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Introduction

Being a graduate student, much like a trapeze artist or a crash test dummy, is full of danger. With constant work, and poor nutrition, it is unsurprising that there is such a high burnout rate from being a graduate student. Luckily there is an insurance market in order to provide the graduate students to take care of themselves during their treacherous profession. However, what about insurance firms have imperfect information about the customers they are providing for? Asymmetric information is the bugaboo of markets, principles, and governments. As this article will show, it can create vast inefficiencies in markets through simply not knowing the true nature of people.

This article set up the problem of asymmetric information, or the problem where one party has more information than another. Here Rothschild & Stiglitz show the problem of providing insurance when customers are privy to more knowledge than insurance providers. In this article they show that under conditions of asymmetric information, insurance markets can become inefficient. Specifically, that the insurance markets may not be able to create pooling or separating equilibriums if there are different types of people in the world. For those who aren’t economists, or do not know about pooling or separating equilibriums, here is a quick review:

- **Pooling equilibrium:** is one where the two types of people choose the same strategy/outcome. In this example, there is a pooling equilibrium if people with high and low probability of injury purchase the same insurance.
- **Separating equilibrium:** is where the two types of people naturally diverge on their actions, creating two separate equilibrium points. One for each type of person (in this example, high & low risk types have their own insurance rates)

In this case, the lack of pooling equilibrium means that only separating equilibrium are possible under certain circumstances. And when a separating equilibrium does occur, they are Pareto suboptimal. This is due to the riskier population creating a negative externality upon the low risk population, and firms’ inability to discern the risk level of the population.
This paper is broken into 5 main sections:

1. The Model
2. Showing an equilibrium of a perfect market full of identical people
3. Showing that a pooling equilibrium cannot exist in a situation where asymmetric information exists regarding people’s risk for injury
4. Showing that a separating equilibrium may not exist in a situation where asymmetric information exists regarding people’s risk for injury
5. Providing robustness checks to show that the conclusions do not stem from peculiarities of the underlying assumptions of the model

There are a few main conclusions that can be derived from this paper.

First is that in markets with imperfect information, competitors may change their actions in order to improve their information over buyers.

Second is that competitive insurance markets may have no equilibrium.

Third when equilibrium do exist, those equilibrium are not Pareto optimal.

Key assumptions underlying the model:
• Players want to maximize expected utility between their healthy state and their non-healthy state
• There is no income effect between the two states, (i.e. players desire the same proportion of goods regardless of income, and adjust how much they consume based on their total income)
• Firms wish to maximize profits
• Firms exist in a scenario of perfect competition, where there is no cost of entry into the insurance market
• Players know their probability of being injured/getting sick
• Firms know the actuarial averages of any person suffering an injury (this assumption is relaxed later in the paper)

Key variables and their meanings:
• $W_1 =$ income with no accident
• $W_2 =$ income where accident occurs
• $p =$ probability of accident
• $p_H =$ probability of accident for high risk people
• $p_L =$ probability of accident for low risk people
• $\alpha_1 =$ cost of insurance policy
• $\alpha_2 =$ payout of insurance policy
• $d =$ loss of income from accident
• $E =$ endowment point
The Basic Model

Individuals have an income of size $W$ if they avoid accident. In the event an accident occurs, the individual’s income decreases by $d$ and he receives $W - d$.

The individual can insure himself against this accident by paying an insurance premium of $\alpha_1$, in return for which he will be paid $\alpha^*_2$ if an accident occurs. Income with insurance is $(W - \alpha_1, W - d + \alpha_2)$, where $\alpha_2 = \alpha^*_2 - \alpha_1$. The vector $\alpha = (\alpha_1, \alpha_2)$ describes the insurance contract.

Demand for Insurance Contracts:

Demand for insurance is represented by individuals who buy insurance contract $\alpha$. An individual purchases an insurance contract so as to alter his pattern of income across states of nature. Let $W_1$ denote his income if there is no accident and $W_2$ his income if an accident occurs.

The expected utility theorem states that his preferences for income in these two states of nature are described by a function of the form:

$$\hat{V}(p; W_1; W_2) = (1 - p)U(W_1) + pU(W_2),$$

where $U(\cdot)$ represents the utility of money income and $p$ the probability of an accident.

A contract $\alpha$ is worth $V(p; \alpha) = \hat{V}(p; W - \alpha_1, W - d + \alpha_2)$. The individual always has the option of buying no insurance, so he will purchase a contract only if:

$$V(p; \alpha) \geq V(p; 0) = \hat{V}(p; W_1; W_2)$$

People are identical in all respects, save their probability of having an accident and are risk-averse ($U'' < 0$); thus $V(p; \alpha)$ is quasi-concave.

Supply of Insurance Contracts

Insurance companies supply the insurance contract. We assume that companies are risk-neutral and are concerned only with expected profits, so that contract when sold to an individual who has a probability of incurring an accident of $p$, is worth
Insurance companies have financial resources that they are willing and able to sell any number of contracts that they think will make an expected profit. The market is competitive in that there is free entry. Thus any contract that is demanded and expected to be profitable will be supplied.

Information about Accident Probabilities

Assume that individuals know their accident probabilities and companies do not. In this scenario, individuals with high accident probabilities will demand more insurance than those who are less accident-prone. In an attempt to get information from customers, firms use a market device that Salop and Salop call a self-selection mechanism to forcing customers to make market choices. This reveals customer’s characteristics and make the choices the firm would have wanted them to make had their characteristics been publicly known.

Equilibrium with Identical Customers

![Figure 1](image-url)

To start the authors assume only one type of identical customers.
In Figure 1, X-axis and Y-axis represent income of without accident and with accident respectively. Point E is the initial endowment point. Initial endowment point is the condition that customer without insurance point.

The diagonal line is $W_1 = W_2$. All the endowment point has to be below the diagonal line since $W_1 > W_2$. The points in the diagonal line are in complete insurance. The complete insurance can cover the risk and there is no income difference between having an accident or not.

Perfect competitive market and free entry makes the insurance companies earn 0 profits, and the line for zero profit is fair-odds line (EF). The slopes satisfies:

$$\frac{(1 - p) a_1 - p a_2}{a_2} = 0 \quad (3)$$

The insurance company will only offer contract satisfies this condition. Since all the insurance customers are risk-averse, equilibrium point $a^*$ maximizes the individual utility within the contracts offer. The point $a^*$ is at the intersection of $45^\circ$ line and fair-odds line. And it is tangency to the utility function. In equilibrium each customer buys complete insurance at actuarial odds.

**Imperfect Information: Equilibrium with Two Classes of Customers**

Now suppose market consists of two type of customers, one with a low risk type with $p^L$ and another with a high-risk type $p^H$ ($p^L < p^H$). The fraction of high type is $\lambda$, and the average accident probability is:

$$\overline p = \lambda p^H + (1 - \lambda) p^L$$

The authors further assume that customers know their own type but companies do not. Firms only know the distribution of the types.
A market under these circumstances has two kinds of equilibrium: pooling and separating. For pooling case, insurance companies offer only one contract to both types of customers. In separating case, they offer two contracts to different type separately.

However, the authors show that under these relatively simple assumptions that there is no pooling equilibrium. The proof is given as follows from the graph.

Point E is initial endowment and assumes there is equilibrium in point ∂. Further, profits should be zero in equilibrium. If \( \pi(p, \bar{p}) < 0 \), firms would not offer it. If \( \pi(p, \bar{p}) > 0 \), there exists an contract that offer slightly more consumption in each state of nature and this contract will get the whole market since it is perfectly competitive. Thus \( \pi(p, \bar{p}) = 0 \). And lies on the market odds line EF (with slope \( \frac{p}{1 - p} \)).

However, under all circumstances, there is a contract \( \beta \) that incentivizes low-risk customers to deviate from \( \partial \). Thus the existence of \( \beta \) contradicts the second definition of equilibrium (2).

Therefore there is no pooling equilibrium exist.

**Figure 3:**
Graph II shows that a pooling equilibrium doesn’t exist; however we still need to check to see if there is a separating equilibrium.
We first prove that \((\alpha^H, \beta)\), where customers with both high and low probabilities of accident get complete insurance, is not an equilibrium. The proof is as follows. If an insurance company offers both \(\alpha^H\) and \(\beta\), the low-risk individuals will pick \(\beta\), where they will receive complete insurance. The high-risk types, however, will also choose \(\beta\) over \(\alpha^H\) where they will receive more income in either state. Therefore this is not an equilibrium. In this situation, the insurance company will lose money on the high risk individuals, which violates the definition that company makes zero profit at equilibrium.

We then prove that for a market with both low and high risk individuals, \((\alpha^H, \alpha^L)\) is the only possible separating equilibrium, where \(\alpha^L\) is the intersection of EL and UH in figure III. At equilibrium, the contract chosen by the low-risk types should not be more attractive than that of the high-risk individuals. Therefore, \(\alpha^L\) should lie on the same indifference curve of high-risk individuals where \(\alpha^H\) lies. Moreover, \(\alpha^L\) should also maximize the utility of the utility of low-risk individuals given that it does not incur negative profit for the insurance company.

Note that \((\alpha^H, \alpha^L)\) is a possible equilibrium. Meaning if an equilibrium set exists, it should be \((\alpha^H, \alpha^L)\). However, this does not guarantee the existence of such an equilibrium.

Finally there may or may not exist an equilibrium. The equilibrium \((\alpha^H, \alpha^L)\) exists when the pooled fair odds line lies below UL, or in other words, the proportion of high risk types is not so small in the total population. Otherwise, there will be no equilibrium. We can see that if the pooled fair odds line lies above UL, there will always be a point \(\gamma\), which is more attractive than \(\alpha^H\) and \(\alpha^L\) to
both high and low risk players. Finally \( y \) incurs a positive profit for the insurance company, which violates our definition of an equilibrium.

To generalize, there are two conditions where an equilibrium does NOT exist.

1. The cost to pool for the low-risk individuals is low. (e.g., relatively small proportion of high-risk individuals, or the probabilities of the two groups are not too different)
2. Costs of separating are high

Welfare implications

The presence of the high-risk individuals exerts a negative externality on the low-risk individuals. However, the high-risk individuals are not better off than they would be in isolation.

Robustness

The second half of the article seeks to show that the findings in the first section are not derived from some peculiar assumptions. To show that the findings stem from perfect competition and asymmetric information, the authors tweak the underlying assumptions to show that the results still hold.

Information Assumptions

One assumption that might be problematic is that customers might not know their exact probability of getting sick, or suffering an accident. Therefore, the authors relax this assumption to see what the effect of changing it is. They show that if the two types of players are not biased in their assessment of their risk, then the analysis does not change.

However, it could be possible that the two types of customers also have two different risk profiles. It is not a stretch to believe that risky customers are most risk-adverse, since they know that they are more likely to suffer an accident/get sick. Therefore the authors assume that lower risk individuals will be more risk adverse than high-risk individuals.

Under these new assumptions, the authors show on Figure IV that a pooling equilibrium cannot exist. If a pooling equilibrium did exist, it would be on the fair odds line where customers receive complete insurance coverage. However, any point in the shaded area would be a preferred
policy point for the low risk customers. Therefore, for the definition of this article, this cannot be a point, since if an insurance firm offered a point on the shaded area, the lower risk customers would no longer purchase insurance at the pooling equilibrium.

Price Competition Versus Quantity Competition

Under price competition, insurance firms set one price ($\alpha_1/\alpha_2$) and customers would be able to buy as much insurance as they desire at that price. Meanwhile under price and quantity competition, insurance firms offer several different contracts with different prices and quantities. However, customers would only be able to purchase one contract at a time.

A graduate student, for example, would not be able to buy insurance against boredom for $1000 from three different insurance firms, and then cash out $3000 if by some strange reason the graduate student suffered from boredom. Instead the three insurance companies would fight out which insurance company would have to pay out the $1000 originally promised. The reason for this is due to moral hazard. If a graduate student could purchase several contracts insuring against boredom, then they might be more likely to take boring classes, such as biology or chemistry compared to political science.

The authors argue that price and quantity competition is a special case of price competition. Under price equilibrium, customers only buy insurance at the lowest price quoted. However, under price and quantity equilibrium, several bundles of priced and quantities of insurance will exist depending on the desires of the customers.
Restrictions on Firm Behavior and Optimal Subsidies

Another previous assumption was that each insurance company could only issue only one contract, and in equilibrium firms make nonnegative profits. The authors relax this assumption, where firms could offer more than one contract.

The benefit of relaxing this assumption is that firms are now not dependent on the policies offered by other firms, and the information generated by the choices of individuals. However, once we change this assumption, the authors show that contracts exist that can break the equilibrium and there is still the optimal subsidy problem. The optimal subsidy problem shows that all separating equilibrium are Pareto inefficient.

In Figure V, EF is the market odds line and a separating equilibrium \( \left( \frac{\bar{\alpha}^H}{\bar{\alpha}}, \frac{\bar{\alpha}^L}{\alpha} \right) \) exists. Suppose a firm offers two contracts \( \alpha^H \) and \( \alpha^L \) where the low-risk contract makes a loss and the high-risk contract makes a profit.

Both types prefer the new contract over the equilibrium contract and if the new contracts are offered by the same firm, the losses are offset by the profits. This upsets the equilibrium.

Optimal subsidy problem: choosing two contracts to maximize the low-risk types utility subject to the high-risk types incentive constraint and the constraint that the two contracts break even.

The optimal high-risk contract will always has a solution where the high-risk contract entails complete insurance so that \( V(p^H, \alpha^H) = U(W - p^H d + \alpha) \), where \( \alpha \) is the per capita subsidy.
of the high risk by the low risk. This subsidy decreases income for each low-risk person by \( \gamma \alpha \), where \( \gamma = \lambda / (1 - \lambda) \)

To find the optimal contract, one chooses \( \alpha \) and \( \alpha_2 \) to maximize

\[
U(X)(1 - p^L) + U(Z)p^L
\]

Subject to:

\[
U(Y) \geq U(X)(1 - p^H) + U(Z)p^H, \quad \alpha \geq 0
\]

Where

\[
X = W_0 - \gamma \alpha - \alpha_2 p^L / (1 - p^L)
\]

\[
Y = W_0 - p^H d + \alpha
\]

\[
Z = W_0 - d - \gamma \alpha + \alpha_2
\]

Sufficient condition:

\[
\frac{(p^H - p^L)\gamma}{p^L(1 - p^L)} > \frac{U'(Y)[U'(Z) - U'(X)]}{U'(X)U'(Z)}
\]

Where \( X, Y \) and \( Z \) are determined by the optimal \( \alpha^*, \alpha_2^* \).

The right-hand side of the equation is always less than

\[
\frac{U'(W_0 - d)[U'(W_0 - d) - U'(W_0)]}{U'(W_0)^2}
\]

so that there exist values of \( \gamma \) large enough to satisfy the sufficient condition.

**Alternative Equilibrium Concepts**

Finally, the authors change their definition of an equilibrium in order to see if it changes their findings. In their model the firm assumes that its actions do not affect the market—the set of policies offered by other firms was independent of its own offering. However, an alternative equilibrium concept changes rationality in the market; individuals who purchase such policies get
informationally consistent equilibrium. This notion was used by Spence's (1973) in the educational signaling literature.

A local equilibrium is a set of contracts such that there do not exist any contracts in the vicinity of the equilibrium contracts that will be chosen and make a positive profit. The set of separating contracts, maximizing the welfare of low-risk individuals, can be an example. A greater degree of rationality is demonstrated by the response of the firm to new contracts supplied by competitors.

A Wilson equilibrium is a set of contracts such that, when customers choose among them so as to maximize profits, (a) all contracts make nonnegative profits and (b) there does not exist a new contract (or set of contracts), which, if offered, makes positive profits even when all contracts that lose money as a result of this entry are withdrawn.

When the separating equilibrium exists, it is a Wilson equilibrium. When it does not exist, the Wilson equilibrium is the pooling contract that maximizes the utility of the low-risk customers, namely $\beta$ in Figure VI. $\beta$ dominates the separating pair $(\alpha_L, \alpha^H)$. Consider $\gamma$, which the low risk prefer to $\beta$. Under the standard equilibrium, it upsets $\beta$. Under Wilson's it does not. When the low-risk switch to $\gamma$, $\beta$ loses money and is withdrawn. The high-risk buy $\gamma$ too, which causes it to lose money. Therefore, $\gamma$ does not successfully compete against $\beta$.

Under Wilson's equilibrium, firms respond to competitive entry by dropping policies, but not by adding new policies. Wilson equilibrium may not exist if groups differ in their attitudes towards risk. The authors consider that Wilson's equilibrium can characterize monopoly (oligopoly) models better than competition ones.