mentioning that risk seeking does not mean preferences are convex. Second, expected utility theory is also violated with respect to the negative prospects. For example, the certainty effect can still be observed within the negative prospects. The only difference is that in the positive domain, the certainty effect leads to a conservative choice, while in the negative domain, this effect drives people to make riskier choices. Third, the reflection effect eliminates the potential explanation of the certainty effect as aversion for uncertainty or variability. As we can see, people prefer the choice with higher variation (-4000,0.8) than the one with certainty (-3000). This rebuts the general opinion that certainty is preferred.

2.4 Probabilistic Insurance

Many people regard the purchase of insurance as evidence for the concavity of the utility function, because people would take a price of insurance that exceeds the expected actuarial cost. However, the purchase of some unconventional forms of insurance imply the opposite argument. Kahneman and Tversky offer two examples in this paper. One is that people often prefer insurance with low deductibles but limited coverage than that of higher coverage with higher deductibles. Another counterexample comes from people’s responses to a specified insurance, which is referred to as probabilistic insurance. This kind of insurance costs half of the regular premium and when you suffer an accident: you have a 50% chance to pay the other half of the premium and get the compensation from the insurance company to cover your losses; and 50% change to get your insurance payment back and thus bear the full cost of the accident.
In order to get people’s responses, the authors took a survey among 95 Stanford students, asking them whether they would take this probabilistic insurance such that there is indifference between taking the regular insurance and leaving the property uninsured. The result shows about 80 percent of interviewees rejected this insurance, which means reducing the probability of a loss from $p$ to $\frac{p}{2}$ is less valuable than reducing the probability of that loss from $\frac{p}{2}$ to 0.

However, according to expected utility theory, the probabilistic insurance is better than the regular insurance. In other words, if one would like to pay $y$ to insure a probability $p$ of losing $x$, then one should definitely be willing to pay a lower price $ry$ to reduce the probability of losing $x$ from $p$ to $(1-r)p$, $0 < r < 1$. It can be proved as follows:

If $u(w - x, p; w, 1 - p) = u(w - y)$, set $u(w - x) = 0$ and $u(w) = 1$, then $u(w - y) = 1 - p$. If and only if $u$ is concave: $u(w - ry) > 1 - rp$. One decent explanation for this phenomenon is that the risk is not adequately captured by the assumed concavity of the utility function.

### 2.5 The Isolation Effect

The last criticism for expected utility theory is that it neglects the effect of different paths to final states, which may produce inconsistent preferences. Kahneman and Tversky argue a pair of prospects can be decomposed into common and distinctive parts in different ways, and different compositions can result in different preferences. They call this phenomenon the isolation effect.

PROBLEM 10: Consider the following two-stage game. In the first stage, there is a probability of 0.75 to end the game without winning anything, and a probability of 0.25 to move into the second stage. If you reach the second stage you have a choice between $(4000, 0.8)$ and $(3000)$. Your choice must be made before the game starts, i.e., before the outcome of the first stage is known.

<table>
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<th>A</th>
<th>B</th>
<th>N = 141</th>
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<td>(4000,0.8) [22]</td>
<td>(3000) [78]</td>
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The result shows 78% of interviewees chose $(3000)$. However, with more careful consideration, we find that prospects in problem 9 have the same expectation as that of problem 4, rather than problem 3. Kahneman and Tversky argue it is because people ignore the first stage, whose outcomes are the same. Thus, problem 10 is automatically “turned” into problem 3, in which most people also choose the latter option. The phenomenon of reversion of preferences due to different approach to final states violates the Asset Integration property. In other words, people’s preference do not depend on the final states.

Graphically this is represented with the following decision tree:

From problem 11 and 12, we can achieve a consistent result that the changes of wealth play a more significant role on people’s choices, rather than the final states—this finding is inconsistent with utility theory.
3 Theory

From this section forward we will discuss the development of prospect theory, which is an alternative way of decision-making under risk. Prospect theory involves two phases in the decision making process: an early phase of editing and a subsequent phase of evaluation. The editing phase is the initial analysis of the prospects offered, which is simplified at this stage. In the second stage, the edited prospects are examined and the prospect with the highest value is chosen. We now go on to provide a detailed description of the editing phase and the evaluation operation that follow.

3.1 Editing

The editing phase is carried out in order to organize and reformulate the options to help simplify the process of making a decision in the evaluation phase. The editing phase involves several operations that transform the outcomes and probabilities related to the offered prospects. The following are the key operations in the editing phase:

Note that the following operations are applied to each prospect separately:

3.1.1 Coding

As previously discussed, people tend to view outcomes as either gains or losses instead of final states of wealth. In order for people to view outcomes as gains or losses a reference point that corresponds to their current asset position is utilized. The amount paid or received relative to this reference point determines whether they incur a gain or loss. However the reference point and gains and losses are different for individuals since it can be affected by the formulation of the offered prospects and the expectations of the decision maker.

3.1.2 Combination

Decision makers often simplify the prospects offered by combining the probabilities with identical outcomes. For instance the prospect (200, 0.25; 200, 0.25) can be simplified to (200, 0.5) and then evaluated.

3.1.3 Segregation

Sometimes several prospects contain a riskless component which can separated from the risky component in the editing phase. For instance, the prospect (300, 0.8; 200, 0.2) can be segregated into a sure gain of 200 since this is the minimum gain. The risky prospect is (100, 0.8) and this is obtained by subtracting 200 (the riskless component) from 300 with the probability of 0.8 from 300. Similarly, with losses. The following prospect (−400, 0.4; −100, 0.6) can be perceived as a definite loss of 100 with a risky prospect of (−300, 0.4).

3.1.4 Cancellation

This is applied to a set of two or more prospects.

The isolation effect described earlier is the discarding of components that are shared by the offered prospects. This is because it is common to both options and the evaluation is based on the prospects in the second stage. Another type of cancellation involves the discarding of common constituents, that is outcome probability pairs. For example the choice between (200, 0.2; 100, 0.5; -50, 0.3) and (200, 0.2; 150, 0.5; -100, 0.3) can be reduced by cancellation to (100, 0.5; -50, 0.3) and (150, 0.5; -100, 0.3) since the prospect (200, 0.2) is common to both offers.

3.1.5 Simplification

This is the process of rounding probabilities/outcomes of a prospect. For example, the prospect (101, 0.49) can be simplified to an even chance of winning 100. Simplification also involves the discarding of extremely unlikely outcomes (they’re seen as unattainable or impossible).
3.1.6 Detection of Dominance

This step involves looking for dominated alternatives in the prospects offered which are rejected without further evaluation.

3.2 Editing, continued

Since editing operations help in the process of making decisions they are assumed to be performed whenever possible. However, not all of the operations can be used at the same time as some operations prevent other editing operations from being applied. For example, \((500, 0.2; 101, 0.49)\) will appear to dominate \((500, 0.15; 99, 0.51)\) if the second part of both the prospects are simplified to \((100, 0.5)\). The final edited prospect would therefore depend on the sequence of editing operations utilized and this will vary according to the structure of the prospect set and the format of display.

Deviations of preferences result from the editing of prospects. With the isolation effect, inconsistencies arise as a result of the cancellation of common components. Generally, the preference order between prospects do not have to be consistent across contexts since the same offered prospects could be edited in different ways (depending on the context in which it appears).

3.3 Evaluation

Once the editing phase is complete the decision maker is expected to evaluate each of the edited prospects and choose the prospect with the highest value. According to Kahneman and Tversky, "The overall value of an edited prospect denoted \(V\), and is expressed in terms of two scales, \(\pi\) and \(v\). The first scale associates with each probability \(p\) a decision weight, \(\pi(p)\), which reflects the effect of the probability on the total value of the prospect. One must not mistake \(\pi\) to be a probability measure, and we should mention that usually \(\pi(p) + \pi(1-p) < 1\). The second scale, \(v\) assigns to each outcome \(x\) a number \(v(x)\), which is the subjective value of the outcome. Since outcomes in prospect theory are defined relative to a reference point \(v\) measures the value of deviations from the reference point (thus \(v\) represents gains or losses).

We are currently examining simple prospects of the form \((x, p; y, q)\) which have at most two non-zero outcomes. For this prospect, one receives \(x\) with probability \(p\), \(y\) with probability \(q\), and nothing with probability \(1 - p - q\) where \(p + q \leq 1\). An offered prospect is strictly positive if and only if all its outcomes are positive \((x, y > 0)\) and \(p + q = 1\); and is strictly negative if and only if all its outcomes are negative. A prospect is regular if it is neither strictly positive nor strictly negative.

3.3.1 Evaluation of Regular Prospects

The basic equation of the theory is to determine the overall value of the regular prospects and utilizes \(\pi\) and \(v\) to obtain this value. If \((x, p; y, q)\) is a regular prospect (either \(p + q < 1\), or \(x \geq 0 \geq y\), or \(x \leq 0 \leq y\), then

\[
V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y) \tag{1}
\]

where \(v(0) = 0\), \(\pi(0) = 0\) and \(\pi(1) = 1\). Just as in utility theory, \(V\) is defined by the prospects, while \(v\) is defined on outcomes. These two scales will have the same value for sure prospects, where \(V(x, 1) = V(x) = v(x)\).

3.3.2 Evaluation of strictly positive or strictly negative prospects

To evaluate strictly positive or strictly negative prospects a different rule is utilized. As mentioned earlier the prospects are segregated into two components in the editing phase: (i) the riskless component, which is the minimum gain or loss; and (ii) the risky component, which is the additional gain or loss that is actually at stake. The following equation is used to evaluate such a prospect:
If $p + q = 1$ and either $x > y > 0$ or $x < y < 0$, then

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)]$$  \hspace{1cm} (2)$$

This equation calculates the value of a strictly positive or a strictly negative prospect by adding the value of the riskless component and the difference in value between the outcomes multiplied by the weight associated with the higher gain or higher loss outcome. For example:

$$V(400, 0.25; 100, 0.75) = v(100) + \pi(0.25)[v(400) - v(100)]$$

Equation (2) essentially applies the decision weight to the difference in value of the outcomes $v(x) - v(y)$. This part of the equation represents the risky component of the prospect but not the riskless part $v(y)$. The right hand side of the equation can be expanded and written as $\pi(p)v(x) + [1 - \pi(p)]v(y)$. As a result, equation (2) can be reduced to equation (1) if $\pi(p) + \pi(1 - p) = 1$. However, this condition will generally not be satisfied and will be shown in the following sections.

There have been many elements of the evaluation model that have appeared in previous attempts to modify the expected utility theory. One of which was to define utility in terms of gains and losses rather than on final wealth. Markowitz observed the presence of risk seeking in preferences among positive as well as negative prospects which led him to propose a utility function that has a convex and concave region in both the positive and negative domains. However, his treatment retains the expectation principle and therefore cannot account for many of the violations that Kahneman and Tversky find in regards to the expectation principle. The concept of decision weights to replace probabilities was proposed by Edwards and was investigated in several empirical studies.

The general bilinear form that is utilized in the expected utility theory is retained in prospect theory. However, to demonstrate the effects illustrated in the first section of the paper the assumption that values are perceived as gains or losses and that decision weights are not related to the stated probabilities must hold. These diversions from expected utility theory will lead to unacceptable consequences such as inconsistencies and violations of dominance. However, these deviations of preferences will be corrected by the decision maker when he realizes that his preferences are inconsistent. At times, the decision maker will not have the opportunity to realize that his decision violates the rules he hopes to obey—and in these situations the deviations suggested by the prospect theory are then expected to occur.

### 3.4 The Value Function

As mentioned previously for prospect theory our “carriers of value” are the changes that occur in wealth as opposed to final states. This is similar to the principle of people’s perception and judgment. We tend to evaluate changes and differences with respect to a reference point as opposed to absolute magnitudes. Think about temperature, loudness, brightness, there is an adaptation factor where we adapt to a certain level and so then stimuli is perceived with respect to that reference point. For example, we perceive something as hot or cold depending on what temperature we have become accustomed to. Moreover, this idea applies to non-sensory things such as prestige, health and wealth. So depending on our current states, same levels of these things (i.e. wealth) could have different implications for different people.

The value function is a function of two arguments:

1. the reference point, and
2. the magnitude of change (positive/negative) from that reference point.

Since preference order of prospects is not greatly altered by small or even moderate changes in asset position, thus the value function can be simply represented as a function of the reference point.

Kahneman and Tversky point out that psychological response is also a concave function of magnitude of physical change. Recall our earlier example of temperature. A 3 to 6 degree change is easier to discriminate than a change from 13 to 16 degrees. Similarly, for monetary values, the difference between $100 to $200
feels “greater” than the difference between $1,100 and $1,200. Note, that these are similar for losses (except if large loss is intolerable). Thus, Kahneman and Tversky hypothesize that the value function for changes of wealth are concave for gains ($v''(x) < 0$ for $x > 0$) and convex for losses ($v''(x) > 0$ for $x < 0$). As shown on the graph, marginal value for both gains and losses are generally decreasing with their magnitude. Although what the authors have presented so far has been in a riskless context, they show that the value functions derived from risky choices display the same properties.

PROBLEM 13: Choose between

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PROBLEM 13': Choose between

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<tr>
<td>-6,000</td>
<td>-4,000</td>
<td>-2,000</td>
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Applying equation (1) to both of the problems above yields:

\[
\pi(.25)v(6,000) < \pi(.25)[v(4,000) + v(2,000)]
\]

Thus, gains are concave.

\[
\pi(.25)v(-6,000) > \pi(.25)[v(-4,000) + v(-2,000)]
\]

Thus, losses are convex. Hence,

\[
v(6,000) < v(4,000) + v(2,000)
\]

and

\[
v(-6,000) > v(-4,000) + v(-2,000)
\]

We should keep in mind that there are critical values for which individuals may have “exceptionally steep rises” when it comes to gains and losses. For example, sometimes individuals are sensitive to certain amounts for which they are trying to save for i.e. needing $100,000 to purchase a house. Thus, the derived value function does not always reflect the pure attitudes towards money. In fact, there could exist small “perturbations” that can cause convex regions for gains and concave regions for losses in the value function. In general, losses are seen as even stronger events than gains. The pain associated with losing a certain amount of money is greater to the pleasure associated with gaining that same amount. This explains why symmetric bets are not equally attractive: \((-x, .50)\) and \((x, .50)\). Thus we can conclude that the value function is steeper for losses than it is for gains. As mentioned in the last section, “Aversiveness increases with the size of the stake”. If \(x > y \geq 0\), then \((y, .50; -y, .50)\) is preferred to \((x, .50; -x, .50)\). This makes sense since we prefer to lose less \((y)\), all else equal. According to equation (1), then we can rearrange the equations we worked out above to be:

\[
v(y) + v(-y) > v(x) + v(-x)
\]

and

\[
v(-y) - v(-x) > v(x) - v(y)
\]

If \(y = 0\), then \(v(x) < -v(-x)\) and letting \(y\) approach \(x\) yields \(v'(x) < v'(-x)\), provided \(v'\) (the derivative of \(v\)) exists.
The graph below shows how the value function for losses is steeper than the value function for gains:

![Graph showing value function for losses and gains](image)

**Figure 3.—A hypothetical value function.**

In sum, the three things we should take away from value function are:

i. The value function is defined on deviations from the reference point (changes not final states)

ii. The value function is generally concave for gains and convex for losses

iii. The value function is steeper for gains that it is for losses.
3.4.1 The Weighting Function

In prospect theory, the value of each outcome is multiplied by a decision weight (as mentioned before this is not a probability measure). These weights, \( \pi \), measure the impact of events on the desirability of prospects and are not merely the perceived likelihood of these events. Kahneman and Tversky show us that for an even gamble, a decision weight (which is derived from choices) \( \pi(.50) \) will be smaller than .50. The decision weight is only equal to the probability of the event if the expectation principle holds—which has not held in the experiments developed by Kahneman and Tversky. These weights are a function of stated probabilities, \( p \) but we should keep in mind that these could be influenced by other things such as vagueness and ambiguity.

Properties of the weighting function:

1. \( \pi \) is an increasing function of \( p \) with \( \pi(0) = 0 \) and \( \pi(1) = 1 \). Scale is normalized so that \( \pi(p) \) is the ratio of the weight associated with the probability \( p \) to the weight associated with the certain event.

2. For small probabilities, \( p \):
   - \( \pi \) is a **subadditive** function of \( p \):
     \[
     \pi(rp) > r\pi(p)
     \]
     for \( 0 < r < 1 \). Recall problem 8, we can rearrange the preferences to show:
     \[
     \frac{\pi(.001)}{\pi(.002)} > \frac{v(3,000)}{v(6,000)} > \frac{1}{2}
     \]
     This holds by the concavity of \( v \).
   - Very low probabilities are generally overweighted:
     \[
     \pi(p) > p
     \]
     for small \( p \). Consider problem 14:

PROBLEM 14: Choose between

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<tr>
<th>A</th>
<th>72</th>
<th>B</th>
<th>28</th>
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<tr>
<td>(5,000, .001)</td>
<td>(5)</td>
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PROBLEM 14': Choose between

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<tr>
<th>A</th>
<th>17</th>
<th>B</th>
<th>83</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5000, .001)</td>
<td>(-5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This problem shows us that people prefer lotteries over the expected value of the ticket (assuming the value function for gains is concave) and prefer paying an insurance premium (small loss) against a large loss (assuming the value function is convex for losses). Markowitz has seen the same in his research with regards to preference for the lottery.

Note that subadditivity need not hold for large values of \( p \)

3. **Overweighting** refers to the overestimation in assessment of rare events. Although this is not the case in the paper since subjects were assumed to adopt stated values of \( p \), in real life situations, overestimation and overweighting both work to increase the impact of rare events.
4. **Subcertainty** Although \( \pi(p) > p \) for low probabilities, evidence suggests: for \( 0 < p < 1 \),

\[
\pi(p) + \pi(1 - p) < 1
\]

As we’ve already seen, Allais’ paradox implies subcertainty for the relevant values of \( p \). Let’s apply it to problems (1) and (2) and we’ll see that equation (1) tells us that subcertainty holds:

\[
v(2, 400) > \pi(0.66)v(2, 400) + \pi(0.33)v(2, 500)
\]

We can rearrange this as:

\[
[1 - \pi(0.66)]v(2, 400) > \pi(0.33)v(2, 500) \\
\pi(0.33)v(2, 500) > \pi(0.34)v(2, 400) \\
1 - \pi(0.66) > \pi(0.34) \\
\pi(0.66) + \pi(0.34) < 1
\]

5. The slope of \( \pi \) in the interval (0, 1) can be viewed as a measure of the sensitivity of preferences to changes in probability. The sum of the weights associated with complementary events tends to be less than the weight associated with the certain event.

6. **Subproportionality** means the ratio of the corresponding decision weights is closer to unity when the properties are low than when they are high. Recall the violation of the substitution axiom, by equation (1):

\[
\pi(p)v(x) = \pi(pq)v(y) \rightarrow \pi(pr)v(x) \leq \pi(pqr)v(y)
\]

So,

\[
\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)}
\]

Note this property holds if and only if \( \log(\pi) \) is a convex function of \( \log(p) \)

7. Subproportionality + overweighting of small probabilities \( \rightarrow \pi \) is subadditive over that range. If \( \pi(p) > p \) and subproportionality holds, then \( \pi(rp) > r\pi(p) \), \( 0 < r < 1 \), given that \( \pi \) is monotone and continuous over (0, 1).

![Figure 4.—A hypothetical weighting function.](image)
Figure 4 graphs a hypothetical weighting function that satisfies: overweighting and subadditivity for small values of $p$, and subcertainty and subproportionality. $\pi$ changes abruptly at the endpoints. This could reflect the editing phase discussed earlier, which leads to people discarding unlikely events (ignored or overweighted) and treat extremely probable events as if they were certain (the difference is either neglected or exaggerated). Thus the endpoints of $\pi$ and not well-behaved.

Zeckhauser illustrates the possibility of nonlinearity of $\pi$ with a Russian roulette example. For those of you who are unfamiliar with this concept, Russian roulette is a lethal game of chance whereby you load a gun with a bullet, spin the gun’s barrel, place the gun against your head, pull the trigger and see what happens. He investigates how much people playing are willing to pay to remove one bullet from the loaded gun. Most people feel they would be willing to pay significantly more for a reduction of the probability of death from $\frac{1}{6}$ to zero than from the reduction of $\frac{4}{6}$ to $\frac{3}{6}$. However, economic considerations would lead people to pay more for the latter case since the value of money is presumably reduced by the considerable probability that one will not live to enjoy it.

Since we assume that the probabilities of identical outcomes are combined in the editing phase, then the nonlinearity of $\pi$ is invalid. Moreover, nonlinearity involves potential violations of dominance. Suppose $x > y > 0$, $p > p'$ and $p + q = p' + q' < 1$; hence $(x, p; y, q)$ dominates $(x, p'; y, q')$. If preference obeys dominance then nonlinearity must be violated.

Up to here we have just considered cases where the individual chooses between two prospects. What if the individual were to bid and we were to see preferences in this manner? Lichtenstein and Slovic have shown that while people may prefer $A$ to $B$ they bid more for $B$ than $A$. Kahneman and Tversky conclude that preference order cannot be recovered by the bidding procedure. This inconsistency with bids in this model of choice (prospect theory) then implies that measurement of values and decision weights should be based on choices between specified prospects rather than on bids or production tasks. This introduces a complication in that production tasks (bidding) are more convenient for scaling than pair comparisons.
4 Discussion

4.1 Risk Attitudes

In this part, the author uses three characteristics of \( \pi \), which are subcertainty, subproportionality and subadditivity to explain how prospect theory violates expected utility theory. Subcertainty violates the independence axiom, and subproportionality and subadditivity both violate expected utility theory.

According to present theory, risk attitudes are determined jointly by \( v \) and \( \pi \), and not solely by the utility function. Therefore, it is necessary to find the conditions of occurrence under risk aversion or risk seeking. Consider the choice between the gamble \( (x, p) \) and its expected value \( (px) \). If \( x > 0 \), risk seeking implies whenever \( \pi(p) > v(px)/v(x) \) which is greater than \( p \) if the value function is concave. Hence over-weighting \( (\pi(p) > p) \) is a necessary but not a sufficient condition for risk seeking (aversion) in the gains (losses) domain. This analysis restricts risk seeking in the domain of gains and risk aversion in domain of losses through small probabilities, where over-weighting will exist. In fact, these are typical conditions under which lottery tickets and insurance policies are sold. In prospect theory, the over-weighting of small probabilities favors both gambling and insurance, but the S-shaped value function tends to inhibit both behaviors.

Even though prospect theory predicts both insurance and gambling for small probabilities, there still exist complex phenomena that cannot be explained by present analysis. One example is the service and medical insurance, where people purchase insurance often beyond the medium range of probabilities, but then small probabilities of disaster are sometimes totally ignored.

4.2 Shifts of Reference

So far in this paper, when we analyze the prospect theory, we use amounts of money that are obtained or paid to define gains and losses, so the reference point was taken as status quo or one’s current assets. But imagine a businessman who has already lost \( $2,000 \) and now is facing a choice between a sure gain of \( $1000 \) rather than an even chance to win \( $2000 \) or nothing. If he hasn’t adapted to his losses yet, he will likely treat \( 2000 \) or \( 1000 \) as just acceptable, then \( (x, y, z, 1 - p) \) is preferred over \( (z, y, 1 - p) \) for \( x, y, z > 0 \) with \( x > z \).

To prove this proposition, note that \( V(x - z, y, 1 - p) = 0 \) iff \( \pi(p)v(x) = -\pi(1 - p)v(-y) \).

Furthermore,

\[
V(x - z, y, 1 - p) = \pi(p)v(x - z) + \pi(1 - p)v(-y - z)
\]

By the properties of \( v \), then the right hand side of the inequality will be:

\[
= -\pi(1 - p)v(-y) - \pi(p)v(z) + \pi(1 - p)v(-y) + \pi(1 - p)v(-z)
\]

by substitution,

\[
= -\pi(p)v(z) + \pi(1 - p)v(-z)
\]

since \( v(-z) < -v(z) \),

\[
> v(-z)
\]

by subcertainty.
This result shows that if a person hasn’t come to terms with his losses he is more likely to accept gambles. For example, you are likely to purchase insurance because you have owned it in the past or your friends purchase it. The decision can be made to pay a premium to protect yourself against a loss of \( x \). So you can code the prospect like this: the choice between \((-x + y, p; y, 1 - p)\) and \((0)\) rather than the choice between \((-x, p)\) and \((-y)\). The authors believe it is a more attractive representation than the latter one.

If you treat the reference point as final assets, the result will be different if the reference point is current assets. In this case, your reference point is set to zero and the value function is likely to be concave everywhere. According to the present analysis, this formulation essentially eliminates risk seeking, except for gambling with low probabilities.

Many economic decisions are where one pays money in exchange for a desirable prospect which can be chosen between a status quo and an alternative state. For example the decision whether to pay 10 for the gamble \((1000,0.01)\) is treated as a choice between \((990,0.01; -10,0.99)\) and \((0)\). In this analysis, readiness to purchase the positive prospect is equated to willingness to accept the corresponding mixed prospect.

In the isolation effect, the prevalent failure to integrate riskless and risky prospects suggests that people are unlikely to subtract the cost from the outcomes in deciding whether to buy a gamble. Thus the gamble \((1000,0.01)\) will be purchased of 10 if \( \pi(0.01) v(1000) + v(-10) > 0 \).

If this hypothesis is correct, the decision to pay for \((1000,0.01)\) is no longer equivalent to the decision to accept the gamble \((990,0.01; -10,0.99)\). Furthermore, prospect theory means if one is indifferent between \((x(1-p),p;-px,1-p)\) and \((0)\) then he will not pay \( px \) to purchase the prospect \((x,p)\). Thus, people are more risk seeking in deciding whether to accept a fair gamble than in deciding whether to purchase a gamble for a fair price. Thus, Kahneman and Tversky conclude, the location of the reference point and the manner we code and edit the choice problems are both critical factors in the analysis of decisions.

4.3 Extensions

Prospect theory can easily be extended to a wider range of choices. We could simply expand equations (1) and (2) to include a higher number of outcomes. We must realize that when number of outcomes is large, additional editing operations may be necessary to simplify evaluation. However, the way in which complex options are reduced to simpler ones is yet to be researched and is beyond the scope of this study.

Prospect theory can be extended to non-monetary outcomes as well—it is readily applicable to choices involving other attributes (quality of life, number of lives saved/lost with policy). Again, we would expect the same properties to extend to these non-monetary prospects, especially in that expected outcomes are coded as gains and losses with respect to a reference point and losses have a “stronger impact” on people than gains.

This theory can also be extended to the typical situation of choice where probabilities are not explicitly given. There, we must attach decision weights and it will be the case that we will have an analogue to subcertainty. Recall decision weights are likely to be subject to bias, although it will primarily depend on the perceived probability of that event. According to Ellsberg and Fellner, vagueness is likely to reduce decision weights. Consequently, we will have more pronounced subcertainty with vagueness than with clear probabilities.

The analysis of choice under risk has developed two important themes here which should be developed further.

1. Editing operations: determine how prospects are perceived
2. Judgmental principals: deal with evaluation of gains/losses and weighting of uncertain outcomes
Although this paper presents hypothetical choices in a university setting to educated individuals, we do believe their examples generalize to the rest of the population. We look at changes with respect to a reference as opposed to a final state and that makes sense for everyone. Furthermore, Allais’ example can be used for individuals, firms or countries to take advantage of. Like the beginning of the theory section mentions, if people aren’t given sufficient time to make decisions they will likely violate expected utility theory in going from certainty to uncertainty. Marketing campaign designers and football coaches who are making football plays need to keep in mind how others perceive choices and the effects of editing in decision making. They can use prospect theory to their advantage by giving people short response times such that they violate expected utility theory.