The Economics of Information

By George Stigler

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Introduction
In economics, the information known by actors is generally taken to be given and is often assumed to be perfect. This paper adds a dimension to information-related problems by considering the search for information, as well as its costs, through an analysis of consumers seeking the minimum price. In fact, this paper provided the foundation for broader theoretical research into search theory. Empirically, several aspects of the theory are supported in laboratory studies. In particular, individuals do appear to engage in less search when the costs of search are higher.

Key Assumptions
- Homogenous goods
- Prices take a uniform distribution between 0 and 1
- Consumers face a cost when engaging in search, particularly in terms of their time
- Search costs will vary between consumers

Key Notation
- \( p \) = Sellers’ asking price
- \( n \) = Number of searches
- \( N \) = Number of potential buyers
- \( F(p) \) = CDF of prices
- \( f(p) \) = PDF of prices
- \( q \) = Quantity of good demanded per buyer
- \( K \) = Constant term; also the percentage of buyers canvassing seller I who buy from seller i

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I. The Nature of Search

In a market, each seller has its own price for a good. Each buyer, however, will not know all of the prices at which the good is available because prices change frequently. Therefore, each buyer does not know the minimum price for which he can purchase a good. As a result, a buyer engages in search. Search is defined as a process of surveying a number of sellers (or buyers, discussed later) to determine the lowest price.

Stigler considers a case of homogenous goods, but it is important to note that price dispersion (unequal prices) can still exist in this market. Note the empirical evidence provided in Table 1 for instance:
It is important to note that while the internet has made search easier, costs to search still exist. Thus, imperfect knowledge on prices still exists which leads to price dispersion. Facing this price dispersion, consumers will find it beneficial to engage in search, assuming their returns from search are greater than the costs of search. This concept is illustrated in Table 2.

As shown at left, even among identical car models (Panel A) and identical tons of coal (Panel B), different sellers charge different prices. The existence of these price differences is termed price dispersion.

Price dispersion derives from two sources:

1) Consumers being unaware of all of the prices at which a good is available (market ignorance)

2) Additional services provided by seller (customer service, variety of stock)

It is important to note that while the internet has made search easier, costs to search still exist. Thus, imperfect knowledge on prices still exists which leads to price dispersion.

With the assumed distribution, as the number of searches increases, the expected value of the minimum price the consumer finds falls. With infinite searches, the consumer will find the minimum price at which the good is available ($2 here).

Assuming prices (p) are uniformly distributed between zero and one, the distribution of minimum prices is given by:

\[ n (1 - p)^{n-1} \]  

(1) [See derivation in the appendix.]
Given the uniform distribution, respectively, the average minimum price and variance of the average minimum price are:

\[
\frac{1}{n + 1} \quad \frac{n}{(n + 1)^2 (n + 2)} \quad \text{[Derivations in the appendix.]}
\]

Note that as the number of searches (n) increases, the average minimum price and its variance both decrease at a decreasing rate. Expected savings are given by the product of the quantity purchased (q) and the absolute value of the expected reduction of minimum prices with each additional search:

\[
q \left| \frac{\partial P_{\min}}{\partial n} \right|
\]

The more price dispersion in the market, the greater the available savings, so the expected reduction of minimum prices with additional search will be greater.

These concepts are illustrated in Figure 1 below: Stigler considers the case of the uniform distribution (Panel B). The uniform distributions are derived from equation 1, based on the number of searches. Note that in both the normal and uniform distribution cases, as n increases, there is a greater density of probability at a lower price.

While Stigler focuses on the uniform distribution case, he references the more general case proposed by Robert Solow in Footnote 4.
The equation $E(n) = \int_0^\infty p(1 - F)^{n-1}F'dp$ gives the expected value of the minimum price after $n$ searches by the expected value formula (since $F'$ is the PDF and $(1-F)^{n-1}$ is the probability that the lowest price found after $(n-1)$ searches is greater than the actual minimum price $p$). [See mathematical appendix]. This function is a decreasing function of $n$.

When we look at the next equation: $[E(n + 2) - E(n + 1)] - [E(n + 1) - E(n)]$, we can see that each of the bracketed terms is negative since the function is decreasing in $n$. Note that the whole expression will be positive since the difference between $E(n+1)$ and $E(n)$ will be larger than the difference between $E(n+2)$ and $E(n+1)$. This indicates that the expected value of the minimum price decreases at a decreasing rate (i.e. The expected value of the minimum price is convex). In essence, there are decreasing returns to search.

**Costs of Search**

**To Note:**

a) The cost of search is proportional to the number of sellers from which the buyer inquires about the good’s price.

b) The main cost is time, so high income buyers face higher search costs

c) At the optimum level of search: (expected marginal return) = (cost of search)

d) For unique goods (there are few buyers or sellers in the market, like a certain baseball card), the cost of search is much higher

**There Are Multiple Outlets for Decreasing Search Costs:**

1) Advertising (i.e.: classifieds)—buyers and sellers are listed, reducing search, but the cost of placing the advertisement must also be considered.

2) “Specialized traders” (i.e.: used car dealerships) centralize trading. Dealerships may be more essential for some goods or in situations more than others. If search costs are very high (antiques, fine art), they may be especially important. They may be less important if the uncertainty inherent in buying from an individual seller is low.
The Case of “Specialized Traders”:

Once a seller chooses a price (p), all buyers for whom this p is the minimum price will buy from this seller. The expected number of buyers for the seller is given by:

\[ N_i = K N_b n (1 - \hat{p})^{n-1} \]  

(3)

K is a constant and N_b is the number of potential buyers in the market. Since n(1-p)^{n-1} is the distribution of minimum prices (given in (1)), it gives the probability that the seller’s asking price is the minimum price. Thus, the fraction of total buyers for which the seller’s price p is the minimum will buy from this seller.

Note, from footnote 9 that as the price falls, the number of buyers increases since the partial derivative of (3) with respect to p is negative. [See mathematical appendix.]

\[ \frac{\partial N_i}{\partial \hat{p}} = -\frac{(n-1) N_i}{(1 - \hat{p})} < 0 \]

Moreover, the number of buyers increases at an increasing rate since the second partial derivative of (3) with respect to p is positive. [See mathematical appendix.]

\[ \frac{\partial^2 N_i}{\partial \hat{p}^2} = \frac{(n-1)(n-2) N_i}{(1 - \hat{p})^2} > 0 \]

if \( n > 2 \).

With constant returns to scale, it can be profitable to buy at a low price and sell at a high price, for the volume of transactions is not essential to earn positive profit. With economies of scale, firms will compete for a higher volume of sales by decreasing their price markups, which will decrease price dispersion. Greater search by buyers will also reduce price dispersion because fewer buyers will pay high prices.
Determinants of Search
In order to make his analysis in this section concrete, Stigler assumes that agents make unique purchases of homogenous products. If these purchases are recurring then there is an effect of search on the volume of goods purchased.

Think Positive
Positive correlation between asking prices will drive buyers to engage in search more heavily in initial periods than in later periods. Positive correlation between prices does not mean increasing prices over time but rather that the prices of goods in the market follow the same pattern over time. The expected savings of the search will be the present value of the discounted future savings on futures purchases over the planning horizon of the buyer. The reason why fewer searches take place in subsequent periods is because the expected savings from search will not change too drastically due to the positive correlation. [In fact, in footnote 13, Stigler shows that expenditures for a set quantity of goods will be reduced by half if the correlation between prices is perfect (=1) because the costs of search (λ) will be reduced.]. Stigler argues that this will hold for the homogenous case because there will be heterogeneity in the spending preference and in the cost of search among buyers, which jointly set the minimum price level they desire.

If prices are uncorrelated across time periods then the search done in each period will be independent of previous experiences. This induced randomness of the price level will not elaborated on further since it does not play an important role in what follows.

Negative correlation between market prices is not considered by Stigler but we can still analyze it in the context of this paper. Consider a market where sellers sell one of two goods, x and y, and where fluctuations in their respective prices are negatively correlated. Hence, prices of the
two goods fluctuate in opposite directions. Recall that buyers are always searching for the best deal, the minimum price level. When the prices of x and y are not equal, one of the goods will be strictly cheaper than the other because of this negative correlation. The agents will only buy the cheaper good and there will be some entry or exit dynamics/incentives with the sellers.

**Seller Behavior**
Sellers who wish to have recurring support from buyers that value their gains from search or have lower search costs – and will thus search more than average – will have to lower their asking prices. [Think about this in terms of deals for discount shoppers who “buy in bulk”.] Stigler defines this “return customer” behavior as goodwill and stipulates that the agents will not search after they have found these sellers other than to occasionally check in on the prices. This provides room for companies to engage in pricing strategies to maximize profit.

When buyers enter into a new market they are, for the most part, inexperienced and ignorant of the market prices and their dispersion. They face the problem of how much search should they undertake before they commit and buy the product. Stigler offers an avenue for extension of his model at this point stating that the buyers must go through some sort of sequential search process until they can pin down the price level.

Stigler also notes that the nature of the price level and its dispersion is a function of the average amount of search which in itself depends on the type of commodity. He summarizes his views into 4 postulates.

1. The larger the fraction of the buyer’s expenditures on the commodity, the greater the savings from search and hence agents will conduct more thorough search

2. The larger the fraction of experienced buyers (“return customers”) in a market with positively correlated prices, the greater the effective amount of search
3. The larger the fraction of repetitive sellers, the higher the correlation between successive prices, hence by 2 above, the larger the amount of accumulated search

4. The cost of search will be larger, the larger the geographical size of the market

Stigler briefly offers an example of these “market initiates” by mentioning the purchasing dilemma that tourists face. They come into a new market with only rudimentary knowledge of where to go and what prices to expect.

$1 + 1 = 4$

Stigler notes that the effect of an increase in the number of buyers in the market is uncertain. An increase in the number of buyers will drive market entry of sellers. As the number of sellers increases, the offer prices they put into the market will become more varied and thus there is more price dispersion.

This leads to interesting buyer behavior. Buyers will begin to pool their knowledge on the price level that they obtain from search. Buyers will canvass sellers and then will share the information they obtain from their search with other buyers. In the simple two buyer case, if each one goes to $s$ sellers (assuming no duplication), then when they pool their results they will have effectively canvassed $2s$ sellers. Stigler remarks that this is less reliable but cheaper in terms of search costs.

**Sources of Dispersion**

Dispersion of prices is an intrinsic factor in markets. Even if all goods are homogenous and there was no cost to sellers to ascertain their rivals’ asking prices, dispersion would persist. The important limitation for this is the volatility of market conditions and the heterogeneity of market participants.
Price differences in the market will persist if they do not reward additional search by the buyers. The optimal search, with perfect intertemporal price correlations, is defined by the equation:

\[ q \times \frac{\partial p}{\partial n} = i \times MC_{search} \]

Where \( i \) is the interest rate in the market. To illustrate this, Stigler assumes that additional search efforts cost $1 per unit of addition time spent searching and with a given interest rate of \( i = 5\% \), the expected reduction in the price will be equal to:

\[ \frac{\partial p}{\partial n} = \frac{i \times MC_{search}}{q} = \frac{0.05 \times 1}{q} \]

In this example, the reduction in the minimum price the buyer is willing to pay is inconsequential. So having some dispersion in prices will be beneficial overall, even if the return to search is small.

**Market Shocks**  
Changing knowledge or information about the market will also lead to price dispersion. As exogenous shocks influence the market equilibrium, the conditions that buyers and sellers face will fluctuate. These perturbations will cause uncertainty on both the buy and sell sides of the market and will force them to investigate, through search, what the new prevailing price level is. This “knowledge degradation” caused by shocks can be used by both sides to take advantage of the other.  
The greater the instability of market prices through supply or demand side shocks, the more likely the existence of price dispersion in the market.

**Agent Heterogeneity**  
In addition to broad market changes, the mix of potential buyers and the mix of sellers are also dynamic. This change in composition on both sides causes further informational heterogeneity as the new entrants will be ignorant of prices. The information set of the old buyers will be
degraded by their presence. For instance, if in the current period only new buyers enter, then, sellers have incentive to raise their offer prices because these new agents will be unaware that in the past prices were lower. The existing buyers that were aware of past price levels will now not know what the new price levels are – their information set has diminished. Hence, we get more search because both new and old buyers are trying to ascertain the price level. Furthermore, it is logical within the model’s framework that some sellers do not alter their prices, in order to maintain “goodwill” with returning buyers, thus we have price dispersion in the market as well.

**Pass Go, Collect $200**

As the market grows in size and the number of market participants, firms will emerge that have monopoly power in the harvesting and dissemination of market information. Stigler references that this will manifest itself in trade journals and specialized information brokers but relevant examples are Facebook and Google. These “information monopolists” bear the costs of collection of information but through scale economies their internal costs of collection fall, leading to them be the centralized information source for the market.

**II. ADVERTISING**

**Advertisements Designed Only to Identify Sellers**

**Model**

- In the first period
  - Total number of potential customers: N
  - The probability of being informed: c
  - The number of potential customers being informed should be: cN
- In the second period
  - Total number of potential customers: N(1-b)
  - The probability of being informed if weren’t informed in first period: c(1-c)
  - The number of potential customers being informed should be: cN(1-b)(1-c)
  - [The probability can be interpreted as the probability not informed in the first period but informed in the second period]
- In the kth period
  - Total number of potential customers: N(1-b)^k-1
The probability of being informed: \( c(1-c)^{k-1} \)

The number of potential customers being informed should be: \( cN[(1-b)(1-c)^{k-1}] \)

Thus the total number of potential customers being informed should be \( cN[(1-b)(1-c)+...+(1-b)^{k-1}(1-c)^{k-1}] \)

[See mathematical appendix for further details]

To Note:
1. In a situation where a seller advertises in the \( k \)th period (\( k \) is very large), then the number of potential buyers informed of the seller’s identity is \( \lambda N \).
2. In a situation where \( r \) sellers advertise in \( k \)th period, then the distribution of potential buyers is \( N(\lambda + (1 - \lambda))^r \) [This is the binomial distribution.]
3. From a buyer’s perspective, the value of the sellers’ information is the amount by which it reduces the expected cost of the purchase to the buyer. The expected value of the information to buyers is:
   
   \[
   \sum_{m=1}^{r} \frac{r!}{m!(r-m)!} \lambda^m (1-\lambda)^{r-m} \Delta C_m
   \]

   since \( \Delta C_m \) gives the savings and the rest of the formula gives the probability of being informed of \( m \) buyers (as this is a binomial distribution). [See mathematical appendix.]
4. Buyers who spend more on a good or who search more will also search more for advertisements.
5. The buyers with more information will undergo more searches, so the value of information will be actually be higher than that shown above.

Alternatives for charging advertisement cost:
1. Charge the buyer
   - Example: Catalogues
   - The seller would supply more information than the buyer wanted. As ads would be published together in one publication, it would be hard for the buyer to effectively express his preferences for advertisement information.
2. Charge the seller
   - Example: Newspaper advertisements
   - Incentivizes sellers to limit information to only what the buyers want in order to reduce their own costs
3. Institute a separate charge for ads, distinct from the good’s price
   - Ads of different sellers would be sold together to capture efficiencies in distributing information together
4. Charge for ads as part of the commodity price
• Appears unfair to the consumer who only wishes to purchase the good and not the ads, but consumers can choose to purchase from firms who advertise only a little and receive a lower price.

5. Advertisement wrapped in entertainment

• Examples: Advertisements on TV
• People are more willing to pay for information if it is supplied in an entertaining way.
• Advertisements and entertainments are complementary goods. This is like consumer demand for air-conditioned stores.

Comparing Seller’s Advertising Decision
We are going to compare the seller’s decision to advertise, given the competition regime in the market (i.e. monopoly vs. competitive market).

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<thead>
<tr>
<th>Comparison</th>
<th>Monopoly</th>
<th>Competitive Market</th>
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<tr>
<td>Amount of potential buyers who know seller’s identity</td>
<td>$\lambda N$ [in the limit]</td>
<td>$T_i = \lambda N \frac{s\lambda_i}{r\lambda} &lt; \lambda N$ [See mathematical appendix]</td>
</tr>
<tr>
<td>Amount of potential buyers who know seller’s identity who will buy from that seller</td>
<td>$\lambda N$ [Since this is a monopoly, all will buy from the seller]</td>
<td>$T_i K = \lambda N \frac{s\lambda_i K}{r\lambda} &lt; \lambda N$</td>
</tr>
<tr>
<td>Seller’s profit function</td>
<td>$\pi = \lambda N pq - \phi(\lambda Nq) - ap_a$ [Revenue – (Non-ad costs) - (Ad costs)]</td>
<td>$\pi = T_i K pq - \phi(T_i K)q - ap_a$ [Revenue – (Non-ad costs) - (Ad costs)]</td>
</tr>
<tr>
<td>Maximum profit conditions (FOC)</td>
<td>$\frac{\partial p}{\partial q} = \frac{p_a}{Nq\phi/\partial}$ (MR = MC)</td>
<td>$T_i \left(K \frac{\partial p}{\partial p} + pq \frac{\partial K}{\partial p}\right) = T_i \phi \left(K \frac{\partial q}{\partial p} + q \frac{\partial K}{\partial p}\right)$ (MR=MC)</td>
</tr>
<tr>
<td>Interpretation of FOC</td>
<td>When $\phi = 0$ (Production costs =0) (Cournot spring), the monopolist advertises up to the point where price = marginal cost of advertising.</td>
<td>This is similar in the competitive case.</td>
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</table>
The monopolist will advertise more, the higher the death rate (b), unless it is very high relative to the contact rate (c), as then ads would not be effective.

<table>
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<tr>
<th>Production cost relationship</th>
<th>$\phi' = p \left( 1 + \frac{1}{\eta_{qp}} \right)$</th>
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<tbody>
<tr>
<td>[See mathematical appendix.]</td>
<td></td>
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</table>

Price exceeds marginal cost by $\frac{1}{\eta_{qp} + \eta_{Kp}}$ which is less than $\frac{1}{\eta_{qp}}$.

It means that price in competitive market is less than in monopoly.

This is due to $\eta_{Kp} = -\frac{(s-1)p}{(1-p)}$

As search $s$ increases, $\eta_{Kp}$ becomes more negative, and price decreases.

**Interpretation of production cost relationship**

Price exceeds marginal cost by $\frac{1}{\eta_{qp}}$, where $\eta_{qp}$ is the elasticity of demand w.r.t. price. (negative sign)

**Sensitivity Analysis:**

(The advertising and search relationship)

\[ \text{sign} \frac{\partial a}{\partial s} \approx \text{sign} (1 + \eta_{Ks}) \]

Elasticity of fraction of buyers who buys seller I w.r.t. amount of search:

\[ \eta_{Ks} = 1 + s \log(1 - p) \]

Increased search by buyers (s) will lead to increased advertising by low-price sellers (low $p \rightarrow 1 + \eta_{Ks}$ positive) and reduced advertising by high price sellers (high $p \rightarrow 1 + \eta_{Ks}$ negative).

**Sensitivity Analysis:**

(The advertising and amount of rival sellers relationship)

\[
\frac{\partial a}{\partial r} \left\{ \frac{\lambda_i \partial^2 \lambda_i}{\partial a^2} + \left( \frac{\partial \lambda_i}{\partial a} \right)^2 \right\} = \lambda_i \frac{\partial \lambda_i}{\partial a} \left( 1 - \frac{r \partial K}{K \partial r} \right)
\]
This means that the amount of advertising by the firm will decrease as the number of rivals increases.

**Price advertising**

**Question:** As search is now very economical, why, for homogenous goods, is there still price dispersion?

**General answer:** Price differences decrease significantly if prices are advertised by a large proportion of sellers. However, price dispersion does not disappear because no advertising will reach all potential buyers within the population.

- **Buyer’s perspective:** the average consumer buys several hundred different items each month; hence the cost of keeping currently informed about all items prices is prohibitively large.
- **Seller’s perspective:** the average seller may sell thousands of items and to advertise each as prices change would be far too expensive.
- **Manufacturer’s perspective:** the manufacturer is harmed by uncertainty about his price. Since buyers must bear search costs, consumption will be less if price dispersion is large, and thus search costs are high. This paper conjectures that uniform prices are set by the manufacturer to remove price variation, even though there is still a dealer’s margin.
- Price advertising has the same results as extensive search by a large fraction of potential buyers → dispersion of asking prices will be much reduced (but not vanish).
- Since price advertising tends to be focused on products for which there is a high marginal value of search, reductions in price dispersion for commodities with large total expenditures will be largest.

**Conclusion**

This seminal paper by Stigler has shown us that there are many things to consider when we discuss market information. The most fundamental component of information, the price level for goods, is not steadfast across the market. Quite on the contrary, it varies and requires search effort from both sellers and buyers to determine it within a reasonable degree of certainty. Buyers search to find the most favorable price, their own internal minimum price at which they are willing to purchase the good. Sellers search to ascertain the prices their competitors are offering in order to tender their own price regime to buyers.
Search is determined by both the behavior of prices and the characteristics of the good being sold. If prices are positively correlated over time then search in earlier periods will be greater. Even in homogenous goods markets, costs of search faced by both sellers and buyers will limit the convergence of prices to a uniform level.

Price dispersion is an innate feature of markets, even those with homogenous goods. Markets are not static and this constant change causes knowledge depreciation. As the supply and demand conditions vary, buyers and sellers will have to search to understand the new market price for goods. Even those experienced buyers will have to engage in search to update their understanding of the market. The greater this instability in the market, the more price dispersion exists. Even with perfectly correlated prices, agents on both sides of the market cannot invest the time to vet all alternatives. Furthermore, changes in the mix of agents in the market will cause sustained price dispersion.

Advertising is a means for sellers to effectively communicate with the broader population. Because the composition of the buyers market is constantly changing, they can use it as a means to identify themselves to new buyers and maintain their relevance among experienced buyers. The buyers in the market value the information provided by the advertisements because it reduced their expected cost of purchases. Depending on whether the sellers act in a monopolistic or competitive manner their advertising decision will be altered. Even though advertising can reach a much broader audience, price dispersion cannot be fully eliminated because of the time cost faced by buyers to remain perfectly informed. It is also interesting to note that if quality of
goods may vary, reputation of a seller may be valuable by reducing the high search costs of a buyer that would be required to guarantee purchase of a quality good.

The model developed by Stigler is a basic one which we think can be built upon in a couple of ways. Firstly, the model could be made dynamic. Second, the assumption that higher income people search less may not be as valid today as it was in 1961. High earners will arguably have better access to information through technology or personal connections (pooling). Stigler’s model can also be extended to explaining the online advertising industry.
Mathematical Appendix

Derivation of (1)
From footnote 2:
\[ [1 - F(p)]^n = \left[ \int_0^1 dx \right]^n \]
where F(p) is the CDF of p and the integral is evaluated from p to 1.

The left hand side gives the probability that prices are greater than p, and thus that p is the minimum price. Evaluating the integral on the right hand side gives:
\[ (1-p)^n \]
Taking the derivative of this function, which is equivalent to the CDF, to get the PDF gives:
\[ n \left(1 - p\right)^{n-1} \]  

(1)

Derivation of Average Minimum Price
From footnote 2, the probability that a price is greater than p (meaning p is the minimum price) is:
\[ \left[ \int_0^1 dx \right]^n \]

Evaluating this integral, we get:
\[ (1-p)^n \]
Taking the derivative, we get:
\[ f(p) = \frac{d}{dp} (1-p)^n = n(1-p)^{n-1} \]
\[ E(p) = \int_0^1 n(1-p)^{n-1} \]

Evaluating the integral by parts, we get:
\[ (1-p)^n \bigg|_0^1 - \int_0^1 (1 - p)^n dp \]
\[ = 0 + \frac{1}{n+1} (1-p)^{n+1} \bigg|_0^1 \]
\[ = \frac{1}{n+1} \]
Derivation of Variance of Average Minimum Price

\[ \text{Var}(p) = \int_0^1 (p - \frac{1}{n+1})^2 * n(1-p)^{n-1} dp \]  
[By variance formula]

\[ = \frac{1}{(n+1)^2} \int_0^1 (np - (1 - p))^2 * n(1-p)^{n-1} dp \]  
[Simplify]

\[ = \int_0^1 (n^2 p^2 - 2np(1-p) + (1-p)^2) * n(1-p)^{n-1} dp \]  
[Simplify]

\[ = \int_0^1 n^3 p^2 (1-p)^{n-1} dp - \int_0^1 2n^2 p(1-n)^n dp + \int_0^1 n(1-p)^{n+1} dp \]  
[Simplify]

Integrate; integrate the first term by parts

\[ = -n^2 p^2 (1-p)|_0^1 + \int_0^1 2n^2 (1-p)^n p dp - \int_0^1 2n^2 p(1-n)^n dp - \left( \frac{n(1-p)^{n+2}}{n+2} \right)_0^1 \]  
[Simplify]

\[ = 0 + \int_0^1 2n^2 (1-p)^n p dp - \int_0^1 2n^2 p(1-n)^n dp + \frac{n}{n+2} \]  
[Simplify]

\[ = \frac{n}{(n+1)^2(n+2)} \]

Footnote 4

\[ E(n) = n \int_0^\infty p (1 - F)^{n-1} F' dp \]  
[Given]

Note: If \( F(p) \) is the uniform distribution (as is generally assumed in the text):

Since:

\[ F'(p) = f(p) \]  
[Derivative of CDF is PDF]

\[ f(p) = \frac{1}{b-a} = 1 \]  
[Because \( f(p) \) is the uniform distribution where \( b=1, a=0 \)]

Plugging these identities into the original equation:

\[ E(n) = \int_0^\infty pn(1-p)^{n-1} \]

By the expected value formula, this gives the expected value of the minimum price since \( n(1-p)^{n-1} \) is the distribution of the minimum price from equation (1).
Derivation of Footnote 9
\[ N_i = K N_b (1-p)^{n-1} \] [Given]
\[ \frac{\partial N_i}{\partial p} = (n-1) K N_b n (1-p)^{n-2}(-1) \] [Take derivative w.r.t \( p \)]
\[ = -\frac{(n-1) K N_b n (1-p)^n}{(1-p)} = -\frac{(n-1) N_i}{(1-p)^2} < 0 \]

Second derivative:
\[ \frac{\partial^2 N_i}{\partial p^2} = (n-1)(n-2) K N_b n (1-p)^{n-2} \] [Take derivative w.r.t \( p \)]
\[ = \frac{(n-1)(n-2) K N_b n (1-p)^n}{(1-p)^2} = \frac{(n-1)(n-2) N_i}{(1-p)^2} > 0 \]

Derivation of Footnote 10
\[ N_i = K N_b n (1-p)^{n-1} \] [Given]
\[ \log(N_i) = \log(K) + \log(N_b) + \log(n) + (n-1) \log(1-p) \] [Take logs of both sides]

Differentiating with respect to \( n \) gives:
\[ \frac{1}{N_i} \left( \frac{\partial N_i}{\partial n} \right) = \frac{1}{n} + \log(1-p) \]
\[ = \frac{1}{n} + 0 - p \]
\[ = (1/n) - p \]

So, the number of buyers will increase with more searches if \( \frac{\partial N_i}{\partial n} > 0 \), which will occur if \( (1/n) > p \) or “if the price is below the reciprocal of the amount of search.”
### The Model of Advertising

#### 1. The Model

<table>
<thead>
<tr>
<th>Illustration</th>
<th>1st period</th>
<th>2nd period</th>
<th>kth period</th>
<th>Sum of potential buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of potential customers</td>
<td>N</td>
<td>N(1-b)</td>
<td>N(1-b)^k-1</td>
<td></td>
</tr>
<tr>
<td>The probability of being informed</td>
<td>c</td>
<td>c(1-c)</td>
<td>c(1-c)^k-1</td>
<td></td>
</tr>
<tr>
<td>The number of potential customers being informed</td>
<td>cN</td>
<td>cN(1-b)(1-c)</td>
<td>cN[(1-b)(1-c)]^k-1</td>
<td>cN[(1-b)(1-c)+...+(1-b)^k-1(1-c)^k-1]</td>
</tr>
</tbody>
</table>

- As k converges to infinity, this approaches \( \frac{cN}{1-(1-b)(1-c)} = \lambda N \)
- Hence, the number of potential buyers informed of a seller’s identity: \( \lambda N \)

#### 2. Finding the number of potential buyers informed of sellers’ identity
- The paper supposed there exist r sellers and N potential buyers.
- The distribution is given by binomial distribution: \( N(\lambda + (1-\lambda))^r \)
- This means the probability of being informed of exactly m sellers’ identities should be \( \frac{r!}{m!(r-m)!} \lambda^m (1-\lambda)^{r-m} \)
- Thus the number of people who have been informed of exactly m sellers’ identities should be \( \frac{Nr!}{m!(r-m)!} \lambda^m (1-\lambda)^{r-m} \)

#### 3. Finding the value of information to buyers
- Suppose the expected reductions are \( \Delta C_1, \Delta C_2, \ldots \), for searches of 1, 2, ..., then the value of the information to buyers is \( \sum_{m=1}^{r} \frac{r!}{m!(r-m)!} \lambda^m (1-\lambda)^{r-m} \Delta C_m \)
Monopoly

1. Sellers’ Profit Function:
   - A monopolist will advertise so as to maximize the profits
   - Profit function: \( \pi = Npq\lambda - \Phi(N\lambda q) - ap_a \)
     - \( P = f(q) \) is the demand curve of individual buyer;
     - \( \Phi(N\lambda q) \) is production costs other than advertising; and
     - \( ap_a \) is advertising expenditures.

2. The condition for maximum profit:
   - \( \frac{\partial \pi}{\partial q} = N\lambda(p + q \frac{\partial p}{\partial q}) - \phi'N\lambda = 0 \)
   - \( \frac{\partial \pi}{\partial a} = Nqp \frac{\partial \lambda}{\partial a} - \phi'Nq \frac{\partial \lambda}{\partial a} - p_a = 0 \)

3. Production cost relationship:
   - Production cost: \( \phi' = p \left( 1 + \frac{1}{\eta_{qp}} \right) \)
     - \( \eta_{q_i} \) is the elasticity of a buyer’s demand curve.
Competitive Market

Under competition, the amount of advertising by any one seller (i) can be determined as follows.

1. Finding the fraction of potential buyers who will canvass the seller i
   - Each buyer will engage in an amount s of search.
   - For one time search (s=1):
     - The amount of sellers that each buyer on average will know: \((r-1)\lambda + \lambda_i\)
     - For buyers who know seller i:
       - the percent of buyers who will canvass seller i: \(\frac{\lambda_i}{(r-1)\lambda + \lambda_i}\)
       - the percent of buyers who will not canvass seller i: \(1 - \frac{\lambda_i}{(r-1)\lambda + \lambda_i}\)
   - For s searches (s=s):
     - assumption: amount of search < amount of sellers that each buyer know: \(s \leq (r-1)\lambda + \lambda_i\)
     - The amount of sellers that each buyer on average will know: \([r-1]_+\lambda_i \] \[s\]
     - For buyer s who know seller i:
       - the percent of buyers will canvass seller i: \(\left(\frac{\lambda_i}{(r-1)\lambda + \lambda_i}\right)^s\)
       - the percent of buyers will not canvass seller i: \(1 - \left(\frac{\lambda_i}{(r-1)\lambda + \lambda_i}\right)^s\)
       - the percent of buyers who will canvass seller i at least once: \(1 - \left(1 - \frac{\lambda_i}{(r-1)\lambda + \lambda_i}\right)^s\)
   - Focus on the percent of buyers who will canvass seller I at least once (with s searches):
     - From the last equation, we will approximate \(\frac{\lambda_i}{(r-1)\lambda + \lambda_i} \approx \frac{\lambda_i}{r\lambda}\)
     - Then we will have: \(1 - \left[1 - \frac{\lambda_i}{r\lambda}\right]^s\). We know that \(1 - \left[1 - \frac{\lambda_i}{r\lambda}\right]^s\) is a binomial form.
     - Take the first two terms of the binomial expansion: \(\left(\frac{S}{0}\right)1^s - \left(\frac{S}{1}\right)1^{s-1}\). We know that \(\frac{\lambda_i}{r\lambda}\)
     - Then we will have: \(1 - \left(\frac{S}{0}\right)1^s - \left(\frac{S}{1}\right)1^{s-1} = \frac{s\lambda_i}{r\lambda}\)
   - The number of buyers canvassing seller i \((T_i)\) is a multiplication of:
     - the percent of buyers who will canvass seller i at least once (with s searches): \(\frac{s\lambda_i}{r\lambda}\)
     - the number of potential buyers who knows seller i: \(\lambda N\)
   - So, \(T_i = \lambda N \frac{s\lambda_i}{r\lambda}\)

2. Finding the fraction of buyers canvassing seller i who will buy from seller i:
   - K will be the percentage of buyers canvassing seller I who will buy from seller i.
   - K depends on relative price, amount of search, and the number of rival sellers.
   - Hence, the fraction of buyers canvassing seller i who will buy from seller i: \(T_i K\)
3. Finding the seller i profit function:
   - Let the income from sales be: \( T_iKpq \)
   - Let the production costs be: \( \phi(T_iKq) \)
   - Let the advertising costs be: \( ap_a \)
   - Then seller i profit function: \( \pi = T_iKpq - \phi(T_iKq) - ap_a \)

4. The conditions for maximum profit:
   - \( \frac{\partial \pi}{\partial p} = T_i\left(K\frac{\partial q}{\partial p} + pq\frac{\partial K}{\partial p}\right) - T_i\phi'\left(K\frac{\partial q}{\partial p} + q\frac{\partial K}{\partial p}\right) = 0 \) ................................. (8)
   - \( \frac{\partial \pi}{\partial p} = Kpq\frac{\partial T_i}{\partial a} - \phi'Kq\frac{\partial T_i}{\partial a} - p = 0 \) ................................. (9)

5. Finding the production costs relationship:
   - Rearranging equation (8), we get:
     \[
     \phi' = \frac{T_i\left(K\frac{\partial q}{\partial p} + K\frac{\partial q}{\partial p} + \frac{\partial K}{\partial p}\right)}{T_i\left(K\frac{\partial q}{\partial p} + K\frac{\partial q}{\partial p} + \frac{\partial K}{\partial p}\right)} \left(\frac{K\partial q + q\partial K}{K\partial q + q\partial K}\right) = \frac{p\left(\frac{\partial q}{\partial p} + \frac{\partial K}{\partial p}\right)}{\left(\frac{K\partial q}{q} + q\partial K\right)} = p\left(1 + \frac{\frac{\partial q}{q} + \frac{\partial K}{K}}{\frac{\partial q}{q} + \frac{\partial K}{K}}\right) = p\left(1 + \frac{1}{\left(\frac{\partial q}{q} + \frac{\partial K}{K}\right)}\right)
     \]
   - Production costs: \( \phi' = p\left(1 + \frac{1}{\eta_q + \eta_k}\right) \)
   - Elasticities of demand q with respect to price: \( \eta_q = \frac{\partial q}{p} \)
   - Elasticities of fraction of buyers K with respect to price \( \eta_k = \frac{\partial K}{K} \)
     - For uniform distribution, \( \eta_k \) is as follows:
     - From equation (3), \( \eta_k = \frac{\partial K}{K} = \frac{\partial N_i}{N_i} \)

6. Finding the advertising and search relationship:
   - \( \text{sign} \frac{\partial a}{\partial s} = \text{sign} \left(1 + \eta_Ks\right) \)
     - For uniform distribution, \( \eta_K \) is as follows:
     - From equation (3), \( \eta_K = \frac{\partial K}{K} = \frac{\partial N_i}{N_i} \)

7. Finding the advertising and rival sellers relationship:
   - \( r \frac{\partial a}{\partial s} \left\{ \lambda_i \frac{\partial^2 \lambda_i}{\partial a^2} + \left(\frac{\partial \lambda_i}{\partial a}\right)^2 \right\} = \lambda_i \frac{\partial \lambda_i}{\partial a} \left(1 - \frac{r \frac{\partial K}{K}}{\partial r}\right) \)
   - The term in brackets on the left side is negative by the stability condition; the right side is positive.