Why is Central Paris Rich and Downtown Detroit Poor?

An Amenity-Based Theory

By Jan K. Brueckner, Jacques-Francois Thisse, and Yves Zenou

Presented by Group Von Neumann Morgenstern
Anita Chen, Salama Freed, Jian Zhai, Liming Zheng

Notation Key:
CBD=central business district
Y=total income
x=distance from CBD
a(.)=function that describes amenity level
t=commuting cost per mile
\( \hat{x} \)= point where bid prices between 2 competing bidders is equal
q=housing consumption
p=price per unit of housing
z=income of consumer’s neighborhood
v(.)=indirect utility function
e= consumption of numeraire nonhousing good

Part 1: Introduction

It has been observed that high-income folks in the United States tend to live in the suburbs outside of the city and away from the CBD (e.g. Detroit, MI) with some rare exceptions like New York City. Contrast this to how Paris and other western European cities are structured, where the rich tend to be clustered in the CBD and the not-so-rich tend to live further away from the city.

Richard Muth\(^1\), William Alonso\(^2\), and other urban economists have previously argued that this pattern can be explained through \( t/q \), the ratio between commuting cost per mile, \( t \) and housing consumption, \( q \). We consider these two variables by first assuming that the rich have higher levels of housing consumption than the poor. If that’s the case, then the rich would value lower housing/land prices than the poor and would congregate in areas where prices are lower (e.g. the suburbs). However, running counter to that effect would be the cost of commuting time. If the rich tend to work in the CBD, then the importance of accessibility may outweigh the effect of housing consumption. Analogously we can say that if \( t/q \) rises with income, then we would see that the effect of higher housing consumption dominates, but if \( t/q \) falls with income, then we would observe a spatial pattern as in Paris.

---

\(^1\) See *Location and land use* (1964).
\(^2\) See *Cities and housing* (1969).
An inconsistency with this theory was discovered by W.C. Wheaton\(^3\), when he found that \(t/q\) is roughly constant across different income groups in the U.S. The purpose of Bruecker et al.’s paper is to explain that inconsistency by introducing an amenity-based theory of location by income.

Amenities are broadly categorized into two groups: exogenous and endogenous.

1. **Exogenous amenity**:
   - **Natural amenity**: Aesthetically-pleasing topographical features of a city. An example would be the River Seine, which runs through Paris.
   - **Historical amenity**: Monuments, parks, museums, or any other well-preserved building from past centuries. Again, consider the historical sites in Paris, like Versailles or the Louvre.

2. **Endogenous amenity**: Amenities that reflect the current economic state of the city
   - **Modern amenity**: Man-made amenities like upscale restaurants, shopping districts, and gyms.

Higher-income areas are generally observed to have more modern amenities.

The key assumption driving the results of this paper is that the marginal value of amenities rises sharply with income, e.g. amenities are increasingly valuable to increasingly richer people. To study this, we consider the case that the rich tends to favor living in the suburbs (or that the effect of \(q\) dominates).

Let’s examine 2 cases—both which consider the “appealing” force of amenities:

**Case 1**: Suppose that the appeal of amenities falls rapidly away from the CBD. In this case, then we see that the amenity advantage overcomes the effect of \(q\) and we observe that the rich congregate in the CBD.

**Case 2**: Suppose that the appeal of amenities is weak or negative, and is unable to overcome the effect of \(q\). Then we would observe that the rich live in the suburbs.

Before formally introducing the model, consider the following definitions:

1. **The bid-price function**, \(p(x)\): In the context of this paper, \(p(x)\) gives us the price a person is willing to pay to live at different distances from CBD in order to achieve a given level of utility.
2. **The “standard” model**: This refers to the monocentric city model, developed by Mills, Muth, and Alonso, which aims to explain the spatial pattern of high income people living in the suburbs.

The rest of this handout is structured as follows: Part 2 will discuss the above two cases considering only exogenous amenities. This is done first with homogenous consumers and then with 2 groups of consumers—rich and poor. Part 3 will modify the model to consider endogenous amenities. Part 4 discusses why exogenous amenity patterns differ across cities.

**Part 2: Model with exogenous amenities**

Assumptions

1. Consumers gain utility from consuming houses, \(q\) and other market goods, \(e\). We consider \(e\) as the numeraire in the budget constraint so we normalize its value to 1.

2. Consumers all have the same level of income (we later modify this to 2 income groups: rich and poor) and have CES preferences.
3. All consumers work in the CBD.
4. Housing prices are higher closer to the center.

Consumers will live at a distance \( x \) from the CBD. Consider distance as creating a disutility. The modification Bruecker et al. make to the standard model is to include exogenous amenities. Because amenities are located in the CBD, we also consider amenities as a function of \( x \). We can then write the consumer’s utility function as \( u(e, q, a) \).

**Homogenous Income Group**

Continuing with the definitions listed above and assuming a homogenous income group, income is denoted as \( y \), while total commuting cost is \( tx \); this means disposable income is \( y - tx \). Recall that disposable income is equivalent to the budget constraint, \( e + pq \), we can rewrite \( e \) and substitute back into the utility function: \( u(y - tx - pq, q, a) \).

Consumers maximize utility subject to their budget constraint

\[
\begin{align*}
\text{Max } & \quad u(y - tx - pq, q, a) \\
\text{s.t.} & \quad e + pq = y - tx,
\end{align*}
\]

which yields the FOC

\[
\begin{align*}
\frac{\partial u}{\partial q} &= pu^e \\
\text{where } & \quad u^q = \frac{\partial u}{\partial q} \text{ and } u^e = \frac{\partial u}{\partial e}
\end{align*}
\]

Because \( p \) must vary to ensure equal utility at all points on a consumer’s indifference curve, we can denote \( \bar{u} \) as the uniform utility level, which can be defined as

\[
u(y - tx - pq, q, a) = \bar{u}.
\]

We can use this combined with the FOCs to determine the optimal bid price (\( p(x) \)) and quantity (\( q(x) \)) as a function of location.

If we think of the housing market as an auction place, we can consider \( p(x) \) as the bid-price function. \( P(x) \) will show us the different bundles of prices per unit of housing and distances in which the consumer is indifferent. The effects of distance from CBD on bid price can be shown by differentiating the uniform utility condition with respect to \( x \). Rearranging and solving for \( p'(x) \) yields

\[
p'(x) = -\frac{t}{q(x)} + \frac{p[y-t x, p(x), a(x)]}{q(x)} a'(x)
\]

Where the first term is the commuting cost effect and the second term is the amenity effect.
Note $v^a$ is the marginal valuation of amenities after optimal adjustment of housing consumption. In other words, it is the indirect utility function after we have optimized our housing consumption. This is a measure of the marginal rate of substitution toward amenities and away from the numeraire non-housing good or the ratio $\frac{\partial u}{\partial a} / \frac{\partial u}{\partial e}$. The standard model proposed by Muth assumes this marginal valuation due to amenities is zero, meaning consumers will not substitute away from the non-housing good in favor of amenities.

However, the slope of the bid price function shows that house prices fall with increasing distance to the Central Business District (CBD), so suburban consumers are compensated for their commuting.

Note:
- If $a'(x)>0$, $p'(x)$ can be either positive or negative, as the magnitude is offset by the commuting cost effect ambiguously.
- If $a'(x)<0$, $p'(x)$ is negative, indicating that amenities decline with distance.
- To simplify this, the authors assume the commuting cost effect (the first term) dominates, so $p'(x)$ is always negative.

**Heterogeneous Income Group**

Suppose we expand the model to include two income groups—rich (with income $y_1$ and commuting cost $t_1$) and poor (with income $y_0$ and commuting cost $t_0$). Reinforcing the definitions of rich and poor, we note that $y_0<y_1$ and $t_1>t_0$. This generates two different bid price functions ($p_0(x)$ and $p_1(x)$).

A group will occupy the area where it outbids the other group for housing.

Let $\bar{x}$ be the boundary between the areas where the bid prices of the two groups are equal, so $p_1(\bar{x}) = p_0(\bar{x})$ at this boundary. The relative bid price slopes at $\bar{x}$ determine where the group’s bid price is maximal.

- If $p_1'(\bar{x}) > p_0'(\bar{x})$, then the poor outbid the rich for urban housing while the rich outbid the poor for suburban housing.
- If $p_1'(\bar{x}) < p_0'(\bar{x})$, then the rich outbid the poor for urban housing while the poor outbid the rich for suburban housing.

The difference between the bid price slopes is

$$\Delta \equiv p_1'(\bar{x}) - p_0'(\bar{x}) = \frac{t_0}{q_0(\bar{x})} - \frac{t_1}{q_1(\bar{x})} + a'(\bar{x})\left[\frac{v^a[y-t_1(\bar{x}), p_1(\bar{x}), a(\bar{x})]}{q_1(\bar{x})} - \frac{v^a[y-t_0(\bar{x}), p_0(\bar{x}), a(\bar{x})]}{q_0(\bar{x})}\right]$$

(5)

When $\Delta < 0$, the rich locate to the center.
Assuming that which was originally proposed by Muth,

$$\Delta \equiv p'_{1}(\tilde{x}) - p'_{0}(\tilde{x}) = \frac{t_{0}}{q_{0}(\tilde{x})} - \frac{t_{1}}{q_{1}(\tilde{x})}$$

The difference between the quantity of housing consumed for rich and poor is a difference in incomes between the two. Since the disposable income for the rich, \(y_{1} - t_{1}\tilde{x} > y_{0} - t_{0}\tilde{x}\), we can say \(q_{1}(\tilde{x}) > q_{0}(\tilde{x})\).

Note:
- If \(t\) rises less rapidly than \(q\) as income increases, then \(\frac{t_{0}}{q_{0}(\tilde{x})} > \frac{t_{1}}{q_{1}(\tilde{x})}\) and the rich live in the suburbs.
- If \(t\) rises more rapidly than \(q\) as income increases, then \(\frac{t_{0}}{q_{0}(\tilde{x})} < \frac{t_{1}}{q_{1}(\tilde{x})}\) and the rich live in the city center.

Now we reintroduce the amenity effect and make two assumptions. First, location effects support the rich living in the suburbs. Second, \(v^a\) rises with income, more dramatically than an increase in housing consumption. Consumers tend to buy more to increase utility rather than consume more housing. Therefore \(\frac{v^a}{q_{1}(\tilde{x})} > \frac{v^a}{q_{0}(\tilde{x})}\).

Note:
- If \(a'(x)<0\) but small in absolute value, then the entire amenity effect will also be negative and close to zero. Then the commuting cost effect will dominate, and \(\Delta > 0\) and the poor live in the center and the rich live in the suburbs.
- If \(a'(x)<0\) and large in absolute value, then the amenity term dominates, and \(\Delta < 0\). If the amenity term dominates, the rich live in the center and the poor live in the suburbs.

Hence, the authors suggest the pattern of location by income can be set based on the city’s exogenous amenities.

The authors make this assumption by holding all other parameters in equation 3 fixed and only varying \(a'(x)\), and this approach has not been justified. Thus, in order to justify this, they compare the amenity patterns in two cities, a reference city and a comparison city.

In the reference city, the rich live in the suburbs, and the amenity function is \(a_{r}(x)\) and the \(\tilde{x}\) value is \(\tilde{x}_{r}\), while for the comparison city, the amenity function is \(a_{c}(x)\) and the \(x\) value is \(x_{c}\). The two amenity functions intersect at \(\tilde{x}_{r}\). The utility, commuting cost and utility are the same between the reference city and the comparison city.

For each group,
In the end, the key constraint is that the amenity function continues to intersect the original $\hat{x}$ as the slope continues to rise, as shown in Figure 1.

\[ \Delta_r = \frac{t_0}{q_0} - \frac{t_1}{q_1} + a_r'(x) \left[ \frac{v^a[y-t_1\hat{x},\hat{p},\hat{a}]}{q_1} - \frac{v^a[y-t_0\hat{x},\hat{p},\hat{a}]}{q_0} \right] \]

\[ \Delta_c = \frac{t_0}{q_0} - \frac{t_1}{q_1} + a_c'(x) \left[ \frac{v^a[y-t_1\hat{x},\hat{p},\hat{a}]}{q_1} - \frac{v^a[y-t_0\hat{x},\hat{p},\hat{a}]}{q_0} \right] \]
No ex-ante reasons can support that amenity-living pattern relation is a causal relationship.

The new model takes into account a new endogenous factor—neighborhood income $z$. Then the utility will be written as $u(e, q, a, z)$ where all other factors remain as in the former model. To simplify the model, distance $x$ can only be 1 or 0 which represent suburb and center respectively, so the commuting cost in suburb will be fixed at $t_0$ for the poor, and $t_1$ for the rich. Also, the dwelling demand will be limited to two quantities with the relation, $q_0 < q_1$, and the larger one belong to the rich people. We assume that poor people do not care about amenities or the neighborhood income while rich people value both.

There might exist two equilibria in which rich people all choose to live in the center or outside the city in the case that exogenous amenity’s role is important but not dominant.

Proof:
As can be seen in the former section, if exogenous amenities are sufficient to compensate the extra expenditure for rich people to live in the center, the living pattern will be that rich people all live inside the city. Under this condition, those people will even get additional benefit from the high neighborhood income resulting from this living pattern. Thus, rich people living in the center could be an equilibrium state. However, under the same condition, if for some reason, rich people all gather in the suburb and the utility they get from the high neighborhood income can offset the benefit from the amenity in the city center, they will not want to move to the center. Thus, rich people living outside the city could also be the equilibrium when the amenity advantage of the center is not too big to offset.

In the other situation, we can conclude from the last section that the living pattern will be that rich people all live in the suburb. A similar discussion can be done as in the last paragraph, and the conclusion is that under this situation, rich people living outside the city will be an equilibrium state, while if rich people can benefit enough from living near each other, then living in the center could also be an equilibrium.

Mathematically, let $\bar{p}_0$ be the poor people’s bid price for living in the suburb and $\bar{p}_0$ be their bid price for urban houses. Respectively, $\bar{p}_1^A$ and $\bar{p}_1^A$ will be the rich people’s bids under pattern A, which is that all rich people live in the center. Then, for poor people:

$$u(y_0 - \bar{p}_0 q_0, q_0, \bar{a}, \bar{z}) = u(y_0 - t_0 - \bar{p}_0 q_0, q_0, \bar{a}, \bar{z}) \quad (7)$$

Since poor people do not care about $a$ and $z$, so it necessarily has

$$\bar{p}_0 - \bar{p}_0 = \frac{t_0}{q_0}$$

For rich people within pattern $A$:

$$u(y_1 - \bar{p}_1^A q_1, q_1, \bar{a}, y_1) = u(y_1 - t_1 - \bar{p}_1^A q_1, q_1, \bar{a}, y_0) \quad (8)$$
Since rich people value both exogenous and endogenous amenity, the first argument at the left hand should less than it at the right hand. So it necessarily has

\[
\bar{p}_1^A - \bar{p}_1^A > \frac{t_1}{q_1}
\]

As in the model with continuous distance, the group with the larger bid-price differential lives in the center. And the discussion of which difference is larger is just as the beginning of the proof. Analysis for pattern B, the reverse spatial pattern, is performed in the same manners pattern A.

**Part 4: Why exogenous amenity patterns differ across cities**

Reasons:

1. Natural geography
2. Political power concentration: Ades and Glaeser\(^4\)(1995) explains that countries with dictatorship regimes tend to agglomerate larger cities than democratic countries.
3. International trade: Krugman and Livas\(^5\)(1992) argues that cities’ sizes are negatively relate to international trade, since prevalence of import goods reduces the benefit of people living close to city.
4. Protection of historical relics.

**Part 5: Extensions**
The classic Tiebout model\(^6\), argued that the free rider question could be solved by “voting with their feet”, which means people will choose to move to a community where they can match their requirements of local expenditure. In the Tiebout model, we have a production function \(f\) and labor force \(N\), which can produce private goods \(x\) and public good \(g\) with decreasing marginal product.

The maximization problem can be written as:

\[
u(g, x) \quad \text{s.t} \quad Nx + g = f(N)
\]

For the optimal solution, we have the F.O.C

\[
Nu_g / u_x = 1
\]

Above is the Samuelson condition which means marginal rate of substitution equals marginal rate of transformation at the production frontier. Under such a condition, individuals have no intention to transfer private goods into public goods.

\[
g = f(N) - Nf'(N)
\]

---


\(^6\) Tiebout (JPE, 1956)
Above is the famous Hendry-George Theorem, which says the support of public goods is equal to the marginal production of labor. In Georgism⁷, public expenditures should only come from a land value tax, and they argue that besides the “single tax on land value other taxes are inefficient.

⁷ Georgism is the school of thought associated with the political economist Henry George (1839-1897).
Deriving Equation (2)

\[ \frac{d}{de} = -\frac{1}{\theta} \left[ \alpha e^{-\theta} + \beta q^{-\theta} + (1 - \alpha - \beta) a^{-\theta} \right]^{-\frac{1}{\theta} - 1} \cdot -\theta \alpha e^{-\theta - 1} \]

\[ \frac{d}{da} = -\frac{1}{\theta} \left[ \alpha e^{-\theta} + \beta q^{-\theta} + (1 - \alpha - \beta) a^{-\theta} \right]^{-\frac{1}{\theta} - 1} \cdot -\theta (1 - \alpha - \beta) a^{-\theta - 1} \]

\[ \frac{d}{da} \frac{du}{de} = \frac{(1 - \alpha - \beta) a^{-\theta - 1}}{\alpha e^{-\theta - 1}} = \frac{(1 - \alpha - \beta) \left( \frac{a}{\alpha} \right)^{(\theta + 1) - 1}}{\alpha} \]

\[ = \frac{(1 - \alpha - \beta) \left( \frac{e}{\alpha} \right)^{\frac{1}{\theta}}}{\alpha} \text{ if } \sigma = \frac{1}{1 + \theta} \]

Demand functions for e and q

1. Do the utility max problem w.r.t. e and q

Max \( u(e, q, a) \) s.t. \( e + pq = y - tx \)

\[ L = u(e, q, a) - \lambda (e + pq) = 0 \]

\[ \frac{\partial L}{\partial e} = -\frac{1}{\theta} u(e, q, a)^{-\frac{1}{\theta} - 1} \cdot -\theta \alpha e^{-\theta - 1} - \lambda = 0 \]

\[ \frac{\partial L}{\partial q} = -\frac{1}{\theta} u(e, q, a)^{-\frac{1}{\theta} - 1} \cdot -\theta \beta q^{-\theta - 1} - \lambda p = 0 \]

\[ \frac{\partial L}{\partial e} = \alpha e^{-\theta - 1} \]

\[ \frac{\partial L}{\partial q} = \frac{\beta \alpha e^{-\theta - 1}}{\beta q^{-\theta - 1}} = \frac{1}{\rho} \]

\[ \frac{\alpha \rho}{e^{\theta + 1}} = -\frac{\beta}{q^{\theta + 1}} \]
Recall that the MRS of a and e is equivalent to their respective price ratios. But the price of e is normalized to 1 so we can set the MRS to $v^a$ if we assume that the uniform utility level is equivalent to indirect utility. So now $\frac{u^a}{q(x)}$ is equivalent to $\frac{v^a}{q}$, which is what we wanted to show. Now whether $\frac{v^a}{q}$ is
increasing or decreasing depends on whether non-housing goods, e and amenities, a are complements or substitutes. So we need to examine the sign on $\sigma$ which measures the elasticity of substitution. With CES preferences, the only case when the ratio is decreasing is when e and a are perfect substitutes.

<table>
<thead>
<tr>
<th>Elasticity of substitution $\sigma$</th>
<th>$-\frac{1}{\sigma}$</th>
<th>$\frac{1-\sigma}{\sigma}$</th>
<th>$-\frac{1-\sigma}{\sigma}$</th>
<th>$\frac{\nu^a}{q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect substitutes $\sigma = 1$</td>
<td>neg</td>
<td>0</td>
<td>0</td>
<td>Dec</td>
</tr>
<tr>
<td>Complements $\sigma &lt; 1$</td>
<td>neg</td>
<td>pos</td>
<td>neg</td>
<td>Inc</td>
</tr>
<tr>
<td>Substitutes $\sigma &gt; 1$</td>
<td>neg</td>
<td>neg</td>
<td>pos</td>
<td>Inc</td>
</tr>
</tbody>
</table>

Consumer utility with Stone-Geary preferences

\[ U = (e - Re)^{\beta_e} (2 - \delta_e)^{\beta_2} (a - \delta_a)^{\beta_a} \]

Then we have

\[ u^e = \beta_e \cdot (a - \delta_a)^{-1} \cdot U \]
\[ u^e = \beta_e \cdot (2 - \delta_e)^{-1} \cdot U \]

\[ e(x) = \frac{1 - Re + p'y}{p + p'} \]
\[ q(x) = \frac{1 - Re + p'y}{p + p'} \]

\[ e(x) = \frac{\nu' I + p'y - p'e - pq}{p + p'} \]

where \( p' = \frac{\beta_e}{\beta_a} p \)

Notice: this is an interior solution.

Put together:

\[ \frac{v^a}{q} = \frac{u^a}{u^e} \frac{1}{2} = \frac{\beta_e}{\beta_e (a - \delta_a)} \cdot \frac{p'(I - Re + p'y)}{I - Re + p'y} = \frac{p'(I - Re + p'y)}{\beta_e (a - \delta_a)} \]

Basically, $\beta_e$, $\beta_a$, $\beta_e$, and $p'$ will all be positive if $p' > 0$.

\[ \frac{I - Re - p'y}{I - Re + p'y} \]

is increasing with $I$.

If $a - \delta_a > 0$, then $\frac{v^a}{q}$ is increasing.